Dear friends,

We appreciate your previous efforts in participating the code comparison project. As you may know, we have published an important joint paper (PRC 93, 044609 (2016)) on transport heavy-ion comparison, and are trying to finish two joint papers on transport box calculation, with cascade and Vlasov mode, respectively. Following this direction, we now move to the comparison on the particle production in a box system, and are mostly interested in the production of pion-like particles. A transport workshop is scheduled in the March of 2017 at MSU (http://www.nucl.phys.tohoku.ac.jp/transport2017/), and at that time we will try to finalize the results and papers for box calculation with cascade and Vlasov mode, and have a general picture of results of pion-like particle production in the box.

In the past decade, the π^-/π^+ ratio, as a probe of the nuclear symmetry energy, has been a hot topic and stimulated lots of theoretical and experimental efforts. However, divergent constraints on the nuclear symmetry energy were obtained based on the same experimental data by using different transport codes. Thus, it is an urgent and extremely important task for the transport community to provide reliable predictions on the production of pion-like particles based on transport model simulations. To achieve this task, we ask you to do the homework on the production of pion-like particles in a simple box system, as detailed in the following.

Prepare the box system with periodic boundary condition (same as in the previous box calculation, with size of the box L_k = 20 fm):

Periodic boundary condition (BC) can be introduced in an elegant way by redefining the position of particles moving out the box and the distance metric between two points (or particles) as it follows:

- In order to keep all particles inside the box, a particle leaving the box has to enter it on the opposite side, keeping the same momentum. Thus, indicating with $r_i(k)$ (k=1,2,3) the original coordinate of the particle i and with $r_{i,new}(k)$ the modified coordinate, this condition implies:
  - if $r_i(k) > L_k$ then $r_{i,new}(k) = r_i(k) – L_k$
  - if $ri(k)<0$ then $r_{i,new}(k) = r_i(k) + L_k$
  In Fortran, this can be written as $r_{i,new}(k)$ = modulo($r_i(k)$, L_k).
- Let us indicate with $dr(k) = r_i(k) - r_j(k)$ the three components of the distance vector between points i and j. Then, when periodic BC are imposed:
  - if $dr(k) > L_k/2$ then $dr_{new}(k) = dr(k) – L_k$
  - if $dr(k) < -L_k/2$ then $dr_{new}(k) = dr(k) + L_k$
  This condition can be implemented as
  $dr_{new}(k)$ = modulo($dr(k) + L_k/2$, L_k) - $L_k/2$.

Initialization:

- Uniform density ρ=0.16 fm^{-3}. With the above size of the box this corresponds to 1280 nucleons. We will study two cases with isospin symmetric matter (640 neutrons and 640 protons, $\delta = \frac{\rho_n - \rho_p}{\rho_n + \rho_p} = 0$) and isospin asymmetric matter (768 neutrons and 512 protons, $\delta = 0.2$). Particle positions are initialized randomly from 0 to L_k.

- The momenta of initial nucleons follow the Boltzmann distribution

  $$f \sim \exp[-\sqrt{m_N^2 + p^2}/T],$$

  with the temperature T = 0.06 GeV and nucleon mass m_N = 0.938 GeV. The momentum distribution thus doesn't depend on the density.
- We ask you to give the system an initial relaxation time of 10 fm/c after sampling the nucleon momentum according to the Boltzmann distribution, i.e., only nucleon-nucleon elastic scatterings are allowed in the first 10 fm/c, while inelastic scatterings are turned on after 10 fm/c.

