Cluster formation in low density matter

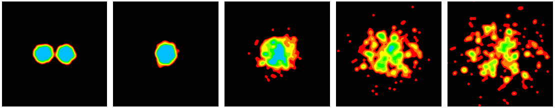
Akira Ono

Tohoku University

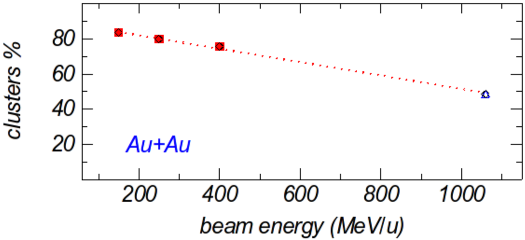
ICNT2013, July 15 – August 9, 2013.

- Clusters in medium and in heavy-ion collisions
- AMD with cluster correlations
- Clusters, collision dynamics and symmetry energy

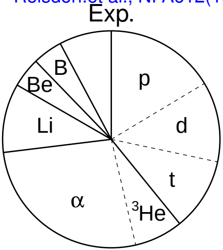
Large fraction of clusters in head-on collisions

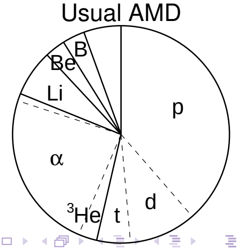


^{197}Au + ^{197}Au at 150 MeV/u



Reisdorf et al., NPA612(1997)493.



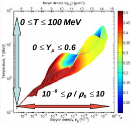


- Clusters (and fragments) are always the important part of the system.
- The actual proton multiplicity is much smaller than the prediction by usual dynamical models.

Week1 discussions by Hermann Wolter

Very low density matter in Astrophysics

In Supernova simulations the Equation-of-State appears for a wide range of Densities, temperatures and asymmetries.



T. Fischer, GSI Darmstadt

In particular also at very low densities, where correlations become important.

Various commonly used EoS's treat this in a phenomenological manner (e.g. Lattimer,Swesty; Shen, Toki; Shen, Horowitz, Teige))
There exists an exact low density limit, the Virial Theorem (Horowitz, Schwenk)

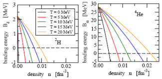
Attempted Improvements: (S.Typel, G. Röpke, T. Klähn, D. Blaschke, HHW, PRC 81, 015803 (2010))
- medium effects on light clusters, quantum statistical approach
- description of low to high density clustered matter in dens.-dep. rel. mean field model (DD-RMF)

Theoretical approach:

Typel, Röpke, Klähn, Blaschke, Wolter,PRC81,015803(2010)

Quantumstatistical model (QS)
- Includes medium modification of clusters (Mott transition)
- Includes correlations in the continuum (phase shifts)
- needs good model for quasi-particle energies in the mean field

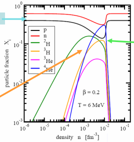
Generalized Rel. Mean Field model (RMF)
- Good description of higher density phase, i.e. quasiparticle energies
- Includes cluster degrees of freedom with parametrized density and temperature dependent binding energies
- Heavier clusters treated in Wigner-Seitz cell approximation (



Global approach from very low to high densities

Particle Fractions

very low density: p,n



Mott density: clusters melt, homogeneous p,n matter;

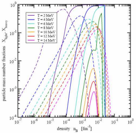
here heavier nuclei (embedded into a gas) become important, not yet fully implemented

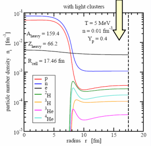
Increasing density: clusters arise: deuteron first, but then α dominates

Calculation in RMF of heavy cluster in Wigner-Seitz cell in beta-equilibrium

S.Typel, G. Röpke, et al.,PRC 81 (2010)

Heavier clusters (nuclei)
(·····) light
(——) heavy fraction



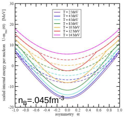


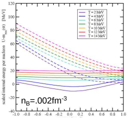
Symmetry Energy in Nuclear and Stellar Matter

Nuclear Matter (w/o clusters) without (-----) and with (——) liquid-gas phase transition

Stellar Matter (with electrons and with clusters) without (------) and with Coulomb contrib removed (——)

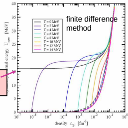
Internal Energy

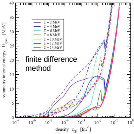




Symmetry Energy

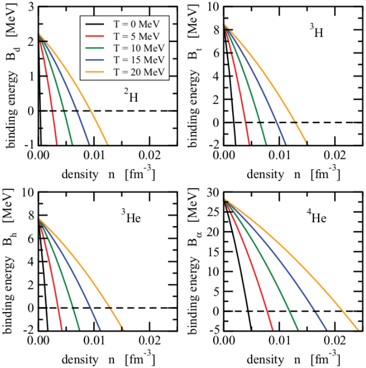
finite at T=0 due to PT





Clusters in infinite nuclear medium

Typel et al, PRC81(2010)015803



- $\mathbf{P} = 0$: Clusters at rest (relative to medium)
- T: temperature of medium

Equation for the deuteron in medium

$$\Big[e(\tfrac{1}{2}\mathbf{P}+\mathbf{p}) + e(\tfrac{1}{2}\mathbf{P}-\mathbf{p})\Big]\tilde{\psi}(\mathbf{p}) + \tfrac{1}{2}\Big[1 - f(\tfrac{1}{2}\mathbf{P}+\mathbf{p}) - f(\tfrac{1}{2}\mathbf{P}-\mathbf{p})\Big]\int \frac{d\mathbf{p}'}{(2\pi)^3}\langle\mathbf{p}|v|\mathbf{p}'\rangle\tilde{\psi}(\mathbf{p}') = E\tilde{\psi}(\mathbf{p})$$

Correlated part of the two-body level density

$$D(E) = \sum_{\mathbf{P},k} g_k\delta(E-E_k) + \sum_{\mathbf{P},l}\frac{g_l}{\pi}2\sin^2\delta_l\frac{d\delta_l}{dE}$$

Q. When do the cluster correlations in the continuum disappear?

Q. What is the effect of the asymmetry δ of the medium?

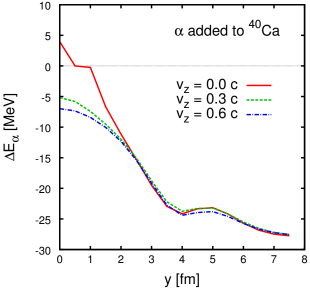
Cluster put into a nucleus (AMD)

