The purpose of this report is to review some simple aspects of heavy ion interactions which may be useful for understanding of sub-barrier fusion processes. The Christensen-Winther (CW) interaction is a typical potential and is a useful starting point for a nucleus-nucleus interaction. It gives good values for the height and radius of the Coulomb barrier for many heavy ion pairs. With this and other similar interactions there is a rather large separation between the top of the Coulomb barrier and the point at which the densities come into contact. When used in conjunction with the coupled channels method potentials like the CW interaction give good fusion cross-sections near and for a few MeV below the top of the barrier. The coupled channels method fails in some cases when the energy is of the order of 5 MeV below the barrier. There is evidence that increasing the surface diffuseness of the ion-ion potential might help to resolve this problem. Cluster radioactivity is an inverse process to sub-barrier fusion, but the energies below the top of the Coulomb barrier are very different.

§1. Heavy ion potentials

The dependence of the interaction potential \( V(r) \) on the separation \( r \) between the centers of two heavy nuclei can be divided into three regions. The outermost (I) is the region outside the Coulomb barrier. It corresponds to \( r > R_B \) where \( R_B \) is the radius of the barrier. The Coulomb repulsion between the nuclei dominates in this region. There is an intermediate region (II) where \( R_0 < r < R_B \) where \( R_0 \approx 1.1(A_1^{1/3} + A_2^{1/3}) \) fm is the radius at which the matter densities of the two nuclei have a significant overlap. The nuclear interaction between the nuclei is important in this region. The Christensen-Winther (CW) potential\(^{1}\) gives a simple prescription for \( V(r) \) in regions I and II. Region III corresponds to \( r < R_0 \) where the matter distributions overlap strongly. A description of the interaction in terms of a potential probably fails in this region especially for heavy nuclei.

Christensen and Winther have given a simplified exponential\(^{1}\) form for their potential which should be a good approximation in regions I and II.

\[
V(r) = S_0 \bar{R}_{12} \exp\left(-\frac{(r - R)}{a}\right) \tag{1.1}
\]

where \( S_0 = 50 \text{ MeV}, \ a = 0.63 \text{ fm}, \ R = R_1 + R_2, \ \bar{R}_{12} = R_1 R_2 / (R_1 + R_2) \) and

\[
R_i = (1.233 A_i^{1/3} - 0.98 A_i^{-1/3}) \text{ fm} \tag{1.2}
\]

The CW potential predicts that the width of the region II, \( R_B - R_0 \), lies in the range 2.4 - 2.7 fm for the typical heavy ion pairs Li + Bi, Ni + Y, O + Zr and C + Pb. For the same pairs the distance \( R_{\text{int}} - R_0 \approx 1.8 \text{ fm} \) where \( R_{\text{int}} \) is the internal turning point for an energy 5 MeV below the top of the Coulomb barrier. Thus for the CW potential which has a surface diffuseness \( a = 0.63 \text{ fm} \) the intermediate region
II is well defined. Even for energies well below the top of the Coulomb barrier the tunneling process is over before the matter distributions of the two nuclei come into contact.

Hagino, Rowley and Dasgupta\textsuperscript{2)} have argued that the surface diffuseness in the CW potential is too small and that a better value would be $a \approx 1.2$ fm. If the potential form is modified by increasing the surface diffuseness and changing the potential strength so the Coulomb barrier height $V_B$ is kept constant then the barrier radius $R_B$ decreases and the region II is less well defined.

Fusion cross sections $\sigma_F$ can be estimated from a potential model potential using the formula

$$\sigma_F(E) = \frac{\pi}{k^2} \sum_{\ell=1}^{\infty} (2\ell + 1) T_\ell(E)$$

where $T_\ell$ is the probability of capture for the partial wave $\ell$, and $k$ is the wave number for the relative motion. The transmission coefficients $T_\ell$ can be calculated by solving the radial Schrödinger equation for the chosen potential with some suitable boundary condition inside the barrier. The agreement with experiment is, however, not satisfactory: the theoretical fusion cross sections fall off too rapidly for incident energies below the Coulomb barrier.

§2. Coupled channels effects

The simple potential model for fusion cross sections fails because strong channel coupling effects modify fusion cross sections. These are due, for example, to the inelastic excitation of collective states and have the effect of replacing the uncoupled single barrier with a distribution of barriers.\textsuperscript{3)} A convenient way to compare experiment with theory is to plot the second derivative $d^2(E\sigma)/dE^2$ against the center of mass energy $E$. The theoretical barrier distribution function for a simple potential model for the fusion cross section has a peak centered at an energy corresponding to the top of the Coulomb barrier and a width proportional to the barrier curvature parameter $\hbar\omega$. The curvature parameter is related to the second derivative $V''$ of the barrier potential and the reduced mass $\mu$ by

$$\omega^2 = V''(R_B)/2\mu.$$  \hfill (2.1)