Common setup: In all the calculations requested in this homework, mean-field potential and Coulomb potential must be turned off for all particles. One should adopt an isotropic constant cross section σ = 40 mb for elastic scatterings between baryons ($N + N \rightarrow N + N$, $N + \Delta \rightarrow N + \Delta$, and $\Delta + \Delta \rightarrow \Delta + \Delta$). Turn off Pauli blocking in all the processes. The production of all the particles other than nucleons, Δ(1232) resonances, and pions should be turned off. We ask you to turn off all the direct pion production from nucleon-nucleon scatterings $N + N \rightarrow N + N + \pi$ and their inverse reactions. For all the elastic and inelastic scatterings, we ask you to remove spurious scatterings, i.e., after a collision happened for a pair of particles (i, j), the same pair should not collide again until one of i and j collides with some other particle (see slides for details), and any other artificial thresholds or cuts on the C.M. energy and distance. **In all the calculation, we ask you to let the system evolve 10 fm/c for relaxation and additional 140 fm/c for real reaction. The time step of 0.5 fm/c or 1 fm/c is recommended. For both BUU models and QMD models, we ask you to present the results for 1000 events for each Hw/case.**

We have Phase I with only production of Delta resonance and Phases II and III with both Delta and pion production. Phase I is simpler with a fixed Delta mass, and Phase II gives Delta resonance a mass distribution. If you already have a code for pion production, Phase III is actually the easiest case to try.

In the following homework description, we give the isospin-averaged cross sections for relevant reaction channels, and also provide detailed information of the cross sections for each isospin-dependent channels. Since isospin degree of freedom is available in almost all the transport codes so far, each code participant may treat the isospin degree of freedom explicitly for both isospin symmetric and asymmetric matter in the real calculation.

Phase I:

The mass of Delta resonances is fixed at 1.232 GeV (option Dc). Turn off the pion production by artificially forbidding the decay of Delta resonances (option P0).

Hw1(Option Dc1P0): only Delta production

In addition to the elastic channels to thermalize the system, consider only the inelastic Delta production channel $N + N \rightarrow N + \Delta$. Adopt a constant isotropic cross section of $\sigma_{NN \rightarrow N\Delta} = 40mb$ (isospin-averaged value) for $\sqrt{s} \geq 2.170 GeV$ and 0mb for $\sqrt{s} < 2.170 GeV$

, with $\sqrt{s}$ the C.M. energy in the two-body scatterings. See below for the isospin-dependent cross sections.

Note the temperature will decrease with the increasing number of Delta resonance, so will the reaction rate. The purpose is to compare the reaction rate and the Delta number as a function of time.

Hw2(Option Dc2P0): Both Delta production and its inverse reaction

Same as Hw1 but introduce the inelastic channel $N+\Delta \to N+N$, and the corresponding isotropic cross section is from the detailed balance condition:

$\sigma_{N\Delta\to NN} = \frac{1}{g}\frac{p_f^2}{p_i^2}\sigma_{NN\to N\Delta}$, with g = 8 the spin-isospin factor, and

$$p_f^2 = \frac{1}{4}s - m_N^2,\ p_i^2 = \frac{(s+m_\Delta^2 - m_N^2)^2}{4s} - m_\Delta^2,$$

and $\sqrt{s}$ the C.M. energy in the two-body scatterings.

The numbers of different species are expected to reach an equilibrium. The purpose is to compare the equilibrated Delta number and the corresponding reaction rate.

We note that the required $N+N \leftrightarrow N+\Delta$ cross sections may be larger than the realistic ones, and one should remove any distance cuts that have been introduced to judge collision attempts.

Isospin dependence of the cross sections:

The isospin states of nucleons and Delta resonances are distinguishable in Hw1 and Hw2, as n, p, Δ^{++}, Δ^{+}, Δ^{0}, and Δ^{-}. The cross sections for elastic channels $N+N \to N+N$, $N+\Delta \to N+\Delta$, and $\Delta+\Delta \to \Delta+\Delta$ are 40mb and isospin independent. The inelastic channels and the corresponding cross sections as well as the spin-isospin factors are:

$p+p \leftrightarrow \Delta^{++}+n$, $n+n \leftrightarrow \Delta^{-}+p$, $\sigma_{NN\to N\Delta}$, g = 4

$p+p \leftrightarrow \Delta^{+}+p$, $n+n \leftrightarrow \Delta^{0}+n$, $\sigma_{NN\to N\Delta}/3$, g = 4

$n+p \leftrightarrow \Delta^{+}+n$, $n+p \leftrightarrow \Delta^{0}+p$, $\sigma_{NN\to N\Delta}/3$, g = 2

For example, the isospin-dependent inelastic cross sections for $p+p \leftrightarrow \Delta^{+}+p$ are $\sigma_{pp\to\Delta^+ p} = 40/3$ mb and $\sigma_{\Delta^+ p\to pp} = \frac{1}{4}\frac{p_f^2}{p_i^2}\sigma_{pp\to\Delta^+ p}$.