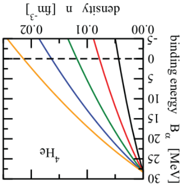
α cluster $\approx |\alpha, \mathbf{Z}\rangle$ = four wave packets with different spins and isospins placed at the same phase space point $\mathbf{Z}$. (Energies are defined relative to $|^{40}\mathrm{Ca}\rangle$.)

$$E_\alpha : \qquad \mathcal{A}\,|\alpha, \mathbf{Z}\rangle|^{40}\mathrm{Ca}\rangle$$

$$E_{\text{nucleon}} : \qquad \mathcal{A}\,|\mathbf{Z}\rangle|^{40}\mathrm{Ca}\rangle \quad (\text{nucleon} = p\uparrow, p\downarrow, n\uparrow, n\downarrow)$$

$$\Delta E_\alpha = E_\alpha - (E_{p\uparrow} + E_{p\downarrow} + E_{n\uparrow} + E_{n\downarrow})$$

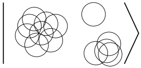




- Due to the shell effect in a finite nucleus, $E_{\text{nucleon}} \approx E_{\mathrm{F}} + \hbar\omega$
  $\Rightarrow$ Lower ΔE_α
- Due to the density dependence of the Skyrme force, the interaction between nucleons in the α cluster is weakened in the nucleus.
  $\Rightarrow$ Dependence on $\mathbf{P}$ or v_z.

$$\frac{\mathrm{Re}\,\mathbf{Z}}{\sqrt{\nu}} = (0, y, 0), \qquad \frac{2\hbar\sqrt{\nu}\,\mathrm{Im}\,\mathbf{Z}}{M} = (0, 0, v_z)$$

Antisymmetrized Molecular Dynamics



AMD wave function

$$|\Phi(Z)\rangle = \det_{ij}\Big[\exp\Big\{-\nu\Big(\mathbf{r}_j - \frac{\mathbf{Z}_i}{\sqrt{\nu}}\Big)^2\Big\}\chi_{\alpha_i}(j)\Big]$$

$$\mathbf{Z}_i = \sqrt{\nu}\mathbf{D}_i + \frac{i}{2\hbar\sqrt{\nu}}\mathbf{K}_i$$

ν : Width parameter = $(2.5 \text{ fm})^{-2}$

χ_{α_i} : Spin-isospin states = $p\uparrow, p\downarrow, n\uparrow, n\downarrow$

Time-dependent variational principle

$$\delta \int_{t_1}^{t_2} \frac{\langle\Phi(Z)|(i\hbar\frac{d}{dt} - H)|\Phi(Z)\rangle}{\langle\Phi(Z)|\Phi(Z)\rangle} dt = 0, \qquad \delta Z(t_1) = \delta Z(t_2) = 0$$

Equation of motion for the wave packet centroids Z

$$\frac{d}{dt}\mathbf{Z}_i = \{\mathbf{Z}_i, \mathcal{H}\}_{\text{PB}} \qquad \text{or} \qquad i\hbar \sum_{j=1}^{A} \sum_{\tau=x,y,z} C_{i\sigma,j\tau} \frac{dZ_{j\tau}}{dt} = \frac{\partial\mathcal{H}}{\partial Z_{i\sigma}}$$

$$\mathcal{H} = \frac{\langle\Phi(Z)|H|\Phi(Z)\rangle}{\langle\Phi(Z)|\Phi(Z)\rangle} + (\text{c.m. correction}),$$

H: Effective interaction (e.g. Skyrme force)

Effective Interaction

- Finite-range effective interaction such as Gogny force

$$v_{ij} = \sum_{k=1,2} (W_k + B_k P_\sigma - H_k P_\tau - M_k P_\sigma P_\tau) e^{-(\mathbf{r}_i - \mathbf{r}_j)^2/a_k^2} + t_\rho (1 + P_\sigma) \rho(\mathbf{r}_i)^\sigma \delta(\mathbf{r}_i - \mathbf{r}_j)$$

$$\langle V \rangle = \frac{1}{2} \sum_{i=1}^{A} \sum_{j=1}^{A} \sum_{k=1}^{A} \sum_{l=1}^{A} \langle ij|v|kl - lk \rangle B_{ki}^{-1} B_{lj}^{-1} \qquad \sim A^4$$

- Skyrme force, in recent calculations.

$$v_{ij} = t_0(1 + x_0 P_\sigma)\delta(\mathbf{r}) + \tfrac{1}{2} t_1 (1 + x_1 P_\sigma)[\delta(\mathbf{r})\mathbf{k}^2 + \mathbf{k}^2 \delta(\mathbf{r})] \qquad \mathbf{r} = \mathbf{r}_i - \mathbf{r}_j$$

$$+ t_2(1 + x_2 P_\sigma)\mathbf{k} \cdot \delta(\mathbf{r})\mathbf{k} + t_3(1 + x_3 P_\sigma)[\rho(\mathbf{r}_i)]^\alpha \delta(\mathbf{r}) \qquad \mathbf{k} = \tfrac{1}{2\hbar}(\mathbf{p}_i - \mathbf{p}_j)$$

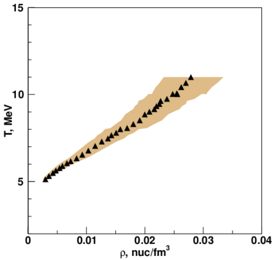
$$\langle V \rangle = \int \mathcal{V}\Big(\rho(\mathbf{r}), \tau(\mathbf{r}), \triangle\rho(\mathbf{r}), \mathbf{j}(\mathbf{r})\Big) d\mathbf{r} \qquad \sim A^2 V$$

$$\rho(\mathbf{r}) = \left(\frac{2\nu}{\pi}\right)^{3/2} \sum_{i=1}^{A} \sum_{j=1}^{A} e^{-2\nu(\mathbf{r} - \mathbf{R}_{ij})^2} B_{ij} B_{ji}^{-1}, \qquad \mathbf{R}_{ij} = \frac{1}{2\sqrt{\nu}}(\mathbf{Z}_i^* + \mathbf{Z}_j)$$

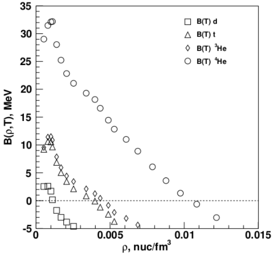
  - Less computational cost for heavy systems.
  - Momentum dependence is not good at high energies.

Clusters at low densities in HIC (NuSYM talk by Natowitz)

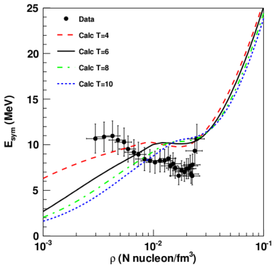
Analyze clusters emitted from the expanding IV source, assuming that 'v_{surf}' of a cluster represents the emission time until which it was in thermal and chemical equilibrium with other clusters in the source.



