

Neutrino-sphere in supernova and symmetry energy

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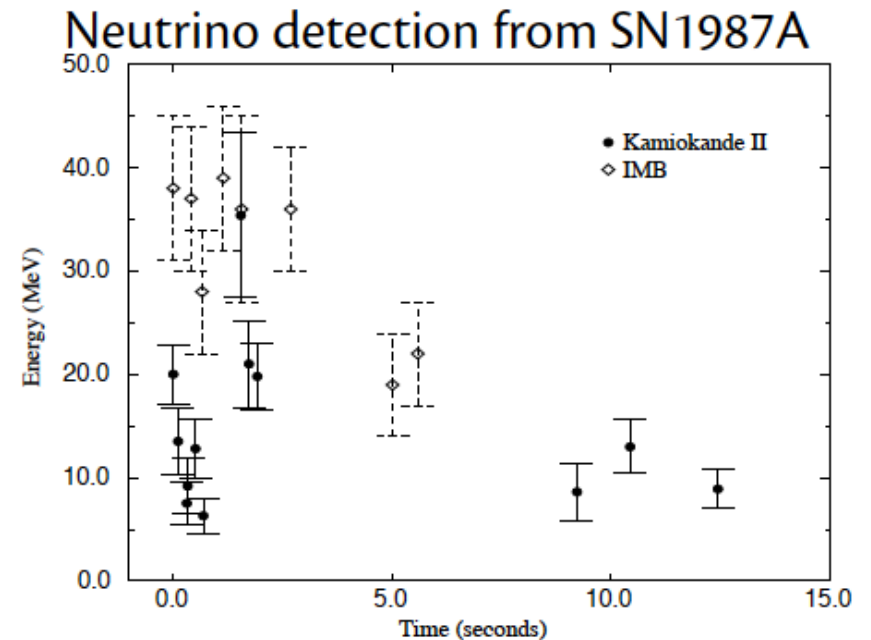
Outline

1. Intro. to neutrino-sphere in supernova
2. Equation of state around neutrino-sphere
3. ν -opacity around neutrino-sphere
4. Summary

Neutrino emissions from proto-neutron star

Single Observation – SN1987A

- ν carries away 99% of GR energy in SN explosion
- ν cooling PNS in ~ 1 minute
- Consistent with ~ 20 events / 20 s observed in SN1987A
- $\bar{\nu}_e$ events from Water Cerenkov detectors
- Interpretation of neutrino spectra - nontrivial



Neutrino spectra and luminosities important for:

- Neutrino oscillations (collective, MSW, vacuum)
- Nucleosynthesis in neutrino driven wind: ν p-process and r-process

Nuclear physics in PNS

Temperature [MeV]	0 ~ 60 (maybe higher)		
Proton fraction Y_p	0 ~ 0.6 (maybe higher in neutrino driven wind)		
Baryon Density [fm^{-3}]	1E-13	1E-2	1E0
Particle spacing [fm]	~1E5	~10	~1
Phases	Gas	Solid (low T)	Liquid

- Challenge: multi-scale, multi-component, multi-phase
- Covering full parameter space – combination of different methods
- Uncertainty in high density: hyperons, excited baryons, quarks
- Uncertainty in high temperature: meson excitations, etc

Neutrino-sphere

- Solar photo-sphere
 - last scattering site of photon
 - a (quasi-static) layer about 100 km thick (compared to the 700,000 km radius of the Sun).
- Neutrino-sphere(s)
 - Last scattering site of neutrinos in proto-neutron star: $\sim 10^{12}$ g/cm³, 5 MeV
 - Different for 6-flavor neutrinos
 - Evolve with time! (~ 1 min)



EOS around neutrino-sphere

Exp: multi-fragmentation in Heavy ion collision: Natowitz et al

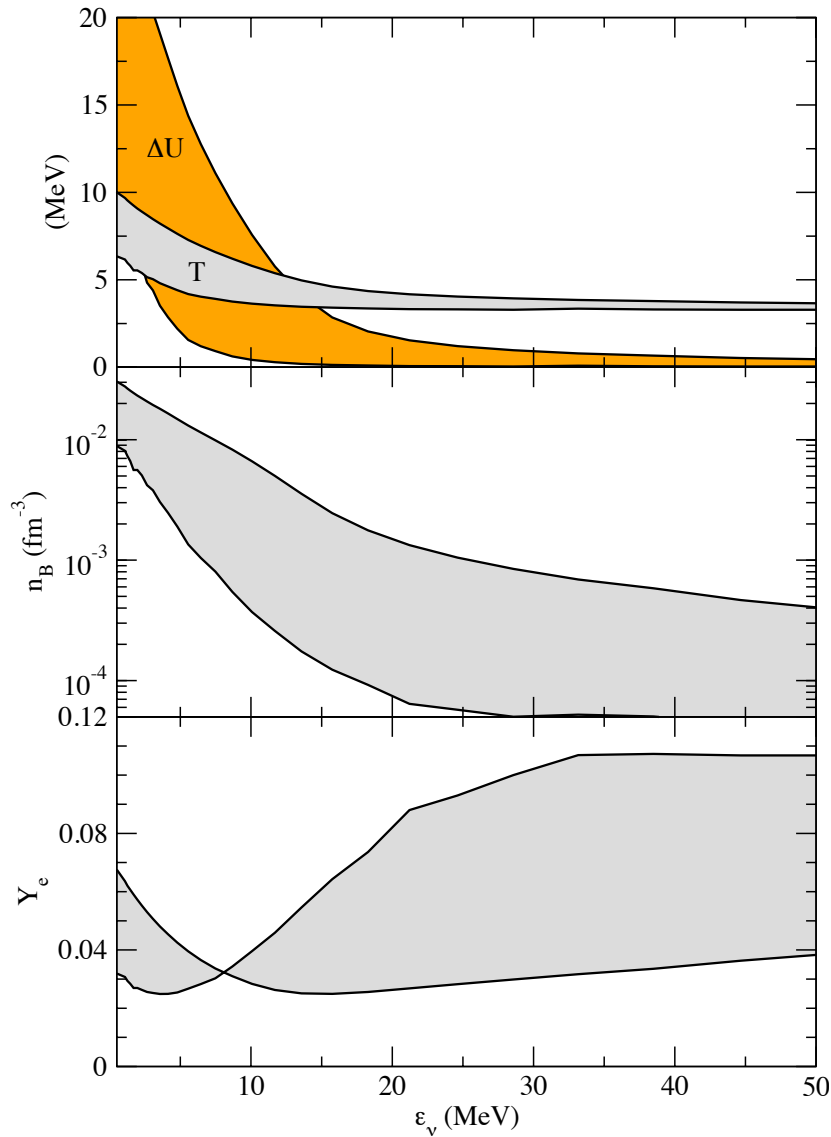
Neutron matter, $T=0$ (benchmark)

