Unraveling the dependence of the Electric Dipole Polarizability on the isovector properties of the nuclear effective interaction

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Table of contents:

INTRODUCTION

The Nuclear Many-Body Problem:

- **Nucleus:** from few to more than 200 strongly interacting and **self-bound fermions**.
- **Underlying interaction** is **not perturbative** at the (low)energies of interest for the study of masses, radii, deformation, giant resonances,...
- **Complex systems**: **spin, isospin, pairing, deformation,** ...
- **Many-body** calculations based on **NN scattering data** in the vacuum are feasible for the **EoS, light and light-medium nuclei**, no extensive calculations for nuclei along the whole periodic table.
- Based on effective interactions, **Nuclear Energy Density Functionals** can be succesfully applied to the whole periodic table (except light systems) for the description of **masses, nuclear sizes, deformations, Giant Resonances,...**

...in the near future:

- **New Radioactive Beam Facilities** will measure nuclear properties far from stability: **new tests for "ab-initio" and EDF calculations**
- **The experimental study of nuclei at the meeting point ($A \sim 40$) between "ab-initio" and EDFs** is now becoming and will become in the near future one of our tools ...
  - ... to **build new EDFs with improved performance** (mainly in interaction channels that are not disentangled by the usual fitting procedures with stable experimental data not from future experiments)
  - ... to **guide "ab-initio" calculations** in the description of **heavy nuclei** well described within the density functional theory.

Approximate realization of an exact Nuclear Energy Density Functional:

Kohn-Sham iterative scheme (static approximation)

- Determine a good $E[\rho]$
- Initial guess ρ_0
- Calculate potential V_{eff} from ρ_0
- Solve single particle (Schrödinger) equation and find single particle wave functions ϕ_i
- Use ϕ_i for calculating new $\rho_1 = \sum_i^A |\phi_i|^2$
- Repeat until convergence

Runge-Gross Theorem: dynamic generalization of the static EDFs.

$$\int dt \{\langle \Phi(t)| i\partial_t |\Phi(t)\rangle - E[\rho(t), t]\} = 0$$

Giant Resonances well described within the small amplitude limit (known as RPA approach)

Nuclear Energy Density Functionals:

Main types of successful EDFs for the description of masses, deformations, nuclear distributions, Giant Resonances, ...

Relativistic mean-field models, based on Lagrangians where effective mesons carry the interaction:

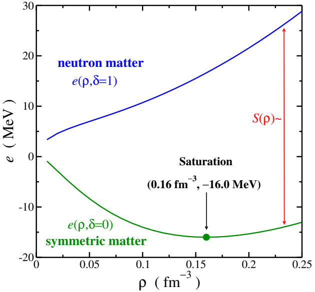
$$
\begin{aligned}
\mathcal{L}_{int} \quad &= \bar{\Psi}\Gamma_\sigma(\bar{\Psi},\Psi)\Psi\Phi_\sigma \quad &+\bar{\Psi}\Gamma_\delta(\bar{\Psi},\Psi)\boldsymbol{\tau}\Psi\Phi_\delta \\
&-\bar{\Psi}\Gamma_\omega(\bar{\Psi},\Psi)\gamma_\mu\Psi A^{(\omega)\mu} \quad &-\bar{\Psi}\Gamma_\rho(\bar{\Psi},\Psi)\gamma_\mu\tau\Psi A^{(\rho)\mu} \\
&-e\bar{\Psi}\hat{Q}\gamma_\mu\Psi A^{(\gamma)\mu}
\end{aligned}
$$

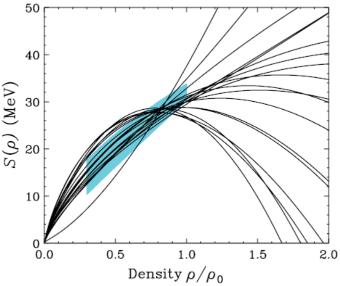
Non-relativistic mean-field models, based on Hamiltonians where effective interactions are proposed and tested:

$$V^{\text{eff}}_{\text{Nucl}} = V^{\text{long-range}}_{\text{attractive}} + V^{\text{short-range}}_{\text{repulsive}} + V_{\text{SO}} + V_{\text{pair}}$$