Qin et al.,
PRL108(2012)172701



Hagel et al.,
PRL108(2012)062702

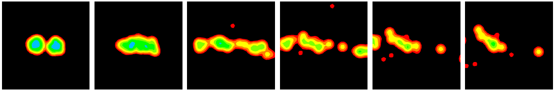


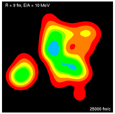
Wada et al.,
PRC85(2012)064618

Justification by dynamical models is desirable?

^{40}Ca + ^{40}Ca, E/A = 35 MeV, b = 0

Furuta and Ono, PRC79 (2009) 014608.



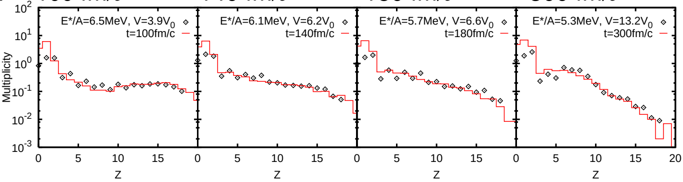


$\left\{\text{States at a reaction time } t\right\} \overset{?}{=\!=\!=}$ An equilibrium ensemble $(E, V, A = 36)$

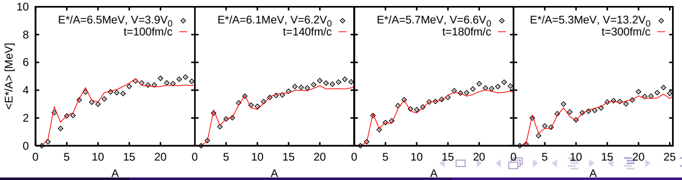
half of Ca + Ca system

t = 100 fm/c 140 fm/c 180 fm/c 300 fm/c

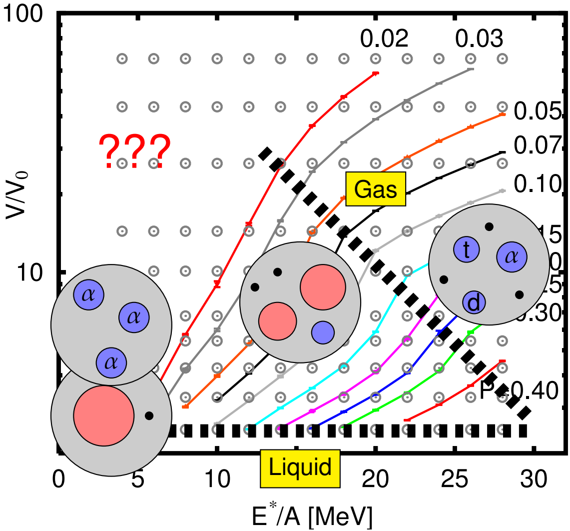
$Y(Z)$

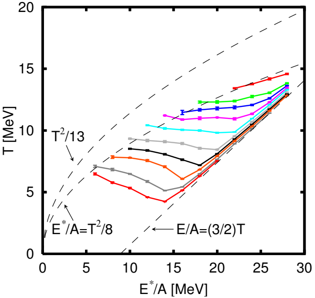


$\langle E^*/A \rangle$



Cluster Gas and Liquid-Gas Transition

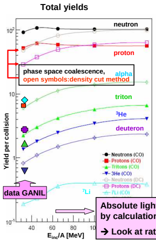


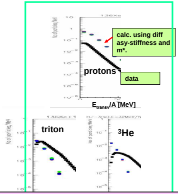


Clusters from HIC at 50 MeV/nucleon (NuSYM talk by Wolter)

Study of Light Fragment Emission: ^{136,124}Xe+^{124,112}Sn, E = 32,..,150 AMeV ,
Yields and spectra in comp. to experiment

136Xe+124Sn (n-rich)



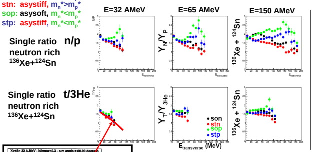


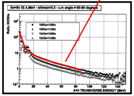
Absolute light fragment yields and spectra not well reproduced by calculations – important to improve!

→ Look at ratios: p/n, t/3He single ratios and double ratios

Comparison to data ^{136,124}Xe+^{124,112}Sn, E = 32,…,150 AMeV
data R. Bougault, A. Chbihi (Ganil, prelim, IWM11)

son: asysoft, $m_n^*>m_p^*$
stn: asystiff, $m_n^*>m_p^*$
sop: asysoft, $m_n^*<m_p^*$
stp: asystiff, $m_n^*<m_p^*$

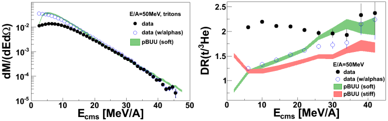




Prelim. data 136+124, 32 AMeV, INDRA favors asy-stiff with $m_n^*>m_p^*$

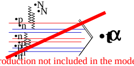
The single t/3He ratio seems to be a promising observable

Similar work at MSU: W.Lynch, INPC, Florence, 2013: ^{124,112}Sn+^{124,112}Sn, 50 AMeV
Calculations using P. Danielewicz code with clusters



- Illustrate with Danielewicz BUU with cluster production.
- Approx. QM description of cluster production up to A=3 (but not beyond)
- $m^*=0.7m_0$, $m^*_p= m^*_n$
- Calculations underpredict the double-ratio

Figure courtesy of Z. Chajecki.

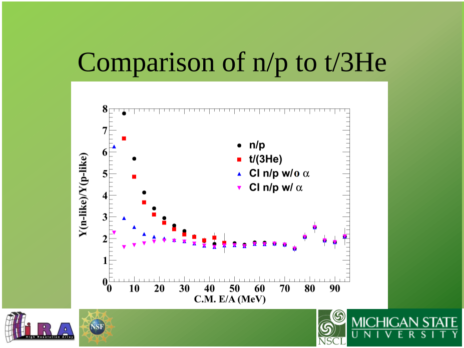


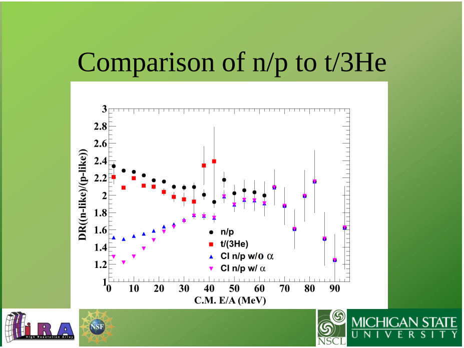
•Alpha production not included in the model => alphas end up being t or ^{3}He

•**Check**: combine experimental alpha spectra with tritons and helium-3 and compare to the model predictions.

