Decay-out probabilities of superdeformed bands in statistical and two-level mixing models

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A recent measurement has established the excitation energy above yrast of a superdeformed band in $^{192}$Pb as only 2 MeV at the point of decay. This value is significantly lower than is generally assumed in models of the decay-out process (but is consistent with several theoretical predictions). Estimates of level densities and $\gamma$ decay widths for levels in the superdeformed band, and for levels at the same energies in the normally-deformed well, are made for this nucleus and the neighbour $^{194}$Pb. Two recent models of the decay are then used to explore the data, and the results of these analyses are compared. Finally, questions are raised about the interpretation of the resulting spreading widths/interaction matrix elements and the implications for the decay-out process.

§1. Introduction

The division between studies of nuclear structure and nuclear reactions is one that is often arbitrary, and occasionally unhelpful: each area complements the other and ultimately one would hope to be able to fully include structure effects in, for example, coupled channels calculations of fusion. Equally, there are several areas of nuclear structure which could learn from approaches and conceptual stances familiar to reaction studies. The study of superdeformed nuclei is one such area.

Superdeformation is the phenomenon whereby a nucleus adopts an extremely elongated, prolate shape, typically with an axis ratio of around 2:1. Superdeformed (SD) shapes are supported by regions of low nuclear level density [as shown in the schematic level diagram in Figure 1(a)] at deformations away from zero, which are associated with deep minima in the nuclear potential energy surface. These “second wells”, which are further stabilised by rotation, are separated from the primary ground-state minimum by a real, angular-momentum dependent potential barrier.

When a nucleus is produced in the excitation energy/spin region associated with the SD shape, there is some probability that it will be trapped in the SD well. It then de-excites via $\gamma$-ray decays along a rotational SD band, until eventually it decays out to states of normal deformation (ND states) in the primary minimum. Somewhat surprisingly, both macroscopic\(^1\) and microscopic\(^2\) calculations predict significant barrier heights at the decay-out spins. Because the change from SD to ND states involves a profound rearrangement of the constituent nucleons, it is an intractable problem for practical microscopic calculations. Instead it is usually described in terms of tunnelling. It is this which offers the possibility of a useful overlap with reaction theory.

The first discrete SD band was observed in 1986.\(^3\) Since then, SD bands have been observed in many different regions of the nuclear chart,\(^4\) with each region
displaying characteristic features depending on the underlying structure (i.e. the nature of the single-particle orbitals driving the nucleus to large deformation). Despite differences in detailed properties, there are some features which are common to all SD bands: (i) each SD band consists of a sequence of γ-ray transitions with very regular energy spacing (indicating highly-collective rotational motion) and (ii) the decay to levels of normal deformation (ND levels) occurs very sharply, over only 2 – 3 SD levels. A schematic picture of the tunnelling decay process is presented in Fig. 1(b): the decaying SD state mixes with one or more ND states at the same energy and spin, allowing a decay branch from SD to lower-lying ND states. However, in the few cases where measurements have been possible, the experimental lifetimes of the SD levels from which the decay occurs indicate that the SD shape is retained to the lowest spins, suggesting that the ND component in the SD wavefunction is small. We must therefore ask, what is it that enhances the probability of decay so dramatically as the spin decreases? Various mechanisms including pairing and chaos have been suggested, but as yet there is no clear solution to the problem.

In order to identify what drives the sudden decay out of the SD well, it is useful to formulate the decay process in a manner such that the unknown factors can be separated from known quantities. Referring to Fig. 1(b), it is natural to formulate the decay-out probability in terms of the widths of the states in the ND and SD wells and some matrix element $V$ which describes the interaction between ND and SD states. This matrix element can itself be related to a spreading width, $\Gamma$.