- Chiral EFT: Schwenk, Hebeler, ...
- QMC: Carlson, Gezerlis, Gandolfi, ...
- QMC/Chiral EFT: Gezerlis et al

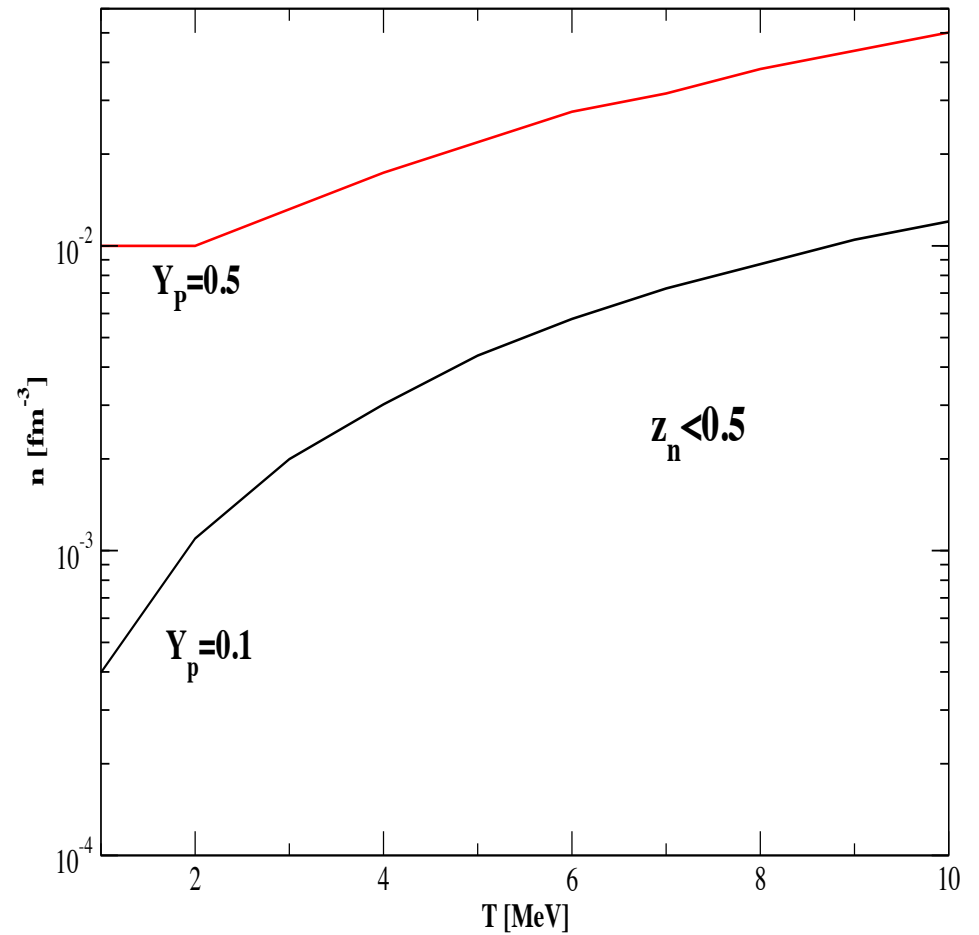
Nuclear matter, finite T

- Statistical method: Röpke, Typel, Wolter, ...
- Virial expansion: Horowitz, Schwenk, Shen ...

Virial expansion is valid around ν -sphere



Courtesy: Luke Roberts



Virial expansion of n, p to 2nd order

Low density: Ensemble of nucleons and nuclei

- Grand partition function in virial expansion

$$\frac{\log Q}{V} = \frac{P}{T} = \frac{2}{\lambda_n^3} [z_n + z_p + (z_p^2 + z_n^2)b_n + 2z_p z_n b_{pn}] \quad \blacktriangleright \text{nucleon-nucleon}$$

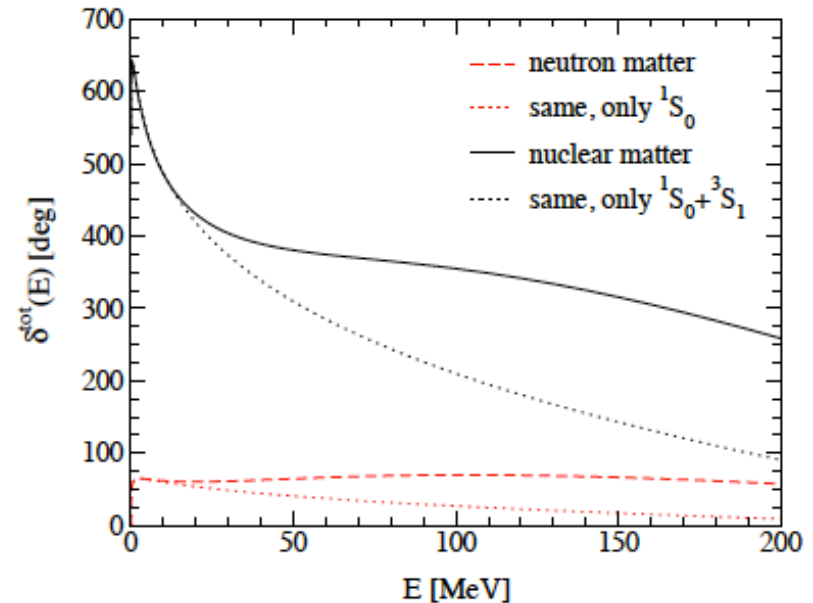
$$+ \frac{1}{\lambda_\alpha^3} [z_\alpha + z_\alpha^2 b_\alpha + 2z_\alpha (z_n + z_p) b_{\alpha n}] \quad \blacktriangleright \text{nucleon-alpha}$$