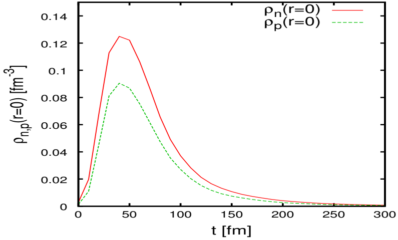
•Need to extend cluster production past A=4.

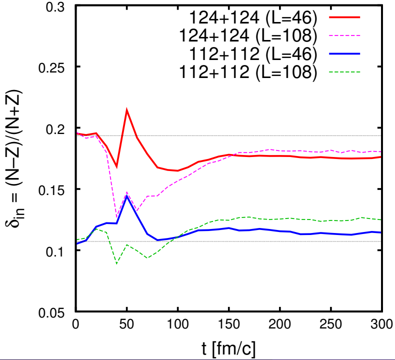
Triton/^{3}He Ratio (MuSYM presentation by Mike Youngs)

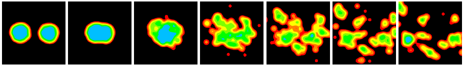




Dynamics of Neutrons and Protons

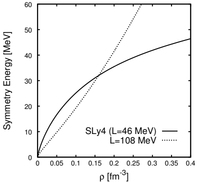






Sn + Sn central collisions at E/A = 50 MeV

- ^{124}Sn + ^{124}Sn
- ^{112}Sn + ^{112}Sn



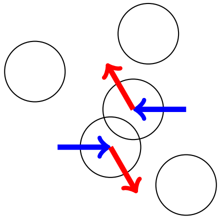
Skyrme force

- SLy4 (L = 46 MeV)
- L = 108 MeV

Observables

- π^-/π^+ yield ratio (at higher energies)
- Neutrons and protons (or ^{3}H and ^{3}He)
- Fragments

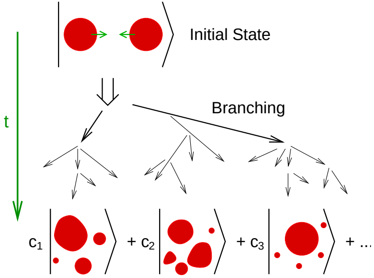
AMD with Two-Nucleon Collisions (very old version)



Stochastic two-nucleon collisions

- Cross section $\frac{d\sigma_{NN}}{d\Omega}(E, \theta)$ in nuclear medium.
- Pauli blocking for the final state.
  (Almost automatic in AMD)

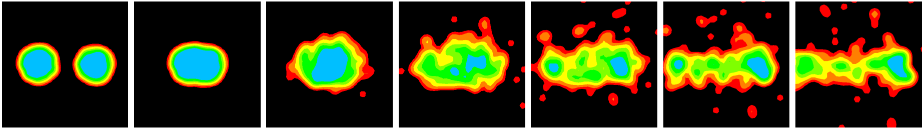
$$W_{i\to f} = \tfrac{2\pi}{\hbar}|\langle\Psi_f|V|\Psi_i\rangle|^2\delta(E_f - E_i)$$



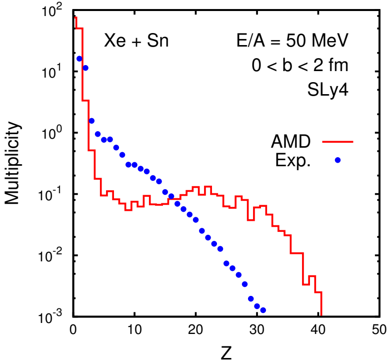
Stochastic equation of motion

$$\frac{d}{dt}\mathbf{Z}_i = \{\mathbf{Z}_i, \mathcal{H}\}_{\mathrm{PB}} + \text{(NN collisions)}$$

Results of AMD with Two-Nucleon Collisions



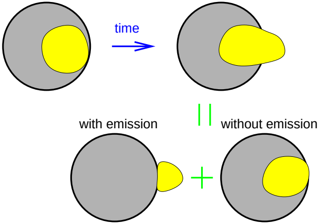
Xe + Sn central collisions at 50 MeV/u



- AMD with NN collisions
- INDRA data, Hudan et al., PRC 67 (2003)

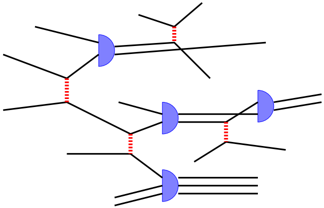
	AMD	INDRA
$M(p)$	40.2	8.4
$M(\alpha)$	2.5	10.1

Two directions of extension of AMD



Wave-packet splitting: Give fluctuation to each wave packet centroid, based on the single-particle motion.

$$\frac{d}{dt}Z = \{Z, \mathcal{H}\}_{\mathrm{PB}} + (\text{NN Collision})$$
$$+ (\text{W.P. Splitting}) + (\text{E. Conservation})$$



At each two-nucleon collision

$\mathrm{N}_1 + \mathrm{B}_1 + \mathrm{N}_2 + \mathrm{B}_2 \to \mathrm{C}_1 + \mathrm{C}_2$

$$\frac{d\sigma}{d\Omega} = F_{\mathrm{kin}}\, |\langle\varphi_1'|\varphi_1^{+\mathbf{q}}\rangle|^2\, |\langle\varphi_2'|\varphi_2^{-\mathbf{q}}\rangle|^2 \left(\frac{d\sigma}{d\Omega}\right)_{\mathrm{NN}\to\mathrm{NN}}$$

$$\frac{d}{dt}Z = \{Z, \mathcal{H}\}_{\mathrm{PB}} + (\text{NN Collision with Cluster})$$
$$+ (\text{Cluster-Cluster Binding})$$

Clusters have to be handled in a special way

Two-nucleon collision:

$$W_{i\to f} = \tfrac{2\pi}{\hbar}|\langle\Psi_f|V|\Psi_i\rangle|^2\delta(E_f - E_i)$$

$$\sum_f |\Psi_f\rangle\langle\Psi_f| = 1$$

What is a suitable complete basis set for the final states of a two-nucleon scattering?

- A usual choice is to change only the two.

