Applications of chiral nuclear forces up to $N^3$LO to nuclear matter and neutron stars

Christian Drischler

The 2017 ICNT Program at FRIB

April 5, 2017

[Credit: ORNL]
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Astrophysics
NS: mass, radius mergers, GWs

Nuclear Forces
(QCD, $\chi$EFT, ...)

Finite Nuclei
stable to exotic

Asym. Matter
EOS, saturation, ...

constrain

predict constrain fits

supported by:
Applications of chiral nuclear forces up to N³LO to nuclear matter and neutron stars

Motivation: Infinite Matter

Energy per particle: \( \frac{E}{A} (n, x, T) \)

based on chiral effective field theory (EFT):

- direct determination of astrophysical quantities: sym. energy, ...
- ideal to test (and to improve) nuclear forces \( \sim n_0 \)
- constrain neutron-star EOS: mass-radius relations, ...

\[ S_2(n) = \frac{1}{2} \frac{\partial^2}{\partial \beta^2} \frac{E}{A}(n, \beta) \Big|_{\beta=0} \]

\[ L = 3n_0 \frac{\partial}{\partial n} S_2(n) \bigg|_{n=n_0} \]
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**Landscape of Nuclear Matter**

**neutron matter** \((Z = 0)\): 
- Krüger et al., PRC 88, 025802 (2013)
- Gezerlis et al., PRL 111, 032501 (2013)
- Roggero et al., PRL 112, 221103 (2014)
- Wlazłowski et al., PRL 113, 182503 (2014)
- Lynn et al., PRL 116, 062501 (2016)
- Dyhdalo et al., PRC 94, 034001 (2016)

**symmetric matter** \((N = Z)\): 
- Hebeler et al. PRC 83, 031301(R) (2011)
- Coraggio, Holt et al. PRC 89, 044321 (2014)
- Wellenhofer et al., PRC 89, 064009 (2014)
- Carbone et al., PRC 90 054322 (2014)
- Coraggio, Holt et al., PRC 89, 044321 (2014)

...
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Constraining Neutron Stars

Improved needed:

• calculate asym. matter directly
• higher orders in the chiral and perturbative expansion

for QMC, see also: Gandolfi et al., Phys. Rev. C 85, 032801(R)
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Hierarchical Nuclear Forces in Chiral EFT

<table>
<thead>
<tr>
<th>Expansion</th>
<th>2N force</th>
<th>3N force</th>
<th>4N force</th>
</tr>
</thead>
<tbody>
<tr>
<td>$Q^0_{LO}$</td>
<td>X H</td>
<td>—</td>
<td>—</td>
</tr>
<tr>
<td>$Q^2_{NLO}$</td>
<td>X</td>
<td>—</td>
<td>—</td>
</tr>
<tr>
<td>$Q^3_{N^2LO}$</td>
<td></td>
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<tr>
<td>$Q^4_{N^3LO}$</td>
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</tbody>
</table>

Many-Body Forces

... and ongoing work at $N^4\text{LO}$ ...

Weinberg, van Kolck, Kaplan, Savage, Wise, Epelbaum, Kaiser, Machleidt, Meißner, ...

see: Epelbaum et al., PRL 115, 122301
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3N forces beyond Hartree-Fock?

Effective NN potentials
by summing one particle over the occupied states of the Fermi sea

» dominant 3N contributions

Holt et al., PRC 81, 024002
Hebeler et al., PRC 82, 014314

so far: only N²LO 3N and \( P = 0 \)

Improved Method

• applicable to all nuclear forces
• N³LO 3N forces due to recent PW decomposition

Hebeler et al., PRC 91, 044001

towards consistent N³LO calculations

some more applications:

Wellenhofer et al., PRC 92, 015801
Holt et al., Progr. Part. Nucl. Phys. 73, 35
Hebeler et al., Ann. Rev. Nucl. Part. Sci. 65, 457–84
...
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MBPT Diagrams

Initial NN Forces
Normal-Ordered 3N Forces
combinatorial factor

$V_{as} = V_{NN} + \xi V_{eff}$

Hartree-Fock

Second Order

+ Higher Orders

Hebeler et al., Phys. Rev. C 82, 014314
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Outline

0. Improved Normal-Ordering Method
1. Isospin-Asymmetric Nuclear Matter
2. Many-Body Convergence?
3. BCS Pairing Gaps in Neutron Matter
$S_2(n) = \frac{1}{2} \frac{\partial^2}{\partial \beta^2} \frac{E}{A}(n, \beta) \bigg|_{\beta=0}$

$L = 3n_0 \frac{\partial}{\partial n} S_2(n) \bigg|_{n=n_0}$

CD, Hebeler, Schwenk, PRC 93, 054314.