For any initial SD level, both the fraction of intensity that remains within the SD band ($F_{SD}$) and the width for γ-decay within the SD minimum ($\Gamma_{SD}$) can be measured. Unfortunately, the ND level density and decay width can only be estimated. In order to make such estimates with any degree of confidence, it is essential that the excitation energy and spin of the SD band at the point of decay are established - and it is precisely this step which has proved a stumbling block for several years. Because of the relatively high excitation energy of the SD states in the decay-out region, ND states at the same energy (and spin) are considerably non-yrast and the density of states $\rho$ in the ND minimum is thus expected to be large. If $\rho$ is sufficiently large,
the SD level of interest may mix with more than one ND level, each of which will have several possible decay paths to the low-lying (easily-identifiable) states. This means that the flux of the SD band (generally less than 1% of the reaction channel) is highly fragmented, making it experimentally difficult to observe discrete, single-step γ-ray transitions from SD to ND states. Without the observation of single-step links, the SD excitation energy cannot be unambiguously determined.

§2. A recent study of $^{192}$Pb

Prior to our experiment, discrete linking transitions had been observed in $^{194}$Hg,$^{7,8}$ $^{194}$Pb,$^{9,10}$ and $^{152}$Dy.$^{11}$ (Analysis of the quasicontinuum component of the SD decay spectrum provided a somewhat less precise measurement of the SD excitation energy in $^{192}$Hg.$^{12}$) In order to start the process of obtaining systematic information across isotope chains, we performed an experiment aimed at measuring the excitation energy of the yrast SD band in $^{192}$Pb at Lawrence Berkeley National Laboratory. For this nucleus, the problems of identifying extremely weak transitions in a complex spectrum are worsened by three factors. Firstly, a high fission cross-section competes with a low fusion cross-section. Secondly, the high fissility of the nucleus limits the high spins to which the band can be observed. Thirdly, the SD γ-rays (used for selecting the data of interest) are buried in the most complex part of the total γ-ray spectrum [see Fig. 2].

These difficulties, however, are outweighed by two main advantages. Firstly, the SD band decays into well-known isomeric levels$^{13}$ in the low-lying ND well, allowing the use of time-correlated γ-ray spectroscopy to select a clean subset of the data [see Fig. 2]. Secondly, the excitation energies of the yrast SD bands in the Hg/Pb isotopes are predicted$^{1,2,14}$ to decrease with decreasing neutron number. Thus the excitation energy above yrast, $U$, at the point of decay will be lower, leading to a smaller expected number of possible decay paths compared to the more neutron-rich neighbours, and less fragmentation in the SD decay.

The experiment was performed using the high efficiency Gammasphere array and a pulsed beam. Through the application of time-correlation techniques, the data allowed the identification of several γ rays linking SD and ND states, thus un-
ambiguously establishing the excitation energy of the SD band as $E_{ex}(10^+) = 4.640$ MeV. Some of the results of this experiment have been published;\textsuperscript{15}) in the following, the measured excitation energy is used in examining the decay-out characteristics.

§3. Approaches to analysing the decay-out of the SD bands

A recent resurgence of theoretical interest has produced several detailed studies of the decay-out problem. In response to some apparent drawbacks and restrictions of earlier approaches\textsuperscript{16}) (such as the unexpected result of spreading widths smaller than widths of the ND states), Gu and Weidenmüller\textsuperscript{17}) proposed a fully statistical treatment of the mixing and decay. An alternative approach, based on a two-level mixing model, has been proposed\textsuperscript{18}) and further refined by Cardamone, Stafford and Barrett.\textsuperscript{19}) Throughout the following, these two approaches will be referred to as GW and CSB models. In the literature, particular attention has been paid to the extraction of spreading widths for the SD states; it is shown below that these two prescriptions yield very different, and perhaps incompatible, results.

The GW theory provides a formulation for the decay of the SD bands which is based on fusion calculations. In Ref.\textsuperscript{17}) the in-band SD intensity, $F_{SD}$, is separated into average and fluctuating parts, and calculated numerically for several cases. The authors suggest an expression for $F_{SD}$ obtained from a fit to these numerical results:

$$F_{SD} = F_{\text{ave}} + F_{\text{fluc}}$$

where

$$F_{\text{ave}} = 1/(1 + \Gamma^\downarrow/\Gamma_{SD})$$  \hspace{1cm} (3.1)

and

$$F_{\text{fluc}} = [1 - 0.9139(\Gamma_{ND}/D)^{0.2172}] \times \exp\left(-0.4343 \ln \left(\frac{\Gamma_{\downarrow}/\Gamma_{SD} - 0.45(\Gamma_{ND}/D)^{-0.1303}}{(\Gamma_{ND}/D)^{-0.1477}}\right)\right)$$  \hspace{1cm} (3.2)

with $D$ the average separation between states in the ND well. In this model, the spreading width is taken to be related to the matrix element $V$ by $\Gamma^\downarrow = 2\pi V^2/D$.