I. Nucleon and alpha: mod. ind. Horowitz, Schwenk '05

2nd virial coef. b_2 determined from elastic scattering phase shifts: model-independent

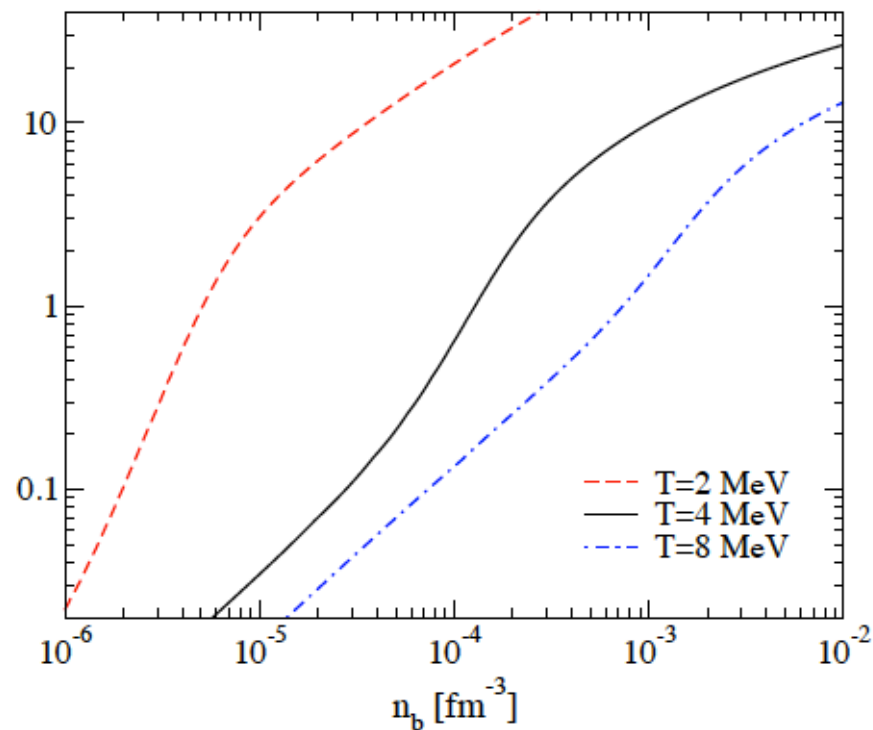
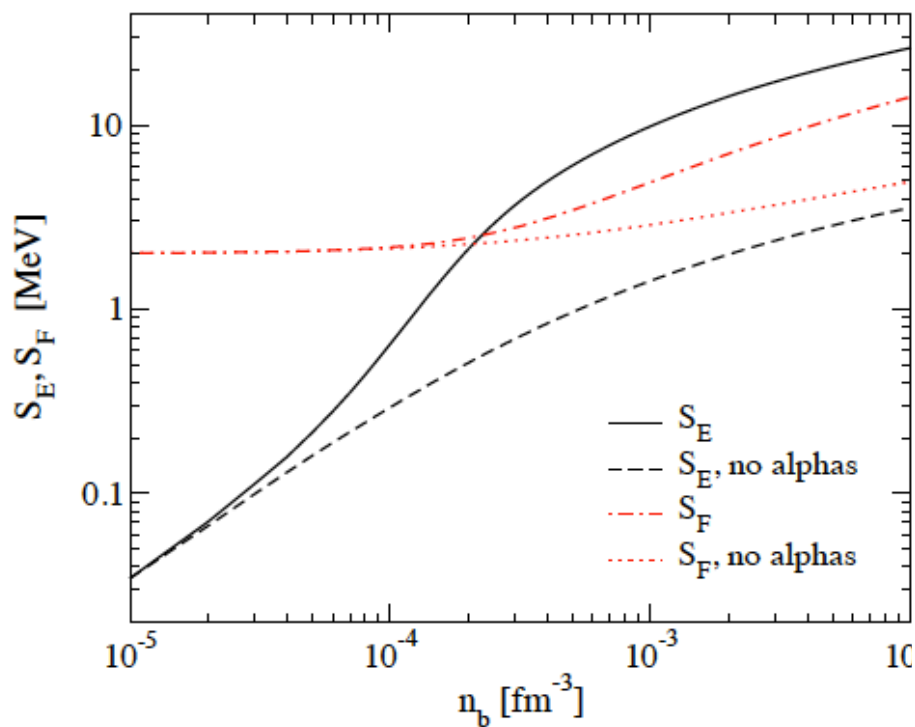
- b_n for n-n (p-p), – b_{pn} for n-p,
- b_α for alpha-alpha, – $b_{\alpha-n}$ for alpha-n

$$b_n(T) = \frac{1}{2^{1/2} \pi T} \int_0^\infty dE e^{-E/2T} \delta_n^{\text{tot}}(E) - 2^{-5/2}.$$



Symmetry energy at finite T

Horowitz, Schwenk '05



$$S_E = \frac{1}{8} \left. \frac{\partial^2 E}{\partial Y_p^2} \frac{E}{A} \right|_{Y_p=1/2}, \quad S_F = \frac{1}{8} \left. \frac{\partial^2 F}{\partial Y_p^2} \frac{F}{A} \right|_{Y_p=1/2}$$

Virial expansion including heavy nuclei

- Grand partition function in virial expansion

Shen, Horowitz, Teige '10

$$\frac{\log Q}{V} = \frac{P}{T} = \frac{2}{\lambda_n^3} [z_n + z_p + (z_p^2 + z_n^2)b_n + 2z_p z_n b_{pn}] \quad \triangleright \text{nucleon-nucleon}$$

$$+ \frac{1}{\lambda_\alpha^3} [z_\alpha + z_\alpha^2 b_\alpha + 2z_\alpha (z_n + z_p) b_{\alpha n}] \quad \triangleright \text{nucleon-alpha}$$

$$+ \sum_i \frac{1}{\lambda_i^3} z_i \Omega_i \quad \triangleright \text{nuclei}$$

