Subbarrier fusion and barrier distributions of $^{48}\text{Ca}+^{90,96}\text{Zr}$


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We report on the measurement of subbarrier fusion cross sections for $^{48}\text{Ca}+^{90,96}\text{Zr}$, with the aim to pin down the contribution of transfer and inelastic excitations in reactions involving $^{90}\text{Zr}$ and $^{96}\text{Zr}$. We also present a re-evaluation of previously published data.

§1. Introduction: a brief history

The relative importance of inelastic and transfer channels in subbarrier fusion is a long standing problem. On a phenomenological basis, a correlation between sub-barrier enhancement and transfer Q-values and/or measured transfer cross sections has been noticed a long time ago$^1$ and has often been taken for granted. However, the arguments were often qualitative on the theoretical side and incomplete from the experimental point of view. The issue received new impulse with the realization that the excitation function contains valuable information in its detailed structure, bearing on the coupled channels implicated in the tunneling process,$^2$ the so called "barrier distribution" method.

The fusion of $^{40}\text{Ca}+^{90,96}\text{Zr}$ represented probably the clearest example of the different importance of transfer and inelastic channels in subbarrier fusion. The conclusion was strengthened by the fact that detailed and very precise data were compared to exact CC calculations. As one can see in Fig. 1, the $^{40}\text{Ca}+^{90}\text{Zr}$ data are well reproduced by CC calculations including only inelastic channels. Not only is the overall cross section reproduced, but also its details as can be seen in the barrier distribution. Yet, despite this success, the same calculation fails miserably in the $^{40}\text{Ca}+^{96}\text{Zr}$ system: the cross section is badly underestimated while the shape of the barrier distribution cannot be reproduced. One has to bear in mind that with $^{96}\text{Zr}$, unlike $^{90}\text{Zr}$, very large positive Q-values are available for transfer channels and that is expected to favour transfer. The conclusion of the paper was that transfer-channel couplings (not included in the CC calculations) were clearly called for.

As a further investigation, a second measurement was performed with $^{36}\text{S}+^{90,96}\text{Zr}$. The projectile $^{36}\text{S}$ is "inert" as was $^{40}\text{Ca}$ in the previous case (its high-energy excitations can be simulated by a renormalization of the nuclear potential) while no

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positive Q-values are available with either target.

Fusion cross sections and barrier distributions could be reproduced by inelastic couplings alone, for both targets, strengthening the argument that the failure of such calculations in the case of $^{40}\text{Ca} + ^{96}\text{Zr}$ must be attributed to transfer coupling. The case was only slightly confused by the non satisfactory quality of the experimental barrier distributions. Despite the good reasons listed so far, the conclusion was to be overthrown in a paper by G.Pollarolo and A.Winther.\textsuperscript{5)} The authors showed it was possible to reproduce the cross sections and the barrier distributions in a semi-classical calculation, together with the transfer cross sections (see ref. 6)). A discussion of the results, including the systems discussed here, the underlying physics and the program GRAZING used in such calculations can be found in a contribution to this conference\textsuperscript{7)}.

All other calculations discussed in the present work have been performed with the code CCFULL\textsuperscript{8)} . The program approximates the excited states with pure harmonic vibrations or pure rotations and in the numerical computation makes use of the isocentrifugal approximation. Apart from that, this code is exact for it solves the coupled Schrödinger equations numerically. In this work, the calculations were limited to inelastic excitation coupling, since transfer may be included only approximately.

§2. A reassessment of the data and of the calculations

The previous discussion underlies a contradiction: the most striking point is the conclusion of 7) that the very large enhancement of sub-barrier fusion, as well as the broad barrier distribution observed in $^{40}\text{Ca} + ^{96}\text{Zr}$ can be attributed to the strong octupole vibration of $^{96}\text{Zr}$: the conclusion of 3) was exactly the opposite. A more detailed analysis is required in order to solve the puzzle.

Concerning the problem at hand, we underline the following approximations and/or assumptions in the calculations of refs. 3) and 4):

\begin{itemize}
  \item[a1)] the projectile ($^{40}\text{Ca}$ or $^{36}\text{S}$) is inert;
  \item[a2)] the CC calculations are limited to 2 phonons in the target nuclei.
\end{itemize}

In the calculations of 5), 7), it is worthwhile to consider that:

\begin{itemize}
  \item[b1)] it is an underlying assumption of the model that the transfer channels do not
modify the barrier distribution, except for a broadening effect; excitation being considered quasi classically, there is no intrinsic limitation to the number of excited phonons.

(Here and elsewhere, an n-phonon state is any state of the type \((2^+)^k(3^-)^i\) such that \(i + k = n\); n-phonon coupling means coupling to all states up to n phonons even if, in practice, only those of the type \((3^-)^k\) contribute significantly).

The item a1) is no real problem: the effect of the high-energy levels of the projectiles effectively amounts to a renormalization of the potential. The only problem is what bare potential to use: coupling both projectile and target excitations using the Akyüz-Winther potential can easily lead to overestimate the cross sections, while the barrier distributions do not change (on the problem of the nuclear potential see 9).

Likewise, item b1) proves a good assumption as long as grazing calculations do a good job in reproducing the data. It is of interest, therefore, to analyze the effect of phonon multiplicity in CC-FULL calculations in greater detail. For the coupling strength of the \(3^-\) state of \(^{90}\)Zr \((\beta_3=0.19)\) one finds that the calculation converges already for \(n=3\) phonons. Adding more phonons does not change the result and already the 3-phonon calculations are not so different from the 2-phonon ones. So, in the case of \(^{90}\)Zr, there is no need for further couplings.