The two-level CSB model provides a simple, closed formula for $F_{SD}$:\textsuperscript{19})

$$F_{SD} = 1 - \frac{\Gamma_{ND}\Gamma^\downarrow/\Gamma_{SD}}{\Gamma_{SD} + \Gamma_{ND}\Gamma^\downarrow/\Gamma_{ND} + \Gamma^\downarrow}$$  \hspace{1cm} (3.3)

where now

$$\Gamma^\downarrow = 2\Gamma_{\text{ave}}V^2/(\Delta^2 + \Gamma_{\text{ave}}^2)$$

where $\Gamma_{\text{ave}} = (\Gamma_{SD} + \Gamma_{ND})/2$ and $\Delta$ is the difference in energy of the two states in the SD and ND wells. (As this cannot be measured, it is estimated by $D/4).\textsuperscript{18}$)

From the above expressions, it can be seen that several parameters need to be established before $\Gamma^\downarrow$ and $V$ can be extracted from the data. These are: $F_{SD}$, $\Gamma_{SD}$, $\Gamma_{ND}$ and the average level spacing in the ND well, $D$. 


A Fermi Gas model density of states can be used to estimate $D$: the usual approach in SD decay studies is to use the cranking model formula, $\rho(U, I) = \frac{\pi^{1/2}}{48} a^{-1/4} U^{-5/4} \exp 2(aU)^{1/2}$, where $a$ is the level density parameter (taken to be 22.58 MeV$^{-1}$). Åberg$^{20}$ offers a different expression for low spins which may be more suitable for the $A \approx 190$ region. Figure 3(a) shows a comparison of the level densities predicted by the low-spin and standard formulae for $U$, $I$ the same as the levels in SD $^{192}$Pb and $^{194}$Pb, calculated using the standard backshift parameter ($2\Delta = 1.4$ MeV). The open(closed) symbols indicate use of the standard(low spin) expression - there is a factor of $\approx 2$ between the two. If either of these sets of numbers is accepted, then the density of ND states with which the SD levels can mix is surprisingly low, giving $D$ of the order of a few thousand eV for $^{192}$Pb and a few hundred eV for $^{194}$Pb at the decay-out spins. However, following analyses of the quasicontinuum component of the SD decay in both nuclei, McNabb et al.$^{21}$ suggest that reduced backshift parameters of 0.4 MeV and 0.95 MeV should be adopted for $^{192}$Pb and $^{194}$Pb respectively. Level densities calculated with these values are shown in Fig. 3(b); corresponding values of $D$ at the decay-out spins are a few hundred eV in both cases.

As can be seen from the above, it is difficult to ascertain the ND level density at the point of decay with a high level of confidence. (We do not address here the additional question of whether a rotational model level density is appropriate for the Pb nuclei.) However, if we wish to proceed, estimates of $D$ are required. For the following, the standard level density formula is used, as a factor of two difference does not affect the broad conclusions, and the reduced backshift parameters are adopted.

