Dynamics Of Fusion in Plasmas

A. Bonasera

Laboratorio Nazionale del Sud, INFN, via Santa Sofia, 62, 95123 Catania, Italy

We investigate the possibility of gaining energy from nuclear fusion reactions using different mixtures of D, T and ^6Li. First, a plasma in equilibrium is studied at different densities and temperatures. In a second, highly non equilibrium case, the plasma is at high densities and excitation energies. While the first case could lead to an energy gain especially when coupled to an accelerator, in the second case the energy given to the system might be larger than the output energy even for a D+T plasma. This is due to the small number of particles which can be treated numerically. Furthermore, there is a possible double counting between the elementary fusion cross section and the exact Coulomb potential used in the calculations.

§1. Introduction

A way of producing energy is by means of nuclear fusion reactions. Such reactions naturally occur in stars and essentially consist of fusion of light elements such as Deuterium (D), Tritium (T), 3He and so on.[1)] Reactions involving D+T have a large efficiency and are mostly studied and proposed as a way to produce energy.[2)] However, there are some shortcomings for such a proposal. While D can be found easily in water, T is radioactive, very dangerous to handle and must be produced via n+Li reactions. Furthermore in D+T reactions, 14 MeV neutrons are produced which are of damage to the reactors and generate large quantities of radioisotopes. Even though, using low activation materials, the neutrons damage could be reduced, it would be better to avoid T as a main fuel ingredient. Alternatives have been proposed[3)] such as $D+^3He$[4)] or $p+^{11}B$ in Colliding Beam Fusion Reactors-CBFR.[5)]

There are essentially two routes studied nowdays to produce energy. In the first one a plasma is confined through strong magnetic fields at a given temperature θ and density ρ. This is like a boiling pot where from time to time some nuclear fusion occurs and energy is produced. In order to get more energy output than input, the θ and/or ρ must be very high. In such conditions, the plasma becomes highly chaotic and even turbulences set in. The confining fields are not strong enough to hold the plasma for a very long time.

A completely different route to fusion is through the use of lasers.[6)] One or more powerful lasers are used to compress and ignite a pellet of fuel.[7)] Problems as confinement are avoided, however the time of compression is short, thus it might be difficult to gain energy from such a method even using the D+T fuel.

Because of the many uncertainties discussed above, theoretical simulations are highly desirable in order to shed some light on the matter. Simulations exist based on hydrodynamics or kinetic approaches.[1),6)] Such approaches deal very often with the *atomic* or macroscopic (from the nuclear point of view) dynamics of the plasma.

The fusion probability is calculated through the relation:

$$R = \rho_1 \rho_2 < \sigma v > \tag{1·1}$$

Where ρ_i are the densities of the fuel components and σ is the measured cross section for fusion as a function of energy and v is the relative velocity. The brackets indicate an averaging over a Maxwellian distribution if the system is in equilibrium or some other distribution if not. The uncertainties in this method are essentially:

The cross section is measured in vacuum, but it might be modified in medium. This is for instance important in stars where, the presence of electrons at high densities might modify the process of fusion.[8)]

-The distributions used in eq.(1.1) usually do not contain correlations which could be important when the system is in extreme conditions, such as in laser or ions induced reactions.

Here we discuss a model which describes fusion microscopically and takes important correlations in the plasma into account at a classical level. We will treat the plasma dynamics with classical molecular dynamics (CMD), i.e. ions and electrons interacting through a two body Coulomb potential, and the quantum mechanical process of fusion with a suitably defined collision term. At the temperatures and densities considered here quantum effects are not important. The approach has been previously applied to study the dynamics of the quark-gluon plasma[9)] and in heavy ion collisions.[10)]

§2. Formalism

We use the Balescu Lenard Vlasov (BLV) equation and classical molecular dynamics. The exact one body classical distribution function $f_1(\mathbf{r}, \mathbf{p}, t)$ satisfies the equation (BBGKY hierarchy):[11)]

$$\partial_t f_1 + \frac{\mathbf{p}}{E} \cdot \nabla_r f_1 - \nabla_r U \cdot \nabla_p f_1 = 0, \tag{2·1}$$

where $U = \sum_j V(\mathbf{r}, \mathbf{r}_j)$ is the exact potential, $E = \sqrt{p^2 + m_i^2}$ is the energy and m_i are the ions, electron masses. Writing the exact one body function as a sum of δ functions in phase-space gives the classical Hamiltonian equation of motion, i.e. CMD . The exact distribution function depends on the initial conditions, which could be changed microscopically by keeping macroscopic quantities such as density and temperature of the system fixed. For each initial condition we obtain one event. We can generate many events and make ensembles averages. Let us now define f_1 and U as sums of an ensemble averaged quantity plus the deviation from this average:

$$f_1 = \bar{f}_1 + \delta f_1; U = \bar{U} + \delta U. \tag{2·2}$$

Substituting these equations in Eq. (2.1) and ensemble averaging gives:

$$\partial_t \bar{f}_1 + \frac{\mathbf{p}}{E} \cdot \nabla_r \bar{f}_1 - \nabla_r \bar{U} \cdot \nabla_p \bar{f}_1 = \langle \nabla_r \delta U \nabla_p \delta f_1 \rangle, \tag{2·3}$$

where one recognizes in the lhs the Vlasov term and in the rhs the Balescu-Lenard collision term.[11] Here, there could be a possible source for an ambiguity in our calculation. Infact, as we will show below, we will solve the exact Coulomb trajectories for the particles. This gives the mean field in eq.(2.3) and the fluctuations in the collision term. However, we will simplify the solution of the collision term and use the experimental fusion cross section. This last term of course include the Coulomb part, thus in some sense we are not consistent in the treatment of the mean field and the collision term which might result in a double counting of the Coulomb term. We will show below estimates of how large the double counting might be.

We solve numerically the set of equations (2.1-2.3), i.e. we generate many CMD events to simulate the plasma, average over the CMD events to obtain the fluctuating and average quantities[9] in order to estimate the collision term. From the collision term we can perturbatively estimate the rate of ion-ion fusion events when those events are rare, i.e. at low energies and/or densities. We follow the mean free path method as discussed in.[10] Briefly, for each event, at each time step dt and for each ion i we search for the closest ion $j(i)$ and define a probability of fusion as:

$$\Pi = \frac{v_{ij}dt}{\lambda} = \sigma\rho(r_i)v_{ij}dt, \tag{2·4}$$

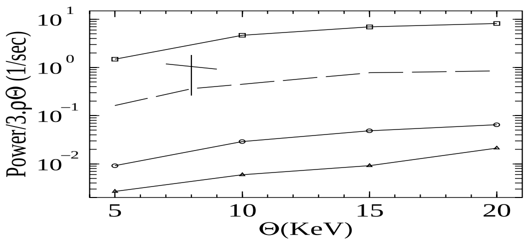

Fig. 1. Normalized power versus temperature at two densities $10^{19}m^{-3}$ (D+T, dashed line) and $9.8 \times 10^{19}m^{-3}$ (full lines) for D+T (squares), D (circles) and D+Li (triangles). The cross indicates the JET result obtained at the lowest density.

where v_{ij} is the relative velocity, σ is the fusion cross section for ions i and j(i) parametrized from data, λ is the local mean free path and $\rho(r)$ the local density. When the probability given by (2.4) is large we simulate in a Montecarlo way the process of fusion with creation of new particles and extra energy is given by the fusion Q-value. Summing over all collisions and averaging over time gives the microscopic counterpart of the rate of fusion events given in eq.(1.1). In our calculations we will restrict to D, T and ^{6}Li. Fusion reactions involving D+Li and n+Li isotopes will create some T ions which could react again and increase the rate of fusion events as well absorb some energetic neutrons. In tokamak-like simulations N_I ions and N_e electrons, adding to zero net charge, are randomly distributed in a box of volume V in coordinate space, while the momenta are chosen randomly according to a Maxwell-Boltzmann distribution at temperature θ. Periodic boundary conditions are enforced. In order to simulate the effect of an ion accelerator, T or Li ions are inserted at the initial time in one side of the box at a given energy. Physically we expect similar conditions in a Tokamak coupled to an accelerator or a CBFR.[5)]

§3. Applications

For illustration let us discuss first the case of Jet and Iter- like tokamak[2)] conditions, fig.1. The calculated power is normalized to the thermal energy-density, where ρ is the ions density (for mixed system we use an equal concentration of ions unless otherwise stated). Fig.1 shows the screening potential for the D+d reaction at $E_{c.m.} = 1keV$ (solid line) and 200 keV (dashed line). The D+T case at two densities (10^{19} and $9.8 \times 10^{19} m^{-3}$) versus temperature, gives an higher power than the D (circles) and D+Li (triangles) cases at the higher density. The experimental value obtained at Jet with a D+T fuel at the lower density is indicated by the cross line. Our calculation underestimate the Jet result of less than a factor 2, which might be due to a non uniform experimental density distribution and the use of accelerated ions to heat the plasma. In the tokamak case, since densities are relatively low, correlations are not so important and the plasma can be considered as an ideal Maxwell-Boltzmann gas. Thus this should be considered as a test to our approach, especially testing the way we calculate fusion probabilities through the collision term, eq.(2.4).