**ISOSPIN-ASYMMETRIC NUCLEAR MATTER**

**Objectives:** equation of state, saturation point, incompressibility, symmetry energy

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*see also:*

Vidaña et al., PRC 80, 045806
CD et al., PRC 89, 025806
Drews, Weise, PRC 91, 035802
Wellenhofer et al., PRC 93, 055802
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Equation of State

11 proton fractions
\[ x = 0.0, 0.05, \ldots, 0.5 \]
up to second order

\[ x = \frac{n_p}{n_n + n_p} \]

Uncertainty bands: Heabler et al., PRC 83, 031301(R)
- 7 Hamiltonians: evolved N³LO NN + bare N²LO 3N
- different combinations of \( \lambda/\Lambda_{3N} \)
- \( c_D, c_E \) fit only to few-body data
- free and Hartree-Fock spectrum
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Saturation Properties

Coester-like correlation
- covers the empirical range due to 3N contributions

\[ n_0 = (0.138 - 0.193) \text{ fm}^{-3} \]
\[ K = (182 - 254) \text{ MeV} \]

empirical saturation point:
max. range of 14 EDF's

CD, Hebeler, Schwenk, PRC 93, 054314

Coester et al., PRC 1, 769

Dutra et al., PRC 85, 035201
Kortelainen et al., PRC 89, 054314
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Symmetry Energy and Slope Parameter

**standard expansion:**
\[ \frac{E}{A}(n, \beta) = \frac{E_{SNM}(n)}{A} + S_2(n)\beta^2 + \ldots \]

\[ S_2(n) = S_v + \frac{L}{3} \left( \frac{n - n_0}{n_0} \right) + \ldots \]

**tight constraints**

\[ S_v = (30.9 \pm 1.4) \text{ MeV} \]

\[ L = (45.0 \pm 7.1) \text{ MeV} \]

in agreement with emp. extractions

Lattimer, Lim, Astrophys. J. **771**, 51

**quadratic expansion is reliable;**

but nonanalytical quartic term: \[ \beta^4 \ln |\beta| \]

Kaiser, PRC **91**, 065201

Wellenhofer *et al.*, PRC **93**, 055802

see also: Hagen *et al.*, Nat. Phys. **12**, 186
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Radius Estimate for a 1.4 $M_{\text{sun}}$ Neutron Star

Universal (empirical) relation by Lattimer, Prakash

Pressure constrained by CC calculations of $^{48}$Ca radii and measurement

Hagen et al., Nat. Phys. 12, 186

Lattimer, Prakash, APJ, 550, 426
Lattimer, Lim, Astrophys. J. 771, 51
CD, Carbone, Hebeler, Schwenk, PRC 94, 054307.

**MANY-BODY CONVERGENCE?**

**Objectives:** test many-body convergence
study impact of N^3LO 3N forces

*Weinberg eigenvalue analysis:* Hoppe, CD, Furnstahl, Hebeler, Schwenk, in prep.

**see also:**
Dickhoff, Barbieri, Prog. Part. Nucl. Phys. 52, 377
Rios *et al.*, PRC 79, 025802
Krüger *et al.*, PRC 88, 025802
Tews *et al.*, PRC 93, 024305
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Testing Many-Body Convergence

- consistent N³LO NN/3N forces
- finite proton fractions need reliable fits of $c_D, c_E$ at N³LO

Golak et al., Eur. Phys. J. A 50 177

Neutron Matter

- use always $c_i$'s recommended for N³LO calculations
- plus many-body uncertainty

Krebs et al., PRC 85, 054006

Uncertainty bands

<table>
<thead>
<tr>
<th>MBPT</th>
<th>SCGF Method</th>
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</thead>
<tbody>
<tr>
<td>Improved Normal-Ordering Method</td>
<td>nonperturbative</td>
</tr>
<tr>
<td>up to third order</td>
<td>full spectral function</td>
</tr>
<tr>
<td>free vs. HF spectrum</td>
<td>Extrapolated to $T=0$ MeV</td>
</tr>
</tbody>
</table>

see also: Carbone et al., PRC 90 054322
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**MBPT vs. SCGF Method**

CD, Carbone, Hebeler, Schwenk, PRC 94, 054307
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Testing Many-Body Convergence

Order-by-order analysis: (at saturation density)

• attractive second vs. repulsive third order
• MBPT well converged for EGM potentials (small third order)
• EM 500 MeV is less perturbative (larger third order)
• small energy shift due to $N^3$LO 3N w.r.t. $N^2$LO 3N contributions

$C_1 = -(0.75 - 1.13) \text{ GeV}^{-1}$
$C_3 = -(4.77 - 5.51) \text{ GeV}^{-1}$
$\Lambda_{3N} = + (2.0 - 2.5) \text{ fm}^{-1}$

