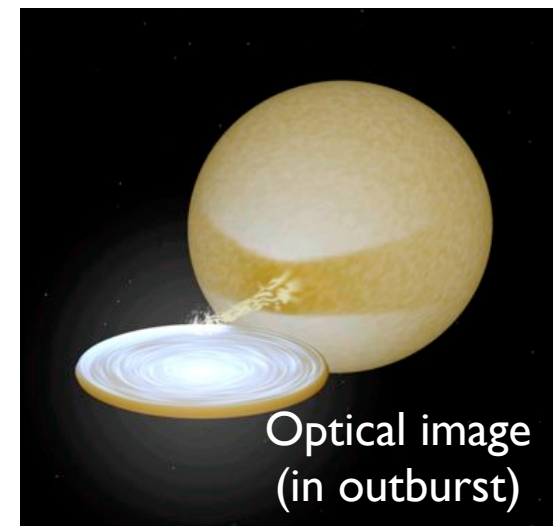
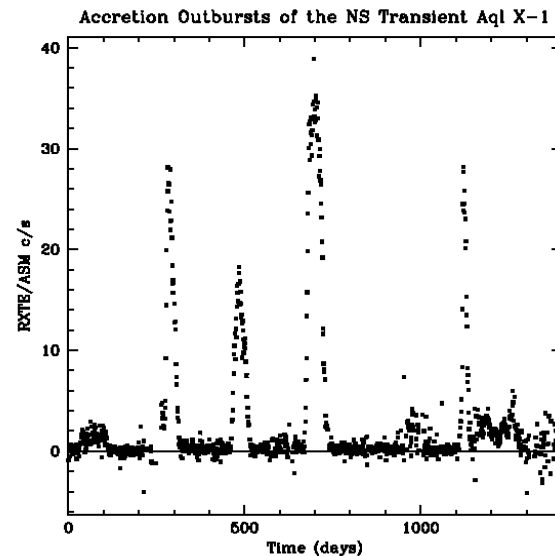
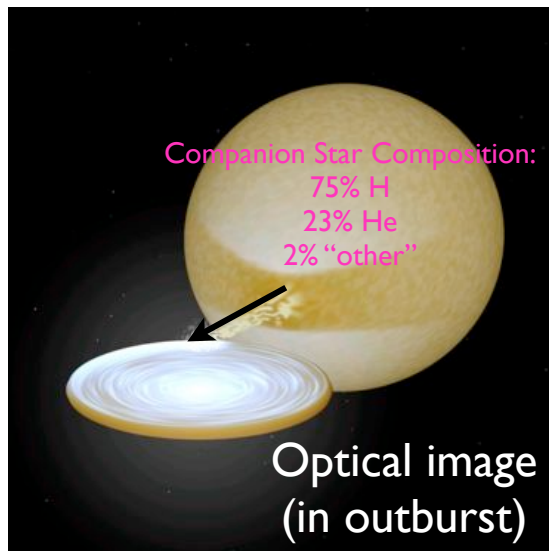


# The Radius of Neutron Stars

Bob Rutledge, Sebastien Guillot (McGill)  
Matthieu Servillat (CNRS), Natalie Webb (Toulouse)

# Quiescent Low Mass X-ray Binaries (qLMXB)

## Outburst

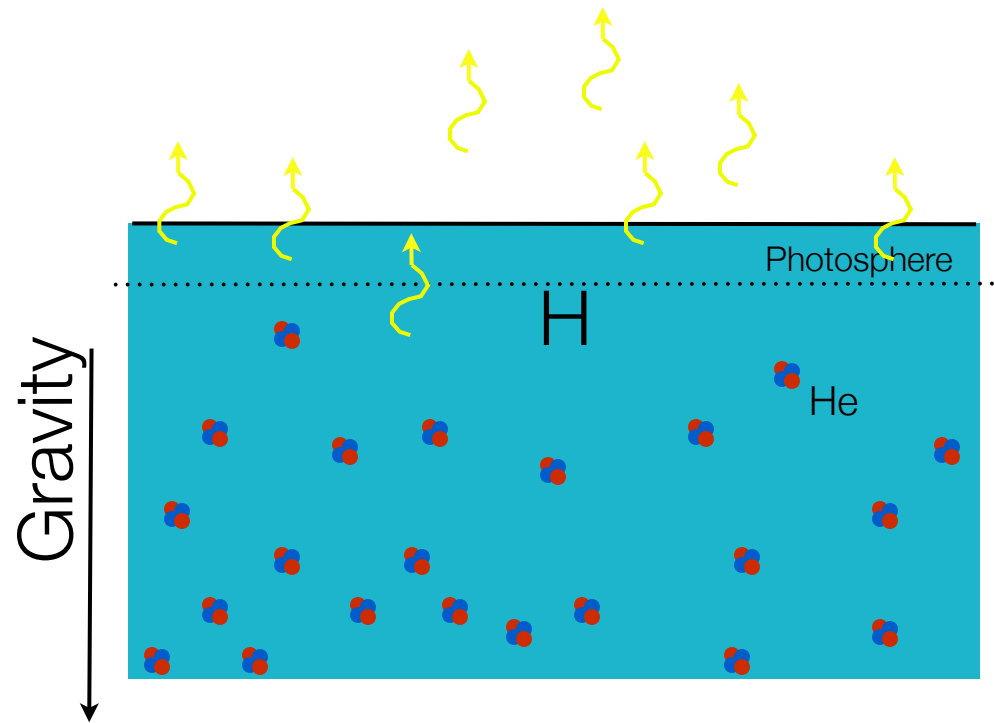


## Quiescence

# qLMXBs, in this scenario, have pure Hydrogen atmospheres

---

- When accretion stops, the He (and heavier elements, gravitationally settle on a timescale of  $\sim 10$ s of seconds (like rocks in water), leaving the photosphere to be pure Hydrogen (Alcock & Illarionov 1980, Bildsten et al 1992).



**Non-Equilibrium Processes in the Outer Crust**  
**Beginning with  $^{56}\text{Fe}$  (Haensel & Zdunik 1990, 2003)**

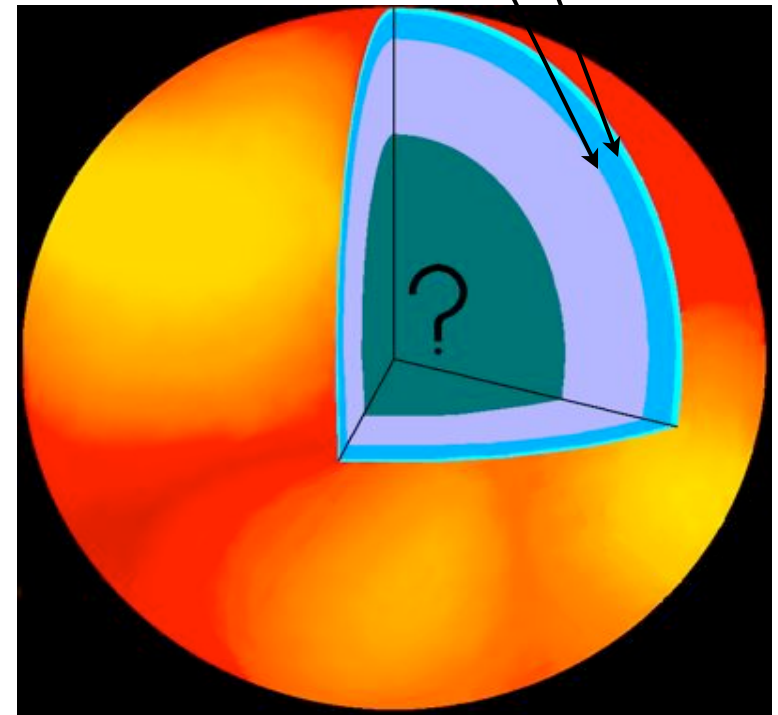
$\rho$ (g cm $^{-3}$ )	Reaction	$\Delta\rho\rho$	Q (Mev/np)
$1.5 \cdot 10^9$	$^{56}\text{Fe} \Rightarrow ^{56}\text{Cr} - 2e^- + 2\nu_e$	0.08	0.01
$1.1 \cdot 10^{10}$	$^{56}\text{Cr} \Rightarrow ^{56}\text{Ti} - 2e^- + 2\nu_e$	0.09	0.01
$7.8 \cdot 10^{10}$	$^{56}\text{Ti} \Rightarrow ^{56}\text{Ca} - 2e^- + 2\nu_e$	0.10	0.01
$2.5 \cdot 10^{10}$	$^{56}\text{Ca} \Rightarrow ^{56}\text{Ar} - 2e^- + 2\nu_e$	0.11	0.01
$6.1 \cdot 10^{10}$	$^{56}\text{Ar} \Rightarrow ^{52}\text{S} + 4n - 2e^- + 2\nu_e$	0.12	0.01

**Non-Equilibrium Processes in the Inner Crust**

$\rho$ (g cm $^{-3}$ )	Reaction	$X_n$	Q (Mev/np)
$9.1 \cdot 10^{11}$	$^{52}\text{S} \Rightarrow ^{46}\text{Si} + 6n - 2e^- + 2\nu_e$	0.07	0.09
$1.1 \cdot 10^{12}$	$^{46}\text{Si} \Rightarrow ^{40}\text{Mg} + 6n - 2e^- + 2\nu_e$	0.07	0.09
$1.5 \cdot 10^{12}$	$^{40}\text{Mg} \Rightarrow ^{34}\text{Ne} + 6n - 2e^- + 2\nu_e$		
	$^{34}\text{Ne} + ^{34}\text{Ne} \Rightarrow ^{68}\text{Ca}$	0.29	0.47
$1.8 \cdot 10^{12}$	$^{68}\text{Ca} \Rightarrow ^{62}\text{Ar} + 6n - 2e^- + 2\nu_e$	0.39	0.05
$2.1 \cdot 10^{12}$	$^{62}\text{Ar} \Rightarrow ^{56}\text{S} + 6n - 2e^- + 2\nu_e$	0.45	0.05
$2.6 \cdot 10^{12}$	$^{56}\text{S} \Rightarrow ^{50}\text{Si} + 6n - 2e^- + 2\nu_e$	0.50	0.06
$3.3 \cdot 10^{12}$	$^{50}\text{Si} \Rightarrow ^{44}\text{Mg} + 6n - 2e^- + 2\nu_e$	0.55	0.07
$4.4 \cdot 10^{12}$	$^{44}\text{Mg} \Rightarrow ^{36}\text{Ne} + 6n - 2e^- + 2\nu_e$		
	$^{36}\text{Ne} + ^{36}\text{Ne} \Rightarrow ^{72}\text{Ca}$		
	$^{68}\text{Ca} \Rightarrow ^{62}\text{Ar} + 6n - 2e^- + 2\nu_e$	0.61	0.28
$5.8 \cdot 10^{12}$	$^{62}\text{Ar} \Rightarrow ^{60}\text{S} + 6n - 2e^- + 2\nu_e$	0.70	0.02
$7.0 \cdot 10^{12}$	$^{60}\text{S} \Rightarrow ^{54}\text{Si} + 6n - 2e^- + 2\nu_e$	0.73	0.02
$9.0 \cdot 10^{12}$	$^{54}\text{Si} \Rightarrow ^{48}\text{Mg} + 6n - 2e^- + 2\nu_e$	0.76	0.03
$1.1 \cdot 10^{13}$	$^{48}\text{Mg} + ^{48}\text{Mg} \Rightarrow ^{96}\text{Cr}$	0.79	0.11
$1.1 \cdot 10^{13}$	$^{96}\text{Cr} \Rightarrow ^{88}\text{Ti} + 8n - 2e^- + 2\nu_e$	0.80	0.01

**Deep Crustal Heating**

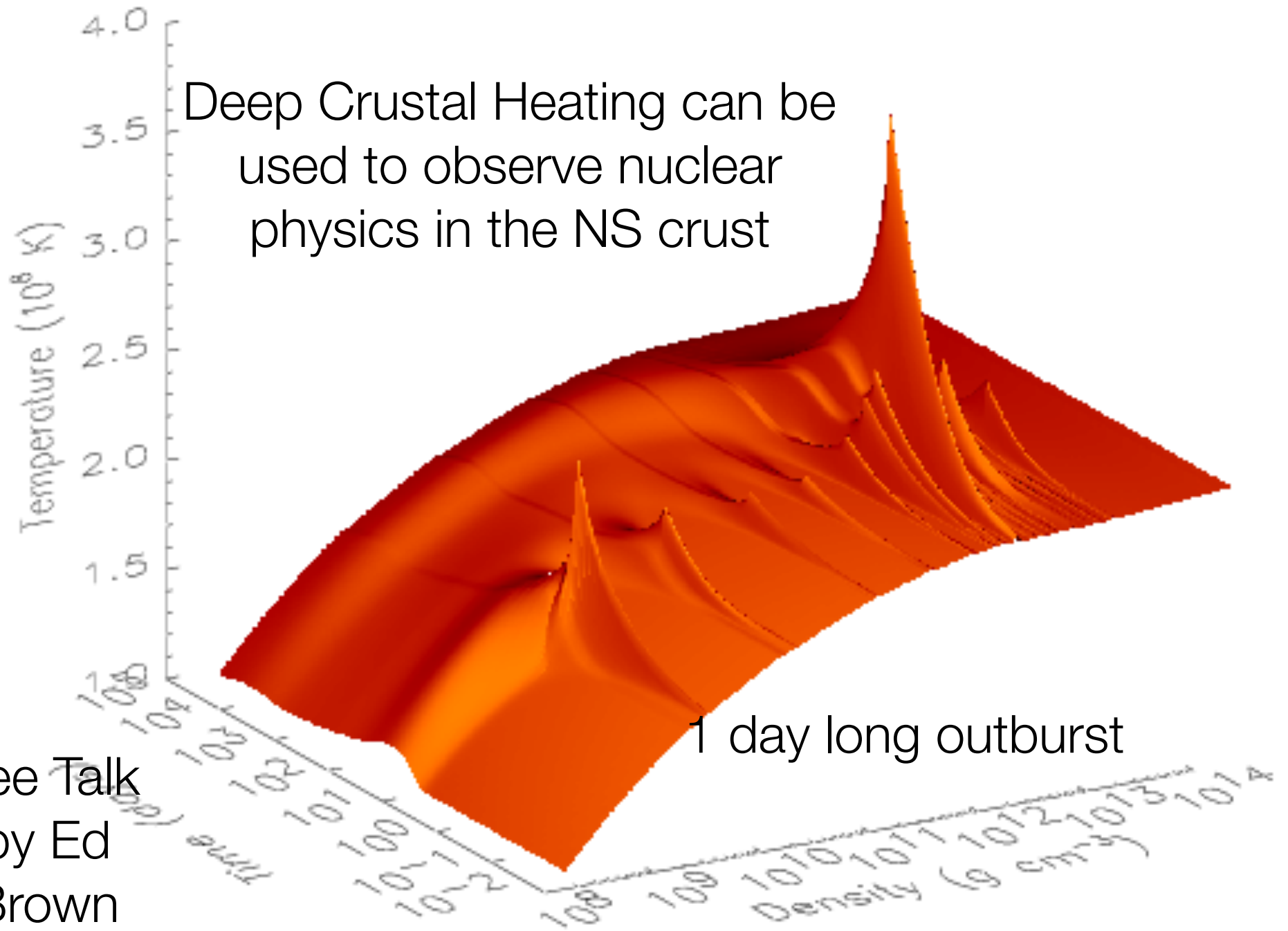
Begins Here  
 Ends Here



**1.47 Mev per np**

Brown, Bildsten & RR (1998)

