### Structure of Hypernuclei and Eos with Hyperon

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- 1. Introduction
- 2. Hyperon-Nucleon interaction and Hypernuclei
- 3. Structure study of hypernuclei
- 4. EoS with strange particle in RMF
- 5. Mass of Neutron stars and three-body forces

# **Major goals of hypernuclear physics**



 $V_{\Lambda N} = V_0 + \boldsymbol{\sigma}_{\Lambda} \cdot \boldsymbol{\sigma}_N V_{\sigma \cdot \sigma} + \mathbf{L} \cdot (\mathbf{s}_{\Lambda} + \mathbf{s}_N) V_{\text{SLS}} + \mathbf{L} \cdot (\mathbf{s}_{\Lambda} - \mathbf{s}_N) V_{\text{ALS}} + S_{12} V_{\text{tensor}} + \cdots$ 





Fig. 5. Kinematical condition of the  $(e, e'K^+)$  reaction for Jlab E89-009.

Jlab Hall A and Hall C experiments energy resolution 0.5MeV high momentum transfer => good to study high spin states Jparc energy resolution more than 1MeV



Table 1: Comparison of  $\Lambda$  Hypernuclear production reactions

$\Delta Z = 0$	$\Delta Z = -1$	comment
neutron to $\Lambda$	proton to $\Lambda$	
$(\pi^+,K^+)$	$(\pi^-,K^0)$	stretched, high spin
in-flight $(K^-, \pi^-)$ stopped $(K^-, \pi^-)$	in-flight $(K^-, \pi^0)$ stopped $(K^-, \pi^0)$	substitutional
$egin{array}{l} (e,e'K^0)\ (\gamma,K^0) \end{array}$	$egin{array}{l} (e,e'K^+)\ (\gamma,K^+) \end{array}$	spin-flip, unnatural parity

# Strategy to determine YN and YY interactions from the studies of light hypernuclear structure



# Single-particle energies of $\Lambda$ G-matrix results vs. experiments

(Y. Yamamoto et al.: Prog. Ther. Phys. S.185 (2010) 72. )



Sd- and fp-shell data are quite Important to extract the  $\Lambda$ behavior in nuclear matter.

Fig. 1. Energy spectra of <sup>13</sup><sub>A</sub>C, <sup>28</sup><sub>A</sub>Si, <sup>51</sup><sub>A</sub>V, <sup>89</sup><sub>A</sub>Y, <sup>139</sup><sub>A</sub>La and <sup>208</sup><sub>A</sub>Pb are given as a function of A<sup>-2/3</sup>, A being mass numbers of core nuclei. Solid (dashed) lines show calculated values by the Gmatrix folding model derived from ESC08a (the Skyrme-HF model). Open circles denote the experimental values taken from Ref. 17).

### Energy levels of core nuclei and hypernuclei



Figure 42: Schematic scheme of low-lying levels of a  $\Lambda$  hypernucleus and  $\gamma$  transitions.  $\gamma_c$  and  $\gamma_s$  are called "core transition" and " $\Lambda$ -spinflip transition", respectively. Excellent resolution of Ge detectors is essential to resolve two levels (called "hypernuclear fine structure") split by the  $\Lambda N$  spin-dependent interactions. See text for details.



 $V_{\Lambda N} = V_0 + \boldsymbol{\sigma}_{\Lambda} \cdot \boldsymbol{\sigma}_N V_{\sigma \cdot \sigma} + \mathbf{L} \cdot (\mathbf{s}_{\Lambda} + \mathbf{s}_N) V_{\text{SLS}} + \mathbf{L} \cdot (\mathbf{s}_{\Lambda} - \mathbf{s}_N) V_{\text{ALS}} + S_{12} V_{\text{tensor}} + \cdots$ 

Millener (p-shell model),Hiyama (few-body)

# Hypernuclear physics



A particle plays a 'glue like role' to produce a dynamical contraction of the core nucleus.

How do we observe nuclear shrinkage effect by experiment?

Theoretical calculation by Hiyama et al. B(E2:  $5/2^+ \rightarrow 1/2^+$ ) =2.85 e<sup>2</sup>fm<sup>4</sup> R<sub> $\alpha$ -np</sub>(<sup>7</sup>Li) is reduced by 22%



B(E2: 3<sup>+</sup>→1<sup>+:6</sup>Li)=9.3 ±0.5e<sup>2</sup>fm<sup>4</sup> →B(E2:5/2<sup>+</sup>→1/2<sup>+:7</sup>Li)<sub>1</sub> = 3.6 ±2.1 e<sup>2</sup>fm<sup>4</sup> B(E2) ∝ R<sup>4</sup>

The shrinkage effect on the nuclear size included by the  $\Lambda$  particle was confirmed for the first time.