- Fitted **parameters contain** (important) **correlations beyond the mean-field**
- Nuclear energy functionals are **phenomenological** → **not directly connected to any NN** (or NNN) **interaction** (opposite to "ab-initio" calculations)

The Nuclear Equation of State: Infinite System



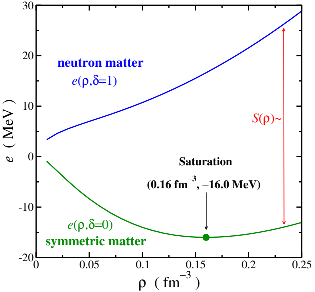


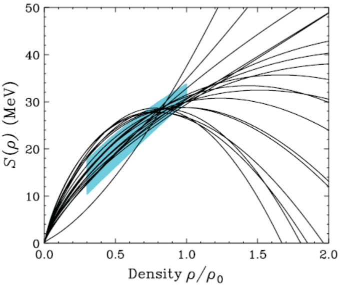
$$\boxed{\frac{E}{A}(\rho,\beta)} = \boxed{\frac{E}{A}(\rho,\beta=0)} + \boxed{S(\rho)\beta^2} + \mathcal{O}(\beta^4)$$

- Nuclear Matter

$$\left[\beta = \frac{\rho_n - \rho_p}{\rho}\right]$$

The Nuclear Equation of State: Infinite System



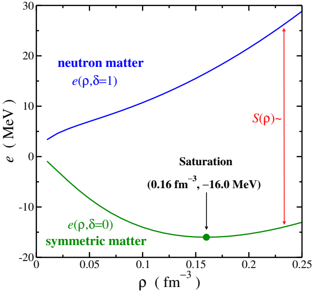


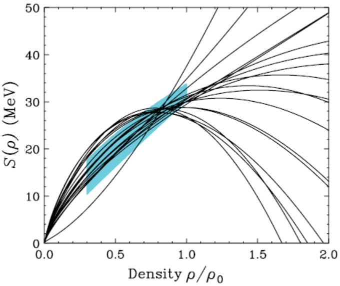
$$\boxed{\frac{E}{A}(\rho,\beta)} = \boxed{\frac{E}{A}(\rho,\beta=0)} + \boxed{S(\rho)\beta^2} + \mathcal{O}(\beta^4)$$

- Nuclear Matter
- Symmetric Matter

$$\left[\beta = \frac{\rho_n - \rho_p}{\rho}\right]$$

The Nuclear Equation of State: Infinite System



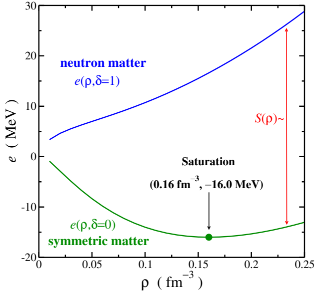


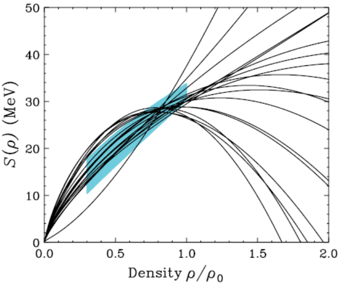
$$\boxed{\frac{E}{A}(\rho, \beta)} = \boxed{\frac{E}{A}(\rho, \beta = 0)} + \boxed{S(\rho)\beta^2} + \mathcal{O}(\beta^4)$$