Deep Crustal Heating can be used to observe nuclear physics in the NS crust

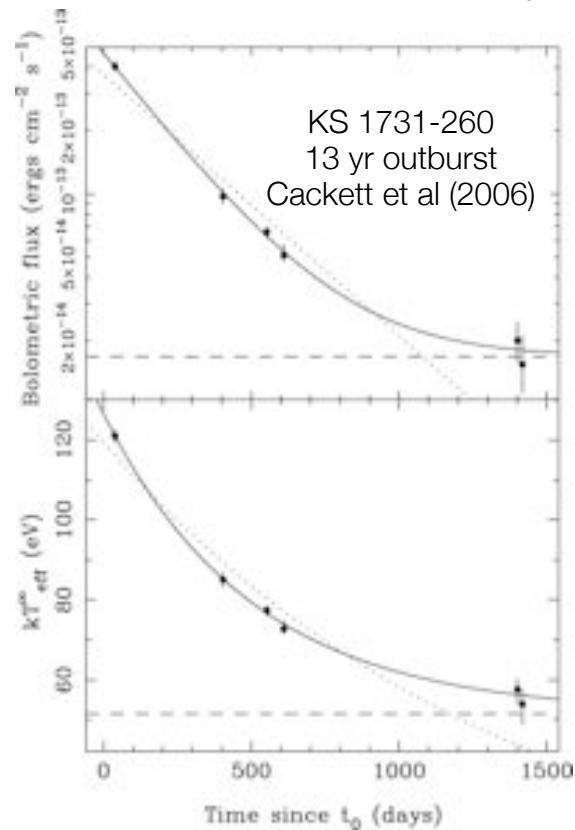
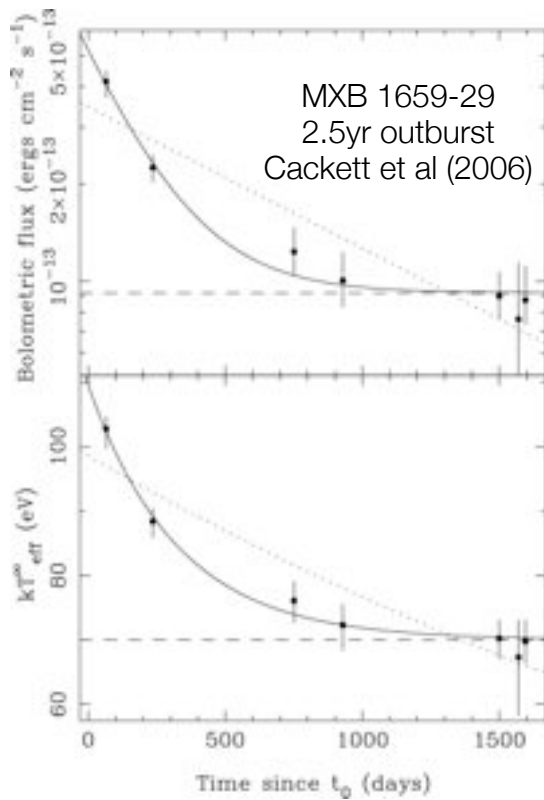
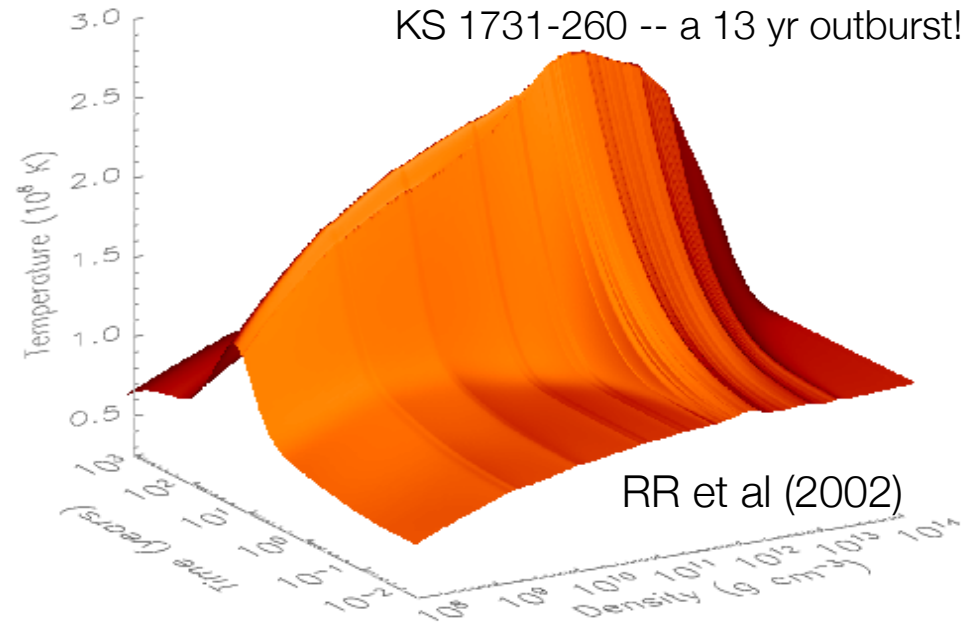


1 day long outburst

See Talk  
by Ed  
Brown  
today!

Ushomirsky & RR (2001)

# Transient Neutron Star Cooling KS 1731-260 & MXB 1659-29



See Talk  
by Ed  
Brown  
today!

Sudden drop x3 in 2013? Cackett et al 2013

# Emergent Spectrum of a Neutron Star Hydrogen Atmosphere

• H atmosphere calculated Spectra are ab initio radiative transfer calculations using the Eddington equations.

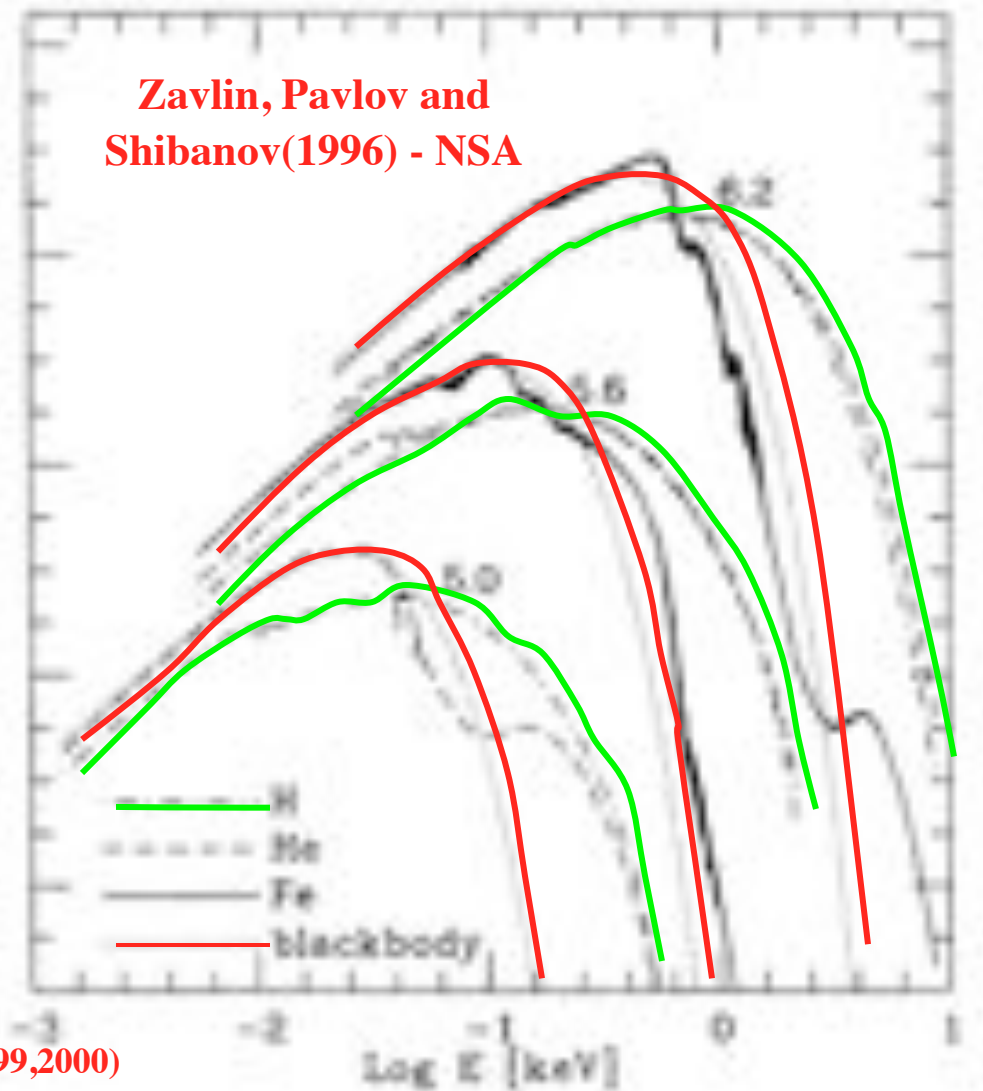
• Rajagopal and Romani (1996); Zavlin et al (1996); Pons et al (2002; Heinke et al (2006) -- NSATMOS; Gaensicke, Braje & Romani (2001); Haakonsen et al (2012)

All comparisons show consistency within ~few % (e.g. Webb et al 2007, Haakonsen 2012).

“Vetted”: X-ray spectra of Zavlin, Heinke together have been used in several dozen works.

$$F = 4\pi T_{eff,\infty}^4 \left( \frac{R_\infty}{D} \right)^2$$

$$R_\infty = \frac{R}{\sqrt{1 - \frac{2GM}{c^2 R}}}$$



## Instruments for measurements of qLMXBs

### Chandra X-ray Observatory

- Launched 1999 (NASA)
- 1" resolution



### XMM/Newton

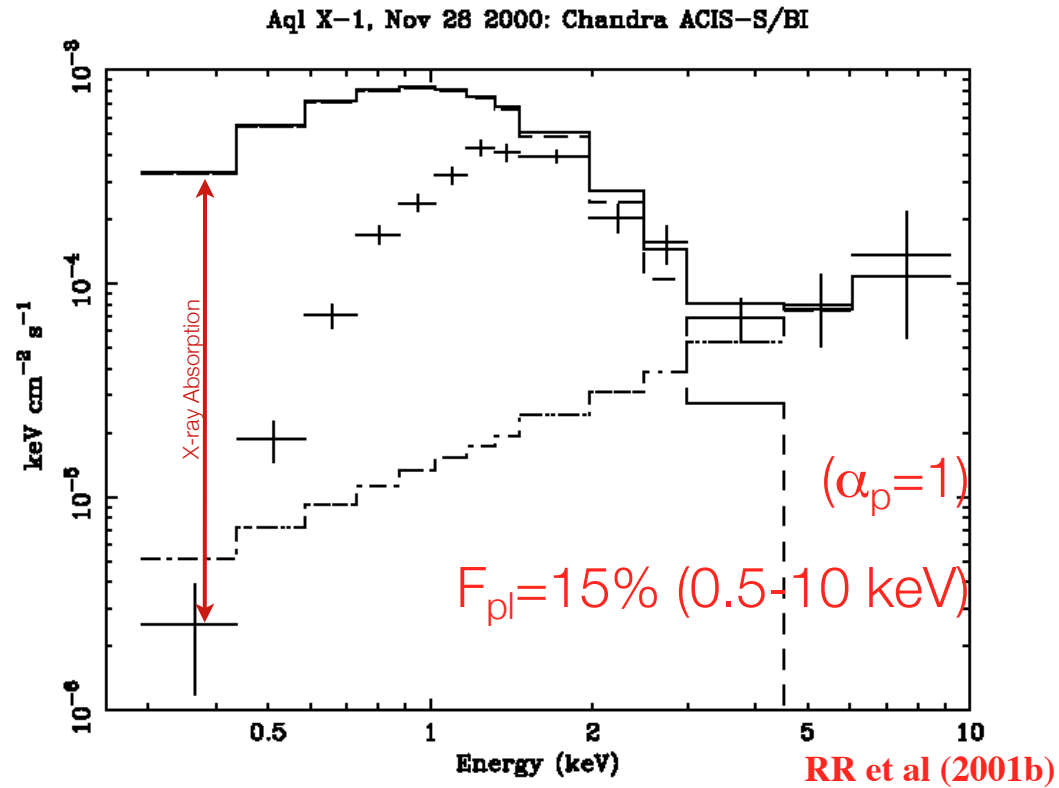
- Launched 1999 (ESA)
- 6" resolution
- ~4x area of Chandra.



Every photon is time tagged (~1 sec), with its energy measured ( $E/\Delta E = 10$ ) with full resolution imaging.



