Charge-exchange scattering to IAS and implication for the nuclear symmetry energy

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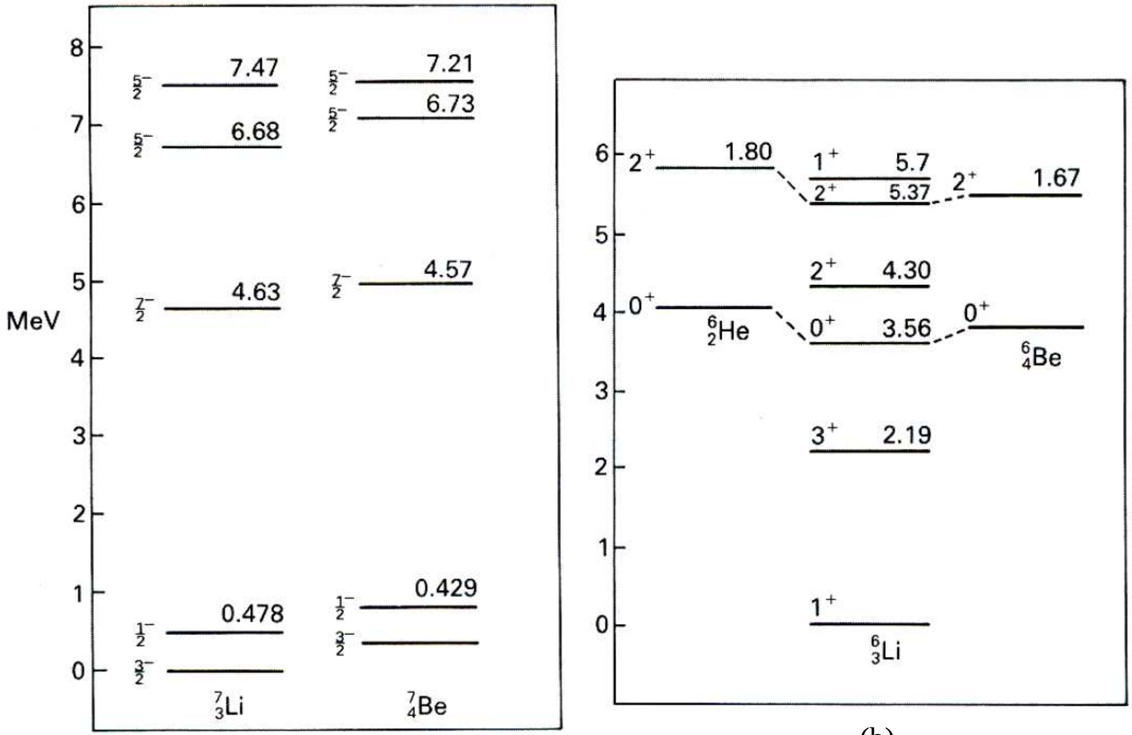

(a) (b)

Figure : Isobaric analog states in the mirror nuclei ^{7}Li and ^{7}Be, and in the isobars $A = 6$

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The Fermi transition ($\Delta L = \Delta S = 0$ and $\Delta T = 1$) between the isobaric analog states (IAS), induced by the charge-exchange (p, n) or $(^3\text{He},t)$ reaction, can be considered as “elastic” scattering of proton or ^{3}He by the isovector part of the OP that flips the projectile isospin

- A.M. Lane, Phys. Rev. Lett. **8**, 171 (1962)
- G.R. Satchler, *Isospin in Nuclear Physics* (Ed. D.H. Wilkinson, North-Holland Pub., Amsterdam, 1969) p390

In the folding model, the isovector part of the OP is determined exclusively by the neutron-proton difference in the nuclear densities and the isospin dependence of the effective NN interaction.

- DTK, W. von Oertzen, A.A. Ogloblin, Nucl. Phys. **A602**, 98 (1996)
- DTK, E. Khan, G. Colò, N.V. Giai, Nucl. Phys. **A706**, 61 (2002)

The isospin coupling links the isovector part of the OP to the cross section of the charge-exchange (p,n) or $(^3\text{He},t)$ scattering to the IAS $\Rightarrow$ The isospin dependence of the OP and that of the effective NN interaction can be tested in the CC analysis of these reactions.

- G.R. Satchler, *Direct Nuclear Reactions* (Oxford Univ. Press, 1983)
- DTK, H.S. Than, D.C. Cuong, Phys. Rev. C **76**, 014603 (2007)

The same isospin- and density dependent NN interaction can be used in a HF calculation to estimate the nuclear symmetry energy $\Rightarrow$ The fine-tuning of the isospin dependence of the NN interaction against the (p,n) or $(^3\text{He},t)$ data allows us to make some realistic prediction of the symmetry energy and its density dependence.

- DTK, B.M. Loc, D.N. Thang, Eur. Phys. J. A (to be published)

Lane potential and isospin coupling in the charge-exchange scattering to the IAS

Given the same isospin $t = 1/2$, the central nucleon-nucleus or ^{3}He-nucleus OP for the elastic scattering on a nonzero-isospin target can be written as

$$U(R) = U_0(R) + 4U_1(R)\frac{\boldsymbol{t}.\boldsymbol{T}}{aA}, \qquad (1)$$

where $\boldsymbol{t}$ is the isospin of the projectile and $\boldsymbol{T}$ is that of the target with mass number A, and $a = 1$ and 3 for nucleon and ^{3}He, respectively. The second term is the **symmetry term** of the OP, and U_1 is known as the **Lane potential** that contributes to both the elastic and charge-exchange scattering. The knowledge of U_1 is of fundamental interest in the study of the isovector mode of direct nuclear reaction

Consider a isospin multiplet with fixed isospins $\boldsymbol{t}$ for the projectile and $\boldsymbol{T}$ for the target $\Rightarrow$ $T_z = (N-Z)/2$. $\tilde{T}_z = T_z - 1$ for the target A and its **isobaric analog** $\tilde{A}$, respectively. In the isospin representation, state formed by adding proton or ^{3}He to A is $|aA>$, and that formed by adding a neutron or triton to $\tilde{A}$ is $|\tilde{a}\tilde{A}>$. $\Rightarrow$ Transition matrix elements of the Lane pot. for the **elastic scattering**

$$\begin{aligned} < aA|4U_1(R)\frac{\boldsymbol{t}.\boldsymbol{T}}{aA}|aA > &= -\frac{2}{aA}T_z U_1(R), \\ < \tilde{a}\tilde{A}|4U_1(R)\frac{\boldsymbol{t}.\boldsymbol{T}}{aA}|\tilde{a}\tilde{A} > &= \frac{2}{aA}(T_z - 1)U_1(R). \end{aligned} \tag{2}$$

and form factor of the **charge-exchange scattering** to the IAS

$$F_{\mathrm{cx}}(R) =< \tilde{a}\tilde{A}|4U_1(R)\frac{\boldsymbol{t}.\boldsymbol{T}}{aA}|aA >= \frac{2}{aA}\sqrt{2T_z}U_1(R). \tag{3}$$

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Total wave function in the two-channel approximation

$$\Psi = |aA > \chi_{aA}(\mathbf{R}) + |\tilde{a}\tilde{A} > \chi_{\tilde{a}\tilde{A}}(\mathbf{R}), \tag{4}$$

where $\chi(\mathbf{R})$ is the relative-motion wave function. The elastic and charge-exchange scattering cross sections are obtained from the solutions of the **coupled-channel Lane equations**

$$[K_a + U_a(R) - E_a]\,\chi_{aA}(\mathbf{R}) = -F_{\rm cx}(R)\chi_{\tilde{a}\tilde{A}}(\mathbf{R}), \tag{5}$$
$$[K_{\tilde{a}} + U_{\tilde{a}}(R) - E_{\tilde{a}}]\,\chi_{\tilde{a}\tilde{A}}(\mathbf{R}) = -F_{\rm cx}(R)\chi_{aA}(\mathbf{R}). \tag{6}$$

Here $K_{a(\tilde{a})}$ and $E_{a(\tilde{a})}$ are the kinetic-energy operators and center-of-mass energies of the $a + A$ and $\tilde{a} + \tilde{A}$ partitions.

G.R. Satchler, *Direct Nuclear Reactions* (Oxford Univ. Press, 1983)

OP in the entrance ($a + A$) and exit ($\tilde{a} + \tilde{A}$) channels are determined explicitly through the isoscalar (U_0) and isovector (U_1) parts of the optical potential (1) as

$$U_a(R) = U_0(R) - \frac{2}{aA} T_z U_1(R), \tag{7}$$

$$U_{\tilde{a}}(R) = U_0(R) + \frac{2}{aA}(T_z - 1) U_1(R). \tag{8}$$

In the CC calculation, U_a and $U_{\tilde{a}}$ are added by the spin-orbital and Coulomb potentials (the Coulomb term in the exit channel is nonzero only if $\tilde{a}$ is triton). IAS's are separated approximately by the Coulomb displacement energy $\Rightarrow$ the charge-exchange transition between them has $Q \neq 0$. To account for this effect (isospin impurity), U_0 and U_1 potentials used to construct $F_{\rm cx}(R)$ are evaluated from the proton or ^{3}He optical potential at the energy of $E = E_{\rm lab} - Q/2$, and those used to construct $U_{\tilde{a}}(R)$ are evaluated from the neutron or triton OP at the energy $E = E_{\rm lab} - Q$.

