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The high-density Symmetry Energy and (the way forward with) transport models

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The Nuclear Symmetry Energy (SE) in "realistic" models

The EOS of symmetric and pure neutron matter in different many-body aproaches



Also momentum dependence of symmetry potential \rightarrow p/n effective mass splitting

Uncertainity in many-body theory: \rightarrow In-medium ρ mass, and short range isovector tensor correlations (B.A. Li; I. Vidana);

→ Investigate in: - HIC in the laboratory and and interpretation with transport models
 - Neutron star observations and modelling of NS structure

Outline:

- 1. Probes of the high-density Symmetry energy in HIC: Yvonne Leifels, very extensive overview here: Promises and Problems
- 2. Transport models Theory and Realizations
- **3. Wish list for transport practitioners**

Transport Theory in Heavy Ion Collisions

Transport theory describes the non-equilibrium aspects of the temporal evolution of a collision.

1. Evolution in coordinate space:

high density

non-equilibrium,

non-sphericity of local



Au+Au, E=1.8AGeV, b=2fm movie thanks to T. Gaitanos, T.Chossy







Momentum distributions, "Flow"



Production of high energy (hard) photons in HIC:



Universal curve, when scaled relative to Coulomb barrier:

- → First chance pn collisions
- \rightarrow medium modification of pny cross section

Particle Production





300

Inelastic collisions: Production of particles and resonances: Coupled transport equations

e.g. pion and kaon production;

coupling of Δ and strangeness channels via collision term

$$\frac{d}{dt}f_{N}(\mathbf{x}_{\mu})=I_{coll}(\sigma_{NN\to NN},f_{N};\sigma_{NN\to N\Delta}f_{\Delta};....)$$
$$\frac{d}{dt}f_{\Delta}(\mathbf{x}_{\mu})=I_{coll}(\sigma_{\Delta N\to NYK}f_{Y}f_{K};....)$$
etc.



Particle production as probe of symmetry energy

Difference in neutron and proton potentials

- 1. "direct effects": difference in proton and neutron (or light cluster) emission and momentum distribution
- 2. "secondary effects": production of particles, isospin partners $\pi^{-,+}$, K^{0,+}



Two limits:

- Isobar model (yield determined by CG-Coeff of $\pi^{-}/\pi^{+} = \frac{5N^{2} + NZ}{5Z^{2} + NZ} \approx \left(\frac{N}{Z}\right)^{2}$ isobar model 1. $\Delta -> N\pi$

2. chemical equilibrium

$$\pi^{-}/\pi^{+} \propto \exp{\frac{2(\mu_{n}-\mu_{p})}{T}} = \exp{\frac{8\delta E_{sym}(\rho)}{T}}$$

-> in principle π -/ π + hould be a good probe!



 π^{-}/π^{+}

Particle production as probe of symmetry energy

Two effects:

G.Ferini et al., PRL 97 (2006) 202301

 1. Mean field effect: U_{sym} more repulsive for neutrons, and more for asystiff
 → pre-equilibrium emission of neutron, reduction of asymmetry of residue

2. Threshold effect, in medium effective masses:

Canonical momenta have to be conserved. To convert to kinetic momenta, the self energies enter

In inelastic collisions, like nn->p Δ^- , the selfenergies may change. Simple assumption about self energies of Δ ..

Yield of pions depends on $\boldsymbol{\sigma} = \boldsymbol{\sigma}_{inel} \left(\sqrt{\mathbf{S}_{in}} - \sqrt{\mathbf{S}_{in}} \right)$

Detailed analysis gives

$$\frac{n}{p} \downarrow \Rightarrow \frac{Y(\varDelta^{0,-})}{Y(\varDelta^{+,++})} \downarrow \Rightarrow \frac{\pi^{-}}{\pi^{+}} \downarrow$$

decrease with asy – stiffness

$$\begin{aligned} \mathbf{J}_{coll} &= \int d\vec{\mathbf{v}}_2 \ d\vec{\mathbf{v}}_{1'} d\vec{\mathbf{v}}_{2'} \mathbf{v}_{12} \sigma_{inel}(\Omega) (2\pi)^3 \delta(\mathbf{p}_1 + \mathbf{p}_2 - \mathbf{p}_{1'} - \mathbf{p}_{2'}) \\ &\times \left[f_{1'} f_{2'} (1 - f_1) (1 - f_2) - f_1 f_2 (1 - f_{1'}) (1 - f_{2'}) \right] \end{aligned}$$

$$\Sigma_i(\Delta^-) = \Sigma_i(n),$$

$$\Sigma_i(\Delta^0) = \frac{2}{3}\Sigma_i(n) + \frac{1}{3}\Sigma_i(p),$$

$$\Sigma_i(\Delta^+) = \frac{1}{3}\Sigma_i(n) + \frac{2}{3}\Sigma_i(p),$$

$$\Sigma_i(\Delta^{++}) = \Sigma_i(p),$$

 $rac{\pi}{\pi^+}$ \uparrow increase with asy – stiffness

Competing effects! Not clear, how taken into account in all studies Assumptions may also be too simple.