- Nuclear Matter
- Symmetric Matter
- Symmetry energy

$$\left[\beta = \frac{\rho_n - \rho_p}{\rho}\right]$$

The Nuclear Equation of State: Infinite System



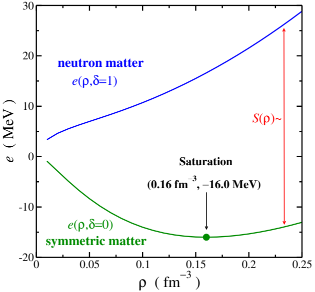


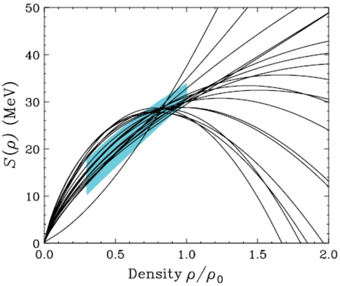
$$\boxed{\frac{E}{A}(\rho,\beta)} = \boxed{\frac{E}{A}(\rho,\beta=0)} + \boxed{S(\rho)\beta^2} + \mathcal{O}(\beta^4)$$

$$= \frac{E}{A}(\rho,\beta=0) + \beta^2\left(J + Lx + \frac{1}{2}K_{sym}x^2 + \mathcal{O}(x^3)\right)$$

$$\left[\beta = \frac{\rho_n - \rho_p}{\rho}; \quad x = \frac{\rho-\rho_0}{3\rho_0}\right]$$

The Nuclear Equation of State: Infinite System





$$\frac{E}{A}(\rho,\beta) = \frac{E}{A}(\rho,\beta=0) + \beta^2\left(\boxed{J} + \boxed{L}x + \frac{1}{2}\boxed{K_{sym}}x^2 + \mathcal{O}(x^3)\right)$$

- $S(\rho_0) = J$
- $\left.\frac{d}{d\rho}S(\rho)\right|_{\rho_0} = \frac{L}{3\rho_0} = \frac{P_0}{\rho_0^2}$
- $\left.\frac{d^2}{d\rho^2}S(\rho)\right|_{\rho_0} = \frac{K_{sym}}{9\rho_0^2}$

$$\left[\beta = \frac{\rho_n - \rho_p}{\rho}; \quad x = \frac{\rho - \rho_0}{3\rho_0}\right]$$

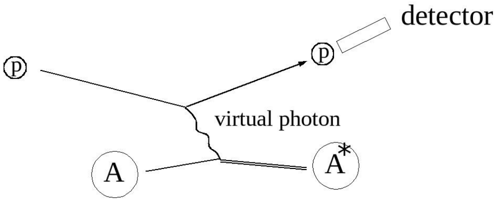
DIPOLE POLARIZABILITY

Recent measurement in ^{208}Pb at RCNP

At the Research Center for Nuclear Physics (RCNP), Osaka ...

- using **polarized protons**
- measuring protons **scattered inelatically**
- excitations via virtual photons (**Coulomb excitation**)
- able to cover a **broad range of excitation energies**
- set up with **high-resolution and efficiency**

Very good agreement with previous measurements is found



A. Tamii et al., PRL107 (2011) 062502

Recent measurement in ^{208}Pb at RCNP

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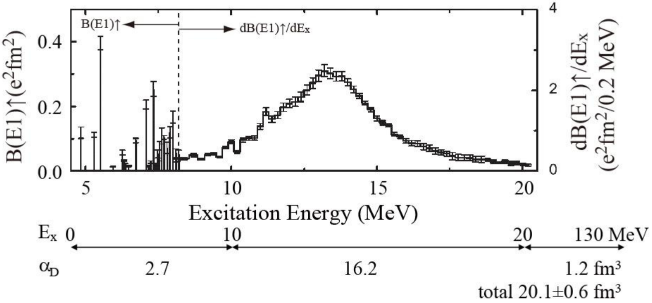


Figure taken from A. Tamii's talk at INPC 2013

Dipole polarizability is determined with high accuracy by taking the average of the RCNP data plus available data in ^{208}Pb† covering a wide range of excitation energies: $\alpha_D = 20.1 \pm 0.6$ $\mathbf{fm}^3$