### Theoretical models for hypernuclei and EoS with hyperons

- RMF (Relativistic Hartree, Relativistic Hartree-Fock models)
- Skyrme Hartree-Fock model with hyperon YN interaction : SU(3) model or phenomenological
- Bruckner Hartree-Fock model with YN scattering data
- quark-coupling model
- chiral model

 Three-body interaction RMF: ωNY σNY BHF: Fujita-Miyazawa-type 3body YNN interaction +repulsive YNN at higher density

# SHF and RMF are phenomenological models

Baryon one-loop approximation (Hartree approximation) makes RMF a phenomenological model.

**NN** part

**YN** part

- $\rightarrow$  We need DATA and AB INITIO results.
  - Saturation point (  $\rho_0$  and E/A( $\rho_0$ )) from mass formula
  - Nuclear binding energies
  - U<sub>v</sub> and U<sub>s</sub> from DBHF results (RMF)
  - P(ρ<sub>B</sub>) from heavy-ion data
  - $\Lambda$  separation energy from single  $\Lambda$  hypernuclear data
  - AΛ bond energy from double Λ hypernuclear data
  - Σ atomic shift
  - $\Sigma$  and  $\Xi$  potential depth from quasi-free production data

Relativistic Hartree-Fock model with pion coupling+rho tensor coupling

# Vector potential in RMF

Vector potential from ω dominates at high density !

$$U_{v}(\rho_{B}) = g_{\omega}\omega \sim \frac{g_{\omega}^{2}}{m_{\omega}^{2}}\rho_{B}$$

- Dirac-Bruckner-Hartree-Fock shows suppessed vector potential at high ρ<sub>B</sub>.
   *R. Brockmann, R. Machleidt, PRC42('90)1965.*
- Collective flow in heavy-ion collisions suggests pressure at high ρ<sub>B</sub>.
   *P. Danielewicz, R. Lacey, W. G. Lynch, Science298('02)1592.*
- Self-interaction of  $\omega \sim c_{\omega} (\omega_{\mu} \omega^{\mu})^2$ 
  - → DBHF results & Heavy-ion data



K. Tsubakihara, H. Maekawa, H. Matsumiya, AO, PRC81('10)065206. ρ<sub>B</sub> (fm<sup>-3</sup>)

**RMF** with Hyperons (Single A hypernuclei)

### RMF for Λ hypernuclei

 $x \sim 1/3$ : R. Brockmann, W. Weise, PLB69('77)167; J. Boguta and S. Bohrmann, PLB102('81)93.  $x \sim 2/3$ : N. K. Glendenning, PRC23('81)2757, PLB114('82)392;

- Tensor: Y. Sugahara, H. Toki, PTP92('94)803; H. Shen, F. Yang, H. Toki, PTP115('06)325; J. Mares, B. K. Jennings, PRC49('94)2472.
- p-dep. coupling: H. Lenske, Lect. Notes Phys. 641('04)147; C. M. Keil, F. Hofmann, H. Lenske, PRC 61('00)064309.

SU(3) or SU(6) (ς, φ): J. Schaffner, C. B. Dover, A. Gal, C. Greiner, H. Stoecker, PRL71('93)1328; Schaffner et al., Ann. Phys. 235('94)35; J. Schaffner, I. N. Mishustin, PRC 53('96)1416. Chiral SU(3) RMF: K. Tsubakihara, H. Maekawa, H. Matsumiya, AO, PRC81('10)065206.



K. Tsubakihara, H. Maekawa, H. Matsumiya, AO, PRC81('10)065206.

#### PHYSICAL REVIEW C 85, 025806 (2012)

#### Hyperon effects in covariant density functional theory and recent astrophysical observations

Wen Hui Long (龙文辉),1,2,\* Bao Yuan Sun (孙保元),1 Kouichi Hagino,2 and Hiroyuki Sagawa3