1. Nucleon and alpha: mod. ind.

Horowitz, Schwenk '05

2. Heavy species: 8980 nuclei

FRDM mass table: Moller et al '97. $\Delta_{\text{rms}} \sim 0.6 \text{ MeV}$

Chemical equilibrium -----

$$\mu_i = Z\mu_p + N\mu_n \quad z_i = z_p^Z z_n^N e^{(E_i - E_i^C)/T}$$

Coulomb correction -----

$$E_i^C = \frac{3}{5} \frac{Z_i^2 \alpha}{r_A} \left[-\frac{3}{2} \frac{r_A}{r_i} + \frac{1}{2} \left(\frac{r_A}{r_i} \right)^3 \right]$$

Nuclear partition function Ω_i ----

eg, Fowler, Engelbrecht, Woosley, '78

Nuclei spacing

$$\frac{4}{3} \pi r_i^3 \left(\sum_j Z_j n_j \right) = Z_i$$

3. Solving: z_n, z_p, r_i :

$$n_B = n_n + n_p + 4n_\alpha + \sum_i A_i n_i = \text{Baryon}$$

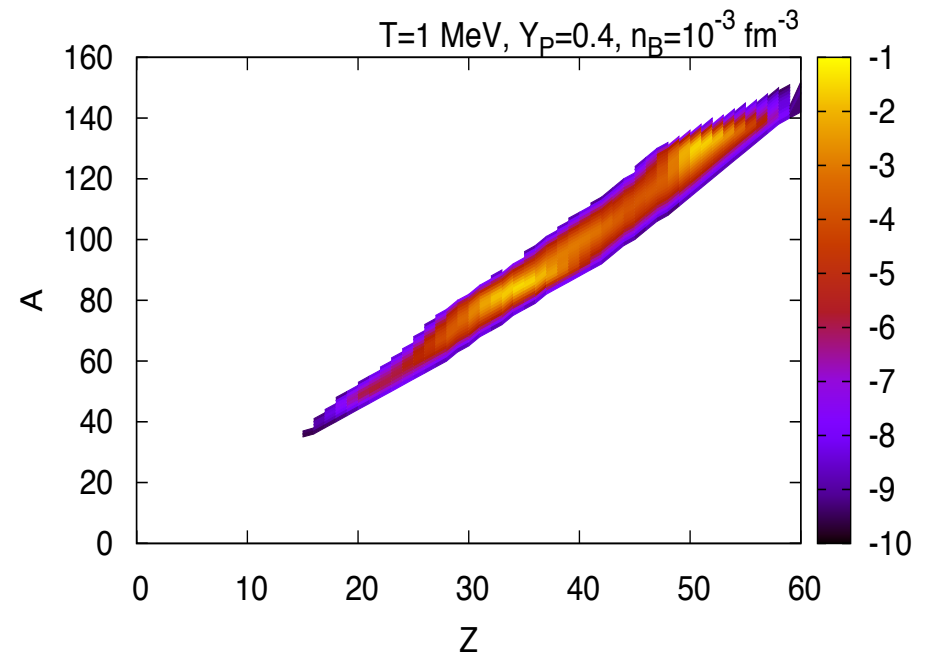
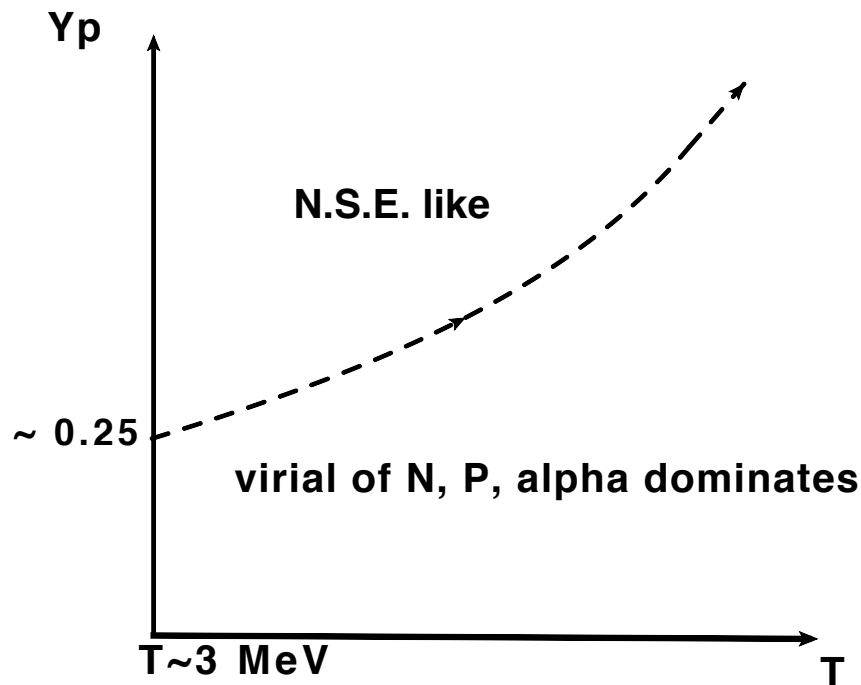
$$Y_P = (n_p + 2n_\alpha + \sum_i Z_i n_i) / n_B = \text{Charge}$$