A. Tamii e…

† K. Schelhaas et al., Nucl. Phys. A 489, 189 (1988); A. Veyssiere, H. Beil, R. Bergere, P. Carlos, and A. Lepretre, Nuclear Physics A 159, 561 (1970)

Isovector Giant Resonances

- In **isovector** giant resonances **neutrons and protons "oscillate" out of phase**
  e.g. within a classical picture: **"e-m interacting probes basically excite protons, protons drag neutrons thanks to the nuclear strong interaction,when neutrons approach too much to protons, they are pushed out"**
- **Isovector** resonances will depend on oscillations of the density $\rho_{iv} \equiv \rho_n - \rho_p \Rightarrow S(\rho)$ will drive such "oscillations"
- The **excitation energy (E_x)** within a **Harmonic Oscillator** approach is expected to depend on the symmetry energy:

$$\omega = \sqrt{\frac{1}{m}\frac{d^2U}{dx^2}} \propto \sqrt{k} \to E_x \sim \sqrt{\frac{\delta^2 e}{\delta\beta^2}} \propto \sqrt{S(\rho)}$$

where $\beta = (\rho_n - \rho_p)/(\rho_n + \rho_p)$

Polarizability, Strength distribution and its moments

- The **linear response** or dynamic polarizability of a **nuclear system excited** from its g.s., $|0\rangle$, to an excited state, $|\nu\rangle$, due to the **action of an external isovector oscillating field** (dipolar in our case) of the form ($Fe^{iwt} + F^{\dagger}e^{-iwt}$):

$$F_{JM} = \sum_{i}^{A} r^J Y_{JM}(\hat{r})\tau_z(i) \text{ } (\Delta L = 1 \rightarrow \text{Dipole})$$

- is proportional to the **static polarizability** for small oscillations

$$\alpha = (8\pi/9)e^2 m_{-1} = (8\pi/9)e^2 \sum_{\nu} |\langle\nu|F|0\rangle|^2/E$$

where m_{-1} **is the inverse energy weighted moment** of the **strength function**, defined as, $S(E) = \sum_{\nu} |\langle\nu|F|0\rangle|^2 \delta(E - E_{\nu})$

- **Isovector energy weighted sum rules (EWSR)** are:

$$m_1 = \frac{\hbar^2}{2m}\frac{NZ}{A}(1 + \kappa_D) \text{ equal to one half of the HF expectation value of } [\hat{F}, [H, \hat{F}]]$$

(Thouless theorem) and where κ is the dipole enhancement factor

Dipole polarizability: Correlations in EDFs

Covariance analysis within a model: theory

Given as set of observables $\mathcal{O}$ used to calibrate the parameters $\mathbf{p}$ of a given model, the optimum parametrization $\mathbf{p}_0$ is determined by a fit with the global quality measure,

$$\chi^2(\mathbf{p}) = \sum_{i=1}^{m} \left(\frac{\mathcal{O}_i^{\text{theo.}} - \mathcal{O}_i^{\text{ref.}}}{\Delta \mathcal{O}_i^{\text{ref.}}} \right)^2$$

Assuming that the χ^2 is a well behaved (analytical) function in the vicinity of the minimum and that can be approximated by an hyper-parabola,

$$\begin{aligned} \chi^2(\mathbf{p}) - \chi^2(\mathbf{p}_0) &\approx \frac{1}{2} \sum_{i,j}^{n} (p_i - p_{0i}) \partial_{p_i} \partial_{p_j} \chi^2 (p_j - p_{0j}) \\ &\equiv \sum_{i,j}^{n} (p_i - p_{0i}) \mathcal{M}_{ij} (p_j - p_{0j}) \end{aligned}$$

where $\mathcal{M}$ is the curvature matrix.