CD, Carbone, Hebeler, Schwenk, PRC 94, 054307
see also:
Srinivas, Ramanan, PRC 94, 064303
Ding et al., PRC 94, 025802
Maurizio et al., PRC 90, 044003
Page et al., “Novel Superfluids”, Oxford University Press

BCS PAIRING GAPS
IN NEUTRON MATTER

Objectives: study subleading 3N contributions
recent (semi-)local NN potentials, new uncertainties
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Chiral NN Potentials and Regularization

<table>
<thead>
<tr>
<th></th>
<th>Short Range</th>
<th>Long Range</th>
<th>Potentials</th>
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<tbody>
<tr>
<td>Nonlocal</td>
<td>Nonlocal</td>
<td></td>
<td>e.g., EM, EGM; Carlsson et al., (PRX\ 6, 011019) (2016)</td>
</tr>
<tr>
<td>Local</td>
<td>Local</td>
<td></td>
<td>Gezerlis et al., (PRL\ 111, 122301) (2013)</td>
</tr>
<tr>
<td>Semilocal</td>
<td>Nonlocal</td>
<td>Local</td>
<td>Epelbaum et al., (EPJ A 51, 53) (2015), (PRL\ 115, 122301) (2015)</td>
</tr>
</tbody>
</table>
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Pairing Gaps: 3N forces in $^1S_0$

- uncertainties: 3N parameter variation (recommended values)
- pairing gap at low densities
  - universal gaps: strongly constrained by phase shifts
  - small 3N contributions: only small suppression for $k_F > 0.8$ fm$^{-1}$
  - almost independent of the energy spectrum
Applications of chiral nuclear forces up to \(N^3\)LO to nuclear matter and neutron stars

Pairing Gaps: 3N forces in \(^3P_2-^3F_2\)

- uncertainties: 3N parameter variation (recommended values)
- pairing gap at high densities
  - 3N forces add attraction: larger max. gap and closure at higher densities
  - effective masses are enhanced due to 3N forces
  - chiral EFT still efficient at \(k_F > 2\) fm\(^{-1}\)?
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(Semi-)Local NN: $^1S_0$ channel

CD, Krüger, Hebeler, Schwenk, PRC 95, 024302

local and semilocal NN forces:
- up to N²LO and N⁴LO
- $R_0 = 0.9, 1.0, 1.1$ and, 1.2 fm

new uncertainties (Epelbaum et al.)
order-by-order analysis in the chiral expansion (LO neglected)

findings:
- $Q(k_F) = \max\left(\frac{p}{\Lambda_b}, \frac{m_\pi}{\Lambda_b}\right)$
- at NLO and beyond gaps agree up to $k_F \sim (0.6–0.8) \text{ fm}^{-1}$
- sensitivity to spectrum is again small

Gezerlis et al., PRC 90, 054323
Applications of chiral nuclear forces up to $N^3\text{LO}$ to nuclear matter and neutron stars

(Semi-)Local NN: $^3\text{P}_2-^3\text{F}_2$ channel

Local and semilocal NN forces:
- up to $N^2\text{LO}$ and $N^4\text{LO}$
- $R_0 = 0.9, 1.0, 1.1$ and, $1.2$ fm

New uncertainties (Epelbaum et al.)
Order-by-order analysis in the chiral expansion (LO neglected)

Findings:
- Large uncertainties: breakdown of the chiral expansion?
Improved Normal-Ordering Method

• applicable to all 3N forces (incl. N^3LO)
• asymmetric matter: results for EOS, symmetry energy, ...

More Applications

• studied **many-body convergence** in neutron matter: N^3LO 3N forces beyond Hartree-Fock and in SCGF method
• **BCS pairing gaps** in \(^1S_0\) and \(^3P_2-^3F_2\):
  – N^3LO 3N contributions to previous NN potentials
  – recent (semi-)local NN potentials, new uncertainties

Extensions – a selection

• finite temperatures, consistently-evolved forces, ...
• **constrain next-gen. potentials: saturation**, … Carlsson et al., PRX 6, 011019
Applications of chiral nuclear forces up to $\text{N}^3\text{LO}$ to nuclear matter and neutron stars

Attractive Interactions: Phase Shifts

![Graph showing phase shifts for different states](image)
Pairing in neutron matter:
New uncertainty estimates and 3N forces

The Gap Equation

\[
\Delta_{lS}^J(k) = - \int_0^\infty \frac{dk'}{\pi} \frac{k'^2}{\sqrt{(\varepsilon_{k'} - \mu)^2 + \sum_{l', l, S, J} |\Delta_{l'S}^J(k')|^2}} \sum_{l'} \frac{i^{l'-l} V_{ll'S}^J(k, k')}{\Delta_{l'S}^J(k')} \]

