Stochastic Mean Field (SMF) description

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Dynamics of many-body system I

\[ H = H_0 + \nu_{1,2} \]

\[ \rho_2(12,1'2') = \rho_1(1,1')\rho_1(2,2') + \delta\sigma(12,1'2') \]

\[ \frac{i\hbar}{\partial t} \rho_1(1,1',t) = \langle 1 | [H_0, \rho_1(t)] | 1' \rangle + K[\rho_1] + \delta K[\rho_1, \delta\sigma] \]

\[ K = F(\rho_1, |v|^2) \]

\[ \delta K = F(v, \delta\sigma) \]

Average effect of the residual interaction

\[ <\delta K> = 0 \]

\[ <\delta K \delta K> \rightarrow \text{Fluctuations} \]
Dynamics of many-body systems II

Collision Integral

\[ \mathbf{K} = g \sum_{234} W(12; 34) \left[ \bar{f}_1 \bar{f}_2 f_3 f_4 - f_1 f_2 \bar{f}_3 \bar{f}_4 \right] \]

\[ \bar{f} = 1 - f \]

Transition rate \( W \) interpreted in terms of NN cross section

-- If statistical fluctuations larger than quantum ones

\[ < \delta K(p, t) \delta K(p', t') > = C \delta(t - t') \]

\[ C(p_a, p_b, r, t) = \delta_{ab} \sum_{234} W(a2; 34) F(a2; 34) \]

\[ F(12; 34) = f_1 f_2 \bar{f}_3 \bar{f}_4 + \bar{f}_1 \bar{f}_2 f_3 f_4. \]

Main ingredients:

- Residual interaction (2-body correlations and fluctuations)
- In-medium nucleon cross section
- Effective interaction (self consistent mean-field) \textit{Skyrme, Gogny forces}
- Effective interactions

Energy Density Functional theories: The exact density functional is approximated with powers and gradients of one-body nucleon densities and currents.
The nuclear Equation of State (T = 0)

Energy per nucleon $E/A$ (MeV)

$E (\rho, \beta) = E (\rho, \beta = 0) + E_{\text{sym}} (\rho) \beta^2 + O(\beta^4)$

symm. matter

symm. energy

$\beta = \text{asymmetry parameter} = (\rho_n - \rho_p)/\rho$

analogy with Weizsacker mass formula for nuclei (symmetry term)!

Symmetry energy $E_{\text{sym}} (\rho)$

$E_{\text{sym}} (\rho) = S_0 + L \frac{\rho - \rho_0}{3 \rho_0} + \ldots$

25 $\leq J \leq$ 35 MeV

20 $\leq L \leq$ 120 MeV
1. **Semi-classical approximation to Nuclear Dynamics**

Transport equation for the one-body distribution function $f$

Semi-classical analog of the Wigner transform of the one-body density matrix

\[
\frac{df(r, p, t)}{dt} = \frac{\partial f(r, p, t)}{\partial t} + \{f, H_0\} = 0
\]

\[
H_0 = T + U
\]

The mean-fielss potential $U$ is self-consistent: $U = U(\rho)$

Nucleons move in the field created by all other nucleons

**Vlasov Equation**, like Liouville equation:
The phase-space density is constant in time

**Semi-classical approximation** $\rightarrow$ **transport theories**

Boltzmann-Langevin
Correlations, Fluctuations
From BOB to SMF …..

- Fluctuations from *external stochastic* force (tuning of the most unstable modes)

\[ f = \overline{I[f]} + \frac{\partial U_{\text{ext}}}{\partial r} \frac{\partial f}{\partial p} \]

Brownian One Body (BOB) dynamics

\[ \lambda = 2\pi/k \]

*Chomaz, Colonna, Guarnera, Randrup*  
*PRL73, 3512 (1994)*

multifragmentation event
From BOB to SMF ……

- Fluctuations from \textit{external} stochastic force (tuning of the most unstable modes)

\[ \dot{f} = \bar{I}[f] + \frac{\partial U_{\text{ext}}}{\partial r} \frac{\partial f}{\partial p} \]

Brownian One Body (BOB) dynamics

\[ \lambda = 2\pi/k \]