We consider two cases with initial isospin asymmetry $\delta = \frac{\rho_n - \rho_p}{\rho_n + \rho_p} = 0$ and $\delta = 0.2$. In the first case the multiplicity of Delta resonance at each isospin state should be the same. In the second case we compare the numbers of Delta resonances at different isospin states, and thus equivalently the π^-/π^+ ratio.

Phase II:

We now adopt an energy-dependent isospin-averaged isotropic cross section for taken from Phys. Rep. 160, 189 (1988) (option Db):

$$\sigma_{NN\to N\Delta} = \begin{cases} \dfrac{20(\sqrt{s}-2.015)^2}{0.015+(\sqrt{s}-2.015)^2}, & \sqrt{s} \geq 2.015 \\ 0, & \sqrt{s} < 2.015 \end{cases}$$

with cross section σ in mb and C.M. energy $\sqrt{s}$ in GeV.

When a Delta resonance is produced based on the above cross section, the mass of the resonance should be sampled according to a distribution function of Breit-Wigner form (option Db):

$$P(m_\Delta) \sim \frac{1}{1+4[(m_\Delta - m_\Delta^0)/\Gamma]^2}$$

with peak mass $m_\Delta^0 = 1.232 GeV$ and constant width $\Gamma = 0.115 GeV$. The range of the mass is limited within $m_N + m_\pi < m_\Delta < \sqrt{s} - m_N$, depending on the C.M. energy $\sqrt{s}$. Pion has a mass of $m_\pi = 0.139 GeV$. The lower limit for the mass of Delta resonance is thus $m_N + m_\pi = 1.077 GeV$.

Hw1(Option Db1P0): only Delta production

Same as that in Hw1 of Phase I except that we now adopt the energy-dependent $N + N \to N + \Delta$ cross section and the Delta mass distribution specified above. The $N + \Delta \to N + N$ channel and pion production are turned off.

Hw2(Option Db2P0): Both Delta production and its inverse reaction

Same as that in Hw2 of Phase I except using the energy-dependent inelastic isotropic cross section for $N + N \to N + \Delta$ as in Hw1 of Phase II and the corresponding cross section for its inverse reaction from the naïve detailed balance condition, i.e.,

$$\sigma_{N\Delta\to NN} = \frac{1}{g}\frac{p_f^2}{p_i^2}\sigma_{NN\to N\Delta}$$, with g = 8 the spin-isospin factor, and

$$p_f^2 = \frac{1}{4}s - m_N^2,\ p_i^2 = \frac{(s+m_\Delta^2 - m_N^2)^2}{4s} - m_\Delta^2,$$

and $\sqrt{s}$ the C.M. energy in the two-body scatterings.

Hw3(Option Db2Pb): turn on pion production and its inverse reaction

Turn on pion production from Delta decay with a constant decay width $\Gamma = 0.115 GeV$, and the isospin-averaged cross section for the inverse reaction is taken from Nucl. Phys. A 379, 553 (1982) (option Pb):

$$\sigma_{N\pi\to\Delta} = \frac{126.2}{1+4[(\sqrt{s}-1.232)/0.115]^2}$$

with cross section σ in mb and C.M. energy $\sqrt{s}$ in GeV.

The purpose is to compare the real pion number, closer to the situation in heavy-ion collisions.

Isospin dependence of the cross sections and decay widths:

The isospin dependence of the inelastic channels $N+N \leftrightarrow N+\Delta$ and the detailed balance condition are the same as those in Phase I except that the energy-dependent inelastic isotropic cross section for $N+N \to N+\Delta$ is now adopted.