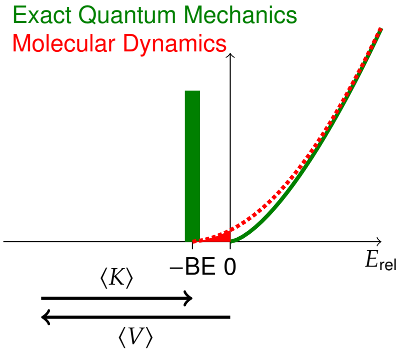
$$\sum_{k_1,k_2}\Big|\varphi_{k_1}(1)\varphi_{k_2}(2)\Psi(3,4,\ldots)\Big\rangle\Big\langle\varphi_{k_1}(1)\varphi_{k_2}(2)\Psi(3,4,\ldots)\Big|$$

- If a deuteron will propagate in medium, a more suitable basis will include

$$\Big|\varphi_{k_1}(1)\psi_{\mathrm{d}}(2,3)\Psi(4,\ldots)\Big\rangle\Big\langle\varphi_{k_1}(1)\psi_{\mathrm{d}}(2,3)\Psi(4,\ldots)\Big|+\cdots$$

Two-body level density (correlated part)

$$D(E) = \sum_{\mathbf{P},k} g_k\delta(E-E_k)+\sum_{\mathbf{P},l}\frac{g_l}{\pi}2\sin^2\delta_l\frac{d\delta_l}{dE}$$

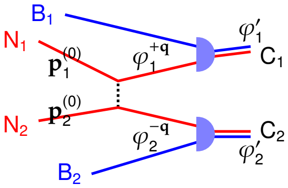


Cluster Formation Cross Section

Similar to Danielewicz et al., NPA533 (1991) 712.

$$N_1 + B_1 \; + \; N_2 + B_2 \;\to C_1 + C_2$$

- N_1, N_2 : Colliding nucleons
- B_1, B_2 : Spectator nucleons/clusters
- C_1, C_2 : N, $(2N)$, $(3N)$, $(4N)$ (up to α cluster)



$$v_{\rm NN}\, d\sigma(\text{NBNB} \to \text{CC})$$
$$= |\langle\varphi_1'|\varphi_1^{+\mathbf{q}}\rangle|^2\; |\langle\varphi_2'|\varphi_2^{-\mathbf{q}}\rangle|^2\; |M|^2\, \delta(\mathcal{H}-E)\, p_{\rm rel}^2 dp_{\rm rel} d\Omega$$

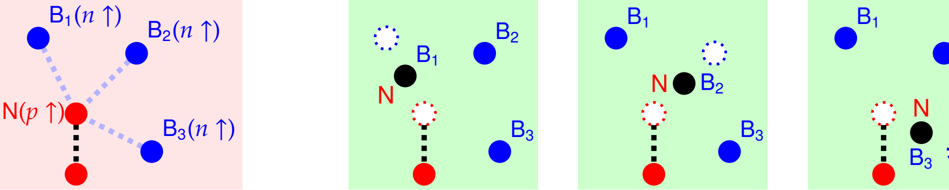
$$\left(v_{\rm NN}\, d\sigma_{\rm NN} = |M|^2\, \delta(\mathcal{H}-E)\, p_{\rm rel}^2 dp_{\rm rel} d\Omega \right)$$

$$\frac{d\sigma}{d\Omega} = F_{\rm kin}\, |\langle\varphi_1'|\varphi_1^{+\mathbf{q}}\rangle|^2\; |\langle\varphi_2'|\varphi_2^{-\mathbf{q}}\rangle|^2 \left(\frac{d\sigma}{d\Omega}\right)_{\rm NN\to NN}$$

$$\mathbf{p}_{\rm rel} = \tfrac{1}{2}(\mathbf{p}_1 - \mathbf{p}_2) = p_{\rm rel}\hat{\mathbf{\Omega}}$$
$$\mathbf{q} = \mathbf{p}_1 - \mathbf{p}_1^{(0)} = \mathbf{p}_2^{(0)} - \mathbf{p}_2$$
$$\varphi_1^{+\mathbf{q}} = \exp(+i\mathbf{q}\cdot\mathbf{r}_{N_1})\varphi_1^{(0)}$$
$$\varphi_2^{-\mathbf{q}} = \exp(-i\mathbf{q}\cdot\mathbf{r}_{N_2})\varphi_2^{(0)}$$

The cross section is given from the NN cross section.

Clusters (in the final states) are assumed to have $(0s)^N$ configuration.



$|\Phi^{\mathbf{q}}\rangle$ | $|\Phi'_1\rangle$ | $|\Phi'_2\rangle$ | $|\Phi'_3\rangle$

After $\mathbf{p}^{(0)} \to \mathbf{p}^{(0)} + \mathbf{q}$ | N + B$_1$ $\to$ C$_1$ | N + B$_2$ $\to$ C$_2$ | N + B$_3$ $\to$ C

Final states are not orthogonal: $N_{ij} \equiv \langle\Phi'_i|\Phi'_j\rangle \neq \delta_{ij}$

The probability of cluster formation with one of B's:

$$\hat{P} = \sum_{ij} |\Phi'_i\rangle N^{-1}_{ij} \langle\Phi'_j|, \qquad P = \langle\Phi^{\mathbf{q}}|\hat{P}|\Phi^{\mathbf{q}}\rangle \qquad \neq \sum_i |\langle\Phi'_i|\Phi^{\mathbf{q}}\rangle|^2$$

$$\begin{cases} P & \Rightarrow \text{Choose one of the candidates and make a cluster.} \\ 1-P & \Rightarrow \text{Don't make a cluster (with any n}\uparrow\text{).} \end{cases}$$

An algorithm to decide cluster formation

decide to do a collision based on $(d\sigma/d\Omega)_{\mathrm{NN}}$

C = N

do for species in $p\uparrow, p\downarrow, n\uparrow, n\downarrow$ (in a random order)

  cycle if C already contains a nucleon of species

  P = probability that C forms a cluster with a nucleon of species

  - taking care of the non-orthogonality
  - taking care of the p_{rel}-dependence of the phase space factors and the overlap probabilities

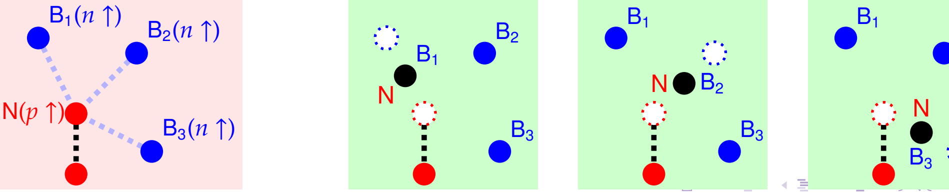
  if rand() $< P$ then

    choose a nucleon B of species

    C = C + B ! put the wave packets at the same phase space point

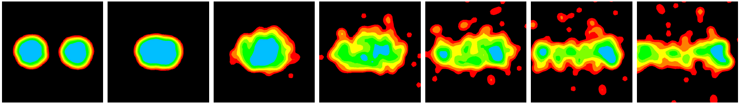
  endif

enddo

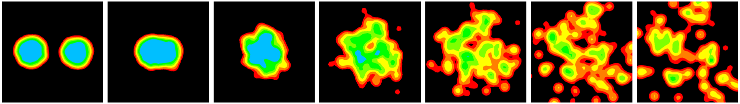


Effect of Clusters on the Density Evolution

Without cluster correlations (AMD with NN collisions)