Folding model

Central nucleon-nucleus or nucleus-nucleus potential U is evaluated as a Hartree-Fock-type potential

$$U = \sum [< ij|v_{\rm D}|ij > + < ij|v_{\rm EX}|ji >], \tag{9}$$

where summation is performed over all nucleon states of the target ($j \in A$) or of both the target and projectile ($i \in a, j \in A$). $v_{\rm D}$ and $v_{\rm EX}$ are the direct and exchange parts of the effective NN interaction. To separate the isovector part of U, the explicit spin-isospin decomposition of the (energy- and density dependent) NN interaction is used

$$\begin{aligned} v_{\rm D(EX)}(E,\rho,s) &= v_{00}^{\rm D(EX)}(E,\rho,s) + v_{10}^{\rm D(EX)}(E,\rho,s)(\boldsymbol{\sigma\sigma}') \\ &+ v_{01}^{\rm D(EX)}(E,\rho,s)(\boldsymbol{\tau\tau}') + v_{11}^{\rm D(EX)}(E,\rho,s)(\boldsymbol{\sigma\sigma}')(\boldsymbol{\tau\tau}'), \end{aligned} \tag{10}$$

where s is the internucleon distance and ρ is the density of nuclear medium around the interacting nucleon pair.

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Using explicit proton (ρ_p) and neutron (ρ_n) densities, nucleon-nucleus or nucleus-nucleus OP is obtained in terms of the **isoscalar** ($U_{\rm IS}$) and **isovector** ($U_{\rm IV}$) parts as

$$U(E,\boldsymbol{R}) = U_{\rm IS}(E,\boldsymbol{R}) \pm U_{\rm IV}(E,\boldsymbol{R}), \tag{11}$$

where - sign pertains to the proton or ^{3}He OP, and $+$ sign to the neutron or triton OP. The folded nucleon-nucleus potentials are

$$\begin{aligned} U_{\rm IS}(E,\boldsymbol{R}) = \int \{ & [\rho_n(\boldsymbol{r}) + \rho_p(\boldsymbol{r})] v_{00}^{\rm D}(E,\rho,s) \\ + & [\rho_n(\boldsymbol{R},\boldsymbol{r}) + \rho_p(\boldsymbol{R},\boldsymbol{r})] v_{00}^{\rm EX}(E,\rho,s) j_0(k(E,R)s) \} d^3r, \end{aligned} \tag{12}$$

$$\begin{aligned} U_{\rm IV}(E,\boldsymbol{R}) = \int \{ & [\rho_n(\boldsymbol{r}) - \rho_p(\boldsymbol{r})] v_{01}^{\rm D}(E,\rho,s) \\ + & [\rho_n(\boldsymbol{R},\boldsymbol{r}) - \rho_p(\boldsymbol{R},\boldsymbol{r})] v_{01}^{\rm EX}(E,\rho,s) j_0(k(R)s) \} d^3r, \end{aligned} \tag{13}$$

where $\boldsymbol{s} = \boldsymbol{r} - \boldsymbol{R}$, $\rho(\boldsymbol{r},\boldsymbol{r}')$ is one-body density matrix of the target, with $\rho(\boldsymbol{r}) \equiv \rho(\boldsymbol{r},\boldsymbol{r})$, and $k(R)$ is determined as

$$k^2(E,R) = \frac{2\mu}{\hbar^2}[E_{\rm c.m.} - V(E,R) - V_C(R)]. \tag{14}$$

The double-folded ^{3}He-nucleus or triton-nucleus potentials are obtained in the same manner

$$U_{\rm IS}(E,\boldsymbol{R}) = \int\int [\rho_1(\boldsymbol{r}_1)\rho_2(\boldsymbol{r}_2)v_{00}^{\rm D}(E,\rho,s) + \rho_1(\boldsymbol{r}_1,\boldsymbol{r}_1+\boldsymbol{s}) \times\rho_2(\boldsymbol{r}_2,\boldsymbol{r}_2-\boldsymbol{s})v_{00}^{\rm EX}(E,\rho,s)j_0(k(E,R)s/M)]d^3r_1d^3r_2, \qquad (15)$$

$$U_{\rm IV}(E,\boldsymbol{R}) = \int\int [\Delta\rho_1(\boldsymbol{r}_1)\Delta\rho_2(\boldsymbol{r}_2)v_{01}^{\rm D}(E,\rho,s) + \Delta\rho_1(\boldsymbol{r}_1,\boldsymbol{r}_1+\boldsymbol{s}) \times\Delta\rho_2(\boldsymbol{r}_2,\boldsymbol{r}_2-\boldsymbol{s})v_{01}^{\rm EX}(E,\rho,s)j_0(k(E,R)s/M)]d^3r_1d^3r_2.$$

$\rho_i = \rho_{in} + \rho_{ip}$ and $\Delta\rho_i = \rho_{in} - \rho_{ip}$, $\boldsymbol{s} = \boldsymbol{r}_2 - \boldsymbol{r}_1 + \boldsymbol{R}$ and $M = aA/(a+A)$.
FF for both the (p,n) and $(^3\text{He},t)$ scattering to the IAS is

$$F_{\rm cx}(R) = \frac{2}{aA}\sqrt{2T_z}U_1(R) = \sqrt{\frac{2}{T_z}}U_{\rm IV}(R) \qquad (16)$$

Isospin- and density dependent CDM3Y6 interaction

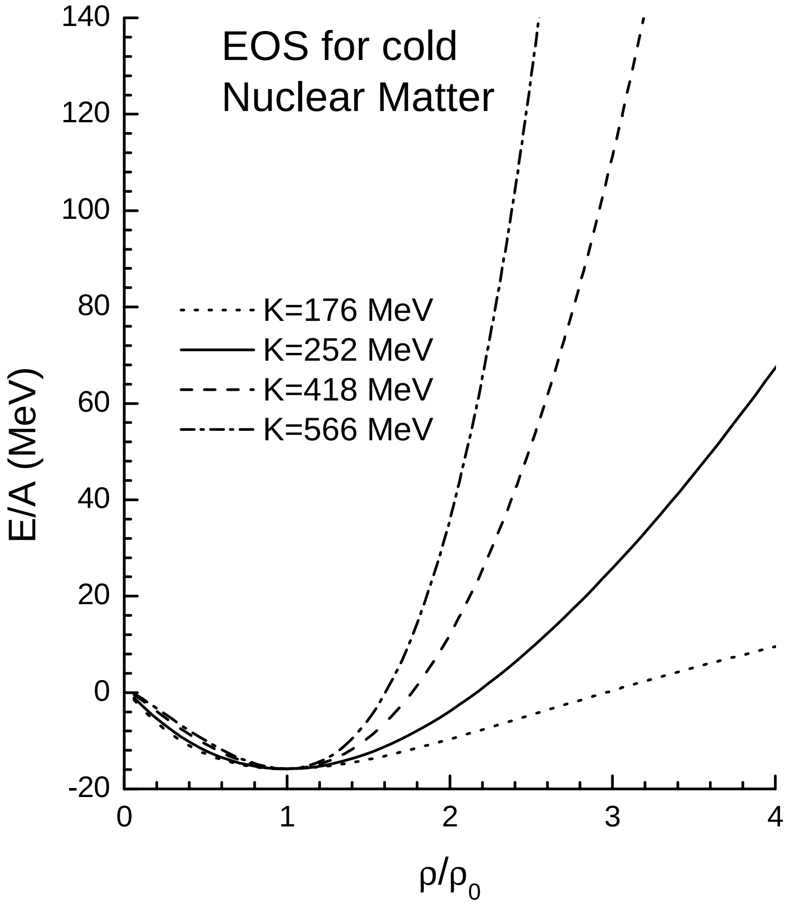
The isoscalar central part

$$v_{00}^{\mathrm{D(EX)}}(E,\rho,s) = F_{\mathrm{IS}}(E,\rho)v_{00}^{\mathrm{D(EX)}}(s), \tag{17}$$

