Cluster production in AMD model

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Clustering phenomena in excited states of nuclear systems

$E^* \sim 80A \text{ MeV}$ Gas of clusters at higher energies

\[ E^* \sim 8A \text{ MeV} \]
excitation energy / temperature

multifragmentation in heavy ion collision

$E^* \sim 8A \text{ MeV}$

molecular resonance
cluster decay

$E^* \sim 8 \text{ MeV}$

collective modes (GR, PR)

threshold decay

weakly bound systems

$E^* \sim 8 \text{ MeV}$

deformation

developed clusters

shell structure

cluster breaking

shell evolution

halo, skin

nn correlation

molecular orbitals

mass number

matter

alpha condensation

liquid–gas
phase transition

Gas of clusters at higher energies

Importance of clusters in heavy-ion collisions

Collisions of two nuclei (e.g., Xe + Sn at 50 MeV/nucleon, $b \approx 0$)

Light-cluster correlations

<table>
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<tr>
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INDRA data, Hudan et al., PRC67 (2003) 064613.


Light-cluster correlations may be important at relatively early times.
Antisymmetrized Molecular Dynamics (very basic version)

**AMD wave function**

\[
|\Phi(Z)\rangle = \det_{ij} \left[ \exp\left\{ -\nu \left( r_j - \frac{Z_i}{\sqrt{\nu}} \right)^2 \right\} \chi_{\alpha_i}(j) \right]
\]

\[
Z_i = \sqrt{\nu} D_i + \frac{i}{2\hbar\sqrt{\nu}} K_i
\]

\( \nu \): Width parameter = (2.5 fm)\(^{-2} \)

\( \chi_{\alpha_i} \): Spin-isospin states = \( p \uparrow, p \downarrow, n \uparrow, n \downarrow \)

**Equation of motion for the wave packet centroids \( Z \)**

\[
\frac{d}{dt} Z_i = \{Z_i, \mathcal{H}\}_\text{PB} + \text{(NN collisions)}
\]

\[\{Z_i, \mathcal{H}\}_\text{PB}: \text{Motion in the mean field}\]

\[\mathcal{H} = \frac{\langle \Phi(Z)|H|\Phi(Z)\rangle}{\langle \Phi(Z)|\Phi(Z)\rangle} + \text{(c.m. correction)}\]

\( H \): Effective interaction (e.g. Skyrme force)

**NN collisions**

\[
W_{i\rightarrow f} = \frac{2\pi}{\hbar} |\langle \Psi_f |V|\Psi_i \rangle|^2 \delta(E_f - E_i)
\]

- \( |V|^2 \) or \( \sigma_{NN} \) (in medium)
- Pauli blocking

Antisymmetrized Molecular Dynamics (very basic version)

Wigner function for the AMD wave function

\[ f_\alpha(r, p) = 8 \sum_{i \in \alpha} \sum_{j \in \alpha} e^{-2\nu(r - R_{ij})^2} e^{-\frac{(p - P_{ij})^2}{2\hbar^2\nu}} B_{ij}^{-1}, \quad \alpha = p \uparrow, p \downarrow, n \uparrow, n \downarrow \]

\[ R_{ij} = \frac{1}{2\sqrt{\nu}} (Z_i^* + Z_j), \quad P_{ij} = i\hbar\sqrt{\nu}(Z_i^* - Z_j), \quad B_{ij} = e^{-\frac{1}{2}(Z_i^* - Z_j)^2} \]

Equation of motion for the wave packet centroids \( Z \)

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NN collisions

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Zero-range effective interaction

Skyrme force

\[ v_{ij} = t_0 (1 + x_0 P_\sigma) \delta(r) + \frac{1}{2} t_1 (1 + x_1 P_\sigma) [\delta(r) k^2 + k^2 \delta(r)] \]

\[ + t_2 (1 + x_2 P_\sigma) k \cdot \delta(r) k + t_3 (1 + x_3 P_\sigma) [\rho(r_i)]^\alpha \delta(r) \]

\[ r = r_i - r_j \]

\[ k = \frac{1}{2\hbar} (p_i - p_j) \]

Spatial integration of the potential energy density which is a function of several kind of densities.