4. Mass fraction:

$$X_a = A_a n_a / n_B$$

- $n < 0.1 - 0.01 n_0$
- $T < 12 \text{ MeV}$

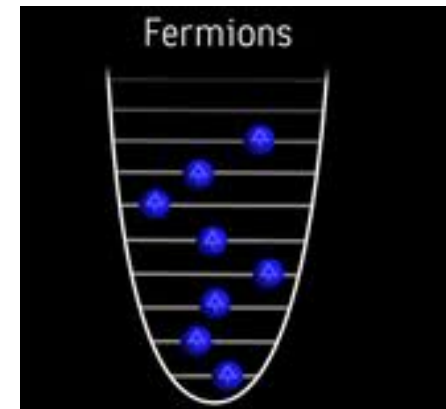
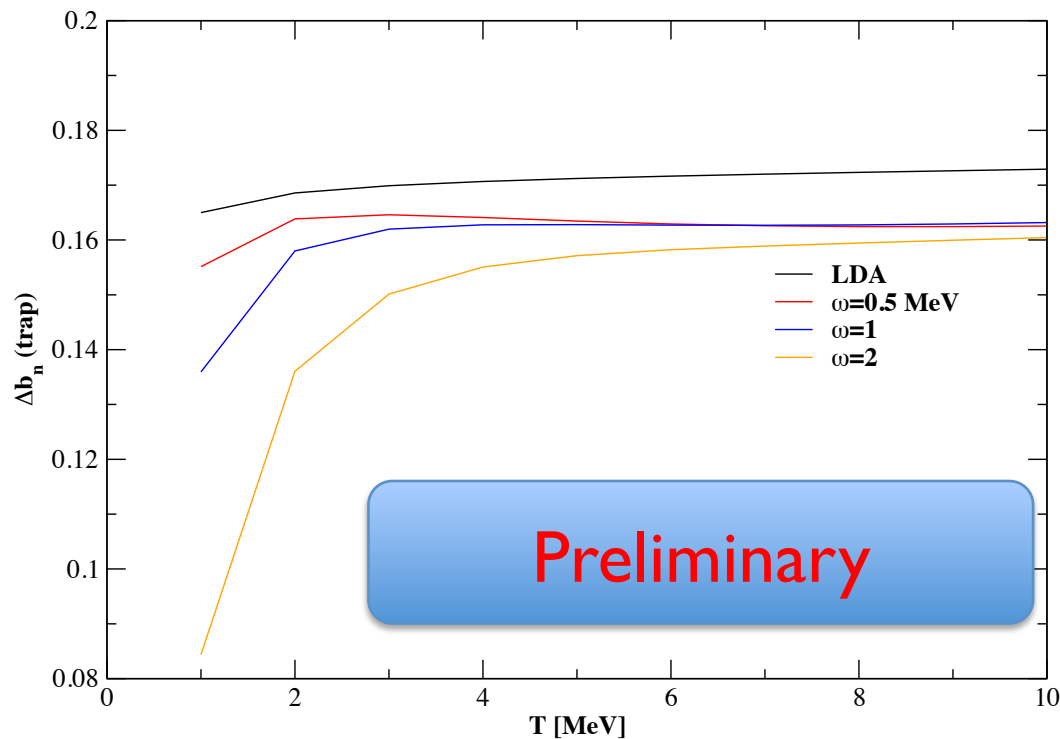
Mass distributions in Virial gas - multi-nuclei distribution



- Exact in low density limit for neutron rich matter
- Match to NSE at large Y_p
- Mass 2, 3 nuclei can be included in virial expansion: O'Connor et al, 2008
- To be improved: model dep. on mass table, nuclear partition function – small effect on thermodynamics, unclear on the dynamics (interactions with neutrinos)

Higher order virial coeff: in trap neutron system

$$\frac{\log Q}{V} = \frac{P}{T} = \frac{2}{\lambda_n^3} [z_n + z_p + (z_p^2 + z_n^2)b_n + 2z_p z_n b_{pn}] + b_3 z^3$$



$V_{HO}(\omega)$

- 2nd order: ~5% within determined from exp. phase shift
- for AV18 potential and Chiral potential
- 3rd order: in progress

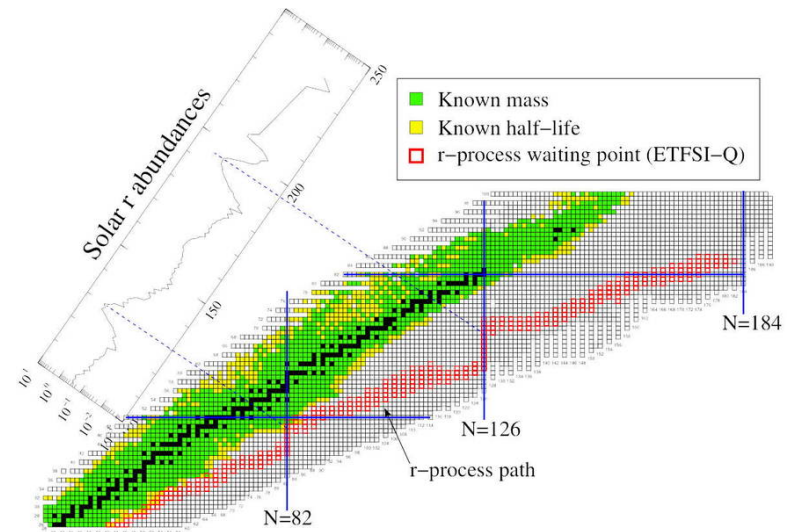
ν -opacity around neutrino-sphere

- Important for neutrino spectra from PNS and key input for r-process: proton (electron) fraction in the wind

- Setting the electron fraction:

$$Y_e \approx \frac{\lambda_{\nu_e}^{-1}}{\lambda_{\nu_e}^{-1} + \lambda_{\bar{\nu}_e}^{-1}} \approx \left(1 + \frac{\dot{N}_{\bar{\nu}_e} (\epsilon_{\bar{\nu}_e} - \Delta)^2}{\dot{N}_{\nu_e} (\epsilon_{\nu_e} + \Delta)^2} \right)^{-1}$$

Neutrino Spectra



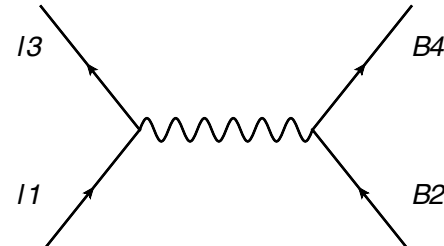
- Effect of symmetry energy and light nuclei

Neutrino-nucleon scattering

- Neutrinos couple to nuclear (isospin) density and (isospin) spin density:

V-A theory: $j^\mu(x) = \bar{\psi}(x)\gamma^\mu(C_V - C_A\gamma_5)(\tau_j)\psi(x)$

Non Relativistic limit: $\rightarrow C_V\psi^\dagger(\tau_j)\psi\delta^{\mu 0} - C_A\psi^\dagger\sigma^i(\tau_j)\psi\delta^{\mu i}$

$$\frac{1}{V} \frac{d^2\sigma}{d\cos\theta dE_3} = \frac{G_F^2}{4\pi^2} E_3^2 (1 - f_3(E_3))$$