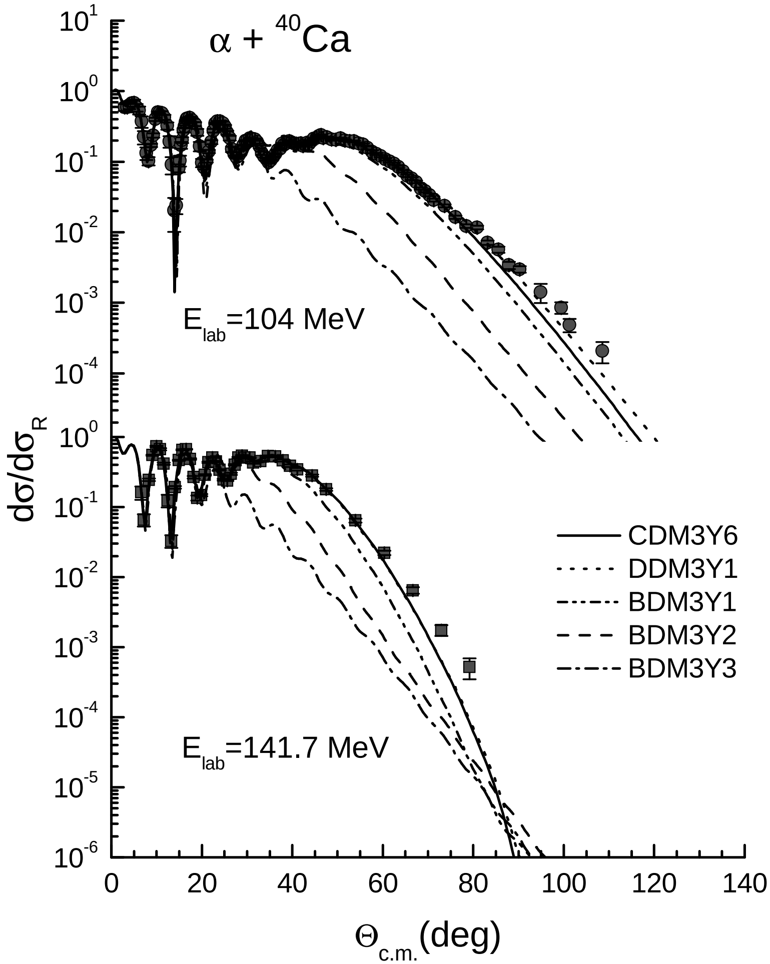
$$\text{where } F_{\mathrm{IS}}(E,\rho) = g(E)C_0[1+\alpha_0\exp(-\beta_0\rho)-\gamma_0\rho]. \tag{18}$$

Parameters of $F_{\mathrm{IS}}(\rho)$ were chosen to reproduce the saturation properties of the symmetric nuclear matter, with $K \approx 252$ MeV, in the HF approximation. $v_{00)}^{\mathrm{D(EX)}}(s)$ were kept as derived in terms of three Yukawas (M3Y-Paris). The CDM3Y6 interaction has been tested in numerous folding model analyses of the elastic, refractive nucleus-nucleus and α-nucleus scattering.

- N. Anantaraman, H. Toki, G.F. Bertsch, Nucl. Phys. **A398**, 269 (1983)
- DTK, G.R. Satchler, W. von Oertzen, Phys. Rev. C **56**, 954 (1997)
- DTK, G.R. Satchler, Nucl. Phys. **A668**, 3 (2000)



DTK, W. von Oertzen, H.G. Bohlen, S. Ohkubo, J. Phys. G **34**, R111 (2007).



Strongly refractive (rainbow) α-nucleus scattering !

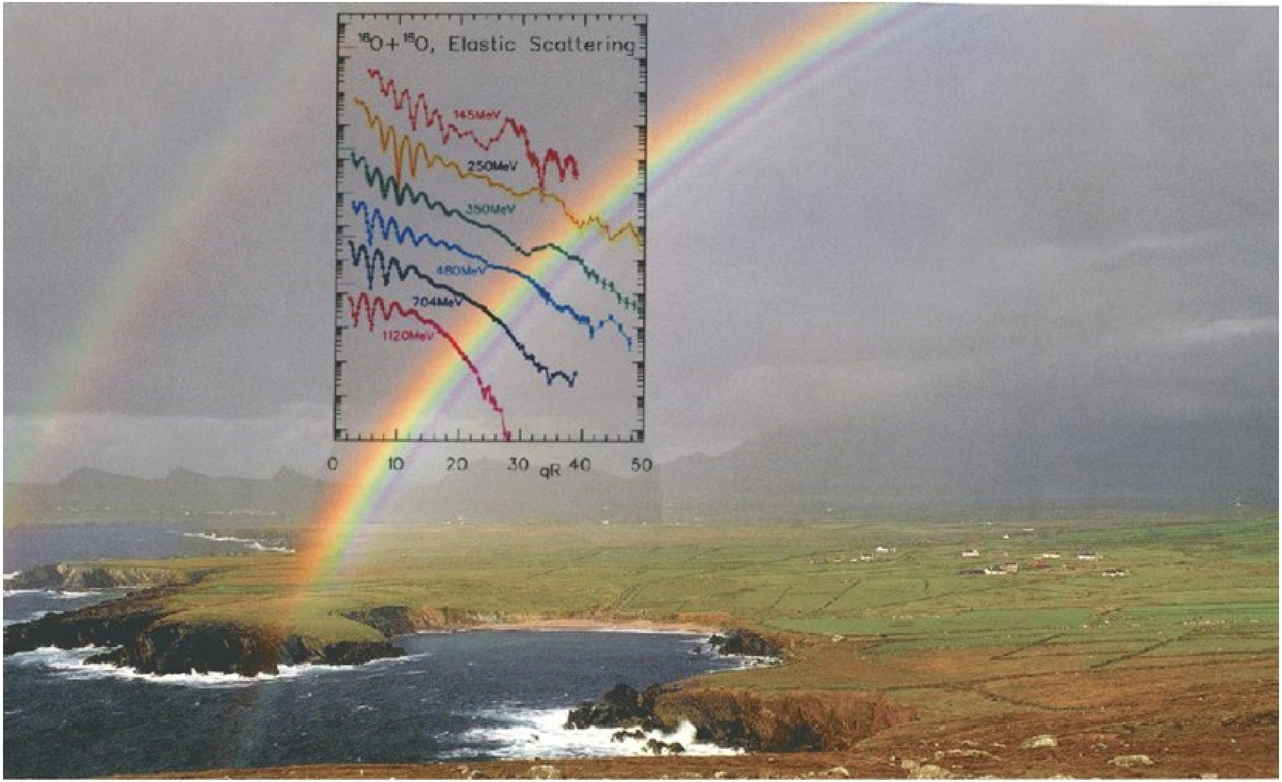


Figure : **At the end of the rainbow – an understanding of nuclear matter** [W. von Oertzen, DTK, H.G. Bohlen, Europhysicsnews **31**, 5 (2000)].

To probe the IV part of the CDM3Y6 interaction in the charge exchange reaction, the same functional has been adopted for the IV density dependence

$$
\begin{aligned}
v_{01}^{\mathrm{D(EX)}}(E,\rho,s) &= F_{\mathrm{IV}}(E,\rho)\, v_{01}^{\mathrm{D(EX)}}(s), && (19)\\
\text{where } F_{\mathrm{IV}}(E,\rho) &= C_1[1+\alpha_1 \exp(-\beta_1\rho)-\gamma_1\rho]. && (20)
\end{aligned}
$$

Parameters of $F_{\mathrm{IV}}(E,\rho)$ were adjusted carefully at each energy E to reproduce in the HF approximation the microscopic BHF results for the nucleon OP in nuclear matter by the JLM group. $v_{01}^{\mathrm{D(EX)}}(s)$ - kept unchanged as derived from the M3Y-Paris interaction. The charge exchange FF obtained with (19)-(20) is used in the CC analysis of the (p,n) and $(^3\mathrm{He},t)$ data.

- DTK, H.S. Than, D.C. Cuong, Phys. Rev. C **76**, 014603 (2007)
- J.P. Jeukenne, A. Lejeune, C. Mahaux, Phys. Rev. C **16**, 80 (1977)