Dipole polarizability: Correlations in EDFs

Covariance analysis within a model: theory

$\mathcal{M}$ provides us access to estimate the errors between predicted observables ($A(\mathbf{p})$),

$$\Delta\mathcal{A} = \sqrt{\sum_{i}^{n} \partial_{p_i} A \mathcal{E}_{ii} \partial_{p_i} A} \tag{1}$$

$\mathcal{E} = \mathcal{M}^{-1}$ and the correlations between predicted observables,

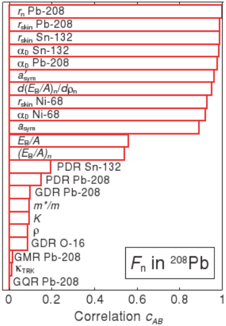
$$c_{AB} \equiv \frac{C_{AB}}{\sqrt{C_{AA} C_{BB}}} \tag{2}$$

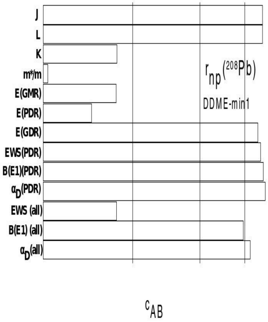
where,

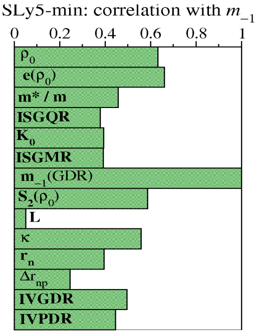
$$C_{AB} = \overline{(A(\mathbf{p}) - \overline{A})(B(\mathbf{p}) - \overline{B})} \approx \sum_{ij}^{n} \partial_{p_i} A \mathcal{E}_{ij} \partial_{p_j} B$$

Dipole polarizability: correlations in EDFs

Covariance analysis within a model: results







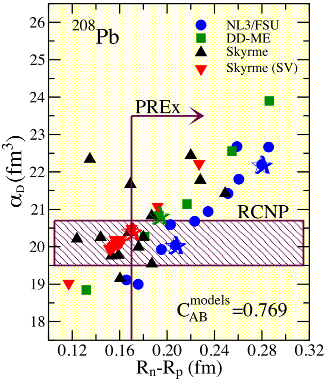
From left to right: SV: P.-G. Reinhard and W. Nazarewicz, Phys. Rev. C 81, 051303(R) (2010); DD-ME1: Nils talk at the INPC 2013; SLy5: X. Roca-Maza

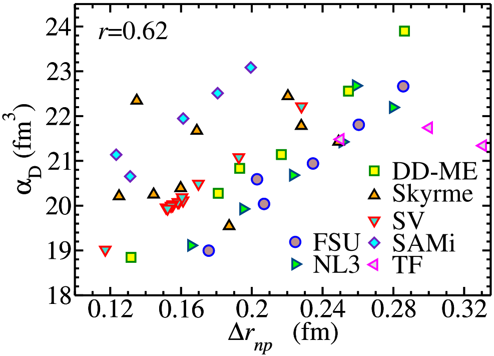
Using the **experimental value** $\alpha_D = 20.1 \pm 0.6$ **fm**^{3} † in ^{208}Pb the **covariance analisis of SV model**, a value $\Delta r_{np} = 0.156^{+0.025}_{-0.021}$ **fm** was found†.

† A. Tamii et al., Phys. Rev. Lett. 107, 062502 (2011)

Dipole polarizability in ^{208}Pb: correlations in EDFs

Systematics for a set of EDFs





J. Piekarewicz et al., Phys. Rev. C 85, 041302 (2012)

X. Roca-Maza et al., in preparation (2013)

From the models of the left panel, using the **experimental value** $\alpha_D = 20.1 \pm 0.6$ **fm**^{3} † in ^{208}Pb a model average for $\Delta r_{np} = 0.168 \pm 0.022$ **fm** was found.