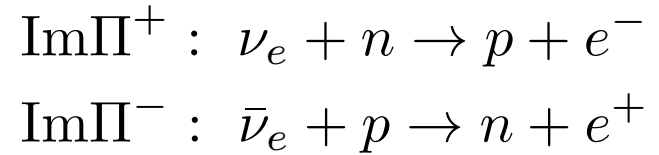
$$\times [C_V^2 (1 + \cos\theta) S_\rho(q_0, q) + C_A^2 (3 - \cos\theta) S_\sigma(q_0, q)]$$

- Free Fermi gas response functions: peaked around $q_0 \sim 0$

$$S_{\rho,\sigma}(q_0, q) = \frac{1}{2\pi^2} \int d^3p_2 \delta(q_0 + E_2 - E_4) f_2(1 - f_4),$$

Dispersion $E_i(p) = M_i + p^2/2M_i$

Medium correction (I): mean field

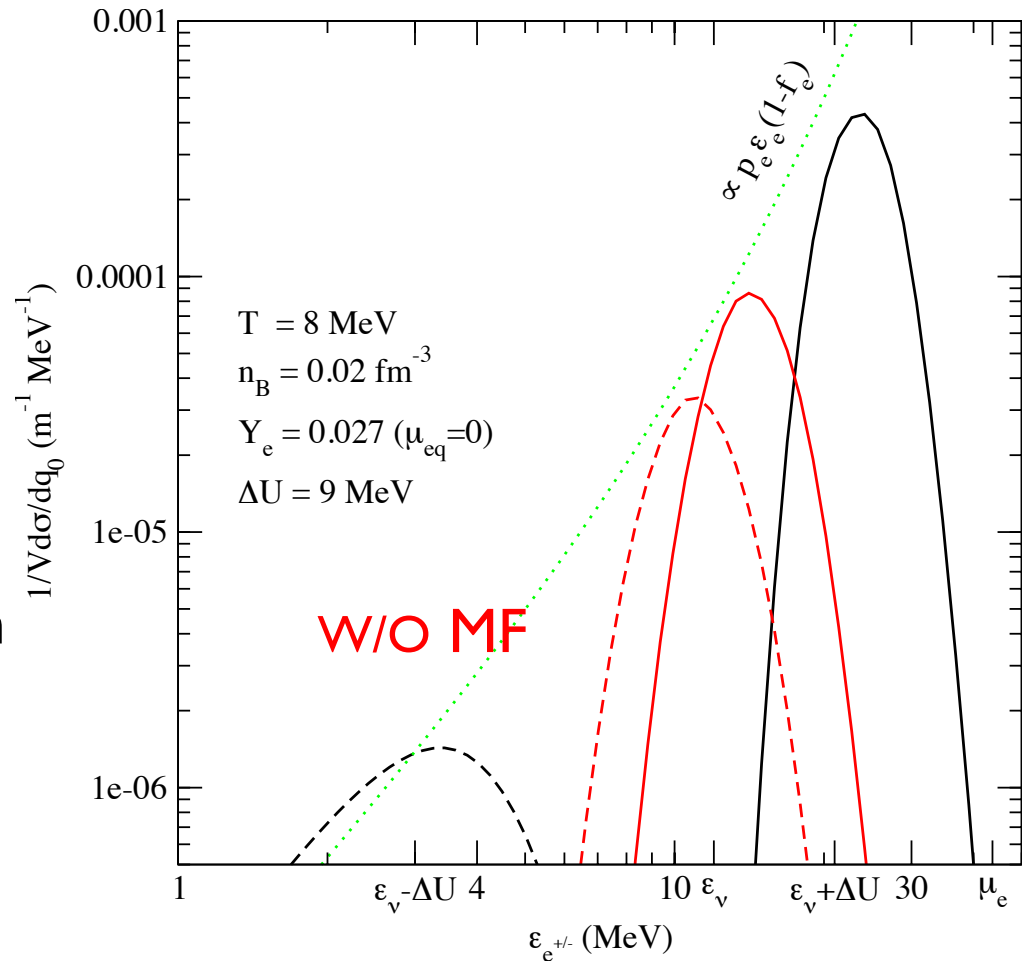


- Dispersion relation of quasi-particle from EOS:

$$E_i(k) = \sqrt{k^2 + M^{*2}} + U_i,$$

$$U_n - U_p = 40 \frac{n_n - n_p}{n_0} \text{MeV}$$

- Response function is peaked around $q_0 \approx U_n - U_p$
- Around neutrino sphere, mean field shift is comparable to temperature.



Roberts (2012)

