Understanding results of box calculation Hw2: nucleon evolution in a mean-field potential

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Box simulations: test of m.f. dynamics

$\lambda = \frac{2\pi}{k}$

$\rho(z, t=t_0) = \rho_0 + a_\rho \sin(k_i z)$

$k_i = n_i \frac{2\pi}{L}$

$a_\rho = 0.2 \rho_0$

Fermi sphere defined as a function of the local density

- Study the time evolution of $\rho(z)$

An example: SMF results

-- Symmetric matter --
- Only mean-field potential
- No surface terms
- Compressibility $K = 240$ MeV
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<th>BUU-type</th>
<th>QMD-type</th>
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*Analysis of the results performed by Yingxun Zhang and Yongjia Wang*
First formulation of Homework #2

BUU-like: 10 runs with 100 test particles
MD-like: 200 runs

Average $\rho(z,t)$

$n = 1$
Zoom at 40 fm/c:
Different oscillation frequency in the different models

![Graph showing different oscillation frequencies in BUU and QMD models at time t=40 fm/c.](image-url)
First formulation of Homework #2
First formulation of Homework #2
Box simulations: test of m.f. dynamics: space Fourier transform

Second formulation of Homework #2:
*Longer final time and results given each 0.5-1 fm/c*

\[ \rho(z,t=t_0) = \rho_0 + a_\rho \sin(k_i z) \]

- \( k_i = n_i \frac{2\pi}{L} \)
- \( a_\rho = 0.2 \rho_0 \)

- Study the time evolution of \( \rho(z) \) on a longer time interval
- Extract the Fourier transform in space

\[ \rho_k(t) = \int dz \sin(kz) \rho(z,t) \]

- Significant contribution only for \( k = k_i \) (to be checked)

*damped oscillations are expected*
\[ \rho_k(t) = \int dz \sin(kz) \rho(z,t) \]

where \( k = n \frac{2\pi}{L} \)

**SMF**

**ImQMD**

**Strong damping**
\[ \rho_k(t) = \int dz \sin(kz) \rho(z,t) \quad k = n_k \frac{2\pi}{L} \]
Output $\rho(z,t)$ with 1 or 0.5 fm/c

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Second formulation of Homework #2:
*Longer final time and results given each 0.5-1 fm/c*

\[ \rho_k(t) = \int dz \sin(kz) \rho(z,t) \quad k = n \frac{2\pi}{L} \]

Larger damping and structureless fluctuations

*Different oscillation frequency in BUU-like*

*In QMD-like*
Box simulations: test of m.f. dynamics: time Fourier transform

\[ \rho(z, t=t_0) = \rho_0 + a_\rho \sin(k_iz) \]

\[ k_i = n_i 2\pi/L \quad a_\rho = 0.2 \rho_0 \]

- Fourier transform in time: 
  \[ \rho_k(\omega) = \int dt \cos(\omega t) \rho_k(t) \]
\( \rho_k(t) = \int dz \sin(kz) \rho(z,t) \)

\( k = n \frac{2\pi}{L} \)

SMF simulations

Fourier transform with respect to time

\( \rho_k(\omega) = \int dt \cos(\omega t) \rho_k(t) \)

\( E = \hbar \omega \)

\( \omega / (k v_F) \sim 1 \quad n = 1, \ E \sim 18 \text{ MeV} \)
Fourier transform with respect to time:

All models
Linearized Vlasov equation $\rightarrow$ stationary solutions (oscillations) $\rightarrow$

extract the oscillation frequency

$\omega = \frac{s}{k v_F F_0}$

$1 + \frac{1}{F_0} = \frac{s}{2} \ln\left(\frac{s+1}{s-1}\right)$

Landau parameter $F_0 = \frac{K}{6 \varepsilon_F} - 1$

$K = 240 \text{ MeV} \Rightarrow F_0 = 0.1$

analytical relation between oscillation frequency and compressibility $K$

$F_0 \approx 0.1$

Fluctuations are amplified $\rightarrow$ fragment formation!

$s \approx 1$

$n = 1$, $E \approx 18 \text{ MeV}$
Evolution of Momentum Distribution

- $t=0 \text{ fm/c}$
- $t=60 \text{ fm/c}$
- $t=140 \text{ fm/c}$

- Vlasov $n=1$

- Plot shows the evolution of momentum distribution over time with different models represented by various lines.
Conclusions

- Model dependence of the oscillation frequency: Induced surface effects? \( F_0 \rightarrow F_0 g(k) \)

  Definition of local density and density-dependent mean-field potential should be checked and compared for all models

- The frequency extracted for BUU-like models is close to the analytical predictions

- Large damping observed for QMD-like models, probably caused by larger surface effects and by fluctuations
Some points to be discussed for HW 2

- Details about the procedure used to evaluate the density, in each model, should be given: how do induced surface effects impact the oscillation frequency?

- The evolution of the momentum distribution in some models needs to be understood.

- More damping in QMD-like models: why?
  The finite number of test particles (1 in this case) may act as a spurious collision term, driving the system towards classical behavior (see Reinhard & Suraud, ‘90)

  Surface effects may also be different.
Possible further investigations for HW 2

- In BUU-like approaches, check the sensitivity of results to test particle number

- Increase the compressibility $K$: more robust oscillations

- Investigate the variance of the density fluctuations at equilibrium:
  Ex. Non-interacting Fermi gas at temperature $T$:
  $$\sigma(V) = \rho / V \ast (3T) / (2 \varepsilon_F)$$

- Investigate unstable conditions: fluctuations will grow
  Investigate growth time and fragment formation

- Switch-on symmetry potential and investigate isovector fluctuations

- Combine mean-field and collision integral in the study of density oscillations
Propagation of fluctuations by the unstable mean-field

Box calculations: $\rho = 0.05$ fm, $^{3}\!\!T = 3$ MeV

Fourier analysis of the density variance $\delta \rho \delta \rho$: rapid growth of density fluctuations

Fragment multiplicity and charge distributions (300 nucleons)