Dynamics of particle production (Δ , π ,**K) in heavy ion collisions**

Pion ratios in comparison to FOPI data



Kaon production as a probe for the EOS Subthreshold, E_{th} =1.58 MeV NN \longrightarrow NA

Nπ

ΛK

ΝΛΚ

Two-step process dominant

In havier systems. Collective effect



C. Fuchs / Progress in Particle and Nuclear Physics 56 (2006) 1-103



messengers from the high density phase



Strangeness ratio : Infinite Nuclear Matter vs. HIC

G. Ferini, et al., NPA762(2005) 147



Pre-equilibrium emission (mainly of neutrons) reduces asymmetry of source for kaon production → reduces sensitivity relative to equilibrium (box) calculation

Kaon production in HIC





G. Ferini, et al., NPA762(2005) 147

- From asy-soft to stiff from lower to upper curves: Stiffer asy-EOS→larger ratio! Opposite to mean field effect!
- Kaons a somewhat more sensitive than pions
- esp. at low energies, close to threshold

- single ratios more sensitive
- enhanced in larger systems

Transport Theory

There are chances and difficulties in the interpretation of HIC experiments --- but, do we completely master the theoretical description???

- Need transport theory to describe non-equilibrium evolution of system
- Foundation of transport theory
- Molecular dynamic approaches, comparisons
- Open problems in transport theory



Classical derivation of a BUU transport equation



Relation of mean field ${\bf U}$ and medium cross section ${\bf \sigma}$ not clear.

 \rightarrow 3 more ways to arrive at the BUU eqn. which clarify its foundations

Quantum Derivation of Vlasov Equation

Start with TDHF

$$\rho(r_1, r_2) = \langle \text{slat.det} | \hat{\psi}^+(r_1) \hat{\psi}(r_2) | \text{slat.det} \rangle$$

$$\frac{\partial}{\partial t} \rho = \frac{1}{i} [h, \rho]_{1,2} \qquad h(1,2) = T(1,2) + \sum_{3,4} (V_{13,24} - V_{14,23}) \rho_3,$$

$$f(r, p) = \frac{1}{(2\pi)^3} \int ds \ e^{-ips} \ \rho(r + \frac{s}{2}, r - \frac{s}{2})$$

Wigner transform Fourier transf wrt fast variation

Equation of motion for Wigner transform f. Use gradient approximation for Wigner transform of products

$$\frac{1}{i}\frac{1}{(2\pi)^3}\int d\mathbf{s} \, \mathbf{e}^{-i\mathbf{p}\mathbf{s}} \left(U(\mathbf{r}+\frac{\mathbf{s}}{2})-U(\mathbf{r}-\frac{\mathbf{s}}{2}) \right) \rho(\mathbf{r}+\frac{\mathbf{s}}{2},\mathbf{r}-\frac{\mathbf{s}}{2}) = 2\sin\frac{\nabla_r U \nabla_p f}{2} U f \approx \nabla_r U \nabla_p f + \dots$$

again Vlasov eq.

$$\frac{\partial f}{\partial t} + \frac{p}{m} \nabla_r f - \nabla_r U(r) \nabla_p f(r, p) = 0 \qquad \Rightarrow I_{coll}$$

Remarks:

• collision term has to be added "by hand" as before

• quantum statistics only contained in initial condition, but is preserved by the evolution (Liouville theorem; for coll. term explicitly via blocking terms)

Can also be done in a relativistic formulation (RMF) $L(\psi;\sigma,\omega,\pi,\eta,\delta,...)$

$$\left[\boldsymbol{p}^{*\mu}\partial_{\mu}^{(r)}+(\boldsymbol{p}^{*}_{\nu}\boldsymbol{F}^{\mu\nu}+\boldsymbol{m}^{*}\partial_{(r)}^{\mu}\boldsymbol{m}^{*})\partial_{\mu}^{(\boldsymbol{p}^{*})}\right]\boldsymbol{f}(\boldsymbol{r},\boldsymbol{p}^{*})=\boldsymbol{I}_{coll}$$