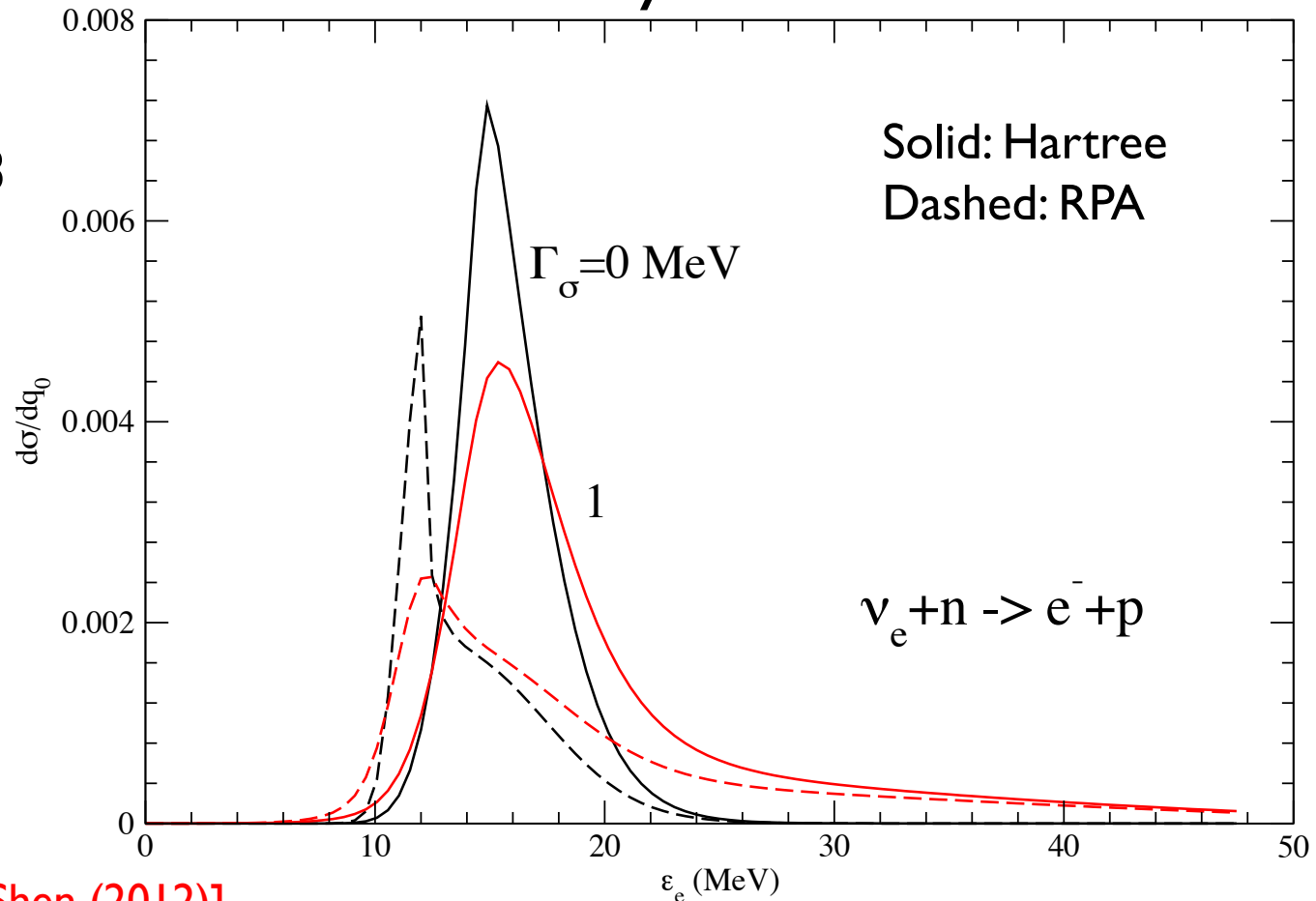
Martinez-Pinedo et al. (2012)

[Roberts, Reddy (2012)]

Medium correction (II)

RPA and beyond

$T=8$ MeV
 $N=0.006$ fm⁻³
 $Y_e=0.03$
 $\mu_e=30$ MeV
 $E_\nu=12$ MeV

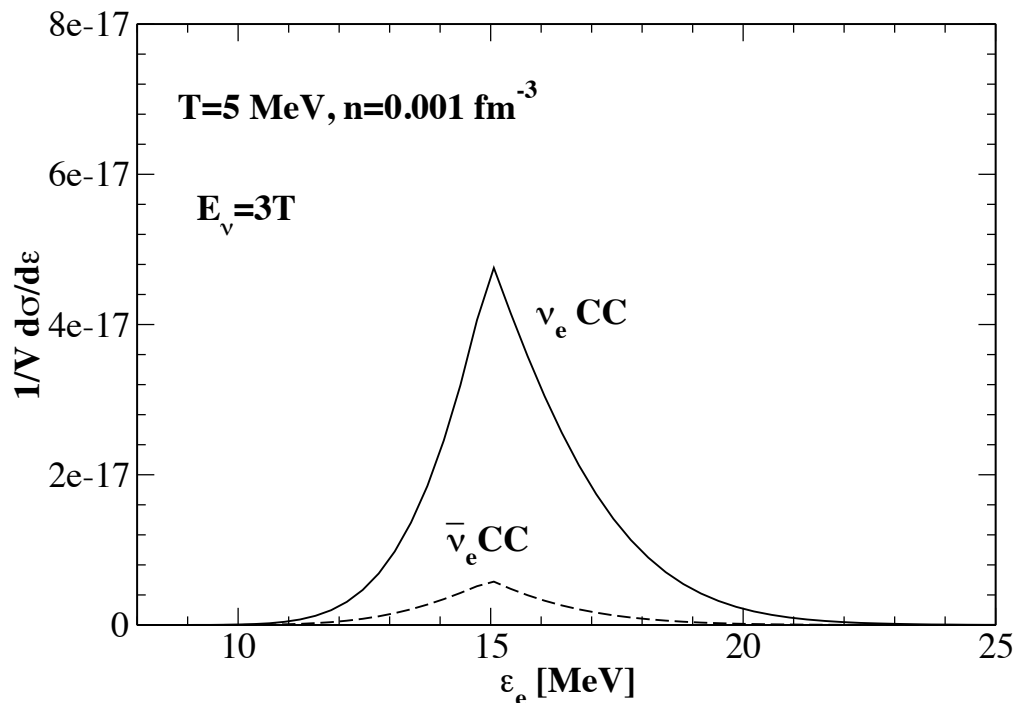


[Roberts, Reddy, Shen (2012)]

- RPA suppresses response and shifts its strength via collective mode. Multi-particle dynamics enhances it.
- Net effect mild suppression. Follow SN trajectory to study details.