The width of the excited ND states for $\gamma$-decay, $\Gamma_{ND}$, is usually estimated using the well-known formula $\Gamma_{ND} = \Gamma_{E1}^{stat} = 0.15 \times 2.3 \times 10^{-11} NZA^{1/3}(U/a)^{5/2}$, which combines the Fermi Gas model level density estimates with the tail of the GDR strength function. This reflects the assumption that the non-yrast ND states will
decay predominantly via fast, statistical E1 transitions rather than via collective E2/M1 or single-particle transitions. As the SD bands observed in $^{192}$Pb, $^{194}$Pb are only 2 – 3 MeV above yrast at the point of decay, it is necessary to ask whether this assumption is valid. A first order justification is provided by a check that $\Gamma_{E1}^{stat} \gg \Gamma_{ND}^{coll}$, where $\Gamma_{ND}^{coll}$ is the width for E2 decay within a rotational band with a moment of inertia given by that of the observed sequence of yrast states. Figure 3(c) shows the behaviour of $\Gamma_{E1}^{stat}$ and $\Gamma_{ND}^{coll}$ for states in the ND well at the same excitation energy and spin as states in the yrast SD bands in the two Pb isotopes. With the reduced backshift parameters, the statistical E1 width at the point of decay ($I = 8 - 12\hbar$ for $^{192}$Pb, $I = 6 - 10\hbar$ for $^{194}$Pb) is almost two orders of magnitude larger than the estimated collective E2 width. The results therefore support the assumption that the ND width can be approximated by the statistical E1 width, provided the reduced backshift parameters are appropriate.

The two remaining parameters are determined experimentally. $F_{SD}$ is simply the measured fraction of intensity that remains in the SD band below the level of interest. The width for $\gamma$ decay within the SD band, $\Gamma_{SD}$, is obtained through measurements of the lifetimes of the SD states, as $\Gamma_{SD} = \hbar/\tau$. The Doppler Shift Attenuation Method (DSAM) has been used to measure lifetimes of the high-spin SD states in $^{192}$Pb$^{22)}$ and $^{194}$Pb$^{23)}$ and the Recoil Distance Method (RDM) to measure lifetimes of the lowest spin states in $^{194}$Pb$^{24)}$. The DSAM measurements do not extend to the decay-out region, but can be used to obtain an average quadrupole moment, $Q_t$. $\Gamma_{SD}$ is then given (in eV) by $\Gamma_{SD} = 8.0 \times 10^{-8} E_\gamma^5 Q_t^2 < IK20I - 2K >^2$, with $E_\gamma$ in MeV, $Q_t$ in $e^2fm^4$. For $^{192}$Pb, values of $\Gamma_{SD}$ have been obtained from the above assuming a quadrupole moment of 19.3 eb, the average of the DSAM results for decays from levels with spins $16\hbar \leq I \leq 26\hbar$. Data for $^{194}$Pb are taken from Krücken et al.$^{25)}$.

§4. Results and Discussion

We now have sufficient information to obtain $\Gamma^4$ and $V$ in both the GW and CSB approaches. The results of these analyses are given in Table I. Data are also included for $^{194}$Hg and $^{152}$Dy; widths and average level spacings for these nuclei are taken from Refs.$^{11),25)}$

It is immediately clear that the two approaches give very different values for the spreading widths. It has been suggested$^{19)}$ that these differences may be explained by consideration of the different definitions of the parameter $\Gamma^4$. In the CSB model, $\Gamma^4$ represents a real, physical rate; that is, the tunnelling rate from the SD state to a single ND state. In the GW model, the spreading width is introduced as a parameter to describe the likelihood of the SD state tunnelling to a large number of ND states. However, for the cases where the level density is small and mixing is therefore predominantly with one ND level, one would hope that the results of the two approaches would converge. In fact, the values of $\Gamma^4_{GW}$ are surprisingly large for all cases except $^{194}$Hg - indeed, the GW analysis yields spreading widths of the order of 1 – 100 eV for the Dy and Pb decays. In contrast, the results for $\Gamma^4_{CSB}$ are
of the order of $\mu$eV for the Pb and Hg decays and meV for the Dy decay.

<table>
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<tr>
<th>Nucleus</th>
<th>$I$ (h)</th>
<th>$F_{SD}$ (eV)</th>
<th>$&lt;D&gt;$ (eV)</th>
<th>$I_{ND}$ (eV)</th>
<th>$I_{GW}$ (eV)</th>
<th>$I_{CSB}$ (eV)</th>
<th>$V_{GW}$ (eV)</th>
<th>$V_{CSB}$ (eV)</th>
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<td>786</td>
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<td>$132 \times 10^{-6}$</td>
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<td>311</td>
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</tr>
<tr>
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<td>455</td>
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<td>16</td>
<td>$4.8 \times 10^{-3}$</td>
<td>$97 \times 10^{-6}$</td>
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<td>$17 \times 10^{-3}$</td>
<td>$10 \times 10^{-3}$</td>
<td>6</td>
<td>$11 \times 10^{-3}$</td>
<td>14</td>
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</table>

Table I. Results of the spreading width analysis for the GW and CBS approaches.