# Aql X-1 with Chandra -- Field Source



$R_\infty$  (d/5 kpc)

$13_{-4}^{+5} \text{ km}$

$kT_{\text{eff},\infty}$

$135_{-12}^{+18} \text{ eV}$

$N_{\text{H}}$

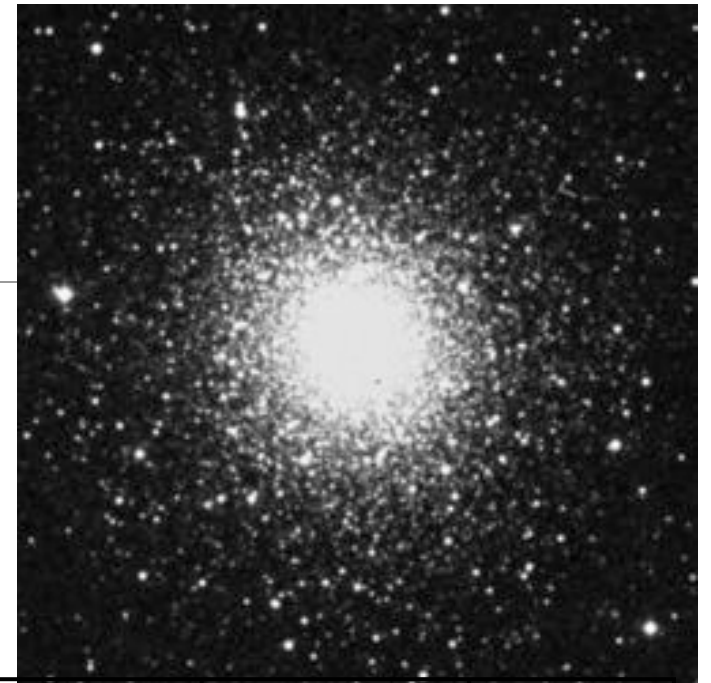
( $1e20 \text{ cm}^{-2}$ )

$35_{-7}^{+8}$

## The LMXB Factories: Globular Clusters

- GCs : overproduce LMXBs by 1000x vs. field stars
- Many have accurate distances measured.

**qLMXBs can be identified by their soft X-ray spectra, and confirmed with optical counterparts.**



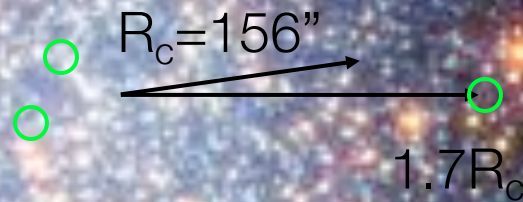
NGC	D (kpc)	+/- (%)
104	5.13	4
288	9.77	3
362	10.0	3
4590	11.22	3
5904	8.28	3
7099	9.46	2
6025	7.73	2
6341	8.79	3
6752	4.61	2

Carretta et al (2000)

# NGC 5139 (Omega Cen)

---

The identified optical counterpart demonstrates unequivocally the X-ray source is a qLMXB.

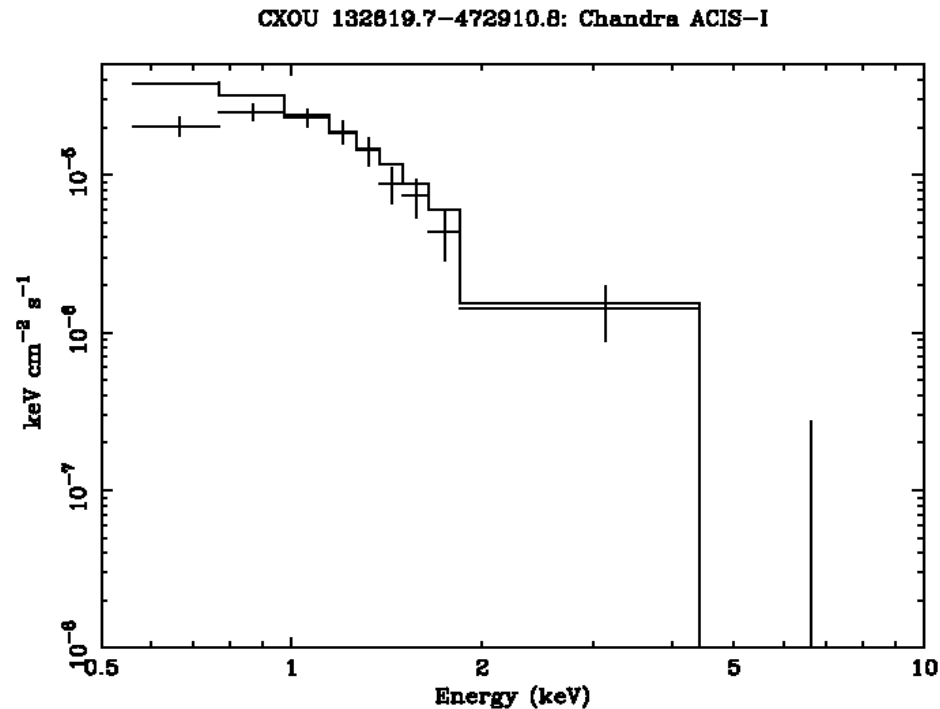


An X-ray source well outside the cluster core  
**Spitzer (Infrared)**

# NGC 5139 (Omega Cen)

X-ray Spectrum is inconsistent with any other type of known GC source (pulsars, CVs, coronal sources).

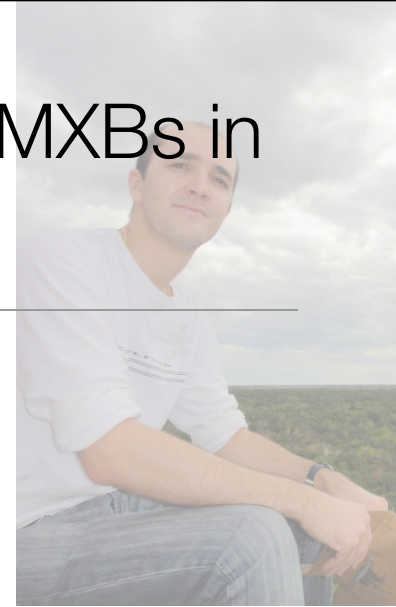
Full confirmation as LMXB requires Hubble photometry (which only exists for this 1 of our 5 sources).



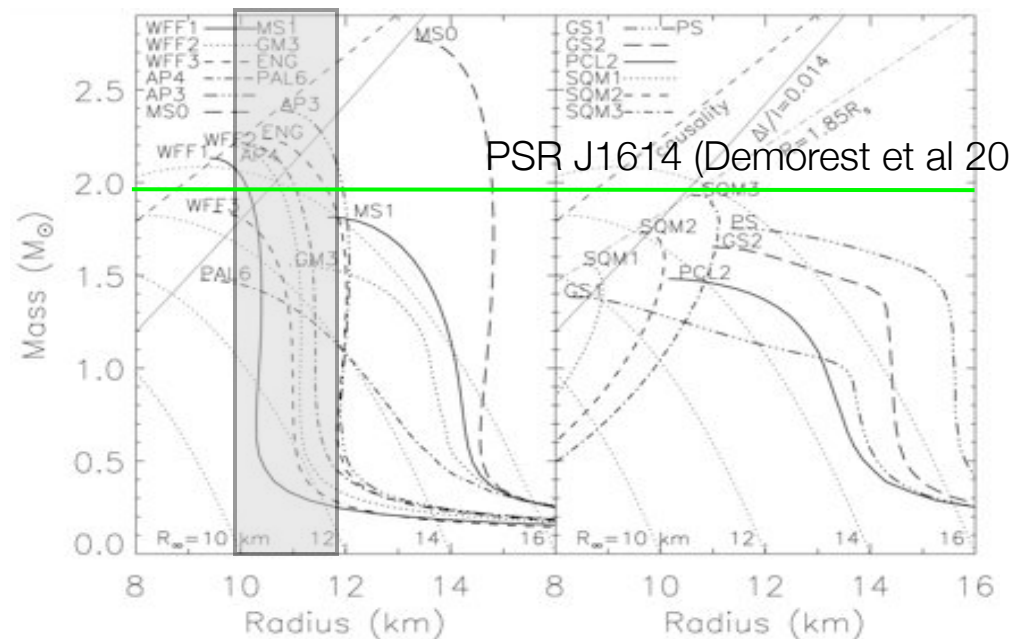
$R_{\infty}$ (d/5 kpc)	$kT_{\text{eff},\infty}$	$N_{\text{H}}$ (1e20 cm <sup>-2</sup> )
$14.3 \pm 2.1$ km	$66^{+4}_{-5}$ eV	(9)

**RR et al (2002)**

# Measuring the Radius of Neutron Stars from qLMXBs in Globular Clusters



- The 1.97(4) solar mass neutron star favors hadronic dEOSs over quark and phase-transition dEOSs. **These have the property of a quasi-constant neutron star radius.**
- Analysis goal: **Using all suitable qLMXB X-ray data sets of targets (there are five) provide the most reliable neutron star radius measurement possible.**
- Assume the radius of neutron stars is **quasi-constant** (a constant, at astrophysically important masses, within measurement error).
- Perform a Markoff-Chain-Monte-Carlo (MCMC) and include all known uncertainties and use conservative assumptions.



# Measuring the Radius of Neutron Stars from qLMXBs in Globular Clusters

---



# Assumptions -- the systematic uncertainties.

---

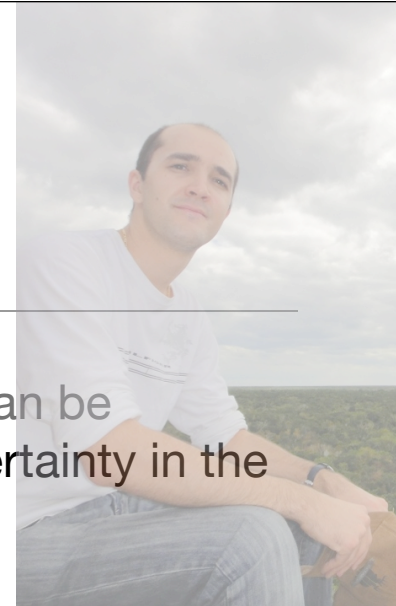
- **H atmosphere neutron stars.** (expected from a Hydrogen companion LMXB; can be proven through optical observations with Hubble, only done in one case, Omega Cen).
- **Low B-field ( $<10^{10}$  G) neutron stars.** (this is true for 'standard' LMXBs as a class, but difficult to prove on a case-by-case basis).
- **Emitting isotropically.** (comes naturally when powered by a hot core).

These assumptions reflect the best knowledge of these systems astronomy has in 2013.  
If you don't like these assumptions: "We find the assumptions not strongly supported  
and therefore ignore this result."

# Accounted-for Uncertainties

---

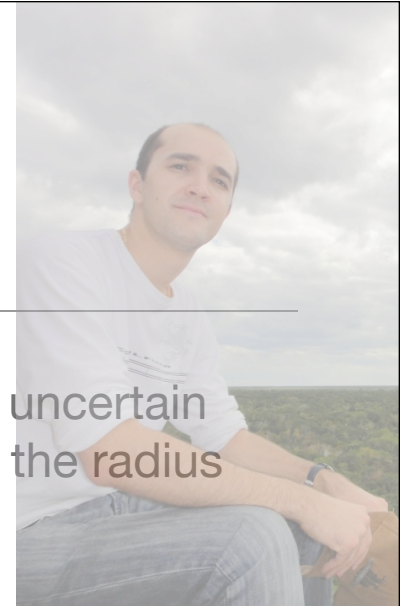
- In all previous works using qLMXB, the distance uncertainty -- which can be 2%-10% for each source -- has been neglected. Reflected in the uncertainty in the measured radius.
- X-ray absorption (due to the Hydrogen column density) is sometimes held fixed at radio-measured values, but is known to be systematically uncertain by x2, unless measured in the X-ray band. Reflected in the uncertainty in the measured radius.
- In some field sources (but no globular cluster sources) excess emission at high energies, not due to a H atmosphere, has been detected. Reflected in the uncertainty in the measured radius.
- Calibration uncertainty is included as a 3% intensity uncertainty.
- There are no remaining known quantified uncertainties.





# Distances

- We used more “uncertain” geometric distances over the systematically uncertain main-sequence fit distances and adopted these larger uncertainties for the radius uncertainty.



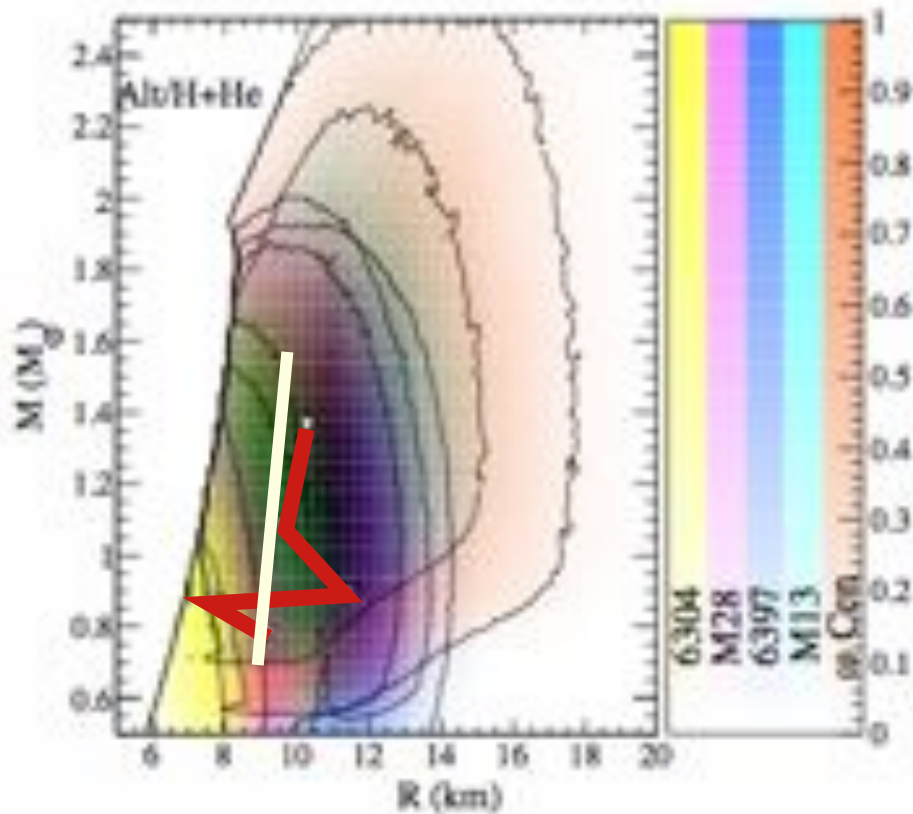
Name	$d_{GC}$ (kpc)	Method	$N_{H,22}$ (HI)	$N_{H,22}$ (X-ray)	Reference
M28	$5.5 \pm 0.3$	Horizontal Branch fitting	0.24	$0.256^{+0.024}_{-0.024}$	Testa et al. (2001)
NGC 6397	$2.02 \pm 0.18$	Dynamical	0.14	$0.096^{+0.017}_{-0.014}$	Rees (1996)
M13	$6.5 \pm 0.6$	Dynamical	0.011	$0.008^{+0.003}_{-0.008}$	Rees (1996)
$\omega$ Cen	$4.8 \pm 0.3$	Dynamical	0.09	$0.182^{+0.043}_{-0.042}$	van de Ven et al. (2006)
NGC 6304	$6.22 \pm 0.26$	Horizontal Branch fitting	0.266	$0.346^{+0.103}_{-0.084}$	Recio-Blanco et al. (2005)

# The major innovation of Guillot et al (2013) is statistical.

---

- All work to date, in combining spectral fits, fit each source individually, then combined the best fit  $M$  and  $R$  afterwards, with error regions.
- Guillot et al (2013) required  $R$  to be the same for all sources.
- This “quasi-constant Radius” should be thought of as a simplified parametric model which can be compared to realistic EoSs.
- The result is an improvement in  $S/N$  over previous work which (for example) would use 5 sources independently, (approximately) as if we had 25 sources.
- A simplified explanation.....