The experimental barrier distribution function is obtained by calculating the second derivative in eq.(2.1) from the data points using finite difference methods. It has a peak at an energy which corresponds quite well with top of the Coulomb barrier in the Christensen-Winther potential for a wide range of heavy ion pairs. Some recent measurements are on $^7\text{Li} + ^{209}\text{Bi}$ (Dasgupta et al\textsuperscript{4)}, $^{12}\text{C} + ^{90}\text{Zr}$ (Newton et al\textsuperscript{5}) and $^{60}\text{Ni} + ^{89}\text{Y}$ (Jiang et al\textsuperscript{6}). On the other hand the width of the experimental peak is always larger than the theoretical value obtained from a simple potential model. Coupled channel calculation which take into account fluctuations in the height of the barrier due to collective effects broaden the distribution and can give a good account of the data near the barrier.\textsuperscript{3)}

Recent experiments suggest the coupled channels method fails in some cases.
For example the fusion cross sections for $^{60}$N + $^{89}$Y (Jiang et al$^6$) exhibit an abrupt decrease at extreme sub-barrier energies. The barrier energy is at about 127 MeV and the abrupt decrease occurs at about 5 MeV below the top of the barrier. This abrupt decrease was discussed in the contribution of Jiang to this conference. It shows up clearly in plots of the logarithmic derivative

$$L(E) = \frac{d[\ln(E\sigma)]}{dE} = \frac{1}{E\sigma} \frac{d(E\sigma)}{dE}$$  

(2.2)

obtained from the experimental fusion cross sections, as a sharp increase in $L(E)$ for $E \sim 122$ MeV. Hagino et al$^2$ have studied this effect in a coupled channels model with potentials which have a surface diffuseness which is larger than the standard value $a = 0.63$ fm in the CW potential. They show that a potential with a much larger value $a \sim 1.3$ fm goes some way to explaining the data. There is also evidence for a large surface diffuseness parameter from fusion measurements above the Coulomb barrier (Newton et al.$^5$). At the present time the physical origin of this large value of $a$ is an open problem. Newton et al.$^5$ also discuss whether a potential with a large surface diffuseness parameter which is required to fit fusion data can also fit elastic scattering. They conclude that it is likely that a potential form can be chosen which fits both.

§3. Cluster radioactivity

Cluster radioactivity is, in a sense, the inverse of sub-barrier fusion. The first example, discovered by Rose and Jones$^7$ in 1984, was a rare decay mode of $^{223}$Ra into $^{14}$C + $^{209}$Pb. Since 1984 numerous other examples have been observed. For example Wang et al$^8$ observed the emission of $^{30}$Mg and $^{32}$Si from $^{238}$Pu. In 2000 Ogloblin et al$^9$ found a cluster decay mode of $^{242}$Cm which emits $^{34}$Si particles. This kind of process involves the penetration of a high potential barrier. The fragments are left in their ground states. One of the favorable conditions for cluster radioactivity is a large $Q$-value. Another is a possible cluster structure in the decaying nucleus. The fragments are left in their ground states. One of the favorable conditions for cluster radioactivity is a large $Q$-value. Another is a possible cluster structure in the decaying nucleus. For example in the decay of $^{223}$Ra the fragment $^{209}$Pb is one nucleon away from the double closed shell $^{208}$Pb. This situation is a reason for a larger-than-average $Q$-value and is favorable for a cluster structure in $^{223}$Ra.

A number of different models have been proposed to describe cluster radioactivity. The agreement provided by a simple square well plus Coulomb repulsion$^9$ with a constant radius parameter $r_0 = 0.98$ fm for all cluster decays is especially remarkable and suggests that the decay probability is not sensitive to the details of the structure of the nuclei involved. Another potential model proposed by Buck et al$^{10}$ provides an effective account of cluster decay as well as the structure of the initial nucleus. The clusters are already present in the initial nucleus in both these models and the spectroscopic factor for the decay is unity. The interaction potential in both of these models has a shorter range and a higher barrier as compared with the CW potential. Another quite, different model, has been developed by Barranco et al.$^{11}$ It is a combination of a pairing model which yields a pre-formation factor and a tunneling model using the CW potential. A larger tunneling probability from
the CW potential is compensated by a small pre-formation factor.

There are some striking differences between sub-barrier fusion and cluster radioactivity. One is the height of the barrier. The initial state is about 25-30 MeV below the top of the barrier for the cluster decay of $^{223}$Ra while for typical fusion reactions it is usually less than 5 MeV. Another is the excitation of the composite nucleus. Cluster decay takes place from the ground state of the composite nucleus, while the final compound state in a fusion reaction might have an excitation energy of 25-30 MeV where the density of compound states is very high.

§4. What happens inside the barrier?

A description of the inter-nuclear interaction in terms of a potential is probably good near the Coulomb barrier. Inclusion of coupled channels can take into account distortions of the nuclei due to the strong forces acting on them near the barrier. When their density distributions overlap a potential description may fail, even when coupled channels effects are allowed for. Inclusion of an imaginary potential can take into account the influence of fusion on elastic scattering, inelastic scattering, direct breakup processes all of which are associated with a short time scale. But an imaginary potential cannot help to describe the fusion process itself.

An anomaly in the fusion cross section at energies far below the barrier could be associated with events which happen after the nuclei have passed the barrier when their densities begin to merge. On the other hand it could be an effect of the barrier. The strong forces which act near the barrier might tear the nuclei apart. This effect could be described in coupled channels if the appropriate channels are included.

References

1) R. A. Broglia and A. Winther, Heavy Ion Reactions (Addison-Wesley, Redwood City, CA, 1991).