	Chemical equilibrium condition
$\mathscr{L}_{\Lambda} = \psi_{\Lambda} (\iota \gamma^{\mu} \partial_{\mu} - M_{\Lambda} - g_{\sigma \cdot \Lambda} \sigma - g_{\omega \cdot \Lambda} \gamma^{\mu} \omega_{\mu}) \psi_{\Lambda},$	$\mu_p = \mu_n - \mu_e,$
$e_{-1}/e_{-} = 0.600$ and $e_{-1}/e_{-} = 0.653$	$\mu_{\Lambda} = \mu_n,$
$s_{\sigma-\Lambda/S_{\sigma}} = 0.000$ and $s_{\omega-\Lambda/S_{\omega}} = 0.000$	$\mu_{\mu} = \mu_{e},$

$$M_{\Lambda} = 1115.0 \text{ MeV}, m_e = 0.511 \text{ MeV}, \text{ and } m_{\mu} = 105.7 \text{ MeV},$$

20.0

FABLE	I.	Details	of	the sel	ected	CDF	ef	fective	Lagrangians.
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	Fock term	Nonlinear $\sigma$ term	Nonlinear $\omega$ term	Density-dependent couplings
PKA1	yes	no	no	yes
PKO3	yes	no	no	yes
NLSH	no	yes	no	no
PK1	no	yes	yes	no
TW99	no	no	no	yes
PKDD	no	no	no	yes

TABLE III. Bulk properties of symmetric nuclear matter at saturation point, i.e., the saturation density  $\rho_0$  (fm<sup>-3</sup>), binding energy per particle E/A (MeV), incompressibility K (MeV), and symmetry energy J (MeV) with its slope L (MeV) and curvature  $K_{sym}$  (MeV). The results are provided by the CDF effective Lagrangians PKA1, PKO3, PKDD, TW99, NL-SH, and PK1.

	ρ	E/A	K	J	L	K <sub>sym</sub>
PKA1	0.160	-15.83	229.96	36.02	103.50	213.23
PKO3	0.153	-16.04	262.47	32.99	83.00	116.56
PKDD	0.150	-16.27	262.18	36.79	90.21	-80.74
TW99	0.153	-16.25	240.27	32.77	55.31	-124.68
NL-SH	0.146	-16.35	355.43	36.12	113.66	79.72
PK1	0.148	-16.27	282.69	37.64	115.88	55.33



A.B. Tsang<sup>1</sup>, J. R. Stone<sup>2</sup>, F. Camera<sup>3</sup>, P. Danielewicz<sup>1</sup>, S. Gandolfi<sup>4</sup>, K. Hebeler<sup>5</sup>, C. J. Horowitz<sup>6</sup>, Jenny Lee<sup>7</sup>, W.G. Lynch<sup>1</sup>, Z. Kohley<sup>1</sup>, R. Lemmon<sup>8</sup>, P. Moller<sup>4</sup>, T. Murakami<sup>9</sup>, S. Riordan<sup>10</sup>, X. Roca-Maza<sup>11</sup>, F. Sammarruca<sup>12</sup>, A. W. Steiner<sup>13</sup>, I. Vidaña<sup>14</sup>, S.J. Yennello<sup>15</sup>

Constraining the Symmetry Parameters of the Nuclear Interaction



# Determining Symmetry Parameters from Nuclear Masses

From liquid drop model:  $E_{\rm sym}(N,Z) = (S_v A - S_s A^{2/3})I^2$  $\chi^2 = \sum_i (E_{ex,i} - E_{sym,i})^2 / \mathcal{N} \sigma_D^2$  $\chi_{vv} = \frac{2}{\mathcal{N}\sigma_D^2} \sum_i I_i^4 A_i^2$  $\chi_{ss} = \frac{2}{N\sigma_p^2} \sum_i I_i^4 A_i^{4/3}$  $\chi_{vs} = \frac{2^{\nu}}{N\sigma_{D}^{2}} \sum_{i} I_{i}^{4} A_{i}^{5/3}$  $\sigma_{S_v} = \sqrt{\frac{2\chi_{ss}}{\chi_{vv}\chi_{ss} - \chi_{sv}^2}} \simeq 2.3 \sigma_D$  $\sigma_{S_{\rm s}} = \sqrt{\frac{2\chi_{\rm vv}}{\chi_{\rm vv}\chi_{\rm ss} - \chi_{\rm sv}^2}} \simeq 13.2\,\sigma_D$  $\alpha = \frac{1}{2} \tan^{-1} \frac{2\chi_{vs}}{\chi_{vv} - \chi_{ss}} \simeq 9^{\circ}.8$  $r_{vs} = -\frac{\chi_{vs}}{\sqrt{\chi_{vv} - \chi_{ss}}} \simeq 0.997$ 