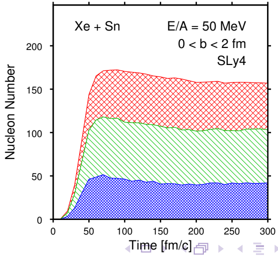


With cluster correlations



During the time evolution, clusters are . . .

- formed at NN collisions.
- propagated by AMD equation. (nothing special)
- broken by NN collisions. (nothing special)



Non-clustered

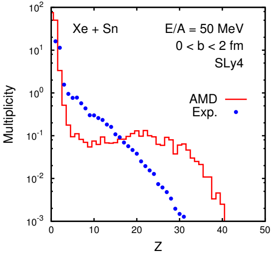
(2N)

(3N)

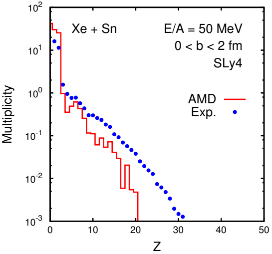
(4N)

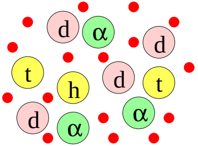
Effects of Cluster Correlations on Fragmentation

Usual NN collisions



With Clusters





Very strong tendency of turning into cluster gas.

	w/o C	with C	INDRA
$M(p)$	40.2	10.9	8.4
$M(\alpha)$	2.5	23.2	10.1
$Z_{\text{gas}}/Z_{\text{tot}}$	55%	78%	(40-50%)

- Gas = $\sum$ (particles of $A \leq 4$)
- Liquid = $\sum$ (heavier fragments)

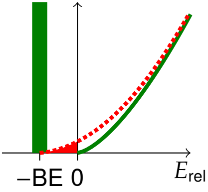
Cluster-Cluster Correlations

Relative motions between clusters should be treated quantum mechanically. In AMD,

- The binding energy a few clusters is reasonably correct,
- but the phase space of bound configuration is too small.

$$\text{e.g. } {}^7\text{Li} = \alpha + t - 2.5\ \text{MeV}$$

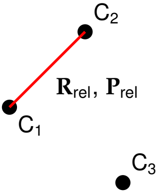
$$|\alpha + t\rangle \rightarrow |{}^7\text{Li}\rangle \text{ with probability} \approx |\langle {}^7\text{Li}|\alpha + t\rangle|^2$$



At every time step, **Clusters** C_1 **and** C_2 **are bound:** $\mathbf{P}_{\text{rel}} \rightarrow 0$,

- **if** C_j is the cluster closest to $\mathbf{C}_i$, $\quad (i, j) = (1, 2)$ or $(2, 1)$,
- **and if** they are moderately separated, $|\mathbf{R}_{\text{rel}}| < R_{\text{max}}$,
- **and if** they are moving slowly away from each other, $|\mathbf{P}_{\text{rel}}| < P_{\text{max}}$ and $\mathbf{P}_{\text{rel}} \cdot \mathbf{R}_{\text{rel}} > 0$.

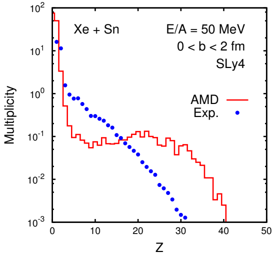
$$P_{\text{max}}^2/2\mu = 8\ \text{MeV}, \qquad R_{\text{max}} = 5\ \text{fm} \qquad \text{(adjustable)}$$



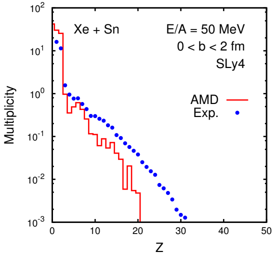
Energy is conserved by scaling the relative momentum between the C_1-C_2 pair and a third cluster C_3.

Effects of Cluster and C-C Correlations on Fragmentation

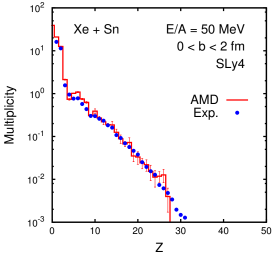
Usual NN collisions



With Clusters



With C & C-C

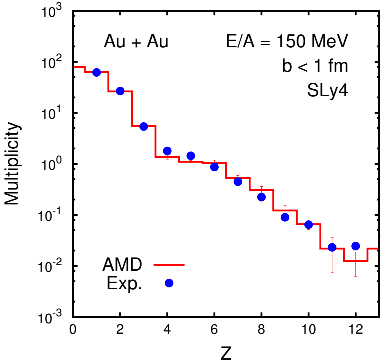


	w/o C	with C	C & C-C	INDRA
$M(p)$	40.2	10.9	10.8	8.4
$M(\alpha)$	2.5	23.2	10.7	10.1
$Z_{\text{gas}}/Z_{\text{tot}}$	55%	78%	43%	(40-50%)

- Gas =
  $\sum$ (particles of $A \leq 4$)
- Liquid =
  $\sum$ (heavier fragments)

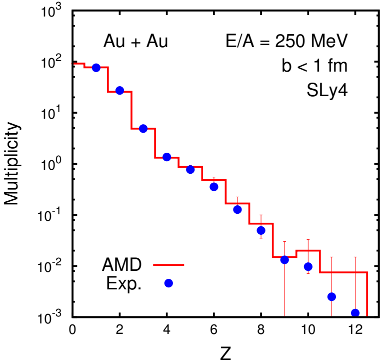
Au + Au Central Collisions at Higher Energies

E/A = 150 MeV



	with C & C-C	FOPI
$M(p)$	32.8	26.1
$M(\alpha)$	20.1	21.0
$Z_{\text{gas}}/Z_{\text{tot}}$	71%	73%

E/A = 250 MeV



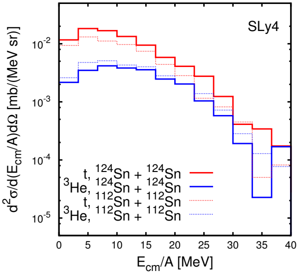
	with C & C-C	FOPI
$M(p)$	42.0	31.9
$M(\alpha)$	19.4	18.2
$Z_{\text{gas}}/Z_{\text{tot}}$	80%	83%

FOPI data: Reisdorf et al., NPA 612 (1997) 493.