The isospin-dependent channels for pion production from Delta decay and its inverse reaction, the decay widths, and the corresponding cross sections are:

$\Delta^{++} \leftrightarrow p+\pi^+$, $\Delta^- \leftrightarrow n+\pi^-$, Γ, $3\sigma_{N\pi\to\Delta}/2$

$\Delta^{+} \leftrightarrow p+\pi^0$, $\Delta^0 \leftrightarrow n+\pi^0$, $2\Gamma/3$, $\sigma_{N\pi\to\Delta}$

$\Delta^{+} \leftrightarrow n+\pi^+$, $\Delta^0 \leftrightarrow p+\pi^-$, $\Gamma/3$, $\sigma_{N\pi\to\Delta}/2$

The initial nucleon system has the isospin asymmetry of $\delta=0$ and $\delta=0.2$. In the first case, the pion numbers at each isospin state should be the same, so are the Delta numbers at each isospin state. In the second case, we can compare the real π^-/π^+ ratio, closer to the situation in heavy-ion collisions.

New Phase II:

The new Phase II is suggested by Pawel Danielewicz with a more consistent relation between cross sections for various channels and thus a good detailed balance condition. The energy-dependent isospin-averaged isotropic cross section for taken from Phys. Rep. 160, 189 (1988) is:

$$\sigma_{NN\to N\Delta} = \begin{cases} \dfrac{20(\sqrt{s}-2.015)^2}{0.015+(\sqrt{s}-2.015)^2}, & \sqrt{s} \ge 2.015 \\ 0, & \sqrt{s} < 2.015 \end{cases}$$

with cross section σ in mb and C.M. energy $\sqrt{s}$ in GeV.

When a Delta resonance is produced based on the above cross section, the mass of the resonance should be sampled according to

$$P(m_\Delta) = p_\Delta m_\Delta A(m_\Delta),$$

where $p_\Delta = \sqrt{\dfrac{(s+m_\Delta^2-m_N^2)^2}{4s} - m_\Delta^2}$ is the Delta momentum in the C.M. frame, and $A(m_\Delta)$ is a distribution function of Breit-Wigner form:

$$A(m_\Delta) = \frac{4 m_\Delta^{0\,2} \Gamma}{\left(m_\Delta^2 - m_\Delta^{0\,2}\right)^2 + m_\Delta^{0\,2}\Gamma^2}$$

with peak mass $m_\Delta^0 = 1.232 GeV$ and constant width $\Gamma = 0.115 GeV$. The range of the mass is limited within $m_N + m_\pi < m_\Delta < \sqrt{s} - m_N$, depending on the C.M. energy $\sqrt{s}$. Pion has a mass of $m_\pi = 0.139 GeV$. The lower limit for the mass of Delta resonance is thus $m_N + m_\pi = 1.077 GeV$.

Hw1(Option Dd1P0): only Delta production

Same as that in Hw1 of Phase I except that we now adopt the energy-dependent $N + N \to N + \Delta$ cross section and the Delta mass distribution specified above. The $N + \Delta \to N + N$ channel and pion production are turned off.

Hw2(Option Dd2P0): Both Delta production and its inverse reaction

Same as that in Hw2 of Phase I except using the energy-dependent inelastic isotropic cross section for $N + N \to N + \Delta$ as in Hw1 of Phase II and the corresponding cross section for its inverse reaction from a more accurate detailed balance condition, i.e.,

$$\sigma_{N\Delta \to NN} = \frac{1}{g} \frac{m_\Delta p_f^2}{p_i(m_\Delta)} \sigma_{NN \to N\Delta} \Big/ \int_{m_N + m_\pi}^{\sqrt{s} - m_N} \frac{dm}{2\pi} m A(m) p_i(m)$$

with $p_f^2 = \frac{1}{4} s - m_N^2$, $p_i(m) = \left[\frac{(s + m^2 - m_N^2)^2}{4s} - m^2 \right]^{1/2}$, and for different isospin channels the coefficients for the cross section and the degeneracy factor g are the same as those in Dc2P0.