In the case of \(^{96}\)Zr \((\beta_3=0.27)\), on the other hand, the calculation does not show signs of convergence up to \(n=5\) phonons! Among the known nuclides, only few double phonon states have been identified and there is almost no evidence of triple phonon states. Besides, a 5-phonon state for \(^{96}\)Zr would lie in the continuum! Nevertheless, the inelastic coupling calculations are doing much better if one couples up to 4 phonons, as seen in Fig. 2. The main result is that, unlike the conclusion of 3), a broad barrier distribution can be obtained from the inelastic coupling, provided a sufficient number of phonons is contributing. However, the cross section is still largely underestimated. The physical meaning of such many-phonons couplings in not clear: maybe they just mimic higher order couplings of different kinds.
§3. $^{48}$Ca + $^{90,96}$Zr data and conclusion

To help settle the problem, we measured the fusion cross section of $^{48}$Ca + $^{90,96}$Zr; this case is similar to $^{36}$S + $^{90,96}$Zr, in the sense that:
i) the projectile is quite "inert" as with $^{40}$Ca and $^{36}$S projectiles, so the inelastic channels are practically the same, while
ii) transfer is inhibited by unfavourable Q-values.
In the experiment, performed at the Laboratori Nazionali di Legnaro, a $^{48}$Ca beam from the XTU Tandem accelerator was accelerated onto thin targets of $^{90}$Zr and $^{96}$Zr.

The evaporation residues (ER) were detected at $0^\circ$ from the beam using an electrostatic deflector for beam rejection. A further separation is required, and is achieved by means of an Energy - TOF telescope consisting of a Micro-Channel Plate and a silicon detector. The quality of the separation can be appreciated in Fig. 3. Four monitor detectors, placed at about $16^\circ$ to the beam, are used for absolute normalization, as well as to check the beam quality and angle. The angular distribution of the ERs was measured at two energies for both targets. The transmission of the electrostatic deflector has been calculated with a Montecarlo code, that reproduces the transmission measured in the $^{40}$Ca + $^{90,96}$Zr reactions.

The experimental cross sections are displayed in Fig. 4 (top) for both reactions together with CC calculations. Lozenges refer to experimental values of $^{48}$Ca + $^{90}$Zr, while the circles correspond to $^{48}$Ca + $^{96}$Zr. Compared to the no-coupling limit (dotted line, $^{48}$Ca + $^{90}$Zr) the fusion enhancement is large, and also the isotopic effect of $^{96}$Zr relative to $^{90}$Zr is large. Unlike with $^{40}$Ca projectile, however, the cross sections can be reproduced by CC calculations with inelastic couplings, provided a large number of phonons is allowed. In Fig. 4, the dashed lines represent calculations in which up to 2-phonon states could be excited. The full lines correspond to 3-phonon states ($^{48}$Ca + $^{90}$Zr) and 4-phonon states (in the case of $^{48}$Ca + $^{96}$Zr).

The barrier distributions are shown in the bottom panels. One sees immediately that the shape of the barrier distributions is very similar to the one observed with a $^{40}$Ca projectile: a structure with two or three barely resolved peaks with $^{90}$Zr and a more complex and broader structure with $^{96}$Zr. The calculated barrier distributions, represented by the full lines (3-phonons for $^{90}$Zr and 4-phonons for $^{96}$Zr) reproduce...
the experimental ones rather well. The dashed lines represent 2-phonon couplings for both targets, whereas the dash-dotted line (only for $^{96}$Zr) corresponds to 3-phonon coupling calculations.

![Graph showing experimental excitation functions and barrier distributions](image)

The comparison in Fig. 5 is very interesting: it shows all six systems mentioned so far in a reduced scale, chosen in order to correct for trivial geometrical differences. All reactions involving $^{90}$Zr practically overlap, and we know from the barrier distributions that also the fine structure of the excitation functions is similar. In all such cases one expects little contribution from transfer couplings due to unfavourable Q-values and the inelastic channels are the same because of the "inert" nature of the projectiles.

When comparing all three measurements with the $^{96}$Zr target, it is interesting to observe the near overlap of the measurements with $^{48}$Ca and $^{36}$S. In both cases, there is a sizable enhancement compared to the $^{90}$Zr target, and also the barrier distribution is similar. Furthermore, in both cases, the Q-values do not favour transfer coupling. One is led to associate their similarity, as well as the difference from the $^{90}$Zr target, to the different structure of $^{96}$Zr, in particular its stronger octupole vibration. On the other hand, the $^{40}$Ca + $^{96}$Zr data clearly stand out from the crowd, showing a
much larger enhancement compared to all other systems, while the barrier distribution is not so different from the other ones observed with a $^{96}$Zr target. It is almost impossible not to conclude that, in the last case, transfer is playing a significant role.

![Graph](image)

**Fig. 5.** Reduced scale comparison of $^{40}$Ca + $^{90,96}$Zr, $^{48}$Ca + $^{90,96}$Zr, and $^{36}$S + $^{90,96}$Zr

In our opinion, this statement is not incompatible with the results of 5) and 6). There are two points to be considered in this respect:

i) those calculations do not separate the inelastic coupling from the transfer coupling, so one cannot rule out that transfer, when coupled with inelastic excitation, may be non negligible;

ii) it is argued in 5) that the barrier distribution is essentially determined by the inelastic couplings, with negligible contribution from transfer. As shown above, it is indeed possible to reproduce the same barrier distributions with pure inelastic couplings, even though the cross section may not be correctly reproduced, as with $^{40}$Ca + $^{96}$Zr.

The barrier distribution should not be confused with the enhancement: a very small contribution to the barrier spectrum, in the low energy region, may have a considerable effect on the sub-barrier enhancement.

**References**

Subbarrier fusion of $^{48}\text{Ca}^{+}\text{Zr}_{76}$

7) G. Pollarolo, this conference.
9) M. Dasgupta, this conference.