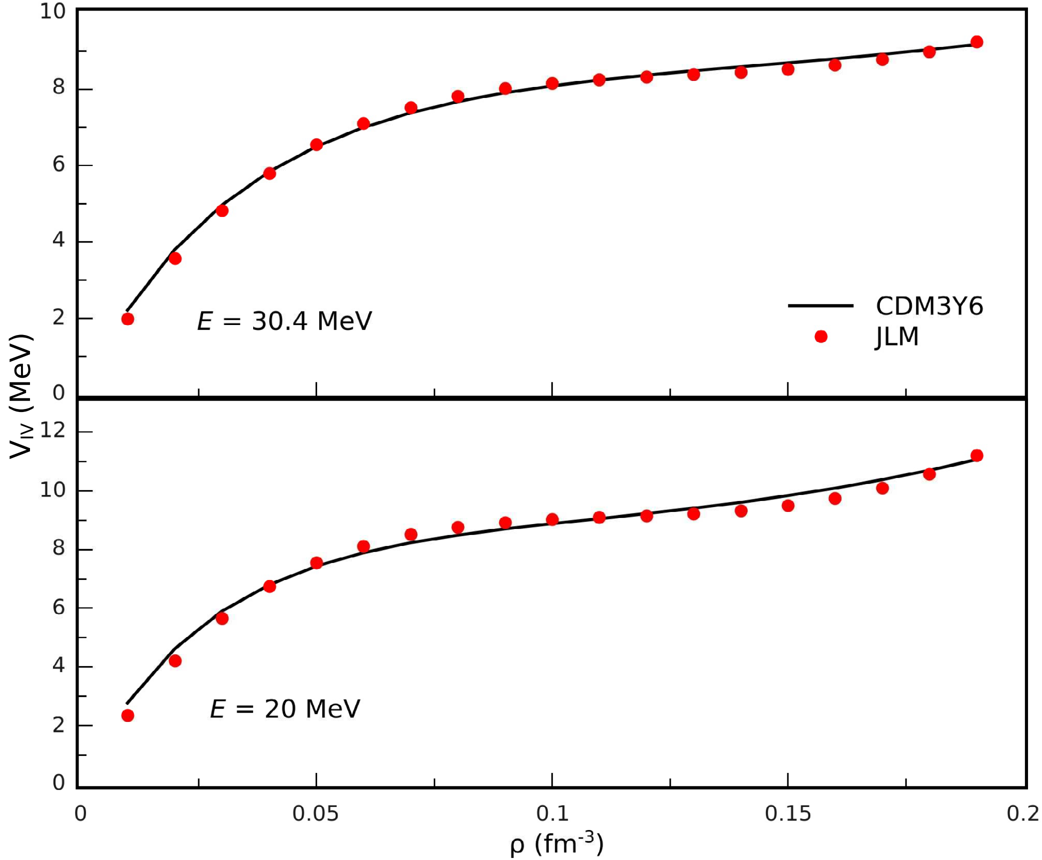


Figure : Real IV nucleon OP in nuclear matter given by the CDM3Y6 interaction, with $F_{\rm IV}(E,\rho)$ adjusted to reproduce the JLM results at $E = 30.4$ and 20 MeV.

Elastic proton scattering

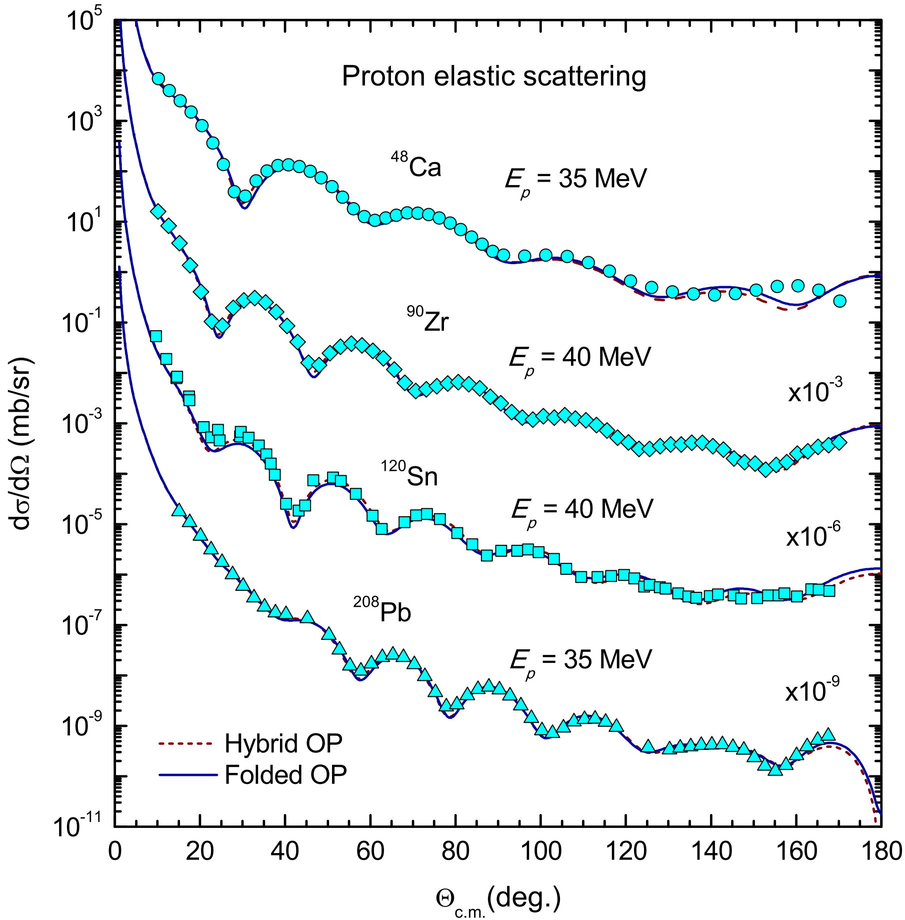
Isospin is not a good quantum number in the repulsive Coulomb field that slows down the incident proton $\Rightarrow$ it is necessary to add the Coulomb correction ΔE_C to the incident proton energy to separate the Coulomb effect and restore the Lane consistency of the OP

R.P. DeVito, DTK, S.M. Austin, U.E.P. Berg, B.M. Loc, Phys. Rev. C **85**, 024619 (2012)

OM analysis of the elastic proton scattering using the complex folded OP given by the CDM3Y6 interaction determined at $E = E_p - \Delta E_C$.

$$U(R) = N_V[V_{\rm IS}(R) - V_{\rm IV}(R)] + iN_W[W_{\rm IS}(R) - W_{\rm IV}(R)]. \qquad (21)$$

The strengths $N_{V(W)}$ of the folded OP were adjusted to the best fit to the data. To ensure a high accuracy of distorted waves, a hybrid OP was also used: real folded pot. + absorptive $W(R)$ pot. based on the CH89 global OP (R.L. Varner *et al.*, Phys. Rep. **201**, 57 (1991).)



Proton elastic scattering
^{48}Ca
E_p = 35 MeV
^{90}Zr
E_p = 40 MeV
x10^{-3}
^{120}Sn
E_p = 40 MeV
x10^{-6}
^{208}Pb
E_p = 35 MeV
x10^{-9}
Hybrid OP
Folded OP
dσ/dΩ (mb/sr)
$\Theta_{c.m.}$(deg.)

(p, n) scattering to the IAS

IV strength of the proton OP is just a few percent of the total OP and it is difficult to probe $U_{\rm IV}$ in the OM analysis. However, $U_{\rm IV}$ can be well fine tuned against the (p, n) data in the CC analysis of the charge-exchange scattering using the complex charge-exchange FF

$$F_{\rm cx}(R) = \sqrt{\frac{2}{T_z}} U_{\rm IV}(R) = \sqrt{\frac{2}{T_z}} [N_{V1} V_{\rm IV}(R) + i N_{W1} W_{\rm IV}(R)], \qquad (22)$$

with $V_{\rm IV}(R)$ and $W_{\rm IV}(R)$ given by the IV part of the CDM3Y6 interaction determined at $E = E_p - \Delta E_C - Q/2$. The best CC fit to the (p, n) data $\Rightarrow N_{V1} \approx 1.3 - 1.5$ and $N_{W1} \approx 1.0 \Rightarrow$ the empirical IV strength of the CDM3Y6 interaction is about $30 \sim 40\%$ stronger than that predicted by JLM.

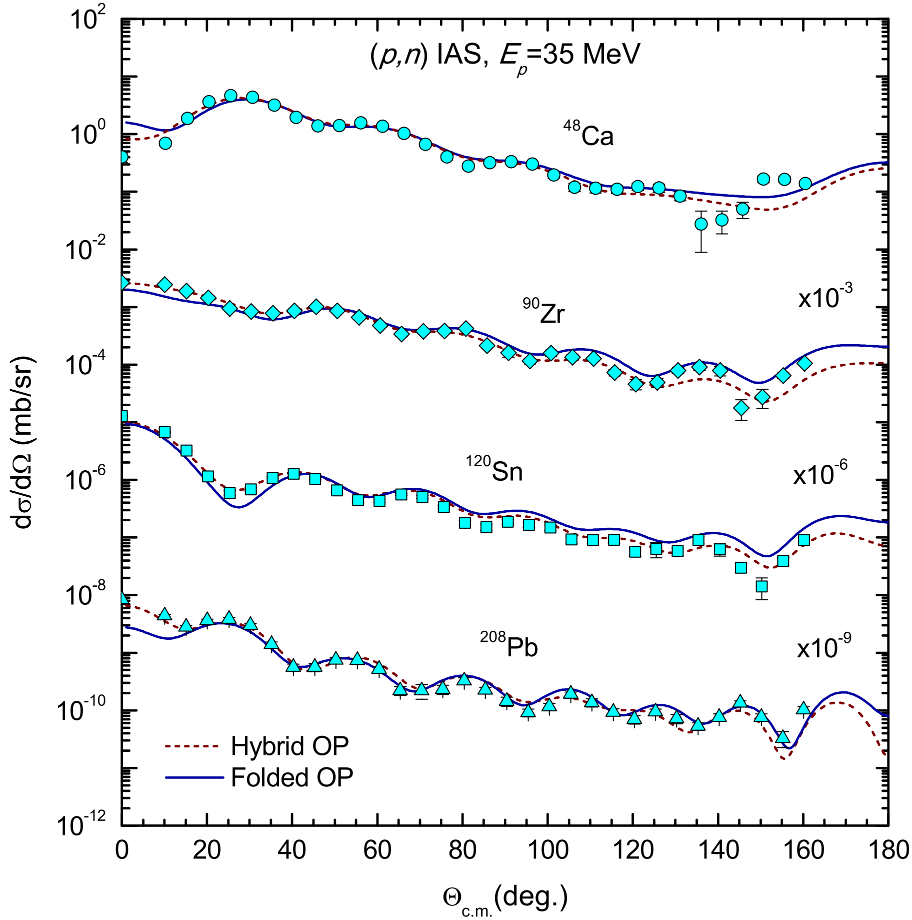


Figure : CC description of the (p, n) scattering to the IAS of different targets at $E_p = 35$ MeV. MSU data by R.R. Doering, D.M. Patterson, A. Galonsky, Phys. Rev. C **12**, 378 (1975).