# “Alt/H+He” (LSI 3)



- Draw your best Neutron Star M-R relationship in your head. Ready?



# “Joint Fits” - the major difference from previous work

$$\frac{\partial \chi^2}{\partial(p1)\partial(p2)} = \begin{array}{c} \text{Source 1} \\ \hline \begin{array}{cccc} \mathbf{R} & \mathbf{M}_1 & \mathbf{T}_1 & \mathbf{N}_{H,1} \end{array} \\ \hline \begin{array}{cccc} \frac{1}{\sigma_R^2} & a_{1,2} & a_{1,3} & a_{1,4} \\ a_{2,1} & \frac{1}{\sigma_{M_1}^2} & a_{2,3} & a_{2,4} \\ a_{3,1} & a_{3,2} & \frac{1}{\sigma_{T_1}^2} & a_{3,4} \\ a_{4,1} & a_{4,2} & a_{4,3} & \frac{1}{\sigma_{N_{H1}}^2} \\ \frac{1}{\sigma_R^2} & a_{5,2} & a_{5,3} & a_{5,4} \\ a_{6,1} & \frac{1}{\sigma_{M_2, M_1}^2} & a_{6,3} & a_{6,4} \\ a_{7,1} & a_{7,2} & \frac{1}{\sigma_{T_2, T_1}^2} & a_{7,4} \\ a_{8,1} & a_{8,2} & a_{8,3} & \frac{1}{\sigma_{N_{H2}, N_{H1}}^2} \end{array} \\ \text{Source 2} \\ \hline \begin{array}{cccc} \mathbf{R} & \mathbf{M}_2 & \mathbf{T}_2 & \mathbf{N}_{H,2} \end{array} \\ \hline \begin{array}{cccc} \frac{1}{\sigma_R^2} & a_{1,6} & a_{1,7} & a_{1,8} \\ a_{2,5} & \frac{1}{\sigma_{M_1, M_2}^2} & a_{2,7} & a_{2,8} \\ a_{3,5} & a_{3,6} & \frac{1}{\sigma_{T_1, T_2}^2} & a_{3,8} \\ a_{4,5} & a_{4,6} & a_{4,7} & \frac{1}{\sigma_{N_{H1}, N_{H2}}^2} \\ \frac{1}{\sigma_R^2} & a_{5,6} & a_{5,7} & a_{5,8} \\ a_{6,5} & \frac{1}{\sigma_{M_2}^2} & a_{6,7} & a_{6,8} \\ a_{7,5} & a_{7,6} & \frac{1}{\sigma_{T_2}^2} & a_{7,8} \\ a_{8,5} & a_{8,6} & a_{8,7} & \frac{1}{\sigma_{N_{H2}}^2} \end{array} \\ \begin{array}{l} \mathbf{R}_1 \\ \mathbf{M}_1 \\ \mathbf{T}_1 \\ \mathbf{N}_{H,1} \\ \mathbf{R}_2 \\ \mathbf{M}_2 \\ \mathbf{T}_2 \\ \mathbf{N}_{H,2} \end{array} \end{array}$$

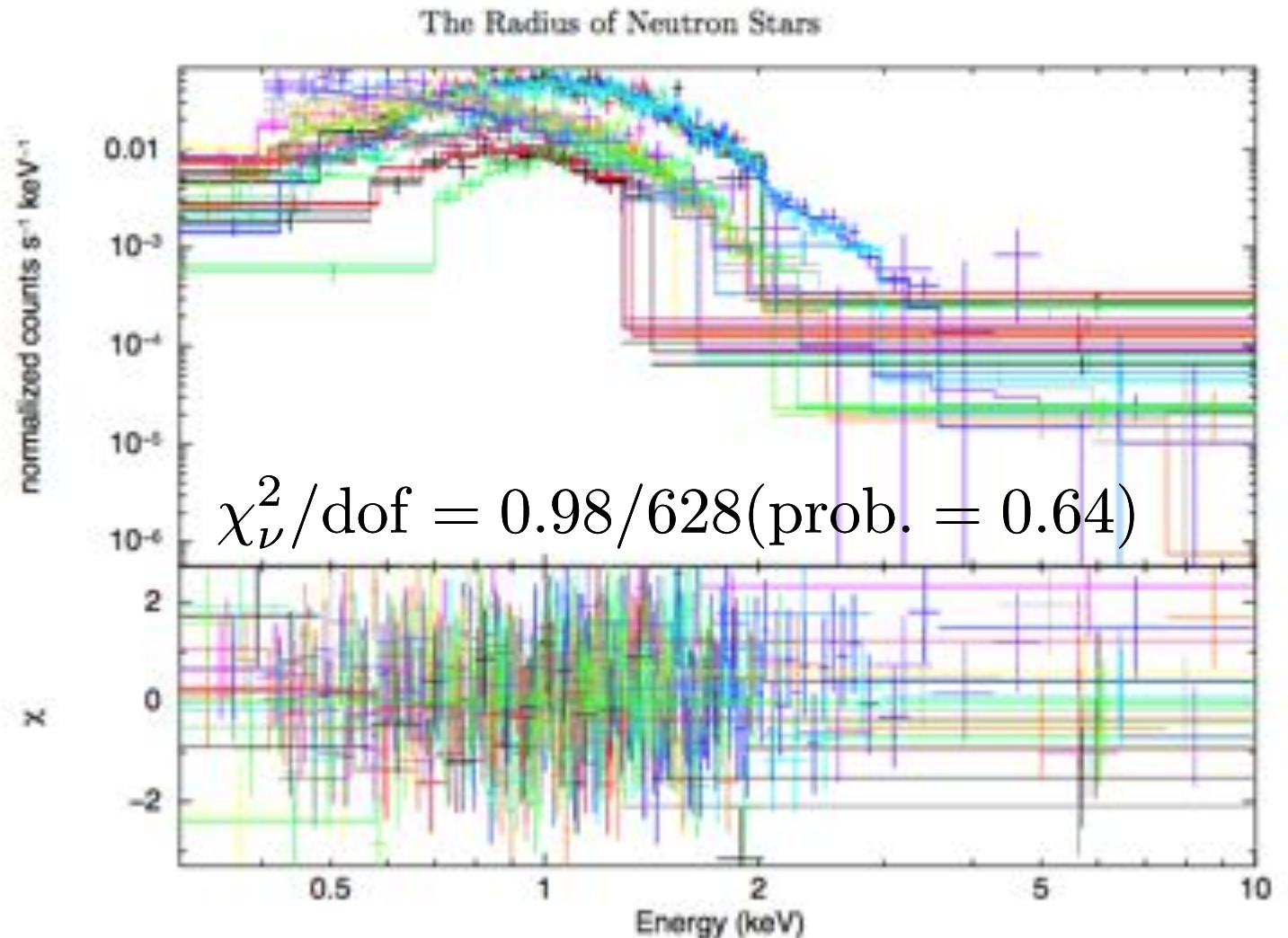
Every parameter (M, R, T, N\_H) of all five sources affects every other parameter of every source

$$\frac{S}{N} \propto \frac{N^2(\# \text{ of Source Matrix Elements})}{N(\# \text{ of sources})} = N$$

In comparison to using the sources “independently”, its as if we have 25 sources, instead of 5 sources.

# Best H atmosphere (+ PL) spectral fit of all 5 qLMXBs

- This model is a statistically acceptable fit to the X-ray spectral data. This is an a posteriori confirmation that the data are consistent with our assumptions.
- After finding the best fit a MCMC method was used to find the uncertainty regions for all parameters - -- the Radius, Mass, Temperature, absorption, distance, and power-law normalization.



# Nh, d fixed, no PL -- Run #1

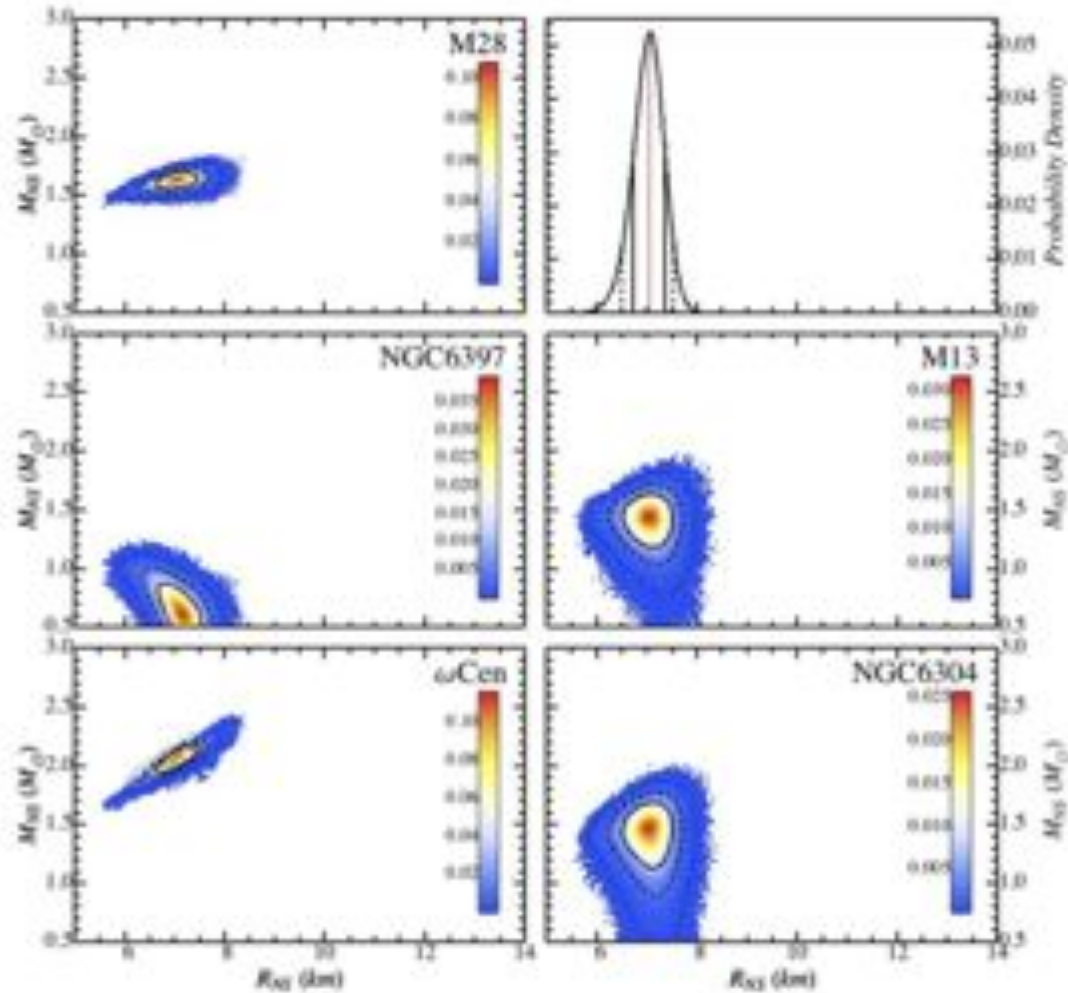


FIG. 9.— Figure showing the marginalized posterior distribution in  $M_{NG}$ - $R_{NG}$  space for the five qLMXBs, in the first MCMC run, when the distance and the hydrogen column density  $N_H$  are fixed and when no PL component is added, corresponding to Run #1. The 1D and 2D posterior probability distributions are normalized to unity. The color scale in the 2D distributions represents the probability density in each bin. The 80%, 90% and 99% confidence contours are shown with solid, dashed and dotted lines in the  $M_{NG}$ - $R_{NG}$  density plots, respectively. The top-right graph is the resulting normalized probability distribution of  $R_{NG}$ , common to the five qLMXBs, with the 80%, 90% and 99% confidence regions represented by the solid, dashed and dotted vertical lines. The radius value is shown by the red line. The measured radius is  $R_{NG} = 7.1^{+1.4}_{-1.2}$  km (90% confidence).

# Nh fixed, d with gaussian bayesian prior, no PL -- Run #2

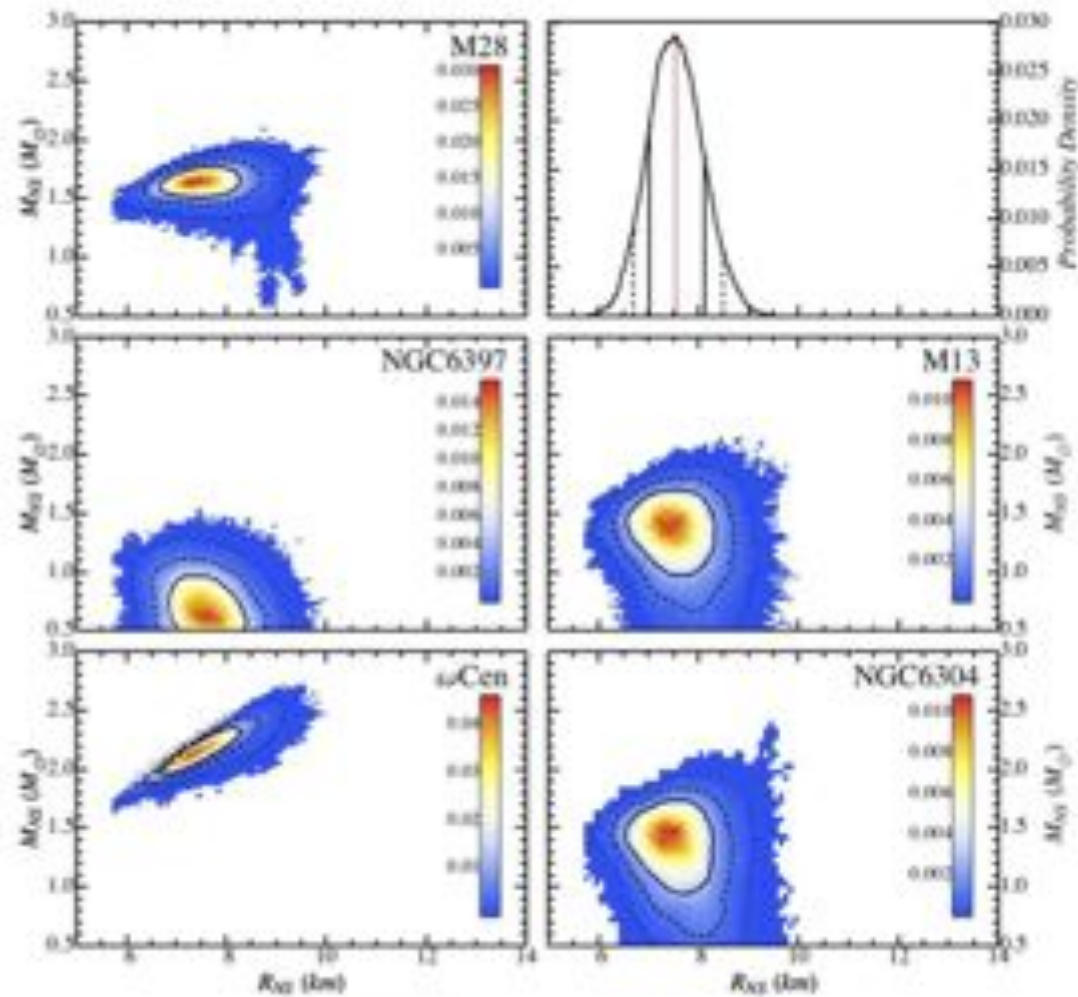


FIG. 10.— Figure similar to the previous one, Fig. 9 but for the MCMC Run #2, where Gaussian Bayesian priors were used for the distances to the five qLMXBs (see Table 3). The resulting radius measurement is  $R_{\text{Gal}} = 7.6^{+0.7}_{-0.4}$  kpc.