\[ \langle V \rangle = \int V(\rho(r), \tau(r), \Delta\rho(r), j(r)) dr \sim A^2 \times \text{Volume} \]

\[ \rho_\alpha(r) = \int f_\alpha(r, p) \frac{dp}{(2\pi\hbar)^3} = \left( \frac{2\nu}{\pi} \right)^{\frac{3}{2}} \sum_{i \in \alpha} \sum_{j \in \alpha} e^{-2\nu(r - R_{ij})^2} B_{ij} B_{ji}^{-1} \]

\[ R_{ij} = \frac{1}{2\sqrt{\nu}} (Z_i^* + Z_j) \]

\[ j_\alpha(r) = \int \frac{p}{M} f_\alpha(r, p) \frac{dp}{(2\pi\hbar)^3} = \left( \frac{2\nu}{\pi} \right)^{\frac{3}{2}} \sum_{i \in \alpha} \sum_{j \in \alpha} \frac{P_{ij}}{M} e^{-2\nu(r - R_{ij})^2} B_{ij} B_{ji}^{-1} \]

\[ P_{ij} = i\hbar \sqrt{\nu} (Z_i^* - Z_j) \]

\[ \tau_\alpha(r) = \int \frac{p^2}{M^2} f_\alpha(r, p) \frac{dp}{(2\pi\hbar)^3} = \left( \frac{2\nu}{\pi} \right)^{\frac{3}{2}} \sum_{i \in \alpha} \sum_{j \in \alpha} \frac{P_{ij}^2 + 3\hbar^2\nu}{M^2} e^{-2\nu(r - R_{ij})^2} B_{ij} B_{ji}^{-1} \]
Failure of fragmentation and cluster production

AMD with usual NN collisions (very basic version)

Central Xe + Sn at 50 MeV/u

Partitioning of protons
(experimental data)

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INDRA data, Hudan et al., PRC67 (2003) 064613.

Two directions of extension of AMD

Wave-packet splitting: Give fluctuation to each wave packet centroid, based on the **single-particle motion**.

\[
\frac{d}{dt} Z = \{Z, \mathcal{H}\}_\text{PB} + (\text{NN Collision})
\]

\[+ (\text{W.P. Splitting}) + (\text{E. Conservation})\]

Ono, Hudan, Chibihi, Frankland, PRC66 (2002) 014603


At each two-nucleon collision, **cluster formation** is considered for the final state.

\[N_1 + B_1 + N_2 + B_2 \rightarrow C_1 + C_2\]

\[W_{i \rightarrow f} = \frac{2\pi}{\hbar} |\langle CC|V_{NN}|NBNB\rangle|^2 \delta(\mathcal{H} - E)\]


Ikeno, Ono et al., PRC 93 (2016) 044612
The system may be composed of many clusters.

Clusters are not only created but also broken by reactions.

I want a transport model which can describe the dynamics for a sufficiently long time (e.g., $t \sim 1000$ fm/$c$).

The decays of the excited fragments at the end of the dynamical calculation are calculated by a statistical decay code.
A cluster in medium & Clusterized nuclear matter

Equation for a deuteron in uncorrelated medium

\[
\begin{align*}
\left[ e\left(\frac{1}{2}P + p\right) + e\left(\frac{1}{2}P - p\right) \right] \bar{\psi}(p) \\
+ \left[ 1 - f\left(\frac{1}{2}P + p\right) - f\left(\frac{1}{2}P - p\right) \right] \int \frac{dp'}{(2\pi)^3} \langle p|\nu|p'\rangle \bar{\psi}(p')
\end{align*}
\]

\[ = E\bar{\psi}(p) \]

Momentum (\(P\)) dependence of B.E.
Röpke, NPA867 (2011) 66.