† A. Tamii et al., Phys. Rev. Lett. 107, 062502 (2011)

Dipole polarizability: Correlations in EDFs

Insights from a macroscopic approach

Given that **only the** m_{-1} moment is **required** for the calculation of the **dipole polarizability**, one may perform a constrained calculation

$$\delta\left\{\langle\mathcal{H}\rangle - \lambda\langle\mathcal{D}\rangle\right\} = 0$$

This defines the *constrained energy* $E(\lambda)$. **The dielectric theorem** establishes that the m_{-1} moment may be computed as

$$m_{-1}(E1) = \frac{1}{2}\left.\frac{\partial^2 E(\lambda)}{\partial\lambda^2}\right|_{\lambda=0}$$

Applying **this procedure** in combination with the **droplet model** approach of Myers and Swiatecki[†] yields the following result[††]:

$$\alpha_D = \frac{8\pi}{9}e^2\frac{A\langle r^2\rangle}{48J}\left(1+\frac{5}{3}\frac{9J}{4Q}A^{-1/3}\right)$$

† W. Myers and W. Swiatecki, Annals of Physics 84, 186 (1974)

†† J. Meyer, P. Quentin, and B. Jennings, Nuclear Physics A 385, 269 (1982)

Dipole polarizability: Correlations in EDFs

Insights from a macroscopic approach

Within the DM model:

$$\Delta r_{np} = \sqrt{\frac{3}{5}}\left[t - \frac{e^2 Z}{70J}\right] + \Delta r_{np}^{\mathrm{surface}}$$

$$t \equiv \frac{3r_0}{2}\frac{J/Q}{1+\frac{9}{4}\frac{J}{Q}A^{-1/3}}(I - I_C)$$

Q is the so-called surface stiffness coefficient, $I \equiv (N-Z)/A$ is the relative neutron excess, $\rho_0 = 3A/4\pi r_0^3$,

$I_C = (e^2 Z)/(20JR)$, $R \equiv \sqrt{3/5}\, r_0 A^{1/3}$, and $\Delta r_{np}^{\mathrm{surf}} = \sqrt{3/5}[5(b_n^2 - b_p^2)/(2R)]$ is a correction caused by the difference in the surface width b_n (b_p) of the neutron (proton) density profile

using these expressions:

$$\alpha_D \approx \frac{\pi e^2}{54}\frac{A\langle r^2\rangle}{J}\left[1 + \frac{5}{2}\frac{\Delta r_{np} + \sqrt{\frac{3}{5}}\frac{e^2 Z}{70J} - \Delta r_{np}^{\mathrm{surface}}}{\langle r^2\rangle^{1/2}(I - I_C)}\right]$$

Adopting a value of $J = 31 \pm 2$ MeV[†] one finds for ^{208}Pb that $I_c \approx 0.028 \pm 0.002$ $\sqrt{3/5}(e^2 Z)/(70J)$ is around 0.042 ± 0.003 fm. $\Delta r_{np}^{\mathrm{surf}}$ for ^{208}Pb is almost constant (0.09 ± 0.01 fm) in EDFs[††]

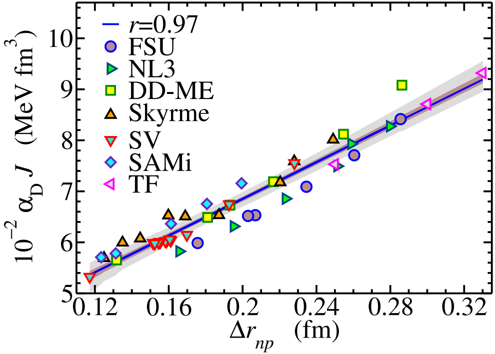
In the DM Δr_{np} is better correlated with $\alpha_D J$ than with α_D alone in a heavy nucleus such as ^{208}Pb

[†] James M. Lattimer, Annu. Rev. Nucl. Part. Sci. 62, 485 (2012)

[††] M. Centelles, X. Roca-Maza, X. Viñas, and M. Warda, Phys. Rev. C 82, 054314 (2010).