# Nh Fixed, D fixed, PL added

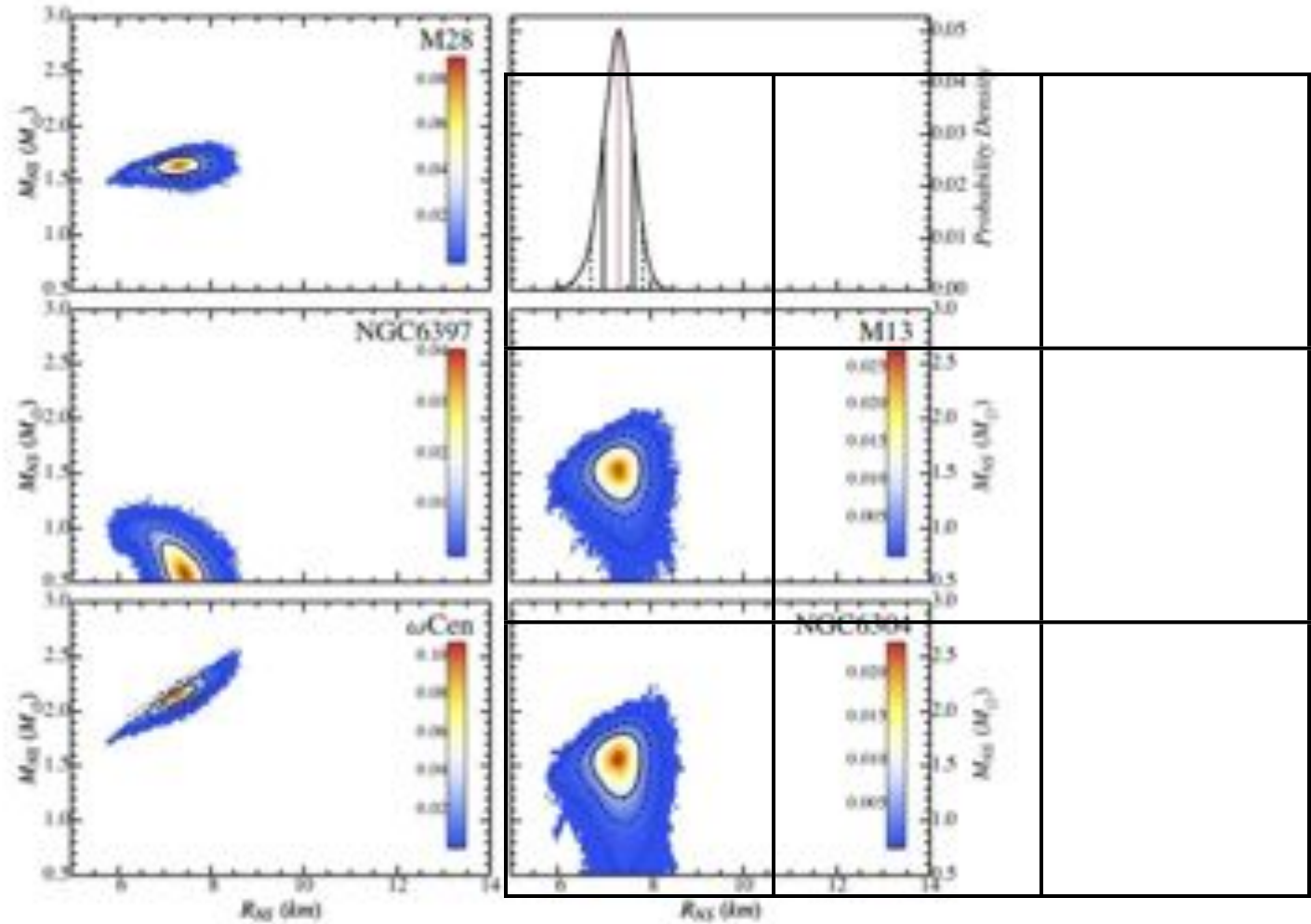


FIG. 11.— Figure similar to Figure 8 corresponding to the results of Run #3. In this run, the datasets are fixed (no priors included), but a PL component (with fixed index  $\Gamma = 1$ ) is added to the spectral model, leading to  $R_{\text{gal}} = 7.3^{+0.7}_{-1.4}$  km.

NH Fixed, Distance with gaussian bayesian priors, PL added

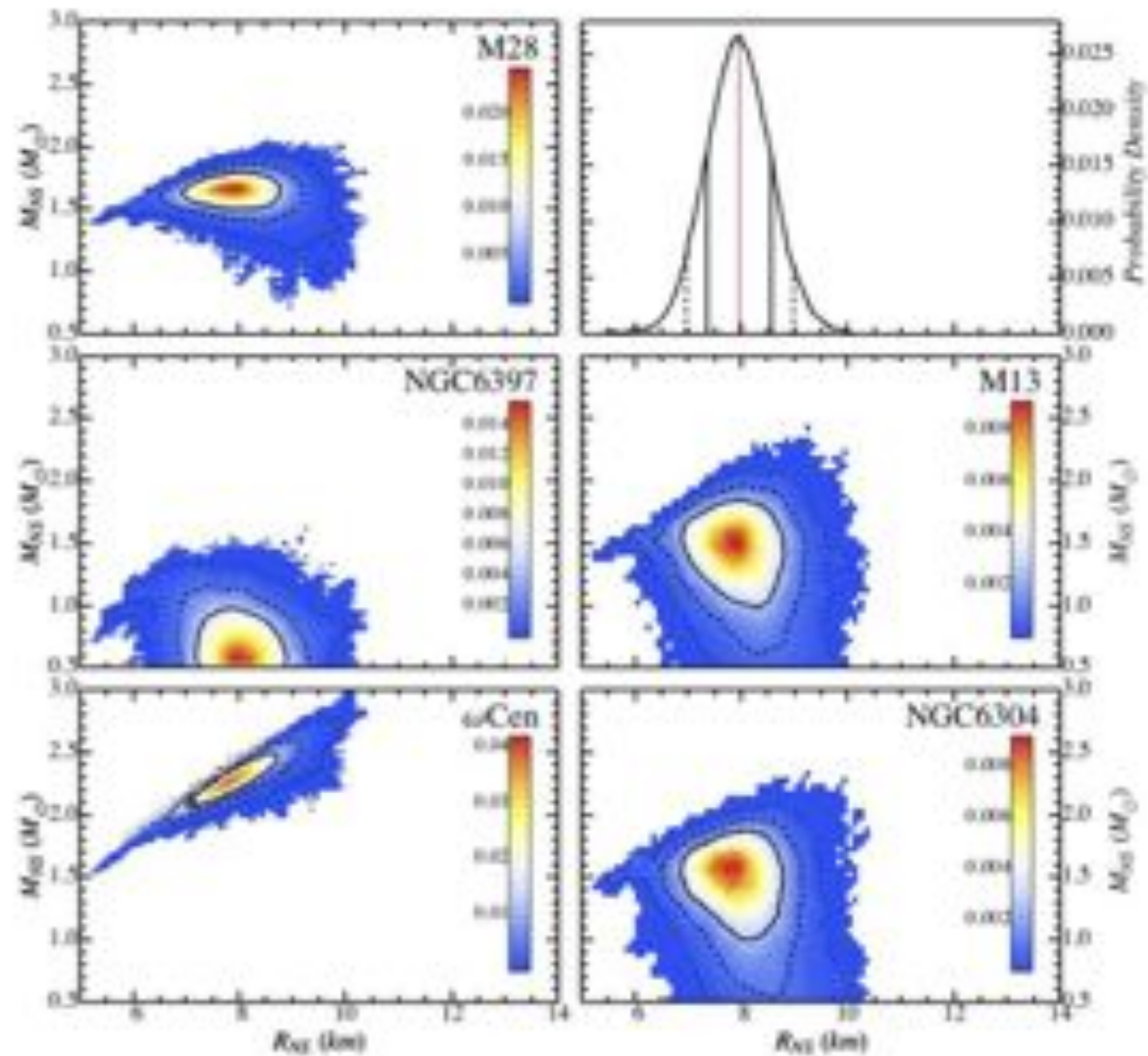


FIG. 12.— Figures similar to Figure 8 but for Run #4. Here, the Gaussian Bayesian priors on the distances are included, as well as the PL spectral component, with  $N_{PL}$  held fixed. The resulting NS radius is  $R_{NS} = 8.0^{+1.0}_{-1.5}$  km.

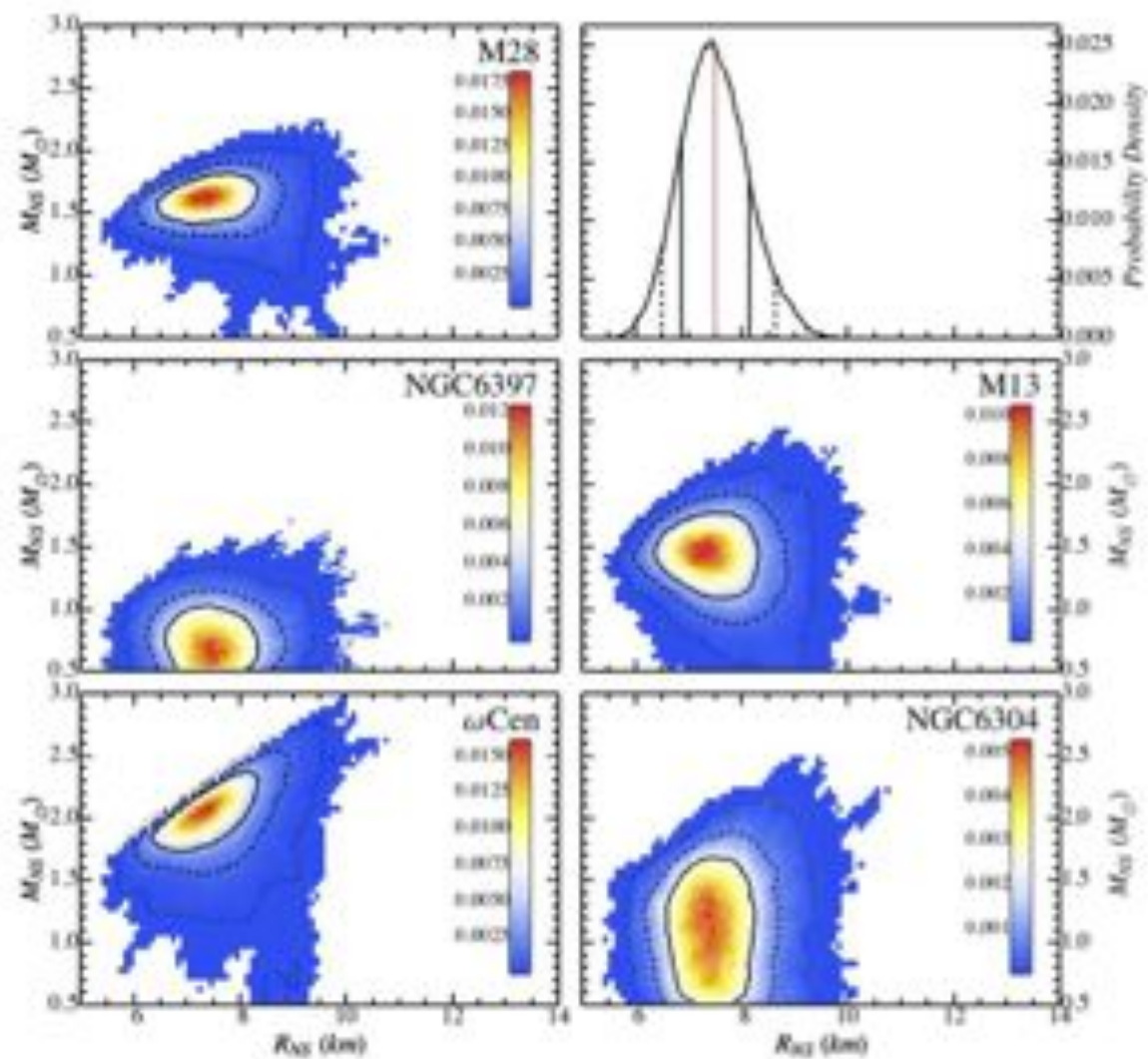


FIG. 13.— Figures similar to Figure 8 showing results of Run #5. The hydrogen column density  $N_H$  is left free in this run, but the distances remained fixed, and no PL component was added. This run produced  $R_{52} = 7.5^{+1.1}_{-1.2}$  km.