QS for symmetric nuclear matter
A cluster put into a nucleus in AMD

\( \alpha \) cluster \( |\alpha, Z\rangle \): Four wave packets with different spins and isospins at the same phase space point \( Z \).

\[
E_{\alpha} : \quad \mathcal{A} |\alpha, Z\rangle^{124}\text{Sn}\rangle
\]

\[
E_N : \quad \mathcal{A} |Z\rangle^{124}\text{Sn}\rangle \quad (N = p \uparrow, p \downarrow, n \uparrow, n \downarrow)
\]

\[
- B_{\alpha} = \Delta E_{\alpha} = E_{\alpha} - (E_{p \uparrow} + E_{p \downarrow} + E_{n \uparrow} + E_{n \downarrow})
\]

(Energies are defined relative to \( |^{124}\text{Sn}\rangle \).)

\[
\frac{\text{Re} Z}{\sqrt{\nu}} = (0, y, 0), \quad \frac{2\hbar \sqrt{\nu} \text{Im} Z}{M} = (0, 0, v_z)
\]

- Distance from the center: \( y \)
- \( \approx \) Dependence on density
- Dependence on \( P_{\alpha} = M_{\alpha} v_z \)
- Due to the density dependence of the Skyrme force, the interaction between nucleons in the \( \alpha \) cluster is weakened in the nucleus.

Energy is OK, but dynamics is . . .
NN collisions without or with cluster correlations

\[ W_{i \rightarrow f} = \frac{2\pi}{\hbar} |\langle \Psi_f | V | \Psi_i \rangle|^2 \delta(E_f - E_i) \]

In the usual way of NN collision, only the two wave packets are changed.

\[ \{ |\Psi_f \rangle \} = \{ |\varphi_{k_1}(1)\varphi_{k_2}(2)\Psi(3,4,...) \rangle \} \]

(ignoring antisymmetrization for simplicity of presentation.)

---

Phase space or the density of states for two nucleon system

Molecular Dynamics

\[ \langle K \rangle \quad \langle V \rangle \]

\[ E_{\text{rel}} \quad -\text{BE} \quad 0 \]
NN collisions without or with cluster correlations

\[ W_{i \rightarrow f} = \frac{2\pi}{\hbar} |\langle \Psi_f | V | \Psi_i \rangle|^2 \delta (E_f - E_i) \]

In the usual way of NN collision, only the two wave packets are changed.

\[ \{|\Psi_f\rangle\} = \{|\varphi_{k_1} (1) \varphi_{k_2} (2) \Psi(3,4,\ldots)\rangle\} \]

(ignoring antisymmetrization for simplicity of presentation.)

Extension for cluster correlations

Include correlated states in the set of the final states of each NN collision.

\[ \{|\Psi_f\rangle\} \ni |\varphi_{k_1} (1) \psi_d (2,3) \Psi(4,\ldots)\rangle, \ldots \]
NN collisions with cluster correlations

\[ N_1 + B_1 + N_2 + B_2 \rightarrow C_1 + C_2 \]

- **N_1, N_2**: Colliding nucleons
- **B_1, B_2**: Spectator nucleons/clusters
- **C_1, C_2**: \( N, (2N), (3N), (4N) \) (up to \( \alpha \) cluster)

**Transition probability**

\[
W(\text{NBNB} \rightarrow \text{CC}) = \frac{2\pi}{\hbar} |\langle \text{CC} | V | \text{NBNB} \rangle|^2 \delta(E_f - E_i)
\]

\[
v d\sigma \propto |\langle \varphi'_1 | \varphi_1^{+q} \rangle|^2 |\langle \varphi'_2 | \varphi_2^{-q} \rangle|^2 |M|^2 \delta(E_f - E_i) p_{\text{rel}}^2 d p_{\text{rel}} d\Omega
\]

\[
|M|^2 = |\langle \text{NN} | V | \text{NN} \rangle|^2: \text{Matrix elements of NN scattering}
\]

\[
\left\langle d\sigma/d\Omega \right\rangle_{\text{NN}} \text{ in medium (or in free space)}
\]

Similar to Danielewicz et al.,

\[
p_{\text{rel}} = \frac{1}{2} (p_1 - p_2) = p_{\text{rel}} \hat{\Omega}
\]

\[
q = p_1 - p_1^{(0)} = p_2^{(0)} - p_2
\]

\[
\varphi_1^{+q} = \exp(+i q \cdot r_{N_1}) \varphi_1^{(0)}
\]

\[
\varphi_2^{-q} = \exp(-i q \cdot r_{N_2}) \varphi_2^{(0)}
\]
For each NN collision, cluster formation is considered.

\[ N_1 + B_1 + N_2 + B_2 \rightarrow C_1 + C_2 \]

\[ W_{i \rightarrow f} = \frac{2\pi}{\hbar} |\langle CC|V_{NN}|NB|B\rangle|^2 \delta(E_f - E_i) \]

- We always have a Slater determinant of nucleon wave packets. A cluster in the final states is represented by placing wave packets at the same phase space point.