Implication for the nuclear symmetry energy

The folding model analysis of the charge-exchange scattering is helpful for the study of the NM symmetry energy, when the same density- and isospin dependent CDM3Y6 interaction is used in the HF calculation of the total NM energy density $\mathcal{E}$

$$\mathcal{E} = \mathcal{E}_{\rm kin} + \frac{1}{2}\sum_{kk'}[\langle \boldsymbol{k}\boldsymbol{k}'|v_{\rm D}|\boldsymbol{k}\boldsymbol{k}'\rangle + \langle \boldsymbol{k}\boldsymbol{k}'|v_{\rm EX}|\boldsymbol{k}'\boldsymbol{k}\rangle],$$

where $|\boldsymbol{k}\rangle$ are plane waves. The total NM energy per particle E

$$\frac{\mathcal{E}}{\rho} = E(\rho,\delta) = E(\rho,\delta=0) + S(\rho)\delta^2 + O(\delta^4) + ...,\ \delta = \frac{\rho_n - \rho_p}{\rho}. \quad (23)$$

The folding model analysis of the (p,n) scattering to the IAS has put a constraint on the nuclear symmetry energy $S(\rho)$ at sub-saturation densities $\rho \lesssim \rho_0$ within the empirical boundaries deduced from experiments.

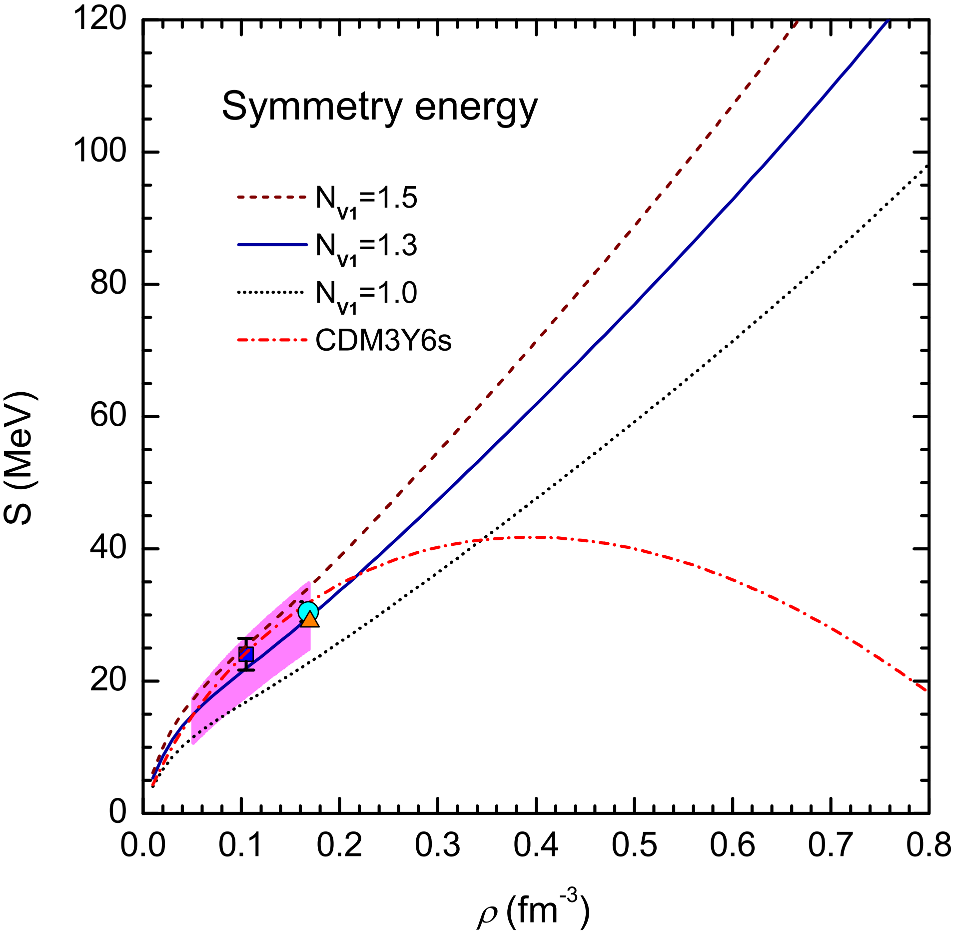


Figure : HF prediction of $S(\rho)$ compared to empirical data by M.B. Tsang *et al.* Prog. Part. Nucl. Phys. **66**, 400 (2011); L. Trippa *et al.* Phys. Rev. C **77**, 061304(R) (2008); DTK, H.S. Than, Phys. Rev. C **71**, 044601 (2005).

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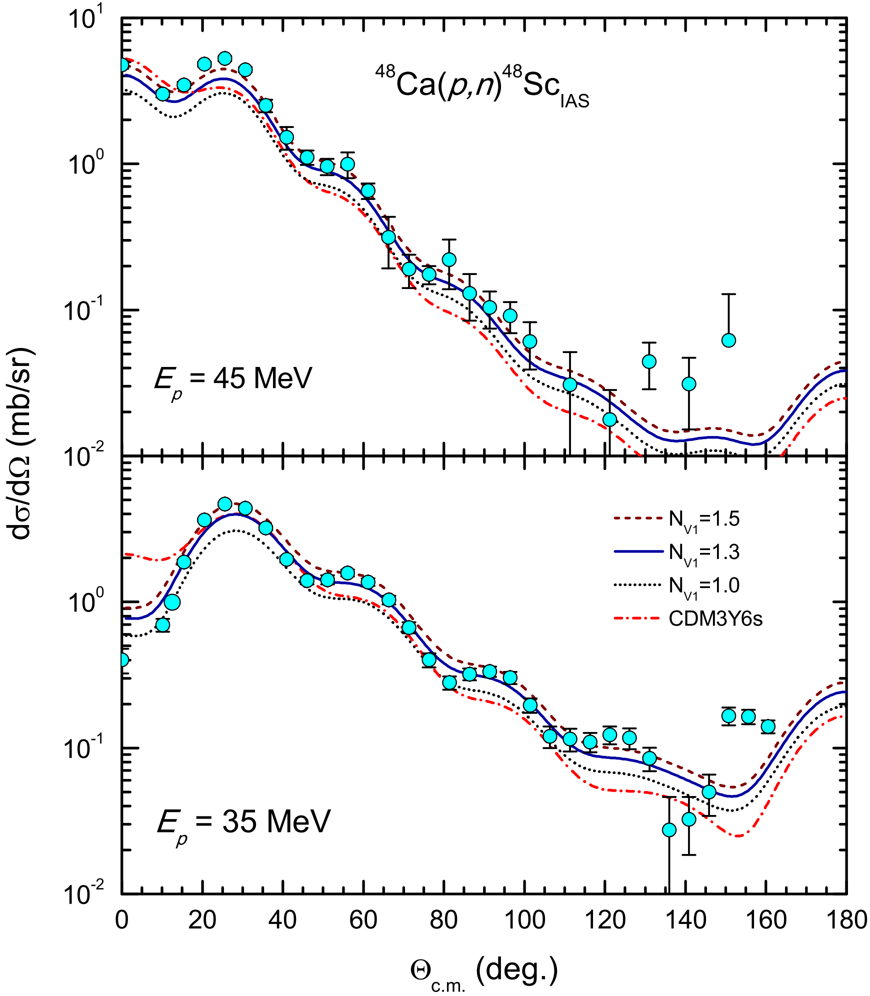
^{48}Ca(p,n)^{48}Sc$_{IAS}$
E_p = 45 MeV
E_p = 35 MeV
N_{V1}=1.5
N_{V1}=1.3
N_{V1}=1.0
CDM3Y6s
dσ/dΩ (mb/sr)
$\Theta_{c.m.}$ (deg.)