It should be noted that the CSB model yields a negative value of $I^\downarrow$ for the $10^+$ state in $^{192}$Pb, which is clearly unphysical. As noted by Cardamone et al.,$^{19}$ the necessity that $I^\downarrow$ is positive imposes the additional constraint that $F_{SD} > \Gamma_{SD}/2\Gamma_{ave}$. The failure of these data to satisfy this constraint could be due to an inappropriate estimate of $I_{ND}$ or the need to consider mixing with a second ND state in the analysis. As described above, many assumptions have been made in estimating $D$ and $I_{ND}$; however, any revision is more likely to decrease the ND width and thus exacerbate the situation. Neither is the alternative explanation appealing: if the two-level model is sufficient in the case of $^{194}$Hg, for example, where the $D$ and $I_{ND}$ are orders of magnitude higher, then it seems unreasonable to require a three- or four-level model for $^{192}$Pb.

We can also compare the calculated values of the matrix element $V$, which represents the interaction between SD and ND states. With the exception of the anomalous $^{192}$Pb $10^+$ state, the values of $V_{GW}$ and $V_{CSB}$ are of the same order of magnitude, but with $V_{CSB}$ two to three times larger than $V_{GW}$. That is, in both approaches, $V^{(192}\text{Pb}) \sim 10 - 100 \times V^{(194}\text{Pb})$; $V^{(194}\text{Pb}) \sim 10 \times V^{(194}\text{Hg})$; and (perhaps most surprisingly) $V^{(152}\text{Dy}) \sim V^{(192}\text{Pb})$. This leads us back to the original question: what is it that initiates the decay out of the SD well? What mechanism results in an interaction showing the above differences between different nuclei? It seems that chaoticity of the ND states cannot be the cause, as the same behaviour is seen in $^{194}$Hg ($U_{\text{decay}} \approx 4$ MeV) and $^{192}$Pb ($U_{\text{decay}} \approx 2$ MeV): the latter cannot possibly be thought to be in the chaotic regime. If low-spin pairing correlations are responsible for the enhanced decay probability, then it is still necessary to understand why the effect is the same for these two nuclei: is the pairing for $I = 12h$ in $^{192}$Pb so much larger than that for $I = 12h$ in $^{194}$Hg, so that it overcomes the lower level density and produces almost the same SD decay-out probability?

It may be argued that the two Pb isotopes should not be treated according to either of the above prescriptions. Their proximity to the ND yrast line means the ND level density is low, and structure effects may be expected to play a significant role. But the analysis of $^{152}$Dy ($U \approx 3 - 4$ MeV) produces similar spreading widths and matrix elements, so for which real SD nuclei are these types of analysis appropriate? Indeed, if the same model cannot be used to describe the decay process for all these nuclei, then we are left facing a further question: why are the decay profiles of SD bands so similar across such a wide range of excitation energies, spins and energies?
above yrast?

In summary: the measured excitation energies of the SD bands in $^{192}$Pb and $^{194}$Pb have been used to estimate level densities and $\gamma$-decay widths in the ND well. It has been shown that a significant uncertainty is involved in these estimates. The statistical GW and two-level CSB approaches have been used to extract spreading widths and interaction strengths for these nuclei and for $^{194}$Hg and $^{152}$Dy. Both models produce somewhat unexpected results, and it is not clear whether they are appropriate descriptions of the decays in the nuclei presented here. However, our primary intention was to find a framework in which the mechanism governing the decay can be understood, and it is therefore essential that any such framework can be generally applied to the experimentally observed cases. Thus it appears that further work, both experimental and theoretical, is required before the mechanisms governing the decay of SD bands can be understood.

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22) A.N. Wilson et al., to be published.