Liquid droplet model:

$$E_{\rm sym}(N,Z) = \frac{S_v A I^2}{1 + (S_s/S_v) A^{-1/3}}$$



J. M. Lattimer Symmetry Energy a

Symmetry Energy and Neutron Star Structure

Hyperon effects in covariant density functional theory and recent astrophysical observations

Wen Hui Long (龙文辉),1,2,\* Bao Yuan Sun (孙保元),1 Kouichi Hagino,2 and Hiroyuki Sagawa3

$$\mathscr{L}_{\Lambda} = \bar{\psi}_{\Lambda} (i\gamma^{\mu}\partial_{\mu} - M_{\Lambda} - g_{\sigma-\Lambda}\sigma - g_{\omega-\Lambda}\gamma^{\mu}\omega_{\mu})\psi_{\Lambda}, \quad (1)$$

where  $M_{\Lambda}$  denotes the mass of the  $\Lambda$  hyperon ( $\psi_{\Lambda}$ ). From the Lagrangian density (1) the contributions to the energy functional as well as the Dirac equation for the  $\Lambda$  hyperon can be determined as for the nucleon [9,11,16,28].

For the  $\beta$ -stable stellar matter containing nucleons,  $\Lambda$  hyperons, electrons, and muons, the chemical equilibrium conditions require that

$$\mu_p = \mu_n - \mu_e, \tag{2}$$

$$\mu_{\Lambda} = \mu_n, \tag{3}$$

$$\mu_{\mu} = \mu_{e},\tag{4}$$

where the chemical potentials  $\mu_n$ ,  $\mu_p$ ,  $\mu_\Lambda$ ,  $\mu_\mu$ , and  $\mu_e$  are determined by the relativistic energy-momentum relation at Fermi momentum  $p = k_F$  [9],

$$\mu_i = \Sigma_0(k_{F,i}) + E^*(k_{F,i}), \quad \mu_\lambda = \sqrt{k_{F,\lambda}^2 + m_\lambda^2}.$$
 (5)







#### To solve Tolman-Openheimer-Volkov Eq.

TABLE II. Maximum mass  $M_{max}$  ( $M_{\odot}$ ), corresponding radii  $R_{max}$ (km), and central density  $\rho_c$  (fm<sup>-3</sup>) for neutron stars, as well as the threshold densities  $\rho_b^{\Lambda}$  (fm<sup>-3</sup>) and  $\rho_b^{\mu}$  (fm<sup>-3</sup>) of  $\Lambda$  and muon emergence. For comparison, the quantities for neutron stars without  $\Lambda$  hyperons are shown in the last three rows. The results are calculated by DDRHF with PKA1 and PKO3 and RMF with PKDD, TW99, PK1, and NL-SH.

	PKA1	РКО3	PKDD	TW99	PK1	NL-SH
M <sub>max</sub>	1.713	1.837	1.849	1.647	1.832	2.213
Rmax	10.425	11.495	11.583	10.333	13.048	13.633
ρ <sub>c</sub>	1.314	1.048	1.024	1.292	0.786	0.700
o <sup>A</sup>	0.272	0.284	0.322	0.368	0.306	0.282
$\rho_b^{\mu}$	0.118	0.122	0.108	0.116	0.110	0.114
M <sub>max</sub>	2.423	2.500	2.329	2.078	2.315	2.802
R <sub>max</sub>	12.354	12.487	11.798	10.632	12.705	13.534
ρ <sub>c</sub>	0.810	0.780	0.888	1.104	0.796	0.650

FIG. 3. (Color online) (a) The mass-radius relations for the neutron stars with  $\Lambda$  hyperons (solid lines) and without  $\Lambda$  hyperons (dashed lines). The symbols denote the neutron stars with maximum masses. The region excluded by causality is shaded. For comparison the observational constraints from three neutron stars in the binaries 4U 160-8248 (light gray/white gray), EXO 1745-248 (dark gray/gray), and 4U 1820-30 (black/black gray) in Ref. [12] are shown as the 1 $\sigma$  and 2 $\sigma$  confidence contours, and the later analyses in Ref. [13] are denoted by the shaded areas with two different models of photospheric radius. (b) The mass and radius for the neutron stars with maximum masses determined with different  $\Lambda$ -hyperon coupling strengths (solid symbols), compared to the ones without  $\Lambda$  hyperons (open symbols). See the text for details. Ref. 12 E. Ozel et al., Phys. Rev. D82, 101301(R) (2010). 13 A. W. Steiner et al., Atrophys. J722, 33 (2010). **Bruckner-Hartree-Fock theory with Hyperons** 

- Microscopic G-matrix calculation with realistic NN, YN potential and microscopic (or phen.) 3N force (or 3B force).
  - Interaction dep. (V18, N93, ...) is large → Need finite nuclear info. E.Hiyama, T.Motoba, Y.Yamamoto, M.Kamimura / M.Tamura et al.
  - NS collapses with hyperons w/o 3BF.