 $m^* = m - \Sigma_s;$ $p^*_\mu = p_\mu - \Sigma_\mu$

+ Mass shell constraint: $(p^{*2} - m^{*2})f(r,p) = 0$

New Feature: two potentials: scalar-vector \rightarrow mom.dep. mean field, "Lorentz- like" forces

Non-equilibrium Transport Theory (Kadanoff-Baym)



Self Energy Approximation

→ Quasi-particle approximation (QPA) $G^{<} = A(r.p)F(r,p)$ A spectral function

$$\frac{A(x,p) \propto \frac{2\Gamma(x,p)}{(p^{*2}-m^{*2})+\Gamma^{2}(x,p)}}{\Gamma(x,p)=m^{*} lm \Sigma_{s}^{+}-p_{\mu}^{*} lm \Sigma^{+\mu}} QPA$$

$$\propto \delta(p^{*2} - m^{*2}) \Theta(p^{*0})$$



reduces no.of variables from 8 to $7 \rightarrow$ particle interpretation possible

$$\begin{bmatrix} \boldsymbol{p}^{*\mu} \partial_{\mu}^{(r)} + (\boldsymbol{p}^{*}_{\nu} \boldsymbol{F}^{\mu\nu} + \boldsymbol{m}^{*} \partial_{(r)}^{\mu} \boldsymbol{m}^{*}) \partial_{\mu}^{(p^{*})} \end{bmatrix} \boldsymbol{f}(\boldsymbol{r}, \boldsymbol{p}^{*}) = \boldsymbol{I}_{coll} \left[\boldsymbol{\sigma}_{NN}^{(in-med)} \right] \qquad \begin{array}{c} \boldsymbol{m} = \boldsymbol{m} - \boldsymbol{\Sigma} \\ \boldsymbol{p}_{\mu}^{*} = \boldsymbol{p}_{\mu} - \boldsymbol{\Sigma} \end{array}$$

The self energy can be taken in the T_Matrix approximation, including exchange and two-body correlations: Brueckner HF theory.

$$\Sigma_{\mathsf{s},\mu} \approx \mathsf{Tr}(\mathsf{T}\mathsf{f}); \quad \sigma_{\mathsf{NN}}^{(\mathit{in-med})} \approx |\mathsf{T}^2|$$



→Now the collision term appears consistently and is obtained on the same footing from the Brueckner T-Matrix.

- → without QPA, "off-shell transport" (particles with widths) (Buss et al., Phys. Rep. 512, 1 (2012) has not been investigated in great detail
- → non-equibrium effects in T-Matrix
 (e.q. 2-Fermi sphere approximation; Fuchs, Sehn, HHW)



Testparticle Solutions of BUU Equation

$$\frac{\partial f}{\partial t} + \frac{p}{m} \vec{\nabla}^{(r)} f - \vec{\nabla} U(r) \vec{\nabla}^{(p)} f(\vec{r}, \vec{p}; t) = \int d\vec{v}_2 \, d\vec{v}_1 \, d\vec{v}_2 \, v_{21} \, \sigma_{12}(\Omega) (2\pi)^3 \, \delta(p_1 + p_2 - p_{1'} - p_{2'}) \\ \left[f_{1'} \, f_{2'} \, (1 - f_1) (1 - f_2) - f_1 \, f_2 \, (1 - f_{1'}) (1 - f_2') \right]$$

non-linear integro-differential equation, no closed solutions

but deterministic !

a) solution on a lattice: has been used for low-dimensional model systems, but too expensive for realistic cases

b) test particle method (Wong 82)
$$f(r,p;t) = \frac{1}{N_{TP}} \sum_{i=1}^{N_{TP}} \delta(r-r_i(t)) \delta(p-p_i(t))$$

where $\{r_i(t), p_i(t)\}$ are the positions and momenta of the TP as a funct. of time, and N_{TP} is the number of TP per nucleon (usually 50 – 200)

→ approximate a (continuous) phase space distribution by a swarm of δ-functions variant: Gaussian TP: smoother distribution with fewer TP

 \rightarrow ansatz into Vlasov eq. \rightarrow Hamiltonian equations of motion:

$$\frac{\partial r_i}{\partial t} = \frac{p_i}{m}; \quad \frac{\partial p_i}{\partial t} = -\nabla U \big|_{r_i}$$



c) the rhs (collision term) is simulated, stochastically; like cascade

→ describes average effect of collisions (→dissipation), NOT Fluctuation

→ if $N_{\tau P} \rightarrow \infty$, exact solution of BUU eqn. !

d) include fluctuations explicitely Boltzmann-Langevin eq.