Let us now discuss the case of a D plasma coupled to accelerated ions of T or Li, similar to the CBFR case discussed in.[5)]

In figure (2) we plot the power normalized to the total kinetic energy density (i.e. plasma plus beam kinetic energies) versus the T (full lines) or Li (dashed lines) beam energies. The plasma density is $4.1 \times 10^{21} m^{-3}$, i.e. two orders of magnitude higher than in figure 1. The beam density is 400 times smaller than the plasma.[5)] Two temperatures are given, 20 KeV (triangles) and 1 KeV (squares). The results at lower temperature give a normalized power higher than for the 20 KeV case. This is very interesting since one can tune opportunely the beam energy to have the maximum gain for a plasma with a relatively high density but low temperature. In the Li induced case at 1 KeV we have a similar output to the T case at 20 KeV plasma temperature for beam energies of the order of 1 MeV. Recall that the channel (Q=2.56 MeV), gives about half

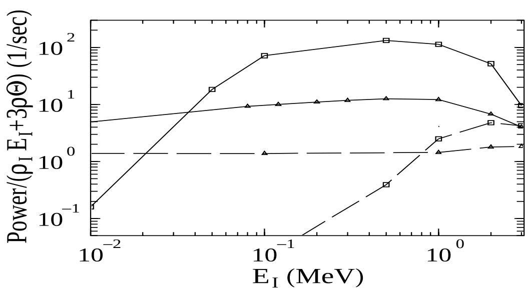

Fig. 2. Normalized power versus T (full lines) and Li (dashed lines) beam energies, at two plasma temperatures 1 KeV (squares) and 20 KeV (triangles).

of the total fusion probability. Hence, in Li induced reactions a large T production is possible with a kinetic energy of the order of 1 MeV, which is in the energy region where the T+D case has a maximum normalized power. A large probability for the secondary process (Q=17.6 MeV) is possible which will increase the energy gain. Also the Li could absorb neutrons giving more T. Fusion can be obtained in a completely out of equilibrium scenario. For instance, a small drop of fuel is highly compressed and excited either by means of laser, ion or electron beams. The reached densities and excitation energies are so high that fusion occurs. To ignite the fuel takes a large amount of energy and the question is if we get enough fusion energy to overcome the input one. In the simulations we prepare the fuel at different initial densities and at very small temperatures. At this stage we give to the drop a radial energy to simulate a second laser or beam injection. We use two prescription for the radial energy:

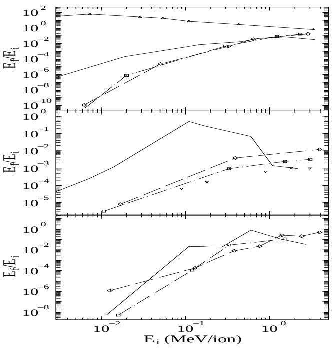

Fig. 3. Normalized fusion energy versus initial radial energy at initial density $10^{28} m^{-3}$ (top) and zero temperature, $4 \times 10^{31} m^{-3}$, 0.8KeV temperature (center) and 1 eV temperature (bottom). D+T (full lines), D+Li (dashed lines) and D (dashed-dotted lines). The pointing down triangles (center) refer to D+Li with initial linear radial energy given to the system, while the (up) triangles in the top frame refer to D+T when the Coulomb force is turned off.

$$E_i = \beta r \tag{3·1}$$

$$E_i = \beta (qr)^2 \tag{3·2}$$

where r is the radial position of the particle, q its charge and β a control parameter. Even though the two prescription are quite different, for fixed energy the energy output is very similar as we demonstrate in figure (3). In the figure, the D+T cases (full lines) give the highest gain defined as the ratio of fusion to initial energy. The D (dashed-dotted lines with squares) and the D+Li cases (dashed lines with rombuses) give comparable values. The triangles (center frame) refer to the D+Li case with linear radial energy,eq.(3.1). This case gives the largest difference for the two choices because of the quadratic charge dependence in eq.(3.2). The results are not so sensitive to the way the energy is given to the system but mostly on how much is given to it. In the top figure the initial density is $10^{28} m^{-3}$ and zero initial temperature. While the density is $4 \times 10^{31} m^{-3}$ and temperature 0.8KeV (center) and 1 eV (bottom) respectively. The energy output increases faster at higher densities as expected, while the dependence on the D to Li ratio is small.