Hw3(Option Dd2Pd): turn on pion production and its inverse reaction

Turn on pion production from Delta decay with a constant decay width $\Gamma = 0.115 GeV$, and the isospin-averaged cross section for the inverse reaction is:

$$\sigma_{N\pi \to \Delta} = (\hbar c)^2 \frac{4\pi}{3 p_{cm}^2} \Gamma A(\sqrt{s})$$

with p_{cm} being the pion or nucleon momentum in their C.M. frame, $\sqrt{s}$ being the nucleon-pion C.M. energy, and $A(\sqrt{s})$ being the Breit-Wigner distribution with Delta mass identical to the C.M. energy.

The purpose is to compare the real pion number, closer to the situation in heavy-ion collisions.

Isospin dependence of the cross sections and decay widths:

The isospin dependence of the inelastic channels $N + N \leftrightarrow N + \Delta$ and the detailed balance condition are the same as those in Phase I except that the energy-dependent inelastic isotropic cross section for $N+N \rightarrow N+\Delta$ is now adopted.

The isospin-dependent channels for pion production from Delta decay and its inverse reaction, the decay widths, and the corresponding cross sections are:

$\Delta^{++} \leftrightarrow p+\pi^{+}$, $\Delta^{-} \leftrightarrow n+\pi^{-}$, Γ, $3\sigma_{N\pi\rightarrow\Delta}/2$

$\Delta^{+} \leftrightarrow p+\pi^{0}$, $\Delta^{0} \leftrightarrow n+\pi^{0}$, $2\Gamma/3$, $\sigma_{N\pi\rightarrow\Delta}$

$\Delta^{+} \leftrightarrow n+\pi^{+}$, $\Delta^{0} \leftrightarrow p+\pi^{-}$, $\Gamma/3$, $\sigma_{N\pi\rightarrow\Delta}/2$

The initial nucleon system has the isospin asymmetry of $\delta=0$ and $\delta=0.2$. In the first case, the pion numbers at each isospin state should be the same, so are the Delta numbers at each isospin state. In the second case, we can compare the real π^-/π^+ ratio, closer to the situation in heavy-ion collisions.

Phase III:

We will compare the box calculations with the cross sections relevant to the production of Delta resonances and pions in each code that are employed in the simulations of heavy-ion collisions. We want to check whether some of the discrepancies observed between the different calculations for heavy-ion collisions are due to the different cross sections used in the different codes.

Hw (Option Da2Pa): default cross sections of each code

For the cross sections and the Delta mass distribution for the $N+N \leftrightarrow N+\Delta$ channel, use any default option of the code that you believe the most realistic (option Da). For the $\Delta \leftrightarrow N+\pi$ channel, also use the default option of the code (option Pa). However, please follow the common setup described in an early part of this document for the other options, i.e., constant elastic cross sections for scatterings between baryons, no mean field or Coulomb, no spurious scatterings, no energy threshold or distance cut, no s-wave pion production, and no other particles than nucleons, Delta resonances, and pions, etc.

For each Hw we ask you to provide data files as follows:

Data files:

1) Collision data files

   NNND.txt for $N+N \rightarrow N+\Delta$ channels and NDNN.txt for $N+\Delta \rightarrow N+N$ channels in the form of:

   Time (in fm/c)

   Lb1, m1, px1, py1, pz1

   Lb2, m2, px2, py2, pz2

   Lb3, m3, px3, py3, pz3

   Lb4, m4, px4, py4, pz4

   for the process of 1+2->3+4, with Lb = 1 for proton, 2 for neutron, 3 for π^-, 4 for π^0, 5 for π^+, 6 for Δ^-, 7 for Δ^0, 8 for Δ^+, 9 for Δ^{++}, m the particle mass in GeV, and px, py, pz the particle momentum in GeV/c in x, y, and z direction.

2) Decay data files

DNPI.txt for $\Delta \to N + \pi$ channels and NPID.txt for $N + \pi \to \Delta$ channels

in the form of:

Time (in fm/c)

Lb1, m1, px1, py1, pz1

Lb2, m2, px2, py2, pz2

Lb3, m3, px3, py3, pz3

for the processes of 1->2+3 and 1+2>3, respectively, with Lb = 1 for proton, 2 for neutron, 3 for π^-, 4 for π^0, 5 for π^+, 6 for Δ^-, 7 for Δ^0, 8 for Δ^+, 9 for Δ^{++}, m the particle mass in GeV, and px, py, pz the particle momentum in GeV/c in x, y, and z direction.