Dipole polarizability in ^{208}Pb: Correlations in EDFs

Insights from a macroscopic approach



X. Roca-Maza et al., in preparation (2013)

Using exp. $\alpha_D = 20.1 \pm 0.6$ $\mathbf{fm}^3$ † in ^{208}Pb on finds the relation

$\Delta r_{np} = -0.156 \pm (0.014)_{\text{theo.}} + \left[1.04 \pm (0.03)_{\text{exp.}} \pm (0.04)_{\text{theo.}}\right] \times 10^{-2} J$

Adopting $J = 31 \pm (2)_{\text{est.}}$ **MeV** †† one obtains

$\Delta r_{np} = 0.168 \pm (0.009)_{\text{exp.}} \pm (0.019)_{\text{theo.}} \pm (0.021)_{\text{est.}}$ **fm**

† A. Tamii et al., Phys. Rev. Lett. 107, 062502 (2011) †† James M. Lattimer, Annu. Rev. Nucl. Part. Sci. 62, 485 (2012)

Dipole polarizability: Correlations in EDFs

Insights from a macroscopic approach

Starting from the DM experessions

$$\alpha_D = \frac{8\pi}{9} e^2 \frac{A\langle r^2\rangle}{48J}\left(1 + \frac{5}{3}\frac{9J}{4Q}A^{-1/3}\right) \ \& \ a_{sym}(A) = \frac{J}{1+\frac{9J}{4Q}A^{-1/3}}$$

one can write

$$\alpha_D \approx \frac{8\pi}{9} e^2 \frac{A\langle r^2\rangle}{48J}\left(1 + \frac{5}{3}\frac{J - a_{sym}(A)}{J}\right)$$

and assuming that the **symmetry energy coefficient of a finite nucleus is very close to that of the infinite system**† at an **appropriate** sub-saturation density ρ_A: $a_{sym}(A) \approx S(\rho_A)$:

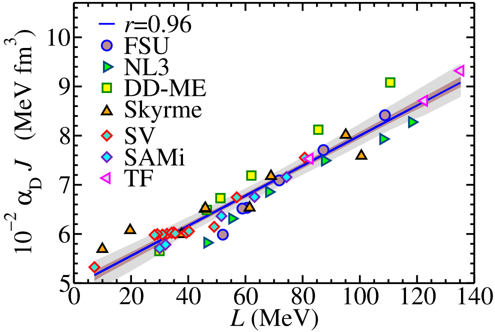
$$\alpha_D \approx \frac{\pi e^2}{54}\frac{A\langle r^2\rangle}{J}\left[1 + \frac{5}{3}\frac{L}{J}\epsilon_A\right]$$

where $\epsilon_A \equiv \dfrac{\rho_0 - \rho_A}{3\rho_0}$ and $\epsilon_{208} = 1/8$ for $\rho_0 = 0.16$ fm-3 for the case of ^{208}Pb

† M. Centelles, X. Roca-Maza, X. Viñas, and M. Warda, Phys. Rev. Lett. 102, 122502 (2009)

Dipole polarizability in ^{208}Pb: Correlations in EDFs

Insights from a macroscopic approach



X. Roca-Maza et al., in preparation (2013)

Using exp. $\alpha_D = 20.1 \pm 0.6$ **fm**^{3} † in ^{208}Pb on finds the relation

$L = -145 \pm (9)_{\text{theo.}} + [6.07 \pm (0.18)_{\text{exp.}} \pm (0.26)_{\text{theo.}}]\,J$

Adopting $J = 31 \pm (2)_{\text{est.}}$ **MeV** †† one obtains

$L = 43 \pm (6)_{\text{exp.}} \pm (12)_{\text{theo.}} \pm (12)_{\text{est.}}$ **MeV**

† A. Tamii et al., Phys. Rev. Lett. 107, 062502 (2011) †† James M. Lattimer, Annu. Rev. Nucl. Part. Sci. 62, 485 (2012)