We notice that we never gain energy apart the D+T case (middle figure) where the ratio is about 1 (we are neglecting the efficiency of energy deposition say from the laser to the plasma). In this case we need initially to compress and heat the system such that before the expansion starts (which is rather quick) the system has enough energy and the particles are close enough for fusion to occur. In the other cases the fuel must be highly compressed to have an appreciable fusion energy. In the experiment of Kodama et al.[7)] a drop of deuterium of density similar to ours, was given a compressional energy of about 10^{-4} MeV/ion. The fusion energy obtained and estimated from the number of neutrons emitted is about 10^{-12} MeV/ion. This gives a difference between input and output energies of about 8 orders of magnitude in agreement with our estimates. We can estimate easily the maximum gain η when the fuel is all burned: $\eta = \frac{N_f Q}{2E_i}$, where E_i is the input energy, and $N_f/2$ is the maximum number of fusions. In the D+T case if we assume an input energy of 100 KeV/ion (where the experimental cross section shows a resonance), we would get a maximum gain of about 100. Other systems like D+D at about 1 MeV/ion give a gain of about 2. The situation is not much more favourable for p+Be at the resonance with a maximum gain of about 8. However, we have seen in our calculations that we get a gain factor of about 1 when giving an input energy as above. It means we need to give a higher energy to overcome the strong Coulomb forces. As we stressed above there could be two sources of ambiguities in our calculations. The first one is purely numerical due to the small number of particles in the calculations, less than 10^4. Since our pellet size is too small, ions leave the system very early when they are still energetic, thus we understerestimate the fusion yield. Infact the important quantity is the ion fusion mean free path contained in eq.(2.4). Such quantity should be smaller than the pellet size for meaningful results. In a following work we will correct this deficiency. The second ambiguity is in the Coulomb term. Infact we treat the dynamics exactly in a classical sense. However, in the collision term we use the experimental fusion cross sections which contain a Coulomb term as well. Thus we have double counting.[10)] In order to demonstrate the importance of the Coulomb force we have performed calculations with the Coulomb force turned off, i.e. with the collision term only. The result is dramatically different than before and it is given in fig. 3 (top) by the full line plus triangles. Now the gain reaches a maximum value of about 100 at low energies and tends to the previous results at very high energies. Thus the Coulomb force is crucial, infact what we see in the calculations is that the electrons being much lighter than the ions reach very quickly the maximum compression and bounce back much before the ions reach their maximum compression. At this stage the plasma is largely positively charged and the strong Coulomb force stops the compression which result in a reduction of the fusion probability. Hydrodynamics calculations usually give large gains in contrast to ours and similar to our results without Coulomb force. This might be due to the use of an EOS obtained in equilibrated and infinitely large system and the assumption of local equilibrium in the calculations. Finite size effects and non equilibration might be the main reason for the discrepancy of our calculations to hydrodynamics.

§4. Summary

In conclusion, the D+T fuel remains the best candidate to give a large output for energy production, however one should take into account the energy needed to produce it and the danger in the handling. The best approach seems that of a D plasma coupled to T opportunely accelerated. If the production and handling of T is a problem (in some countries the use of T could even be forbidden for political reasons), than the D plasma coupled with a Li beam at about 1 MeV/ion could be a valid alternative. Laser induced fusion to produce energy seems a long way ahead from our calculations. The strong Coulomb repulsion and the small size of the pellet are the main reasons for this. In order to gain much energy, one has to compress the fuel slowly in several steps, in order that a chain reaction occurs which burns most of the fuel. Using a fuel containing Li might be successful because of the high probability of T production through Li, both from D and n induced reactions. We stress that our results are valid for very small pellet sizes in laser induced fusion reactions. Preliminary calculations for macroscopic pellet sizes show a very high gain thus supporting the feasibility of energy production using lasers.

We thank profs. S.Cherubini, A. Musumarra, C. Spitaleri and L.Torrisi for stimulating discussions.

References

1) F. F. Chen, Introduction to Plasma Physics and Controlled Fusion (Plenum, New York, 1988), Vol. 1
2) J. Glanz, Science **274**, 1600 (1996).
3) G. L. Kulkinski and Santarius J. F., Nature **396**, 724 (1998).
4) B. Coppi, et al., Fusion Tech. **25**, 361 (1994).
5) N. Rostoker, et al., Science **278**, 1419 (1997); W. M. Nevins and A. Carlson, Science **281**, 307a (1998).
6) J. Meyer-ter-Vehn, Plasma Phys. Control. Fusion **43A**, 113 (2001).
7) R. Kodama, et al., Nature **412**, 798 (2001); R. Kodama, et al., Nature **418**, 933 (2002).
8) S. Kimura, et al., Phys.Rev. **C67**, 022801(R) (2003).
9) A. Bonasera, Nucl.Phys. **A681**, 64 (2001).
10) A. Bonasera, J. J. Molitoris, and F. Gulminelli, Phys.Rep. **243**, 1 (1994).
11) L. D. Landau and E. M. Lifshitz, Physical Kinetics (Pergamon Press, Oxford, 1981), Vol. 10, p.1.