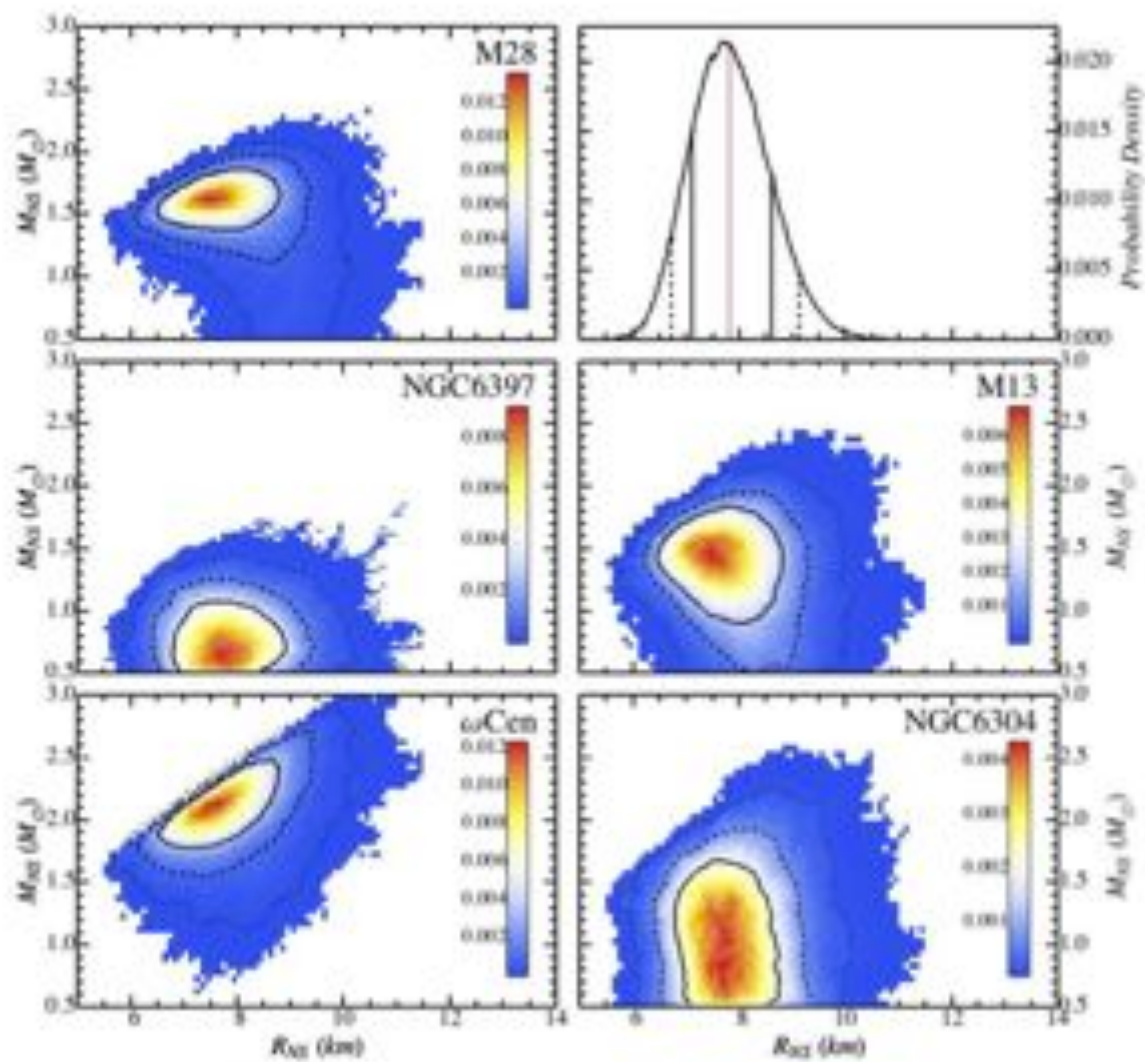
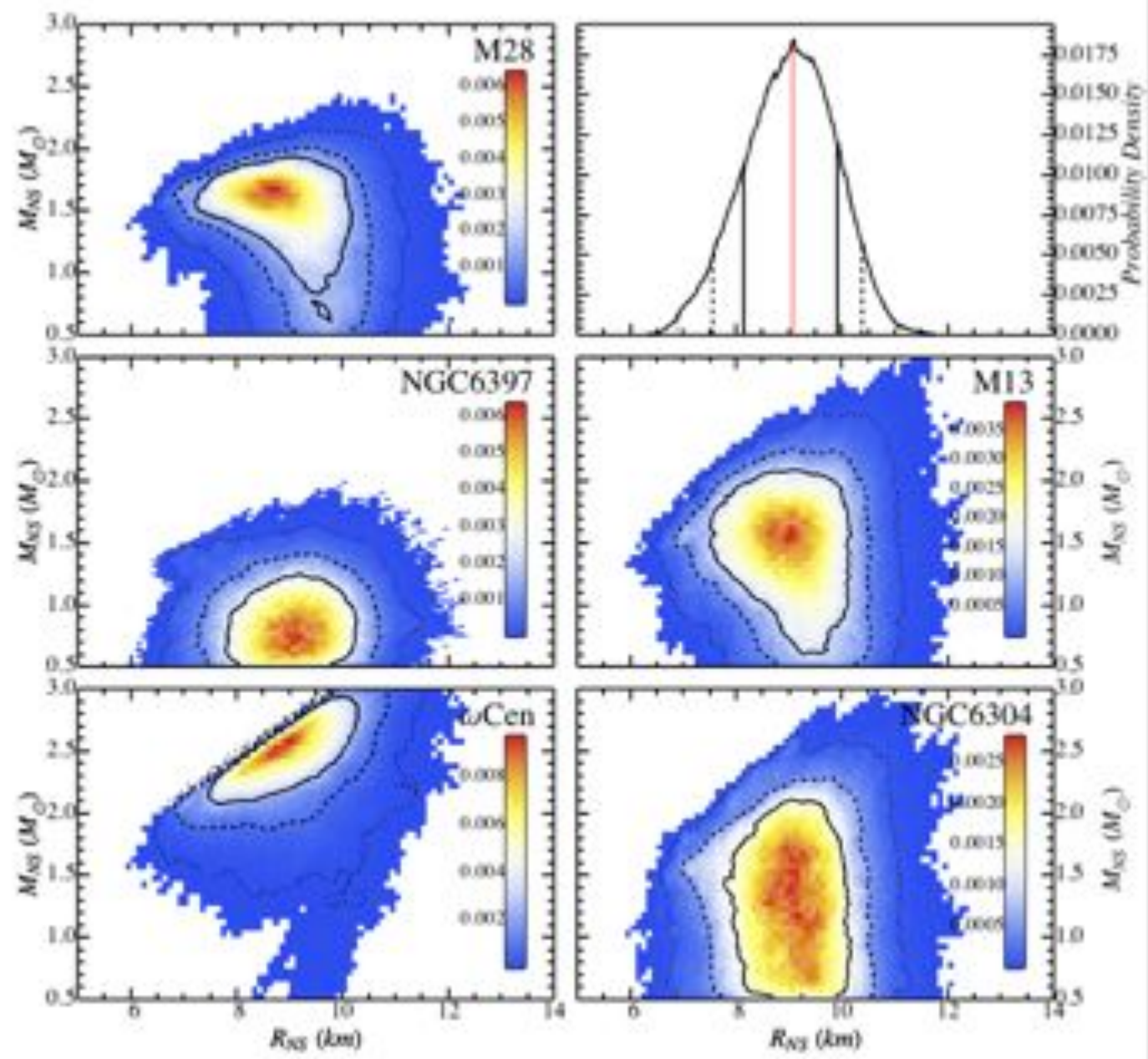
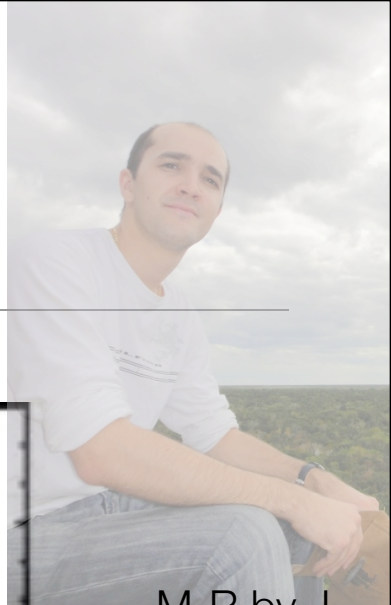
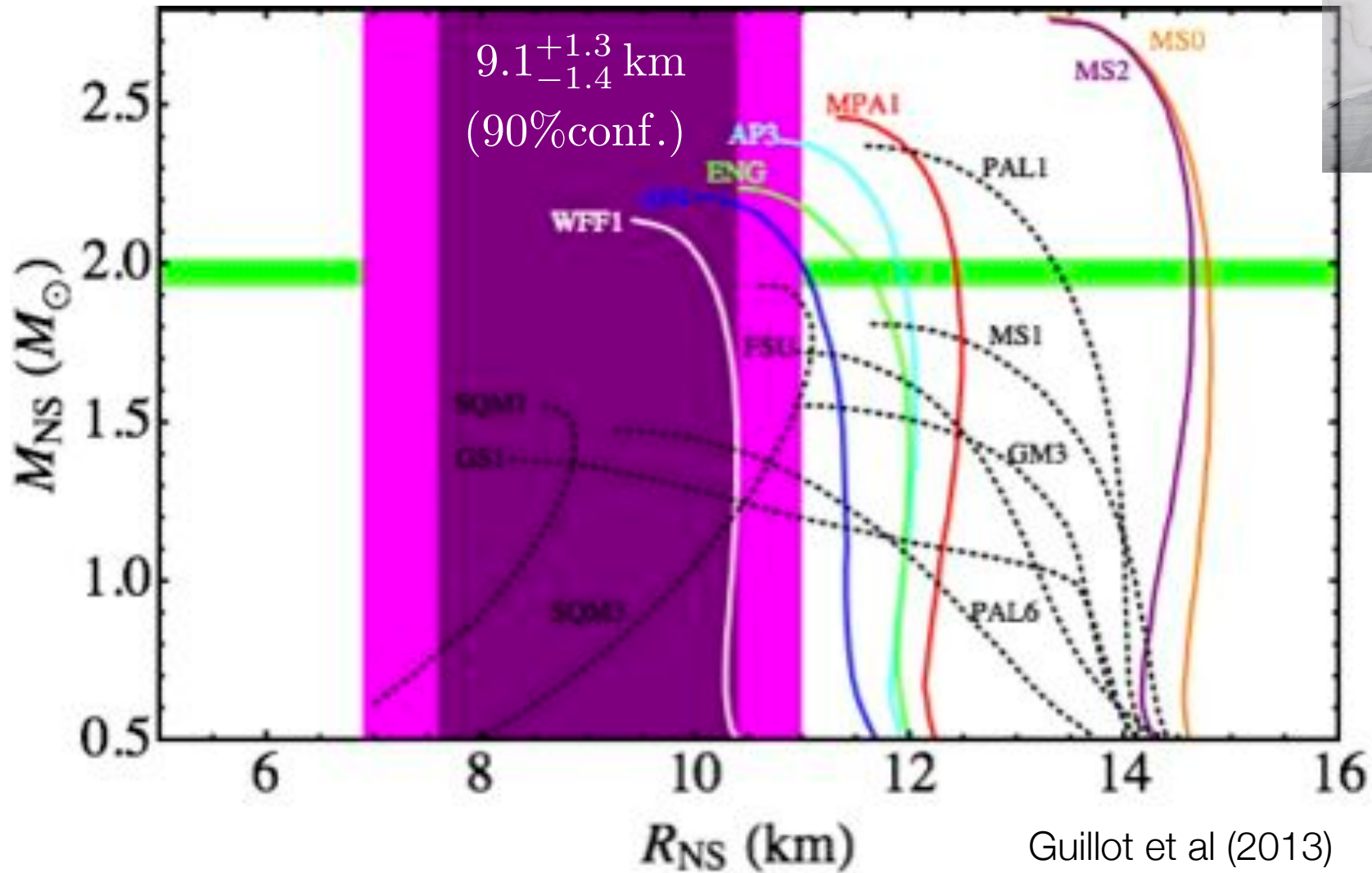


FIG. 14.— Figure similar to Figure 13 but for the MCMC Run #6. The characteristics of this run include  $N_{\text{it}}$  values free to vary in the fit, and the presence of the Gaussian Bayesian priors for the distances. No PL component was included in run. A value  $R_{500} = 7.8^{+1.7}_{-1.7}$  km was found.



# The Neutron Star Radius

<11 km (99% conf).



M-R by J. Lattimer

WFF1=  
Wiring, Fiks  
and Fabrocini  
(1988)

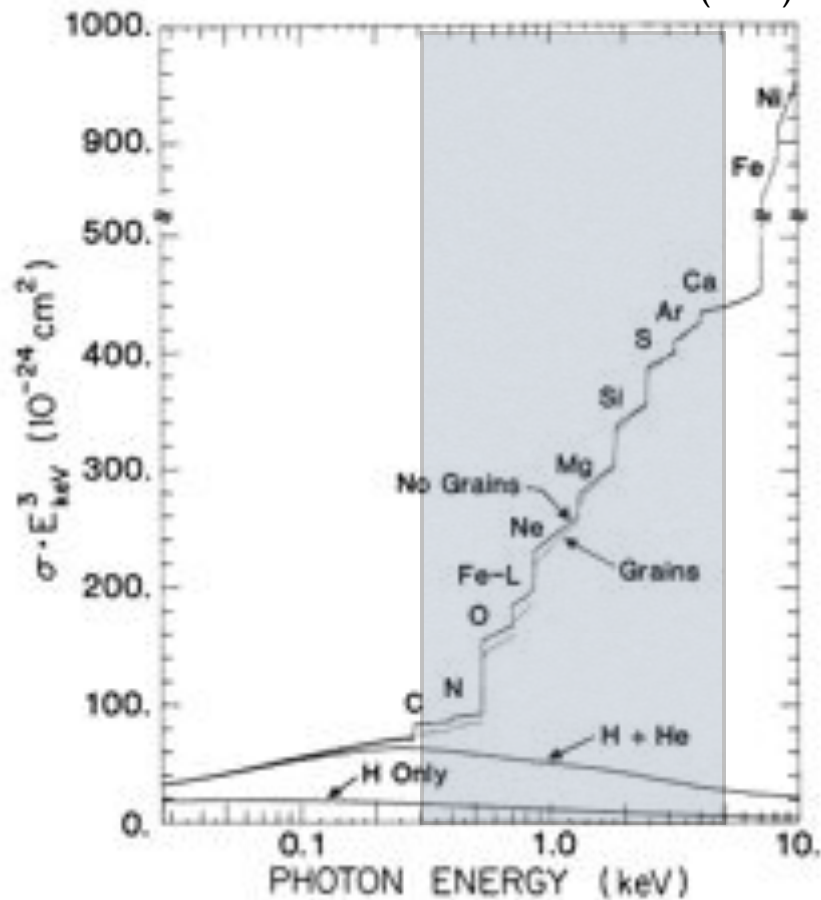
Contains  
uncertainties from:  
Distance  
All spectral  
parameters  
Calibration

# What I would like nuclear physicists to do.

- Take our value of RNS. Take the  $M_{\min} > 1.97 \pm 0.04$  value (Demorest et al 2010). Produce a viable EOS to ....