- Consequently the processes such as \( d + X \rightarrow n + p + X' \) and \( d + X \rightarrow d + X' \) are automatically taken into account.

- No parameters have been introduced to adjust individual reactions. But the cluster formation may be artificially weakened (nnchange_gamma).

- There are many possibilities to form clusters in the final states. Non-orthogonality of the final states should be carefully handled.
Construction of Final States

Clusters (in the final states) are assumed to have \((0s)^N\) configuration.

\[ |\Phi^q\rangle \]

After \(p^{(0)} \rightarrow p^{(0)} + q\)

\[ |\Phi_1'\rangle \]
\[ N + B_1 \rightarrow C_1 \]

\[ |\Phi_2'\rangle \]
\[ N + B_2 \rightarrow C_2 \]

\[ |\Phi_3'\rangle \]
\[ N + B_3 \rightarrow C_3 \]

Final states are not orthogonal: \(N_{ij} \equiv \langle \Phi_i'|\Phi_j' \rangle \neq \delta_{ij}\)

The probability of cluster formation with one of \(B\)'s:

\[
\hat{P} = \sum_{ij} |\Phi_i'\rangle N_{ij}^{-1} \langle \Phi_j'|, \quad P = \langle \Phi^q|\hat{P}|\Phi^q\rangle \neq \sum_i |\langle \Phi_i'|\Phi^q\rangle|^2
\]

\[
\begin{cases} 
P \quad \Rightarrow \text{Choose one of the candidates and make a cluster.} \\
1 - P \quad \Rightarrow \text{Don’t make a cluster (with any } n\uparrow). \end{cases}
\]
An algorithm to decide cluster formation

decide to do a collision based on \((d\sigma/d\Omega)_{NN}\)

\(C = N\)

do for \textbf{species} in \(p \uparrow, p \downarrow, n \uparrow, n \downarrow\) (in a random order)

\(P = \text{probability that } C \text{ forms a cluster with a nucleon of species}\)

- taking care of the non-orthogonality
- taking care of the \(p_{rel}\)-dependence of the phase space factors and the overlap probabilities

if \(\text{rand()} < P\) then

choose a nucleon \(B\) of \textbf{species}

\(C = C + B\) ! put the wave packets at the same phase space point

endif

endo
Correlations to bind several clusters

Clusters may form a loosely bound state.

\[ \text{e.g., } ^7\text{Li} = \alpha + t - 2.5 \text{ MeV} \]

Need more probability of \( |\alpha + t\rangle \rightarrow |^7\text{Li}\rangle \)

**Step 1**
Clusters (and nucleons) \( C_i \) and \( C_j \) are linked,
- if \( C_i \) is one of the 3 clusters closest to \( C_j \), and \( (i \leftrightarrow j) \),
- and if the distance is \( 1 \text{ fm} < |R_{ij}| < 7 \text{ fm} \),
- and if they are slowly moving away,
  \[ P_{ij}^2 / 2\mu_{ij} < 10 \text{ MeV} \text{ and } R_{ij} \cdot P_{ij} > 0. \]

**Step 2**
Linked clusters (CC) are identified.
Following steps are taken only for CC with mass number \( 6 \leq A \leq 9 \) or \( 19 \leq A \leq 23 \).

**Step 3**
Transition of the internal state of CC by eliminating the (radial component of) internal momentum

\[ P_i \rightarrow 0 \text{ for } i \in \text{CC in the c.m. of CC} \]

with some care of the momentum conservation.