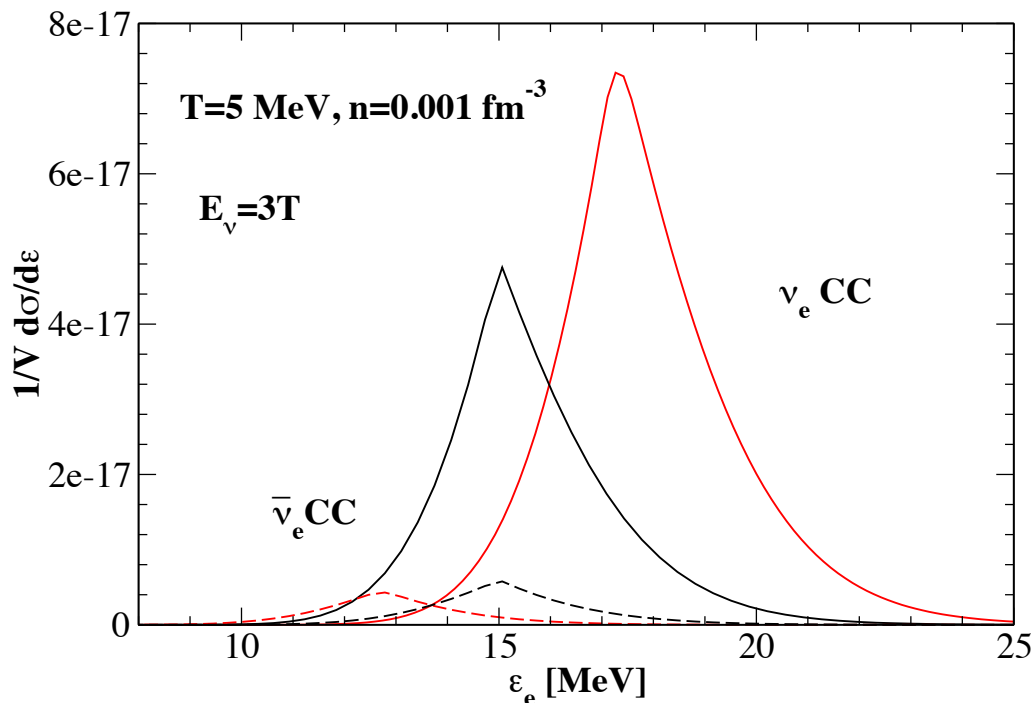
Medium corrections (III): light nuclei in virial expansion for MF shifts

- Virial EOS of n, p:
$$P = \frac{2T}{\lambda^3} \left\{ z_n + z_p + (z_n^2 + z_p^2) b_n + 2z_p z_n b_{pn} \right\},$$
 - 2nd virial coeff. b_n, b_{pn} from mod. ind. scattering phase shifts
 - Suitable for matter around neutrino-sphere
- Single particle energy (MF) shift:
$$U_n = \mu_n - \mu_n^f = -\lambda^3 T (n_n \hat{b}_n + n_p b_{pn}) + O(n_i^2).$$



- W/O MF shift:
 $d\sigma \sim$ density of n or p

- Virial EOS of n, p:
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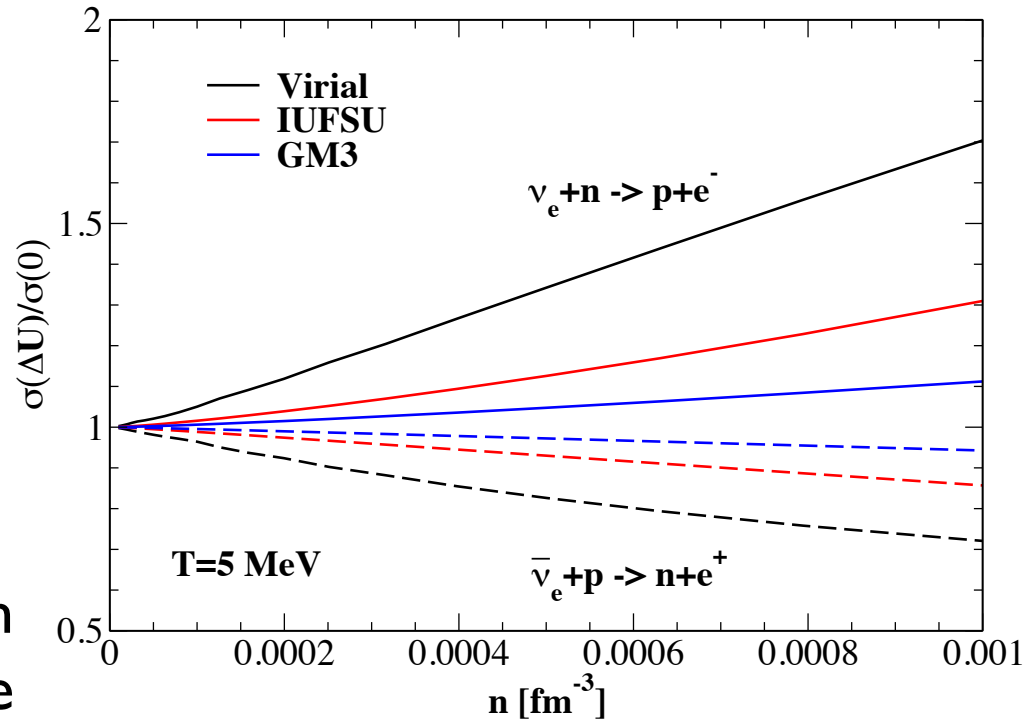
- W/O MF shift:
 $d\sigma \sim$ density of n or p
- W MF shift

- ratio of cross sections

$$\frac{\sigma_{\nu_e}(\Delta U)}{\sigma_{\nu_e}(0)} = \frac{(E_\nu + \Delta U)^2 [1 - f(E_\nu + \Delta U)]}{E_\nu^2 [1 - f(E_\nu)]}$$

$$\frac{\sigma_{\bar{\nu}_e}(\Delta U)}{\sigma_{\bar{\nu}_e}(0)} = \frac{(E_{\bar{\nu}} - \Delta U)^2 \Theta(E_{\bar{\nu}} - \Delta U)}{E_{\bar{\nu}}^2}$$

- effect larger than MF, due to n-p correlation (D)
- May reduce electron fractions in neutrino driven wind compared to MF
- Feedback: larger Y_e in ν -sphere from larger E_{sym}
- Need consistent simulation with EOS and virial response



$Z < 0.16$

Summary

- EOS and neutrino response at neutrino-sphere important for neutrino spectra and r-process in SN
- Strongly related to symmetry energy at sub-nuclear density: nuclear physics known in principle – relatively clean problem !
- Controlled theo. methods possible:
 - 3rd order in virial expansion of nuclear matter (or, extend EFT,QMC to finite T, Y_p)
 - eliminate MF model dep. and converge
 - Chiral EFT/QMC: neutrino response ?
- Q: How could exp. help pin down E_{sym} and response function at these densities: fragmentation in HI, etc
- Q: Role of bound states in neutrino response: more light nuclei (especially at late times)