# X-ray Absorption and the neutral Hydrogen column density ( $N_H$ )

Morrison & McCammon (1983)



- H is what LS13 believes X-ray absorption is due to. So they take the 21-cm line emission measurements from Dickey & Lockman (1990) and assume that the exact value is the correct one.
- In fact, the absorption in X-ray is due to a series of absorption edges in atoms (mostly, O and Fe). There exists no other way by which atomic O and Fe can be measured in the inter-stellar medium than looking for the effects of absorption in the X-ray band. Also, ~20% of the ISM is ionized, and also molecular  $H_2$  (neither contribute 21-cm line, but both increase O +Fe column density).
- Standard procedure I: For low signal to noise data, assume Dickey & Lockman measured 21 cm-line value for  $N_H$ ; also, assume a “cosmic abundance” of metals.
- Standard procedure II: For high Signal-to-noise data you allow  $N_H$  to “float” as a parameter, and it will find the “equivalent  $N_H$ ”, the amount of  $N_H$  needed to produce the amount of absorbing O and Fe observed.
- See Wilms et al (2000), Willingale et al (2013).



# Mass Measurements with Continuum Spectra

$$R_\infty \rightarrow$$

Normalization

$$T_\infty \rightarrow$$

Peak of the spectrum

$$z = \frac{GM_{NS}}{c^2 R_{NS}} \rightarrow \text{Second Derivative at the Peak of the Spectrum}$$

$$I(E_\gamma) \propto \left(\frac{E_\gamma}{kT}\right)^3 \frac{1}{e^{\frac{E_\gamma}{kT}} - 1}$$



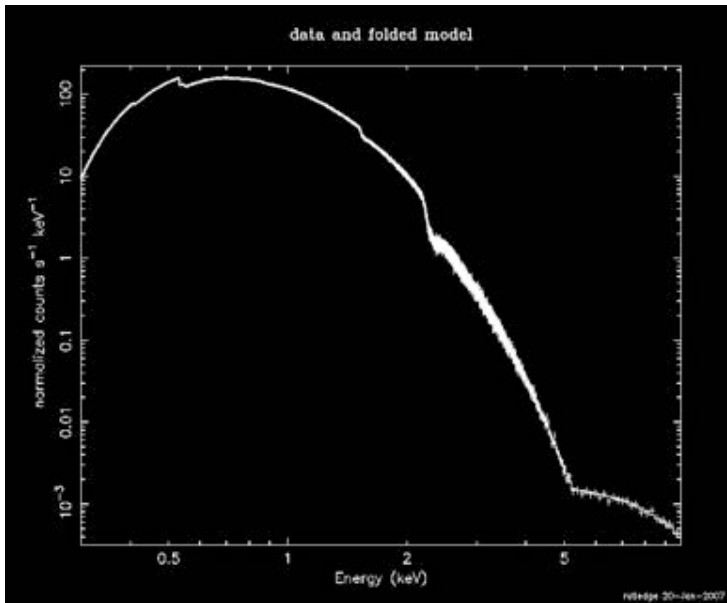
$$\left(R_\infty = \frac{R_{NS}}{\sqrt{1 - \frac{2GM_{NS}}{c^2 R_{NS}}}}, z = \frac{GM_{NS}}{c^2 R_{NS}}\right) \rightarrow (R_{NS}, M_{NS})$$

$$I(E_\gamma) \propto \left(\frac{E_\gamma}{kT}\right)^3 \frac{1}{e^{\frac{E_\gamma}{kT}} - 1} K_{ff,o} \left(\frac{E_o}{E_\gamma}\right)^3$$



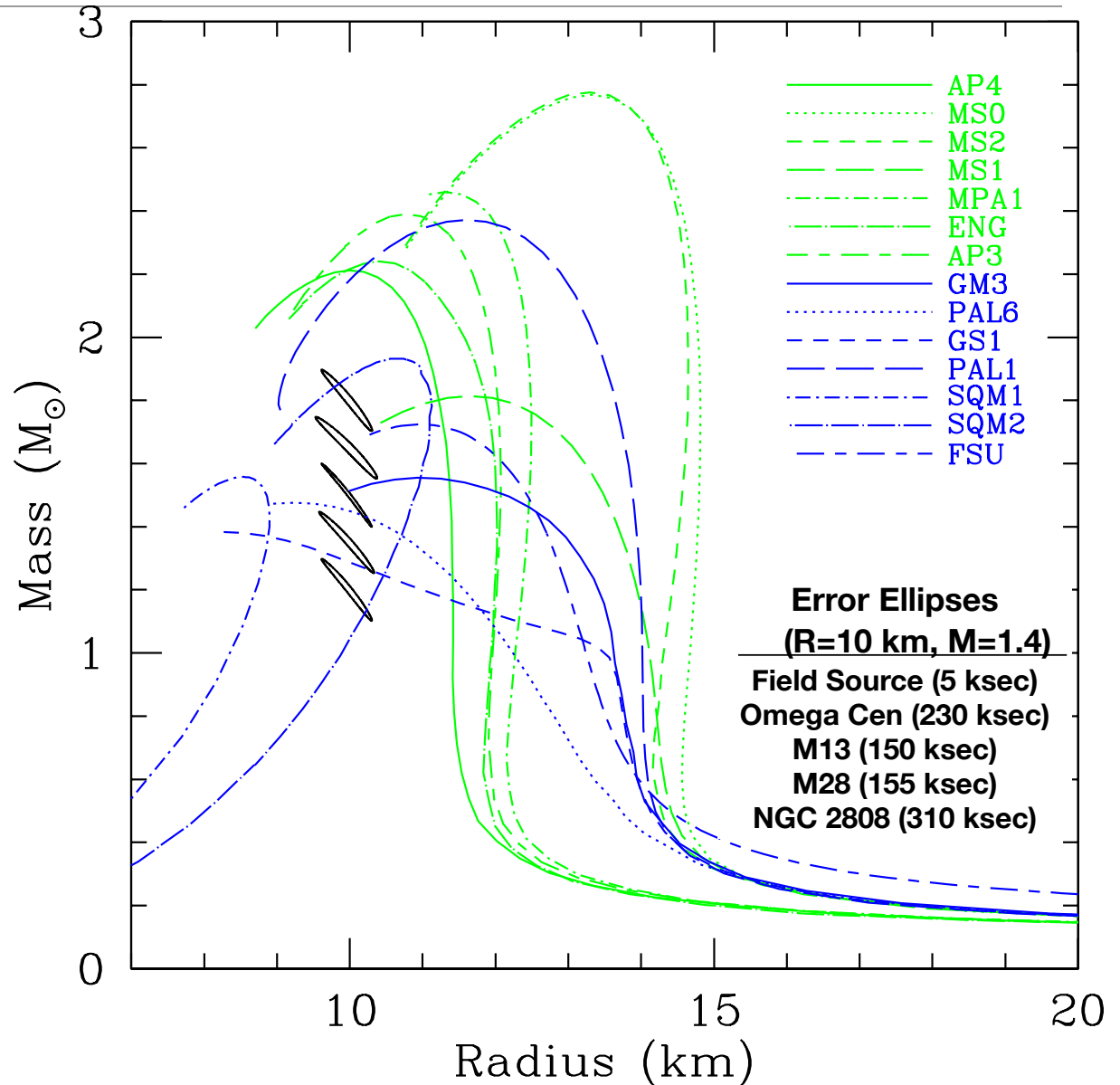
# Calorimeter response curves

## Simultaneous Mass and Radius Measurement



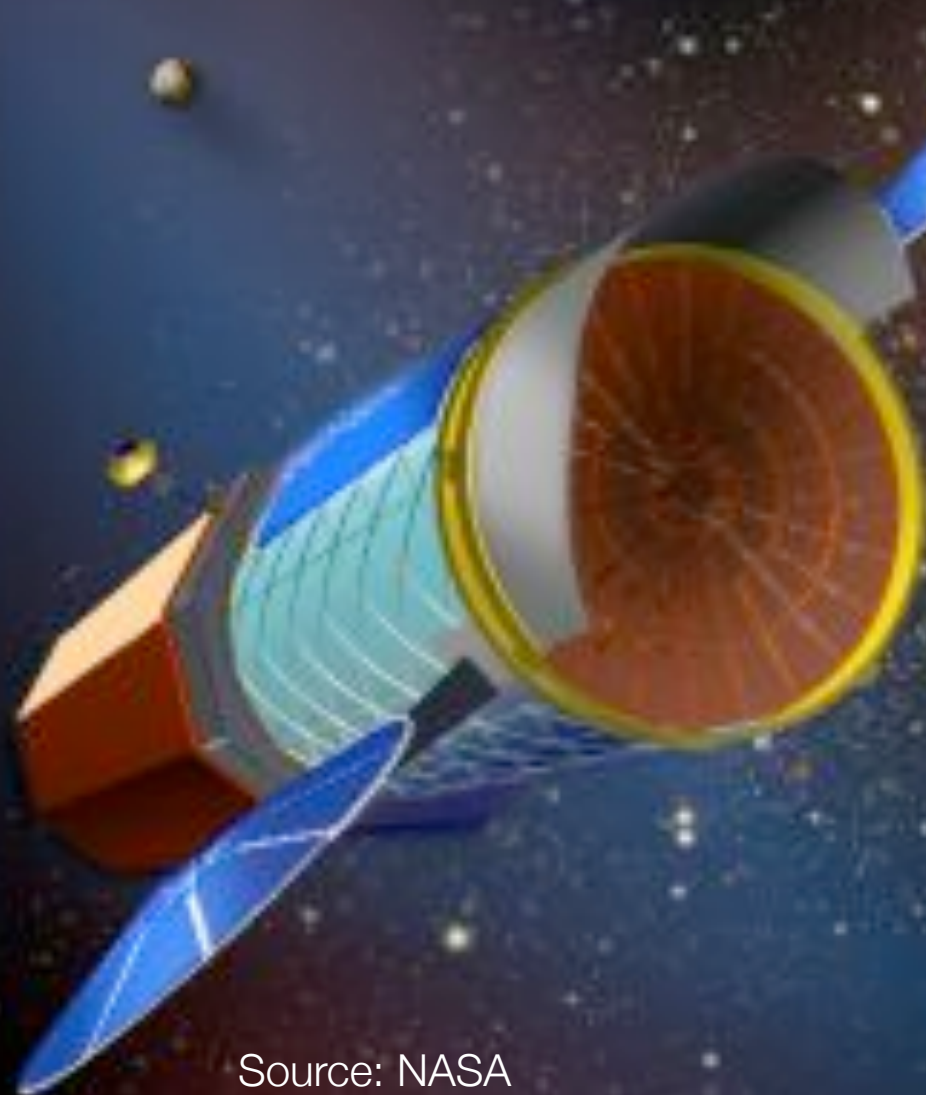
Requirement: 250k-300k counts with **calorimeter** (2.5 eV) energy resolution.

(At XMM/pn-like resolution: 1M -2M counts)



CANCELLED

# The Proposed International X-ray Observatory

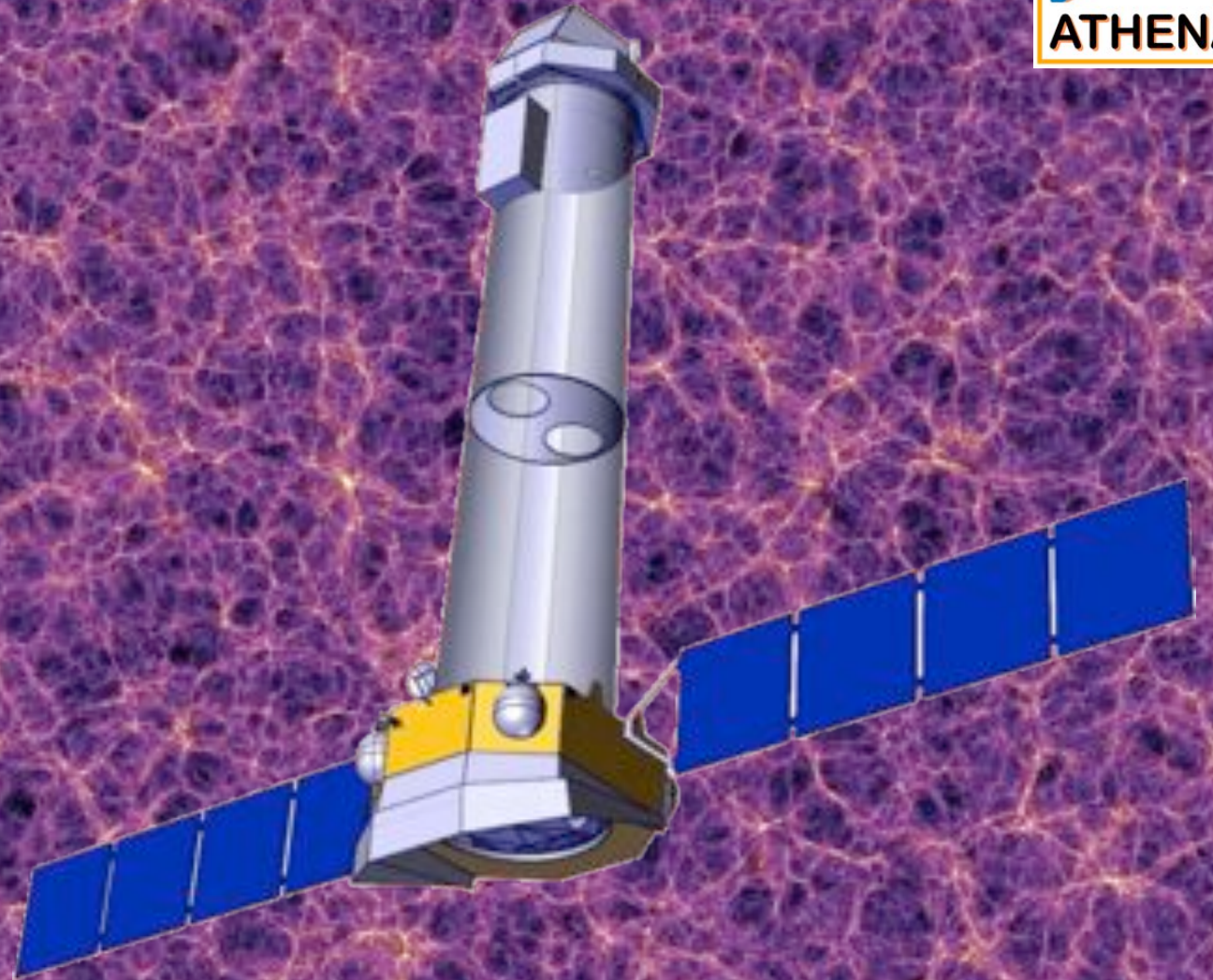


- Science Objectives
  - Matter Under Extreme Conditions
  - Neutron Star Equation of State
- NASA, ESA and JAXA proposal
- Basic specifications
  - Area  $\approx 1 \text{ m}^2$  @ 125 keV
  - Galaxy Cluster Evolution
  - 150 mDy (Dark Energy)
  - Energy resolution  $\approx 25 \text{ eV}$  @ 0.5-2 keV
- Spatial Resolution  $\approx 15 \text{ arcsec}$  half-power diameter  
Observatory”, White, Parmar, Kunieda, Nandra, Ohashi and Bookbinder.  
Launch: 2021