Dipole polarizability in exotic nuclei

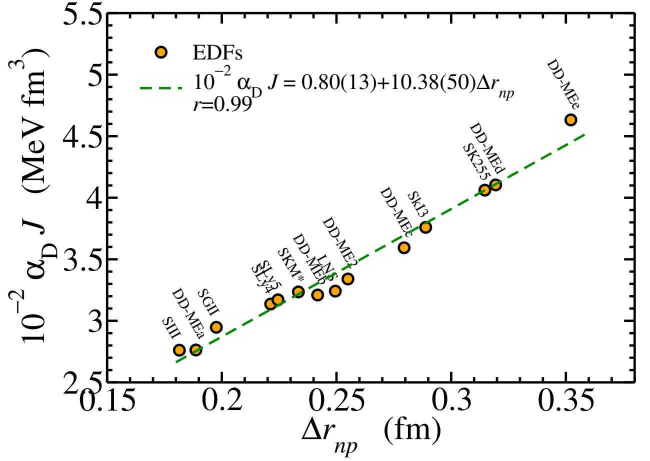
SCRIT: a unique experimental tool for the study of fundamental properties of exotic nuclei[†]

- The **e-m charge distribution of unstable Sn (Z=50) isotopes** will be measured at the **SCRIT (RIKEN) facility next year via electron elastic scattering**.
- If measuring the $E1$ response from inelastic electrons at forward angles **becomes feasible using SCRIT**[††], the **neutron skin of exotic nuclei** and L might be **extracted experimentally from the same facility** using the correlation between $\alpha_D J$ **and** Δr_{np}.

† http://www.riken.jp/en/research/labs/rnc/instrum_dev/scrit/

†† T. Suda et al. Prog. Theor. Exp. Phys. 2012, 03C008

Dipole polarizability in the exotic ^{132}Sn nucleus



X. Roca-Maza in preparation (DD-ME calculations provided by Nils Paar)

CONCLUSIONS

Conclusions:

For medium-heavy and heavy mass nuclei we expect:

- the **macroscopic model** presented here contains **relevant physics** for the description of the **dipole polarizability**

  (accurate within a 10% when compared with self-consistent calculations)

- $\alpha_D J$ is strongly correlated with the Δr_{np} and L in EDFs.

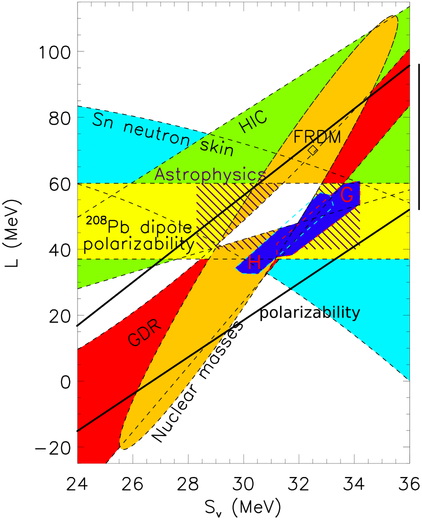
For the case of ^{208}Pb with exp. value $\alpha_D = 20.1 \pm 0.6$ fm^3:

- $\Delta r_{np} = -0.156 \pm (0.014)_{\text{th.}} + \left[1.04 \pm (0.03)_{\text{exp.}} \pm (0.04)_{\text{th.}}\right] 10^{-2} J$
- $L = -145 \pm (9)_{\text{theo.}} + \left[6.07 \pm (0.18)_{\text{exp.}} \pm (0.26)_{\text{theo.}}\right] J$

... and assuming $J = 31 \pm (2)_{\text{est}}$ MeV:

- $\Delta r_{np} = 0.168 \pm (0.009)_{\text{exp.}} \pm (0.019)_{\text{theo.}} \pm (0.021)_{\text{est.}}$ fm
- $L = 43 \pm (6)_{\text{exp.}} \pm (12)_{\text{theo.}} \pm (12)_{\text{est.}}$ MeV

Conclusions



Note that new band and the original yellow band have been derived from the same experimental value and using EDFs in the analysis Figure modified from James M. Lattimer, Annu. Rev. Nucl. Part. Sci. 62, 485 (2012)

Co-workers:

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