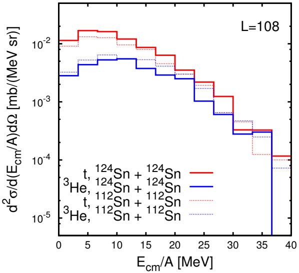
Energy Spectra of Clusters

^{124}Sn + ^{124}Sn and ^{112}Sn + ^{112}Sn central collisions at 50 MeV/nucleon

$\Rightarrow$ Energy spectra of tritons and ^{3}He emitted to transverse directions



SLy4 (L = 46 MeV)

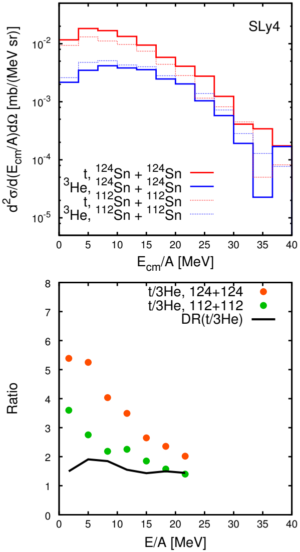


L = 108 MeV

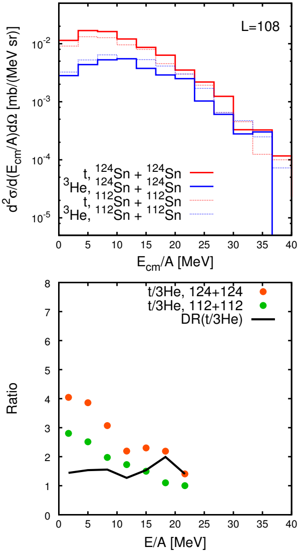
AO, J.Phys.Conf.Ser. 436(2013)012068

- Triton/^{3}He difference is consistent with the gas part of fractionation.

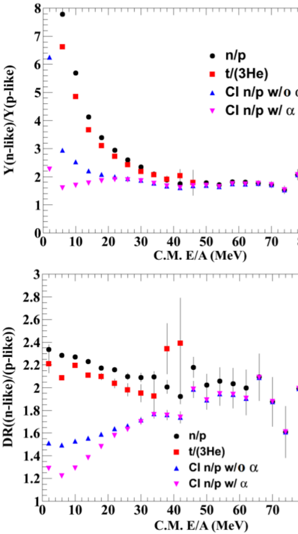
Single and Double Ratios of Cluster Energy Spectra



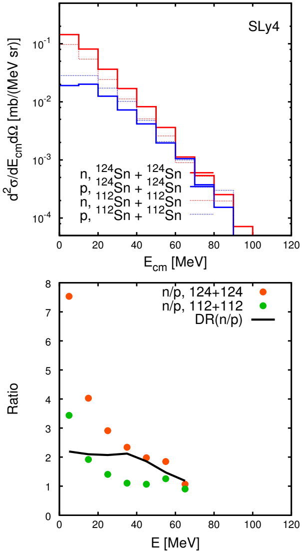
SLy4 (L = 46 MeV)



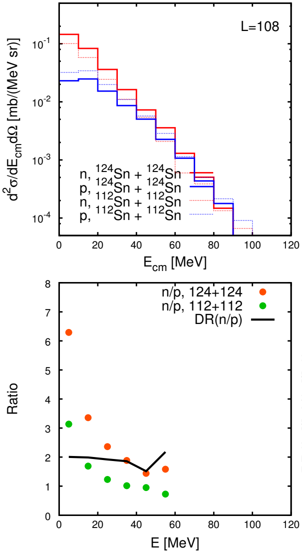
L = 108 MeV



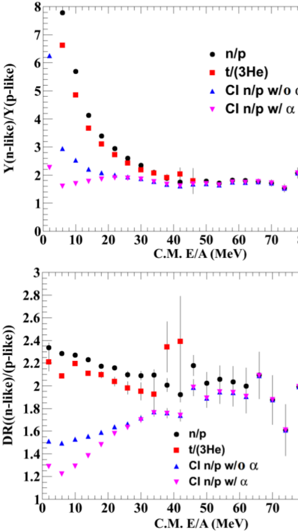
Single and Double Ratios of Nucleon Energy Spectra



SLy4 (L = 46 MeV)

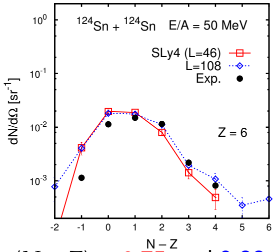


L = 108 MeV

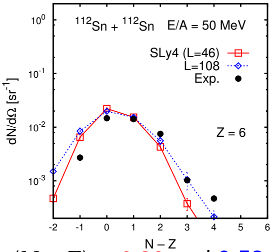


Fragment Isotope Distributions

MSU Data: T.X. Liu et al., PRC 014603 (2004).



$\langle N - Z\rangle$ = 0.75 and 0.99

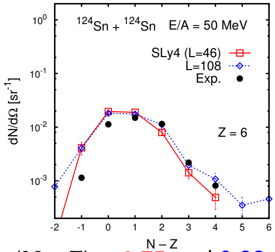


$\langle N - Z\rangle$ = 0.36 and 0.53

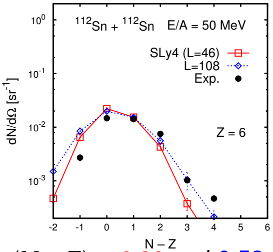
- The average asymmetry and the width are sensitive to the symmetry energy.
- Compared to data, $Z \geq N$ fragments are overproduced.

Fragment Isotope Distributions

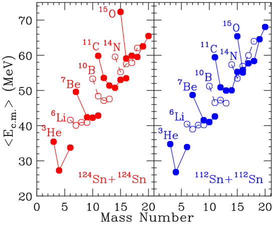
MSU Data: T.X. Liu et al., PRC 014603 (2004).



$\langle N - Z \rangle =$ 0.75 and 0.99



$\langle N - Z \rangle =$ 0.36 and 0.53



Data @NSCL/MSU
Liu et al., PRC86(2012)024605
Kinetic energies of proton-rich fragments are anomalously large.
(generalized ^{3}He puzzle)

- The average asymmetry and the width are sensitive to the symmetry energy.
- Compared to data, $Z \geq N$ fragments are overproduced.