Source: NASA

# Athena+: Revealing the Hot and Energetic Universe

A proposed observatory



Launch: 2028

Kirpal Nandra  
On behalf of The Athena Co-ordination Group

# Science Requirements

	Requirement	Driver
Effective Area	2m <sup>2</sup> @ 1 keV (goal 2.5m <sup>2</sup> ) 0.25m <sup>2</sup> @ 6 keV (goal 0.3m <sup>2</sup> )	Hot Baryons Black hole evolution Accretion Physics
Angular Resolution	5'' (goal of 3'')	Black Hole Evolution Hot Baryons
Fields of view	WFI: 40' diameter (goal 50') XMS: 5' x 5' (goal 7' x 7')	Hot Baryons Black Hole Evolution
Spectral resolution	150 eV @ 6 keV (WFI) 2.5 eV (X-IFU) goal 1.5 eV	Black Hole Evolution Hot Baryons
Count rate capability	>1 Crab	Accretion Physics
Timing resolution	50 μs	Accretion Physics
TOO response	8 hours (2 hours goal)	Hot Baryons

Credit: K. Nandra

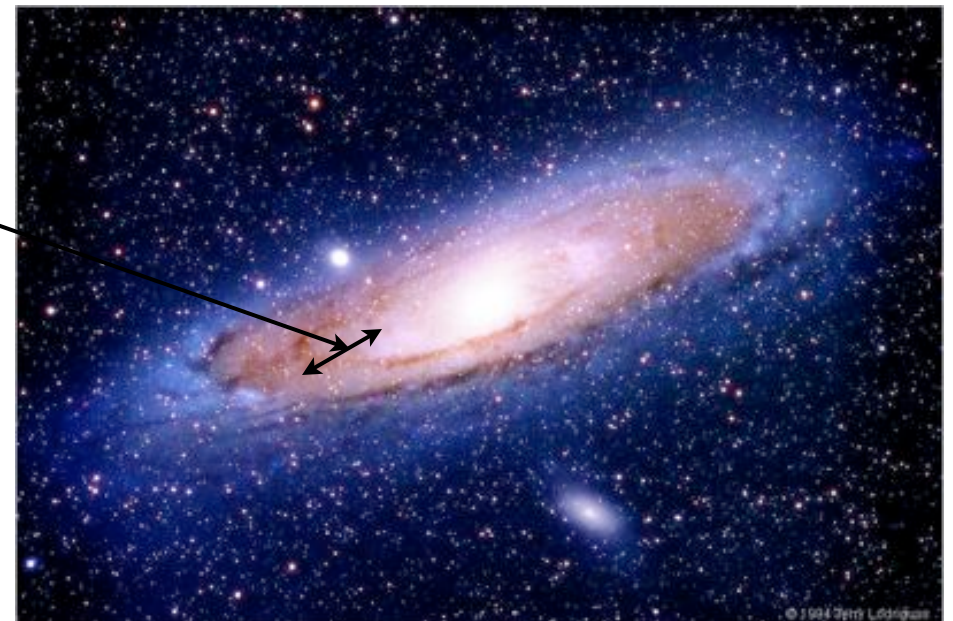
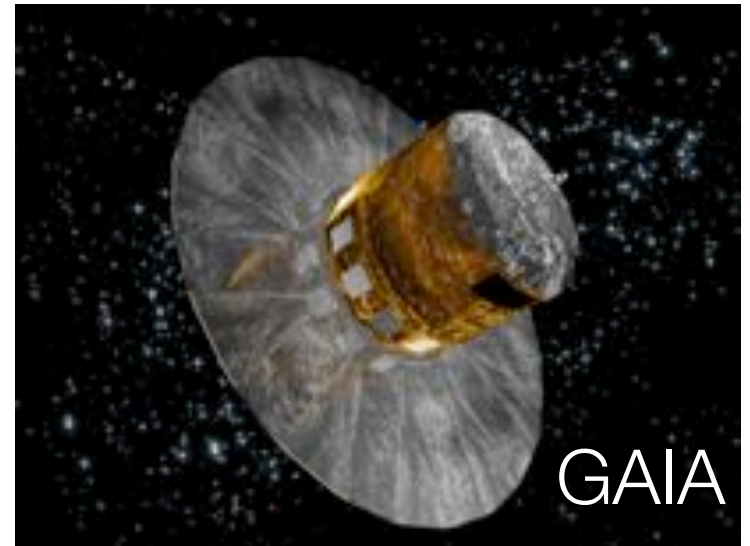


# Measuring Distances: GAIA Mission Capabilities

- An European Space Agency Cornerstone Mission, with a launch (to L2) THIS year.

V	# (millions)	$\sigma_{\mu}$ -arcsec	3% Distance (kpc)
10	0.34	7	4.2
15	26	22	1.4
20	1000	250	0.12

Are there enough qLMXBs within this distance?



# LSI 3 (submitted)

## Neutron Star Masses and Radii from Quiescent Low-Mass X-ray Binaries

James M. Lattimer

*Department of Physics and Astronomy, State University of New York at Stony Brook, Stony Brook, NY  
11794-3800, USA*

`james.lattimer@stonybrook.edu`

Andrew W. Steiner

*Institute for Nuclear Theory, University of Washington, Seattle, WA 98195, USA*

`steiner3@uw.edu`

### ABSTRACT

A recent analysis (Guillot et al. 2013) of the thermal spectra of 5 quiescent low-mass X-ray binaries in globular clusters, in which it was assumed that all neutron stars have the same radius, determined the radius to be  $R = 9.1^{+1.3}_{-1.5}$  km to 90% confidence. However, the masses of the sources were found to range from  $0.86 M_{\odot}$  to  $2.4 M_{\odot}$ , and a significant amount of the predicted  $M - R$  region violates causality and the existence of a 2 solar mass neutron star. The study determined the amount of Galactic absorption along the lines-of-sight from fitting the X-ray spectra and assumed all sources possessed hydrogen atmospheres. We argue, from a Bayesian analysis, that different interpretations of the data are strongly favored. Our most-favored model assumes i) the equation of state of neutron star crusts is well-understood, ii) the high-density equation of state is consistent with causality and the existence of neutron stars at least as massive as  $2 M_{\odot}$ , iii) that the Galactic absorption is determined either from the fits in Guillot et al. (2013) or from independent HI surveys, and iv) that these objects are well-described by either hydrogen or helium atmospheres. With these assumptions, the 90% confidence radius range for  $1.4 M_{\odot}$  stars is 11.4 to 12.8 km, and the allowed range for radii of all neutron stars between  $1.2 M_{\odot}$  and  $2.0 M_{\odot}$  is 10.9 to 12.7 km. This result is in much greater agreement with predictions of the equation of state from both nuclear experiments and theoretical neutron matter studies than the smaller radii deduced by Guillot et al. (2013).

rXiv:1305.3242v1 [astro-ph.HE] 14 May 2013

# LSI 3:

## How it should be done.

- Download all X-ray data from the NASA Archive [heasarc.gsfc.nasa.gov](http://heasarc.gsfc.nasa.gov). All observations are freely available, as are all standard analysis tools. This is done 1000s of times every year by astronomers and results in 1000s of papers annually. That said, it is not idiot-proof.
- Extract X-ray photon spectra from each source. (Hereafter: “Data” means “X-ray photon spectra”, and nothing else.)
- Perform a (for example) Chi-square minimization (or other figure of merit) comparison between the proposed photon spectral model and the X-ray photon spectral data. Is the chi-square “acceptable”? Is it “better” than alternative models?



# LS 13:

## What is actually done

- LS13 gives the impression that our group gave them photon spectral data. **We did not.** We provided numeric values for our best (M,R,T,NH) fits to each source, and their error regions -- which anyone could read from figures in our papers.
- LS13 uses an (unpublished) semi-analytic model for the spectrum (A minor issue: why not use the heavily vetted and widely distributed models NSATMOS (Heinke 2006) or NSA (Zavlin et al 1996)?)
- Normalizes this model against G13 best-fit values and uncertainties, and then compares a Bayesian likelihood of this best fit model to an extrapolated model using different assumptions applied to their analytic model.
- LS13 is not “data analysis” in any sense at all. This is “data modeling theory”. It answers the question: **“If the data look like our model, this is what the results would be.”** Also: **“If someone were to do our analysis with the data, and the data are described by our analytic model, then this is what the results would be.”** **It does not say what the data are actually saying.**

# from LSI3

TABLE 4  
PROPERTIES THE BAYESIAN MODELS

Model	$M (M_{\odot})$	$R$ (km)	$R_{\text{in}}$ (km)	$z$	$I$ (Figure of merit)
Base	$1.31 \pm 0.40$	$11.09 \pm 0.39$	$13.8 \pm 1.1$	$0.26 \pm 0.12$	$(7.32 \pm 0.37) \times 10^{-8}$
Exo	$1.31 \pm 0.43$	$9.68 \pm 0.64$	$12.6 \pm 1.8$	$0.31 \pm 0.16$	$(9.60 \pm 0.57) \times 10^{-8}$
Alt	$1.17 \pm 0.26$	$11.02 \pm 0.33$	$13.2 \pm 0.7$	$0.21 \pm 0.07$	$(5.83 \pm 0.36) \times 10^{-7}$
Exo/Alt	$1.17 \pm 0.26$	$9.81 \pm 0.44$	$12.1 \pm 0.9$	$0.26 \pm 0.08$	$(8.19 \pm 1.35) \times 10^{-7}$
H+He	$1.42 \pm 0.41$	$11.21 \pm 0.76$	$15.0 \pm 1.3$	$0.26 \pm 0.11$	$(1.46 \pm 0.08) \times 10^{-6}$
Exo/H+He	$1.47 \pm 0.51$	$11.24 \pm 0.55$	$14.5 \pm 2.1$	$0.29 \pm 0.15$	$(5.58 \pm 0.39) \times 10^{-7}$
Alt/H+He	$1.34 \pm 0.31$	$12.02 \pm 0.58$	$14.6 \pm 0.8$	$0.23 \pm 0.09$	$(1.55 \pm 0.06) \times 10^{-6}$
Alt/Exo/H+He	$1.34 \pm 0.33$	$11.48 \pm 0.68$	$14.1 \pm 1.2$	$0.24 \pm 0.09$	$(1.84 \pm 0.07) \times 10^{-6}$

NOTE.—The first column is the model label, columns 2 through 5 give the mean and standard deviation for all five neutron stars, and column 6 is the integral for computing the Bayes factor.

- “I” is a “Bayes Integral” - their “goodness” statistic. LSI3 claims it is from comparison with data. It is not. This is not a valid “data analysis” method.
- Which of your “Bayesian Preferred” models are consistent with the observed X-ray spectra for the five sources, and which are not? *This is answered in every data analysis paper ever written. It is not answered in LSI3.*

# Question for LSI 3

- Which of your “Bayesian Preferred” models are consistent with the observed X-ray spectra for the five sources, and which are not? In short, what are their “null hypothesis probabilities”? *This is answered in every data analysis paper ever written. It is not answered in LSI 3.*

TABLE 4  
PROPERTIES THE BAYESIAN MODELS

Model	$M (M_{\odot})$	$R$ (km)	$R_{\infty}$ (km)	$z$	$I$
Base	$1.31 \pm 0.40$	$11.09 \pm 0.39$	$13.8 \pm 1.1$	$0.26 \pm 0.12$	$(7.32 \pm 0.37) \times 10^{-9}$
Exo	$1.31 \pm 0.43$	$9.68 \pm 0.64$	$12.6 \pm 1.8$	$0.31 \pm 0.16$	$(9.60 \pm 0.57) \times 10^{-9}$
Alt	$1.17 \pm 0.26$	$11.02 \pm 0.33$	$13.2 \pm 0.7$	$0.21 \pm 0.07$	$(5.83 \pm 0.36) \times 10^{-9}$
Exo/Alt	$1.17 \pm 0.26$	$9.81 \pm 0.44$	$12.1 \pm 0.9$	$0.26 \pm 0.08$	$(8.19 \pm 1.35) \times 10^{-9}$
H+He	$1.42 \pm 0.41$	$11.21 \pm 0.76$	$15.0 \pm 1.3$	$0.25 \pm 0.11$	$(1.46 \pm 0.08) \times 10^{-9}$
Exo/H+He	$1.47 \pm 0.51$	$11.24 \pm 0.55$	$14.5 \pm 2.1$	$0.29 \pm 0.15$	$(5.58 \pm 0.39) \times 10^{-9}$
Alt/H+He	$1.34 \pm 0.31$	$12.02 \pm 0.58$	$14.6 \pm 0.8$	$0.23 \pm 0.09$	$(1.55 \pm 0.06) \times 10^{-9}$
Alt/Exo/H+He	$1.34 \pm 0.33$	$11.48 \pm 0.68$	$14.1 \pm 1.2$	$0.24 \pm 0.09$	$(1.84 \pm 0.07) \times 10^{-9}$

NOTE.—The first column is the model label, columns 2 through 5 give the mean and standard deviation for all five neutron stars, and column 6 is the integral for computing the Bayes factor.

# Partial List of Problems with LSI3

- LSI3 does not produce a statistical comparison between X-ray photon spectral data and their model. This is the only means by which any model can be tested. MAJOR
- LSI3 assumes specific absorptions (NH values) and constrains them to be fixed. This is an inferior approach to leaving this a free parameter for the data fit (as done by G13). MINOR.
- Uncertainties in all parameters don't contain distance uncertainty, possibility of hard power-law contribution (G13 accounts for both). MINOR.