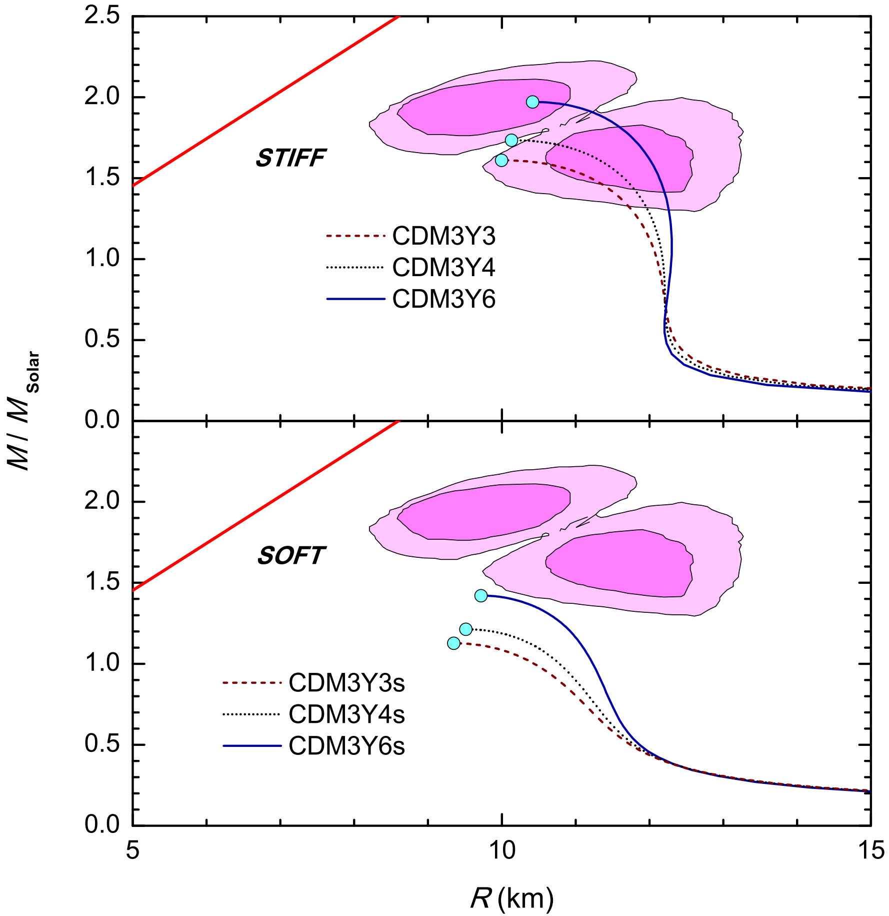

Figure : NS gravitational mass vs. its radius obtained with the EOS's given by the stiff- and soft type NN interactions. HF results from D.T. Loan, N.H. Tan, DTK, J. Margueron, Phys. Rev. C **83**, 065809 (2011), empirical data (x-ray burster 4U 1608-52) deduced by A.W. Steiner, J.M. Lattimer, E.F. Brown, Astrophys. J. **722**, 33 (2010).

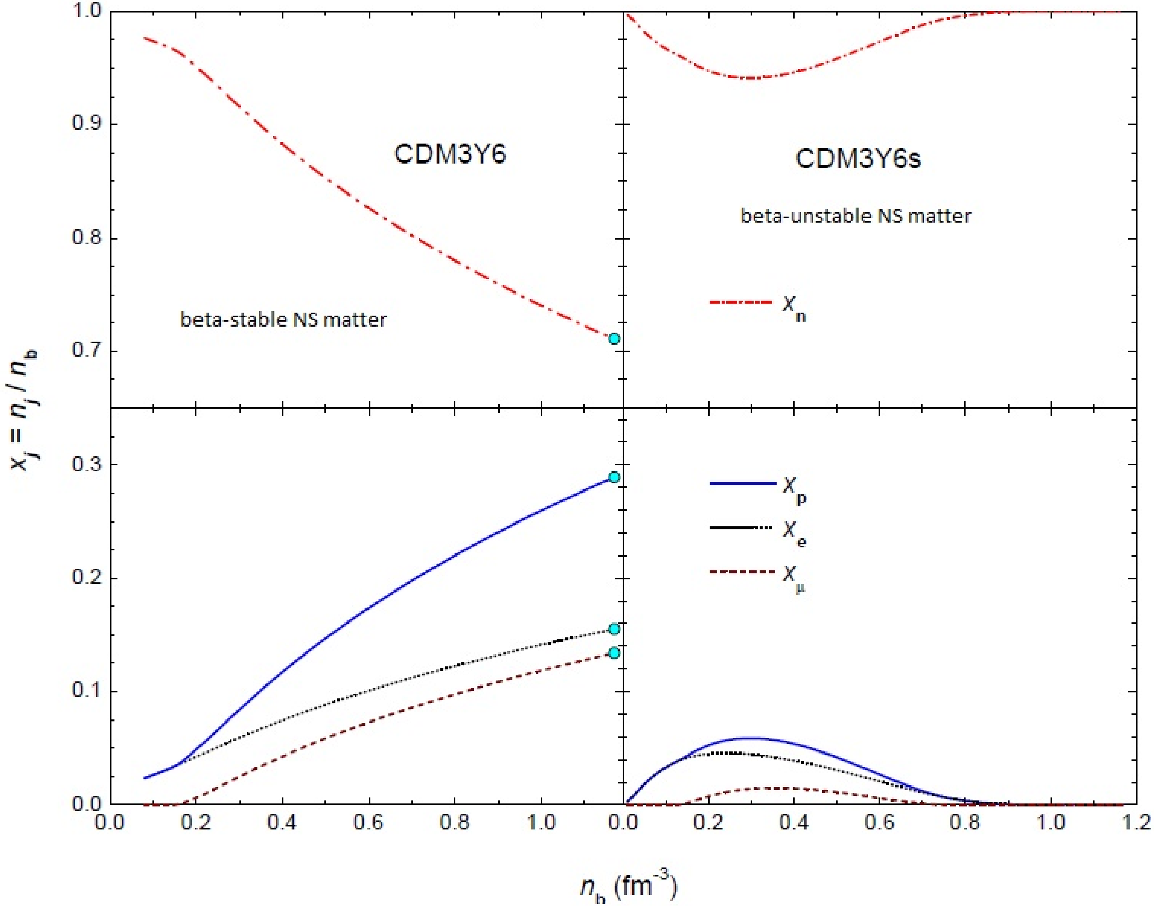

Figure : fractions of constituent particles of the NS matter obtained from the charge-neutrality condition [D.T. Loan, N.H. Tan, DTK, J. Margueron, Phys. Rev. C **83**, 065809 (2011)]

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PHYSICAL REVIEW C **83**, 035802 (2011)

New equation of state for astrophysical simulations

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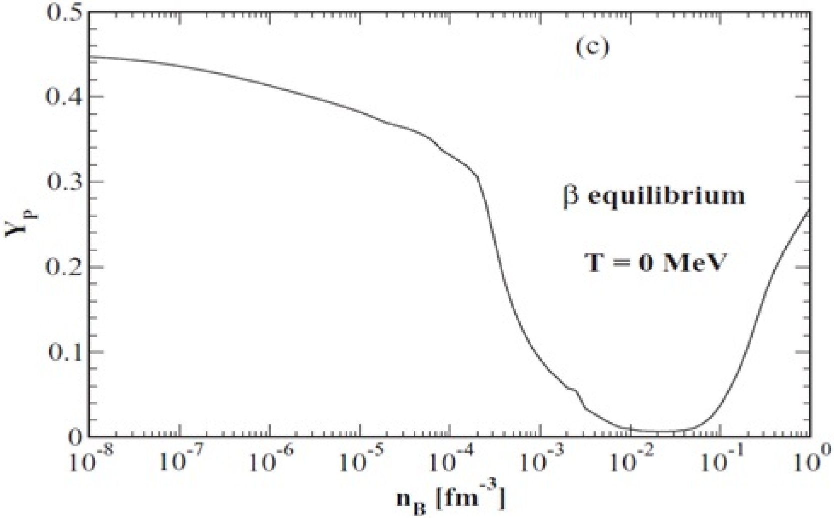

Figure : The same behavior of the proton fraction as that given by the mean-field calculation using CDM3Y6 interaction!

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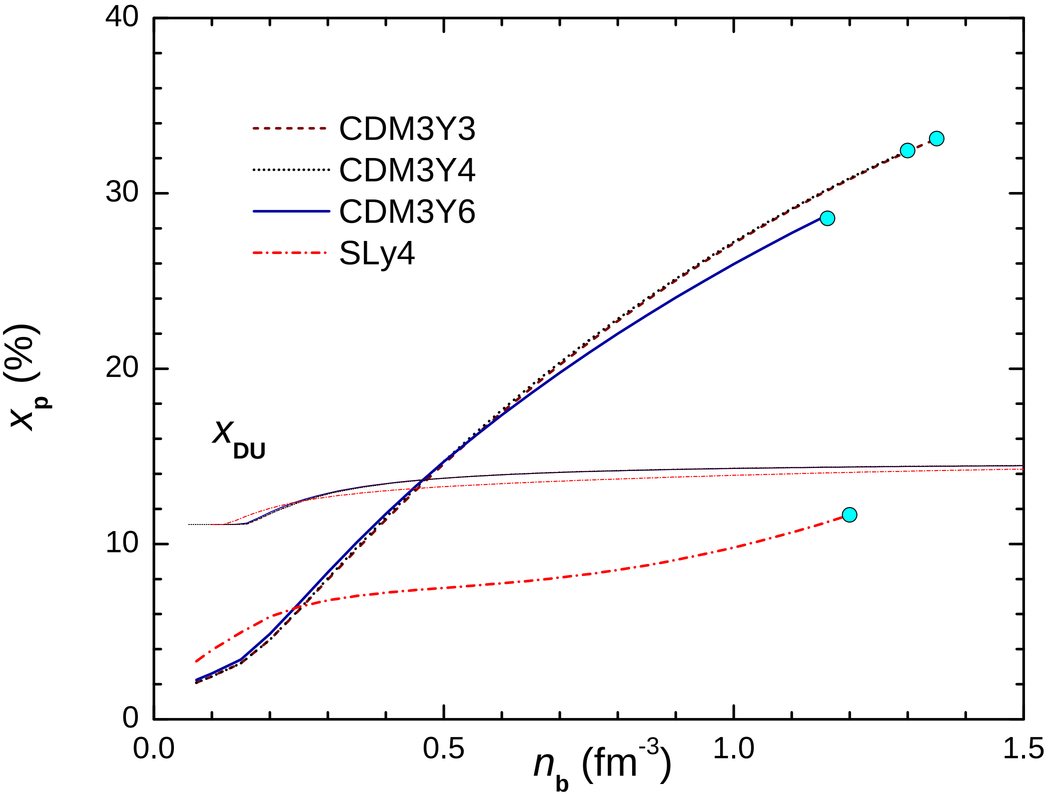