$$\frac{\partial f}{\partial t} + \frac{\vec{p}}{m} \vec{\nabla}^{(r)} f - \vec{\nabla} U(r) \vec{\nabla}^{(p)} f(\vec{r}, \vec{p}; t) = I_{coll} + \delta I_{fluct}$$

different approx. treatments simplest: adjust no. of TP to most unstable mode

Second family: Molecular Dynamics

Classical solution of the many-body problem with assumptions of 2-body interaction instead of MF depending on density

- 1. Classical Molecular dynamics CMD point particles, deterministic, but possibly chaotic behaviour because of short range repulsion
- 2. Quantum molecular dynamics QMD

Gaussian particles with large width to smooth fluctuations, not a wave packet, no antisymmetrization, collision term as in BUU, but between nucleons, origin not clear, and also not cross section (thus similar to BUU with N_{TP} =1) but event generator.

3. Antisymmetrized MD (AMD), Fermionic MD (FMD),

TDHF with Slaterdeterminant of s.p. wave functions in terms of Gaussian wave packets with psition and momentum as dynamical variables i.e. antisymmetrization included Collision term: "wave packet splitting", reduction of wp (Variant: Constrained MD, CoMD, like QMD but correction for self-interaction term)

4. fluctuations: larger in MD approaches, since collision moves nucleon (and not TP), parameter for fluctuations: width of "wave packets". clusterization: additional physical source of fluctuations

$$rac{\mathrm{d}}{\mathrm{d}t} oldsymbol{r}_i = \{oldsymbol{r}_i, \mathcal{H}\}, \qquad rac{\mathrm{d}}{\mathrm{d}t} oldsymbol{p}_i = \{oldsymbol{p}_i, \mathcal{H}\},$$

where the many-body Hamiltonian is of the form

$$\mathcal{H}\{\boldsymbol{r}_n, \boldsymbol{p}_n\} = \sum_{i=1}^{A} \frac{\boldsymbol{p}_i^2}{2m_i} + \sum_{i < j} V(|\boldsymbol{r}_i - \boldsymbol{r}_j|).$$



Comparison of simulations: SMF-AMD: (Rizzo, Colonna,Ono, PRC76(2007); Colonna et al., PRC82 (2010))



BUU(BNV)/SMF	← Comparison→	QMD/CoMD/AMD
	Mean field evolut	ion →very similar!
Semiclass approx to TDHF Solved with inf. no. of TP		TD-Hartree with product wf of sp Gaussians of large width (AMD TDHF)
Collision te	erm and med cross	sect→similar in principle, often not implement
Consistently derivable from KB approach, good approx. BHF		Not consistently derivable but empirically the same,
full ens.: TP coll→ small fluct parall. ens.: average after each →Too little fluct by collision, onl (improve: Bertsch method (Colo	collisions timestep, same ly av. dissip. nna): move N _{TP} TP)	 →different effect of collisions →different effect of collisions →Generates large fluct in phase space depending on width of Gaussian affects also Pauli blocking
Inital state co	Fluctuations prrelations, similar, ini	→diff mainly by effect of collision term tial wf not realistic
BUU eq. should be replaced by Boltzmann-Langevin eq.	,	higher order corr due to localized (packet) we but averaged out by smearing AMD: cluster dynamics
Frag Fluct as seeds of fragmen Early/late rec	gment formation an nts, amplified by mf, (a cognition does not affe	nd recognition at least for not too large inc. energy.) ect dynamics; a-posteriori
Can implement TP sampling and thus also MST,SACA,etc me	ethods	MST, SACA/ECRA methods natural, clustering in AMD
not well o Be	Small clusters (d described in BUU and est treated explicitely (, t,3He,α) I QMD (better in AMD) (but include α!)

Open Problems in Transport Theory and Calculations:

- 1. Ensure that codes are internally consistent (same physical input → same output); resp., determine theoretical systematic error from remaining differences
- 2. Standardization, logical naming of versions (code numbers, QMD 2.1, etc.)
- 3. Consistency of mean field (self energy) and in-medium cross sections (e.g. DBHF)
- 4. Momentum dependence of isoscalar and isovector forces (effective masses)
- 5. Control and study of role of fluctuations (diff. in BUU and QMD type codes)
- 6. Relativistic transport codes for higher energy: scalar and vector fields, Lorentz force, no 3-body terms
- 7. Investigation of "off-shell" transport, esp. for subthreshold production of particles, development of reliable approximations for propagation of particles with finite width, e.g. Δ , but in principle every particle
- 8. Treatment of clusters: dynamical generation of clusters with medium modifications, in-medium amplitudes for cluster production
- 9. Inelastic amplitudes for in-medium production of particles

Thank you