Pairing in neutron matter: New uncertainty estimates and 3N forces

New Uncertainties

\[ \Delta X^{N^3LO}(p) = \max \left( Q^5 \times |X^{LO}(p)|, \right. \]
\[ Q^3 \times |X^{LO}(p) - X^{NLO}(p)|, \]
\[ Q^2 \times |X^{NLO}(p) - X^{N^2LO}(p)|, \]
\[ Q \times |X^{N^2LO}(p) - X^{N^3LO}(p)| \right), \]

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References for Asymmetric Matter Calculations

Calculations (variational, BHF, SCGF, IM-ChPT...)
Fiorilla, Kaiser, Holt, Weise, (2002-12)
Frick et al., PRC 71, 014313 (2005)
Vidaña et al., PRC 80, 045806 (2009)
Drischler et al. PRC 89, 025806 (2014)
Drews, Weise PRC 91, 035802 (2015)
Wellenhofer et al., PRC 93, 055802
Kaiser, PRC 91, 065201 (2015)
...

...
neutron stars are of extremes:

- $R \sim (10 - 14) \text{ km}$, $M \sim 2 M_{\text{sun}}$
- most densest objects we observe

outer core: $n \sim n_0$
- homogeneous, infinite

nuclear matter: well-suited system to apply/check
- nuclear forces
- many-body approaches
Nuclear Matter interacts via the Strong Interaction
(not considering Coulomb)

- fundamental theory is known
- QCD is non-perturbative at low energies of interest
- modern approach: chiral EFT
  - relevant degrees of freedom instead of quarks/gluons
  - use e.g., nucleons and pions
  - pion exchanges and short-range contact interactions
  - expand most general Lagrangian in powers of $Q = \max( p, m_\pi ) / \Lambda_b \sim 1/3$

Weinberg, NP B 363, 3 (1991)
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Neutron Stars in $\beta$ Equilibrium: $0 < x << 0.5$

Such calculations are more involved: less symmetries

$$x = \frac{n_p}{n_p + n_n} \quad \text{or,} \quad \beta = \frac{n_n - n_p}{n_n + n_p} \quad \text{with} \quad \beta = 1 - 2x$$

Obtaining the equation of state:

- parametrizations (fits to PNM plus empirical properties)
- empirically constrained coefficients

$$\frac{E}{A}(\beta, n) \overset{\text{Taylor}}{=} \sum_{i,j} C_{ij} \beta^i \left(\frac{n - n_0}{n_0}\right)^j$$

e.g., $C_{00} \sim -16$ MeV, $C_{20} \sim 31$ MeV, ...
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Normal-Ordering at Finite $T$

CD, Carbone, Hebeler, Schwenk, PRC 94, 054307

Improved Normal-ordering works also at finite $T$
### Applications of chiral nuclear forces up to $N^3$LO to nuclear matter and neutron stars

**Hierachy of Chiral EFT**

<table>
<thead>
<tr>
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<th>2N Force</th>
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<th>5N Force</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>LO</strong> $(Q/\Lambda_{\chi})^0$</td>
<td></td>
<td><img src="image" alt="Diagram" /></td>
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<td><strong>NLO</strong> $(Q/\Lambda_{\chi})^2$</td>
<td><img src="image" alt="Diagram" /></td>
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<tr>
<td><strong>NNLO</strong> $(Q/\Lambda_{\chi})^3$</td>
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<td><strong>$N^3$LO</strong> $(Q/\Lambda_{\chi})^4$</td>
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<tr>
<td><strong>$N^4$LO</strong> $(Q/\Lambda_{\chi})^5$</td>
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<tr>
<td><strong>$N^5$LO</strong> $(Q/\Lambda_{\chi})^6$</td>
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Eff. NN potential vs. full 3N force

Compare HF energies:

significant improvement: $P$-av. method, in particular for $n >> n_0$
Applications of chiral nuclear forces up to \( N^3LO \) to nuclear matter and neutron stars

Full \( N^3LO \) calculations

\[ \frac{E}{N}(n_0) = (16.7 - 21.1) \text{ MeV} \]

\[ \frac{E}{N}(n_1) = (15.7 - 18.4) \text{ MeV} \]

\[ \frac{E}{N}(n_0) = (14.7 - 17.0) \text{ MeV} \]

\[ \frac{E}{N}(n_0) = (14.7 - 21.1) \text{ MeV} \]

\[ c_1 = -(0.75 - 1.13) \text{ GeV}^{-1} \]

\[ c_3 = -(4.77 - 5.51) \text{ GeV}^{-1} \]

\[ \Lambda_{3N, 4N} = +(2.0 - 2.5) \text{ fm}^{-1} \]
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Constraints on the EOS

Hebeler et al., Astrophys. J., 773, 11