Figure : Direct Urca is allowed by CDM3Yn interaction!

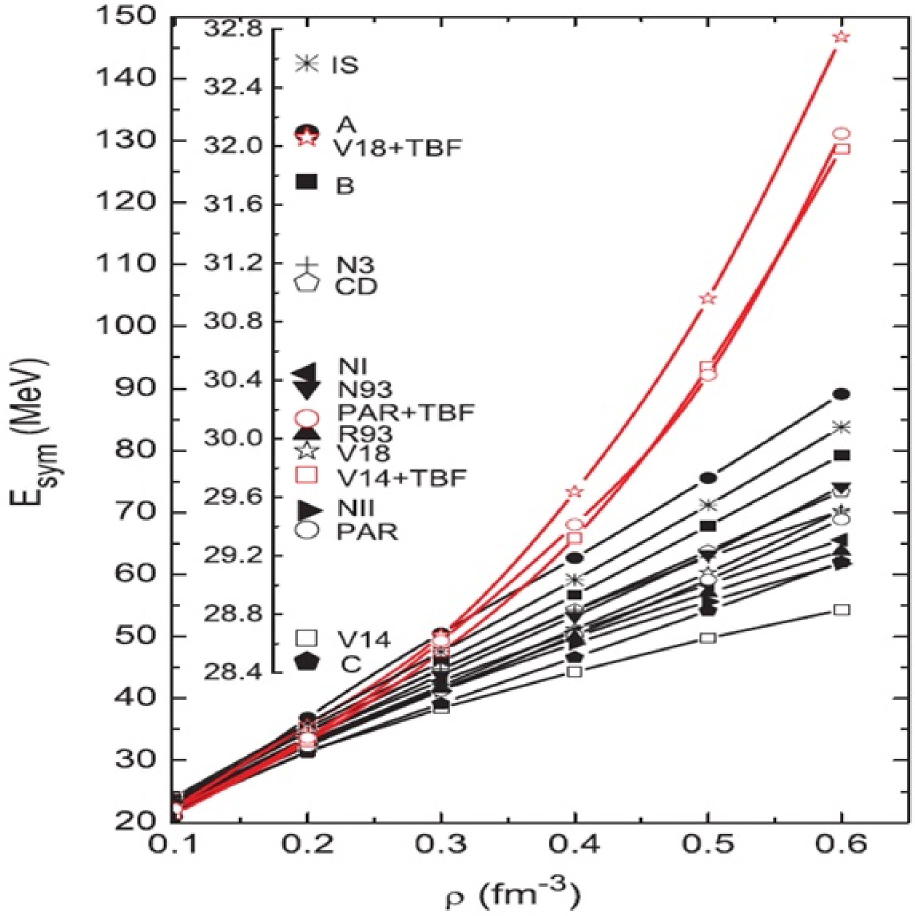

Figure : BHF calculation of asymmetric NM by U. Lombardo & Co.

Analysis of the (^{3}He,t) scattering to the IAS

Given the same spin and isospin of proton and ^{3}He, similar structures of the final states have been observed in the (p, n) and (^{3}He,t) reactions. However, the experimental resolution of (^{3}He,t) reaction is nowadays much higher than that of (p, n) reaction [Y. Fujita, B. Rubio, W. Gelletly, Prog. Part. Nucl. Phys. **66** (2011) 549].

Due to a composite projectile, the isospin dependence of the ^{3}He-nucleus OP is much less known compared to that of the nucleon OP, and the IV term of the real ^{3}He-nucleus OP could not even be established in a recent global OP [D.Y. Pang *et al.*, Phys. Rev. C **79**, 024615 (2009)]. $\Rightarrow$ the Lane consistency of the ^{3}He-nucleus OP ? $\Rightarrow$ Need to carry out a consistent folding model study of both the (p, n) and (^{3}He,t) scattering to the IAS using the same effective NN interaction.

Difficulty: high-quality data of (^{3}He,t) scattering to IAS are scare.

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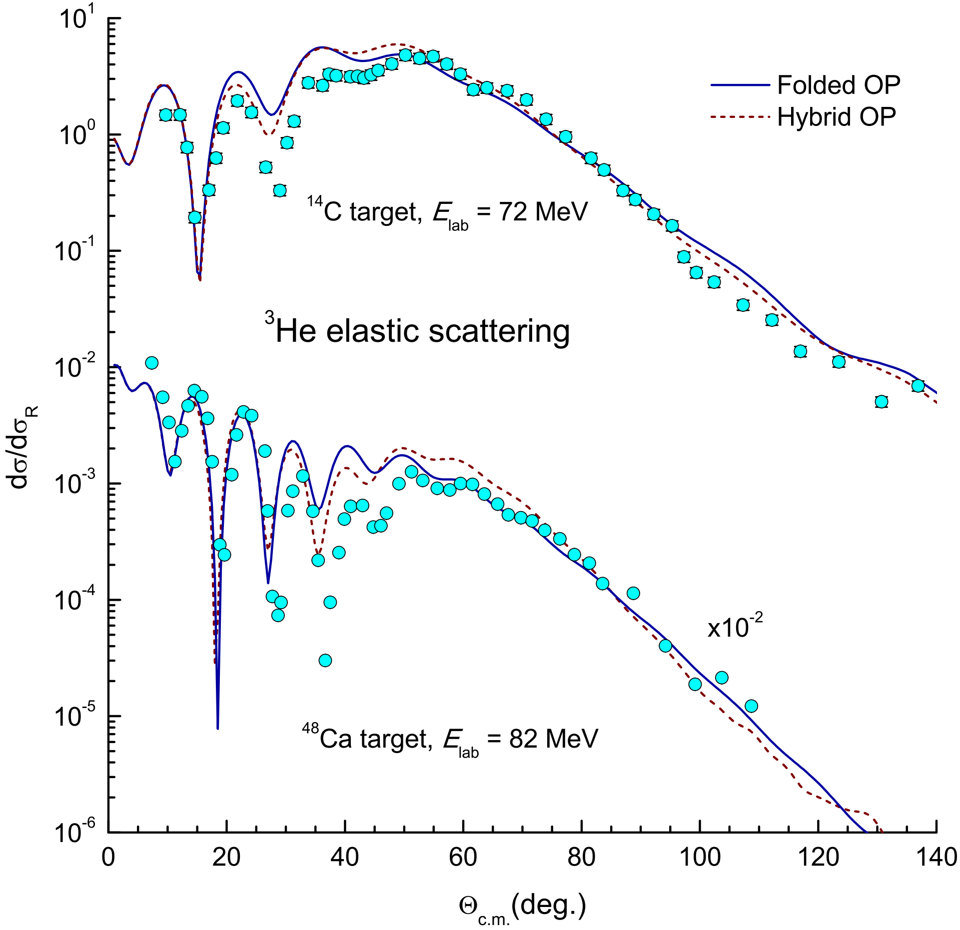

Folded OP
Hybrid OP
^{14}C target, E_{lab} = 72 MeV
^{3}He elastic scattering
x10^{-2}
^{48}Ca target, E_{lab} = 82 MeV
dσ/dσ$_R$
Θ$_{c.m.}$(deg.)