H.J.Schulze, A.Polls, A.Ramos, I.Vidana, PRC73('06),058801.

S. Nishizaki, T. Takatsuka, Y. Yamamoto, PTP108('02)703.

Courtesy of Akira Onishi

# Phenomenological modeling of Three-Body Repulsion in ESC04



Three-body force due to triple-meson correlation

**Reduction of meson mass** in medium  $M_v(\rho)=M_v \exp(-\alpha \rho)$  for vector mesons

**Courtesy of Yamamoto** 

Medium-Induced Repulsion



# Univ. 3-body force $\rightarrow$ higher $\rho_t(Y)$



### Maximum Mass problems in Neutron Stars S. Nishizaki, Y. Yamamoto and T. Takatsuka



Figure 43: Three-body force caused by  $\Lambda N$ - $\Sigma N$  coupling. It is not incorporated in the effective two-body  $\Lambda N$  interaction and has to be studied for each hypernucleus.





Dramatic softening of EOS → Necessity of "Extra Repulsion"

TNI3 → TNI3u: Universal inclusion of TNI3 repulsion

NNY repulsive interaction

# RMF with 3BF

Three-baryon coupling term

The e-baryon coupling term  

$$L = L_{B}^{\text{free}} + L_{M}^{\text{free}} + L_{BM} + L_{M}^{\text{Int}} + \delta L$$

$$\delta L = -U_{\sigma}(\sigma) - \frac{1}{2}c_{\sigma\omega}\sigma\omega_{\mu}\omega^{\mu} - \frac{1}{4}c_{\omega\omega}(\omega_{\mu}\omega^{\mu})^{2}$$

$$-\sum_{B} \bar{\psi}_{B} \Big[ g_{\sigma\sigma B}\sigma^{2} + g_{\sigma\omega B}\sigma\omega_{\mu}\gamma^{\mu} + g_{\omega\omega B}\omega_{\mu}\omega^{\mu} \Big] \psi_{B}$$

$$\mathbf{v} = 3 \text{ terms}$$

- BBMM terms are ignored in standard RMF. (They can be absorbed in other terms by field re-definitions.) R.D.Furnstahl, B.D.Serot, H.-B. Tang, NPA615 ('97)441
- But field re-definition modifies the order of NDA. Naïve dimensional analysis (NDA) v = B/2 + M + d

(B, M, d=# of baryon and non-NG boson field, derivatives to NG fields)

• Higher v terms are found to be suppressed at  $\rho \sim \rho 0$ , but they will contribute more at high densities.

**Courtesy of Akira Onishi** 

## *RMF with 3BF + SU(3)<sub>f</sub> "violation"*

Tsubakihara, AO, in prep.

### Two types of modification

- Solution → EOS becomes stiff gradually at high density. (Fitting meson mass (E325) and Uv in RBHF)
- $R = g_{\omega \Lambda} / g_{\omega N} \sim 0.8 ~(\sim 2/3 ~(SU(3)))$
- $\rightarrow M_{max} \sim 1.7~M_{\odot}$  with hyperons (~ 1.4  $M_{\odot}$  w/o 3BF, violation)

T. Tsubakihara, A. Ohnishi / Nuclear Physics A 00 (2013) 1-4



e 3. Neutron star matter EOSs as functions of  $\rho_B$  and neutron star maximum masses as functions of the central density.

### Summary

- YN interaction is now studied well in light hypernuclei A<30
- Lambda-nucleon interaction gives medium strong attractive potential:
- 1/2~2/3 of nuclear mean field potential (SU(3)<sub>f</sub> might work)  $g_{Y\sigma} / g_{N\sigma} = g_{Y\omega} / g_{N\omega} = 2/3$
- realistic BHF YN interaction is now available
- •various RMF Lagrangian: Hartree and Hartree-Fock level
- hyperon makes softer EoS and does not support 2M of neutron stars
- need a repulsive three-force at high density EoS

**Future perspectives** 

- how we can justify repulsive YNN interaction at high density? (theoretically and experimentally)
- YY interaction: too weak to make hyperon condensation in neutron star ? need more data
- quark degree of freedom => softer EoS?

crossover to baryon to quarks => hard EoS (Hatsuda)

cooling process of neutron star: hyperon effect?