3) Phase-space data files

PS0.txt, PS10.txt, PS30.txt, PS50.txt, PS70.txt, PS90.txt, PS110.txt, PS130.txt, PS150.txt

at time t = 0, 10, 30, 50, 70, 90, 110, 130, and 150 fm/c, respectively,

in the form of:

NP (Number of particles)

Lb1, m1, x1, y1, z1, px1, py1, pz1

Lb2, m2, x2, y2, z2, px2, py2, pz2

… (NP lines)

with Lb = 1 for proton, 2 for neutron, 3 for π^-, 4 for π^0, 5 for π^+, 6 for Δ^-, 7 for Δ^0, 8 for Δ^+, 9 for Δ^{++}, m the particle mass in GeV, x, y, z the particle space coordinate in fm, and px, py, pz the particle momentum in GeV/c in x, y, and z direction.

Reading program and uploading

a) We have provided a few simple Fortran programs to read your collision data files, decay data files, and phase-space data files, respectively, i.e., NNND.for for NNND.txt, DNPI.for for DNPI.txt, and PS.for for the phase-space data, etc. Please check these programs indeed work so that your data files are in the desired format.

b) **It is convenient to use the provided Fortran programs by compiling them with 'ifort' on the server, and in this way you can easily get the result files from your data files as follows:**

Result files:

1) The numbers of π^-,π^0, π^+, Δ^-, Δ^0, Δ^+, and Δ^{++} as a function of time (from 0 to 150 fm/c, with a step of 10 fm/c, or generated from the phase-space files with the provided analysis code) for both the cases of $\delta = 0$ and $\delta = 0.2$.

   The first column is the time in fm/c, and the other columns from left to right are the numbers of π^-,π^0, π^+, Δ^-, Δ^0, Δ^+, and Δ^{++}, respectively.

2) The reaction rates dN_{coll}/dt as a function of time for $N + N \to N + \Delta$, $N + \Delta \to N + N$, $\Delta \to N + \pi$ and $N + \pi \to \Delta$ channels summing over isospin (from 0 to 150 fm/c, with a step of 1 fm/c, i.e., dt = 1 fm/c) for only the case of $\delta = 0$.

   The first column is the time in fm/c, and the other columns from left to right are the reaction rates dN_{coll}/dt for $N + N \to N + \Delta$, $N + \Delta \to N + N$, $\Delta \to N + \pi$, and $N + \pi \to \Delta$ channels, respectively.

3) The collision energy distributions $dN_{coll}/d\sqrt{s}$ as a function of the C.M. energy $\sqrt{s}$ with bin size d$\sqrt{s}$ =0.01 GeV for $N+N \rightarrow N+\Delta$ and $N+\Delta \rightarrow N+N$ in the interval from 2.0 to 2.6 GeV, and for $\Delta \rightarrow N+\pi$ and $N+\pi \rightarrow \Delta$ in the interval from 1.0 to 1.6 GeV, respectively, summing over isospin, for only the case of $\delta = 0$.
The first column is the C.M. energy $\sqrt{s}$ in GeV, and the other columns from left to right are the collision energy distributions $dN_{coll}/d\sqrt{s}$ for $N+N \rightarrow N+\Delta$, $N+\Delta \rightarrow N+N$, $\Delta \rightarrow N+\pi$, and $N+\pi \rightarrow \Delta$ channels, respectively.

We appreciate your previous efforts in doing Phase I, II, and III for symmetric and asymmetric matter. Since the new Phase II needs only minor modifications on the code for the old Phase II, we hope you can finish it ASAP during the Transport2017/ICNT2017 workshop. In that case we may try to finalize the pion-production results in the box calculation for the paper to appear, and have a deeper understanding of pion-like particle production in transport model simulation.

Many thanks for your efforts,
and many greetings,
the Box Simulation Organizing Committee
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