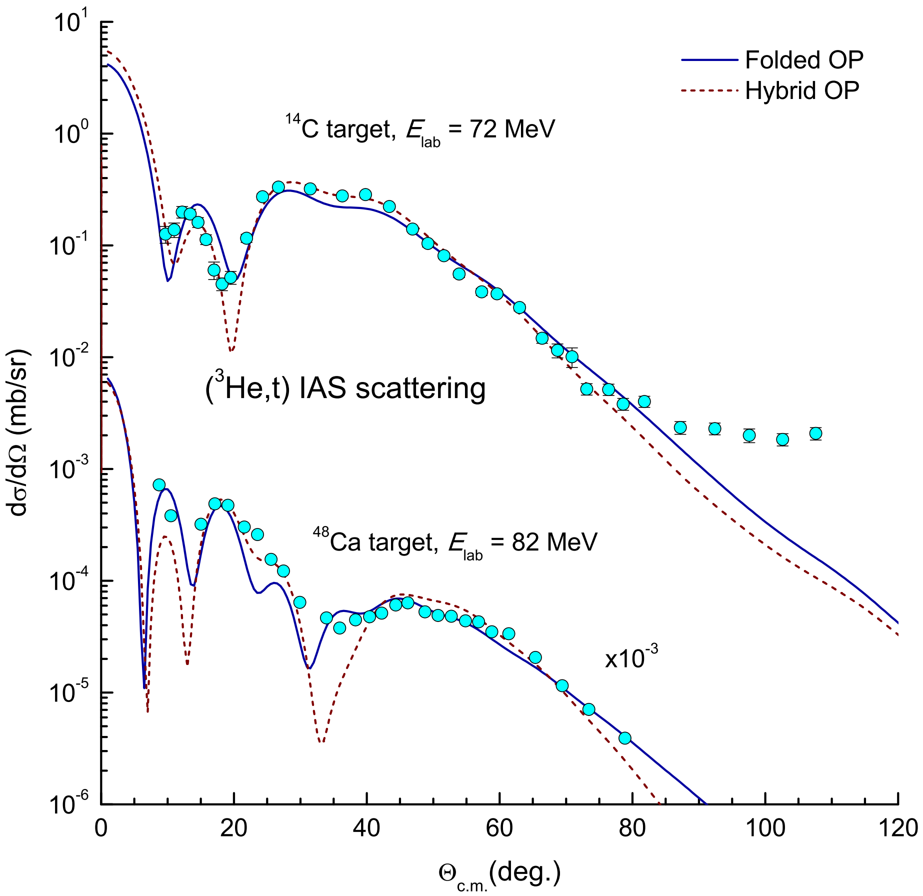

Figure : Data by A.S. Demiyanova *et al.*, Phys. Rev. C **38**, 1975 (1988); T. Tanabe *et al.*, Nucl. Phys. **A311**, 38 (1978)

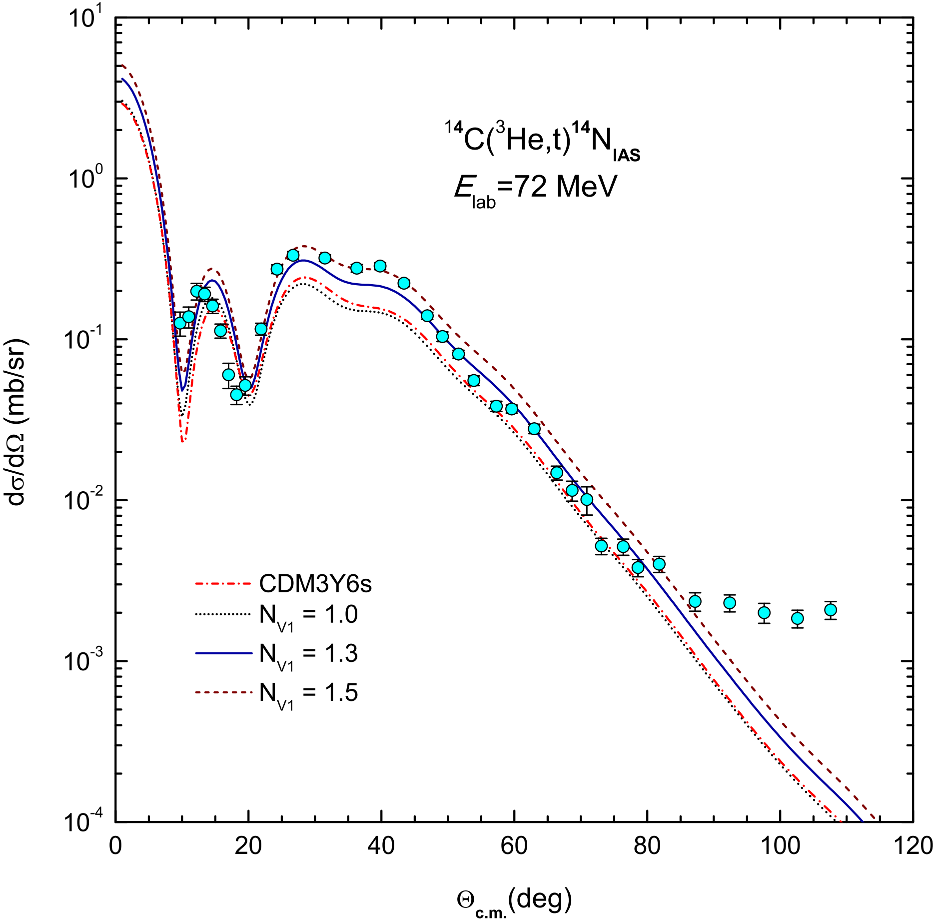

Figure : about the same conclusion on the IV strength of the interaction.

Conclusion

- The first consistent folding model study of the charge-exchange scattering to the IAS has been done using the same effective NN interaction to calculate the isospin dependent OP and charge-exchange FF in both (p, n) and $(^3\text{He}, t)$ cases.
- The CC analysis of the (p, n) and $(^3\text{He}, t)$ data has shown that the (real) IV strength of the CDM3Y6 interaction (based on the JLM results) needs to be enhanced by about $30 \sim 40\%$.
- The JLM-based isospin dependent CDM3Y6 interaction has been used in the HF calculation of the nuclear symmetry energy, and the agreement with the empirical values is reached only if the IV strength is scaled by a factor $N_{V1} \approx 1.3 - 1.6$, similar to that found in the folding model analysis of the charge-exchange data.
- A more definitive conclusion has been made on the slope of the symmetry energy.
- It would be of great interest to have the experiments on the charge-exchange scattering to the IAS pursued at the modern rare isotope beam facilities.

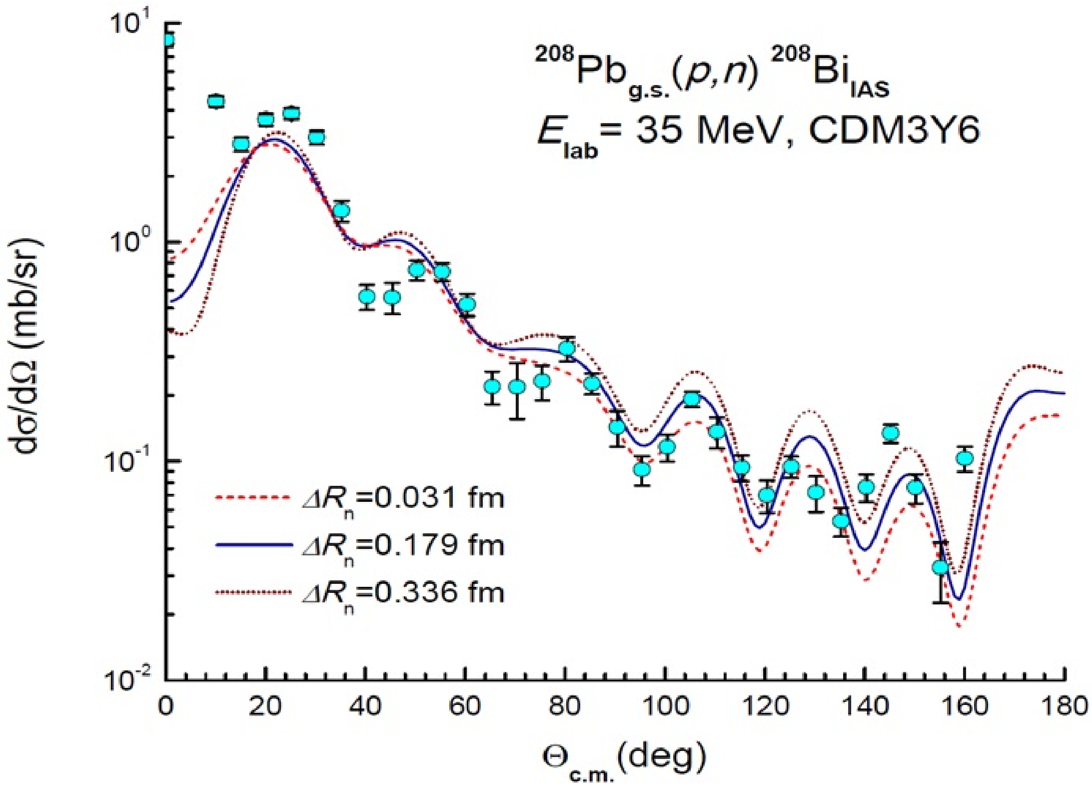

Figure : **Probing the neutron skin !** [emirical 208Pb density taken from (p,p) 800 MeV L. Ray et al., Phys. Rev. C **18**, 1756 (1978)]

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Dedicated to the memory of Ray Satchler (1926-2010)