Figure 43: Three-body force caused by  $\Lambda N \cdot \Sigma N$  coupling. It is not incorporated in the effective two-body  $\Lambda N$  interaction and has to be studied for each hypernucleus.



 $V(ND-soft) \rightarrow 0.5V(ND-soft)$ ; Hiyama

Takahashi et al., PRL 87 (2001) 212502

**RMF** with Hyperons (Double A hypernuclei)

Nagara event ΔB<sub>ΛΛ</sub>~ 1.0 MeV (weakly attractive)

TM & NL-SH based RMF

*H. Shen, F. Yang, H. Toki, PTP115('06)325.* Model 1:  $\mathbf{x}_{\sigma} = 0.621$ ,  $\mathbf{x}_{\omega} = 2/3$  (no  $\varsigma$ ,  $\varphi$ ) Model 2:  $\mathbf{R}_{\varsigma} = \mathbf{g}_{\varsigma\Lambda} / \mathbf{g}_{\sigma N} = 0.56 - 0.57$ ,  $\mathbf{R}_{\varphi} = \mathbf{g}_{\varphi\Lambda} / \mathbf{g}_{\omega N} = -\sqrt{2/3}$ 





#### Three-body couplings in RMF and its effects on hyperonic star equation of state\*

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$$\mathcal{L}_{R=3}^{\sigma\omega} = -\frac{1}{f_{\pi}} \sum_{B} \bar{\psi}_{B} \left[ g_{\sigma\sigma B} \sigma^{2} + g_{\omega\omega B} \omega_{\mu} \omega^{\mu} - g_{\sigma\omega B} \sigma \omega_{\mu} \gamma^{\mu} \right] \psi_{B} - c_{\sigma\omega\omega} f_{\pi} \sigma \omega_{\mu} \omega^{\mu} . \qquad (1)$$

The first and second terms modify the effective mass of nucleons,  $M_N^* = M_N - (g_{\sigma N}\sigma - g_{\sigma \sigma N}\sigma^2/f_\pi - g_{\omega \omega N}\omega^2/f_\pi)$ , where  $\omega$  represents the temporal component. These n = 3 terms modify the effective mass from the n = 2 coupling with  $\sigma$ . The third term modifies the vector potential of nucleons at high density,  $U_v = (g_{\omega N} - g_{\sigma \omega N}\sigma/f_\pi)\omega$ , and the fourth term represents the  $\omega$  meson mass shift at finite density. In RBHF calculations, the vector potential at low densities is almost proportional to the baryon density, but the vector potential to baryon density ratio  $U_v/\rho_s$  is suppressed around  $\rho_0$  or at higher densities. This suppression in RBHF is sometimes simulated by the  $\omega^4$  term [8] or by the density dependent coupling [9]. When we simulate the suppression only with the  $\omega^4$  term, the ratio  $U_v/\rho_s$  is monotonically decreasing with increasing  $\rho_s$ . With a large coefficient of  $\omega^4$ , EOS at high density is thus too softened to support the massive neutron star [7]. The density dependent vector coupling is usually chosen to decrease and be saturated with increasing  $\rho_s$ . With the present n = 3 coupling terms in Eq. (1) in addition to the  $\omega^4$  term, we try to simulate the decreasing and saturating behavior of the  $U_v/\rho_s$  ratio found in the RBHF results.



Figure 1. Calculated vector potential and EOS in symmetric nuclear matter with TB-a and TB-b parameter set in comparison with SCL3 results.



Figure 2. Calculated binding energies and A separation energies (SA) based on TB-a and TB-b parameter sets.





Figure 3. Neutron star matter EOSs as functions of pg and neutron star maximum masses as functions of the central density.

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Figure 47: All the hypernuclear  $\gamma$  transitions observed and identified so far for  ${}^{4}_{\Lambda}$ H [33],  ${}^{4}_{\Lambda}$ He [34],  ${}^{7}_{\Lambda}$ Li [35, 25],  ${}^{9}_{\Lambda}$ Be [35, 40],  ${}^{10}_{\Lambda}$ B [36, 108],  ${}^{11}_{\Lambda}$ B [43],  ${}^{15}_{\Lambda}$ N [108], and  ${}^{16}_{\Lambda}$ O [41]. Level schemes of these hypernuclei with level energies (in MeV) and assigned spin-parities are shown together with reactions to populate excited states in  $\gamma$  spectroscopy experiments.