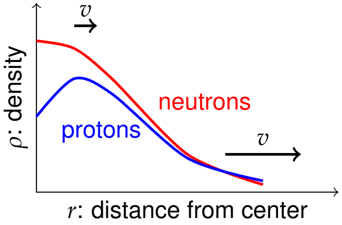
Possible reasons of the difference between calculation and experiment

In order to explain the experimental data, it seems both liquid and gas (low energy part) should be more neutron rich.

- Neutrons have to be more slowly expanding.
  - Momentum dependence of the symmetry potential (m_n^* v.s. m_p^*) should be studied.
- More low-velocity α clusters.
  - Decay from cluster-like excited states of nuclei.

$$^{A}\mathrm{C}^* \to \mathrm{Be} + \alpha$$

$$^{A}\mathrm{O}^* \to {}^{A-4}\mathrm{C} + \alpha \qquad \text{(produce neutron-rich C)}$$

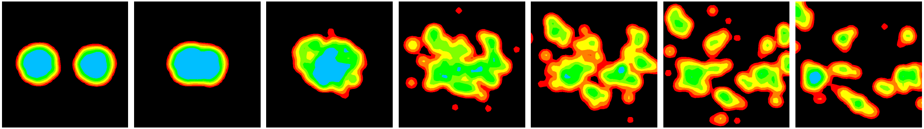


Summary

- Low-density nuclear matter is highly correlated and full of clusters.
- Gas part of the expanding systems can have large n/p and t/^{3}He ratios
  - due to isospin fractionation, and
  - due to the large α-cluster composition in gas.

  Studies of more neutron-rich systems may be interesting.
- It is important to develop methods to describe both the collision dynamics and the nuclear matter properties with cluster correlations.
- AMD with cluster and cluster-cluster correlations (currently works for $E/A \geq 50$ MeV/nucleon)
- Clusters (^{3}H and ^{3}He) in central collisions at 50 MeV/u.
  - Predictions by AMD with Skyrme SLy4 force are not very far from experimental data, but
  - Low-velocity part should be more neutron rich, or more α clusters should be emitted from fragments.

Bulk Properties and Correlations



An event of central collision of Xe + Sn at 50 MeV/nucleon (AMD calculation)

Bulk properties and dynamics
e.g. EOS $E(\rho)$

$\Longleftarrow\Longrightarrow$
interplay

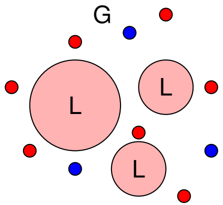
Correlations
e.g. clusters and fragments

$\Downarrow$ $\Downarrow$

Isospin dynamics, Symmetry energy
$\rho_n - \rho_p$, n/p, $t/{}^3\mathrm{He}, \ldots$

Liquid-Gas Separation

At a late stage of reaction

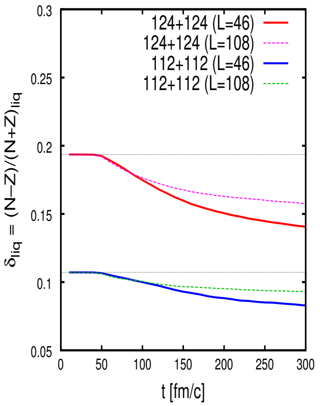


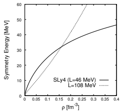
Fractionation/Distillation

$$\delta(\text{liquid}) < \delta(\text{gas})$$

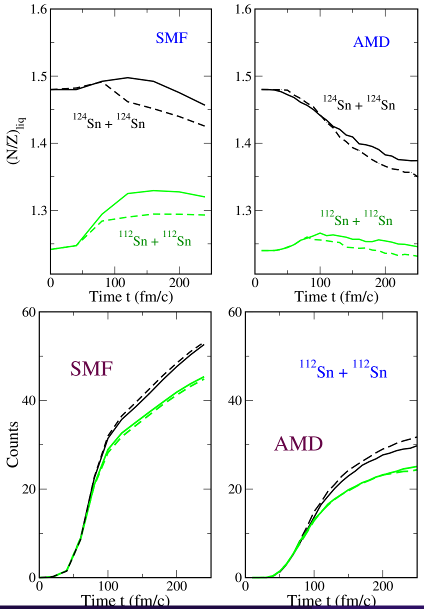
- Gas = $\sum(A \leq 4$ particles)
- Liquid = $\sum(A > 4$ fragments)
- Total = Gas + Liquid

Neutron-proton asymmetry of liquid part





Comparison of AMD and SMF: liquid-gas separation



Colonna, Ono, Rizzo, PRC82 (2010) 054613.

Neutron-Proton Ratio of Liquid

$N_{\text{liq}} = N_{\text{tot}} - N_{\text{gas}}, \; Z_{\text{liq}} = Z_{\text{tot}} - Z_{\text{gas}}$

- Dependence on the symmetry energy $E_{\text{sym}}(\rho)$ (soft or stiff)
- Dependence on models

$N_{\text{gas}}, Z_{\text{gas}}$: Number of Nucleons in Gas

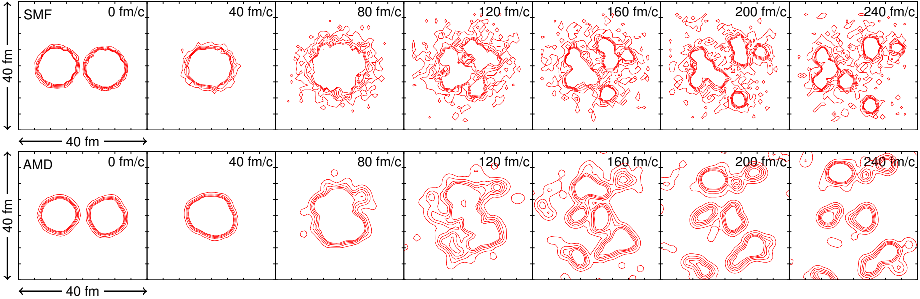
(Emitted nucleons and clusters with $A \leq 4$)

- Black line: neutrons
- Green line: protons

Comparison of AMD and SMF: expansion & fragmentation

Rizzo, Colonna, Ono, PRC76 (2007) 024611.
Colonna, Ono, Rizzo, PRC82 (2010) 054613.

- SMF = Stochastic Mean Field model
- AMD = Antisymmetrized Molecular Dynamics



Central Collisions of ^{112}Sn + ^{112}Sn at 50 MeV/nucleon

Used the same σ_{NN} and very similar effective interactions in both models.