**Next**
Energy conservation.
Correlations to bind several clusters

Clusters may form a loosely bound state.

\[ E_{\text{rel}} \]

\[ 0 \]

Step 4 Search a third particle for E-conservation

- A cluster \( C_k \) is selected, depending on the distance and momentum (\(|R_k|\) and \(|P_k|\)) relative to \( \text{CC} \).
- If the selected \( C_k \) already belongs to a \( \text{CC}' \), this whole \( \text{CC}' \) is treated as the third particle for E-conservation.

Step 5 Scale the radial component of the relative momentum between \( \text{CC} \) and \( C_k \) for the total energy conservation.

\[ P_k = P_{k\|} + P_{k\perp} \rightarrow \beta P_{k\|} + P_{k\perp} \]

Clusters may form a loosely bound state.

- e.g., \( ^7\text{Li} = \alpha + t - 2.5 \text{ MeV} \)
- Need more probability of \( |\alpha + t\rangle \rightarrow |^7\text{Li}\rangle \)

Akira Ono (Department of Physics, Tohoku University)
Transition from a wave packet to a plane wave

Each wave packet has a momentum width. E.g., it is an important part of the Fermi motion.

\[ f(p) = Ne^{-(p-P)^2/2\Delta p^2} \]

\[ p = P + \Delta p \]

The momentum fluctuation \( \Delta p \) is given to a wave packet when it is ‘emitted’, following Ono and Horiuchi, PRC53 (1996) 845 [a simple version of wp splitting].

- For a formed cluster, the momentum fluctuation is given to its center-of-mass motion.
- Total momentum and energy conservation.
- A particle is regarded as ‘emitted’ when there is no other particles around it in phase space within the radius \((\Delta r, \Delta v) = (3.5 \text{ fm, } 0.25c)\).
- Consistency with the method of the zero-point energy correction.
Effect of cluster correlations: central Xe + Sn at 50 MeV/u

Without clusters

With clusters

Cluster production in AMD model
Results for multifragmentation in central collisions

Xe + Sn

Ca + Ca at 35 MeV/u

Au + Au at 250 MeV/u

Multiplot with data points and lines for various energies and systems.

An improvement related to $\alpha$ clusters

Changes:

- Link two clusters only if at least one of them is $\alpha$.
- Don’t produce $\alpha$ clusters at high densities $\rho > \rho_0$. 

**Old**

**New**
Effect of cluster correlations: $p + \text{Al}$ at 180 MeV

The result is sensitive to the inmedium two-nucleon cross sections.

$$\sigma_{NN} = \sigma_0 \tanh(\sigma_{\text{free}}/\sigma_0), \quad \sigma_0 = y \rho^{-2/3}, \quad y = 4.$$  


c.f. Coupland et al., PRC84(2011)054603
$N/Z$ Ratio in $^{132}\text{Sn} + ^{124}\text{Sn}$ at 300 MeV/u (AMD with clusters)

\[
\left(\frac{N}{Z}\right)_{\text{gas}} = \frac{Y_n(v) + Y_d(v) + 2Y_t(v) + Y_h(v) + 2Y_\alpha(v)}{Y_p(v) + Y_d(v) + Y_t(v) + 2Y_h(v) + 2Y_\alpha(v)}
\]

$N/Z$ of spectrum of emitted particles is similar to the neutron-proton density difference at the compression stage.
\[ \left( \frac{N}{Z} \right)_{\text{gas}} = \frac{Y_n(v) + Y_d(v) + 2Y_t(v) + Y_h(v) + 2Y_\alpha(v)}{Y_p(v) + Y_d(v) + Y_t(v) + 2Y_h(v) + 2Y_\alpha(v)} \]

\( N/Z \) of spectrum of emitted particles is NOT similar to the neutron-proton density difference at the compression stage.
Summary of ratios, for $^{132}\text{Sn} + ^{124}\text{Sn}$ at 300 MeV/nucleon

Ikeno’s talk; Ikeno, Ono, Nara, Ohnishi, PRC 93 (2016) 044612
Recent developments of AMD

- Cluster correlations in the final states of NN collisions
- Binding of several clusters (production of Li, Be, ...)
- Treatment of the wave-packet momentum width
- Test particles sampled from $f(r, p)$ of AMD
  - Comparison with other models
  - Combining with another model (pion production)
  - Improvement of NN collision procedure