\textit{multifragmentation event}

- Stochastic Mean-Field (SMF) model:
  Thermal fluctuations (at local equilibrium) are projected on the coordinate space by agitating the spacial density profile

\[ \text{M. Colonna et al., NPA642(1998)449} \]
Details of the model

\[ f(r, p, t) = \frac{C\hbar^3}{4} \sum_i g_r(r - r_i) g_p(p - p_i), \]

triangular functions in \( r \) space (for \( g_r \)) and \( \delta \) functions in momentum space (for \( g_p \))

\[ g(x^j - x_i^j) = 2l - |x^j - x_i^j|, \]

Total number of test particles: \( N_{\text{tot}} = N_{\text{test}} \ast A \)

System total energy (lattice Hamiltonian):

\[ E_{\text{tot}} = \sum_i p_i^2 / (2m) + N_{\text{test}} \int dr \ \rho(r) E_{\text{pot}}(\rho_n, \rho_p) + \int dr \ \rho_p(r) E_{\text{pot}}^{\text{Coul}}(\rho_p)/2, ] \]

\( l = 1 \ \text{fm} \), lattice size
Potential energy

\[ E_{pot}(\rho) = \frac{A}{2} \tilde{\rho} + \frac{B}{\sigma + 1} \tilde{\rho}^\sigma + \frac{C_{surf}}{2\rho} (\nabla \rho)^2 + \frac{1}{2} C_{sym}(\rho) \tilde{\rho} \beta^2, \]

where \( \tilde{\rho} = \rho / \rho_0 \) (\( \rho_0 \) denotes the saturation density),
\( A = -356 \text{ MeV}, B = 303 \text{ MeV}, \sigma = 7/6. \) \( \beta = \) asymmetry parameter

\[ C_{surf} = -6 / \rho_0^{5/3} \text{ MeV fm}^5. \]

Symmetry energy parametrizations:

<table>
<thead>
<tr>
<th>asy-EoS</th>
<th>( E_{sym}/A )</th>
<th>L(MeV)</th>
</tr>
</thead>
<tbody>
<tr>
<td>asysoft</td>
<td>30.</td>
<td>14.</td>
</tr>
<tr>
<td>asystiff</td>
<td>28.</td>
<td>73.</td>
</tr>
<tr>
<td>asysupstiff</td>
<td>28.</td>
<td>97.</td>
</tr>
</tbody>
</table>

New Skyrme interactions (SAMi-J family) recently introduced

Hua Zheng et al.

the Coulomb potential is determined solving the Laplace equation:

$$\nabla^2 E_{pot}^{Coul} = -4\pi e^2 \rho_p = -18.1 \rho_p$$

Initialization and dynamical evolution

- Ground state initialization with Thomas-Fermi
- Test particle positions and momenta are propagated according to the Hamilton equations (non relativistic)
Details of the model: Collision Integral

- **Mean free path method:**
  
  *each test particle has just one collision partner*

\[
\tau_{col} = \frac{\lambda}{v_{kl}} = \frac{1}{\rho \sigma_{NN} v_{kl} }, \quad P_{col}(\Delta t) \approx \frac{\Delta t}{\tau_{col}}
\]

\(\Delta t = \text{time step}\)

- Blocking factors, defined as \(P_{Pauli} = (1 - f_l)(1 - f_k)\),

we now take a \(\Theta\) function in \(r\) space and a gaussian function, with \(\sigma = 29 \text{ MeV}/c\), in momentum space. The \(\Theta\) function is defined as: \(\Theta(r - r_i) = \Theta(R - |r - r_i|)\), with \(R = 2.53 \text{ fm}\). The new definition makes the occupation number smoother (though less local), reducing fluctuations which may induce spurious collisions.

- Free n-p and p-p cross sections, with a maximum cutoff of 50 mb
When local equilibrium is achieved:

\[ \sigma_f^2 = f(1-f) \]

Fluctuation variance for a fermionic system at equilibrium

\[ \sigma^2_{\rho}(\mathbf{r}, t) = \frac{1}{V} \int \frac{d\mathbf{p}}{\hbar^3/4} \sigma_f^2(\mathbf{r}, \mathbf{p}, t). \]

\[ \sigma^2_{\rho} = \frac{\rho}{V} \frac{3T}{2\varepsilon_F} \left[ 1 - \frac{\pi^2}{12} \left( \frac{T}{\varepsilon_F} \right)^2 + \ldots \right]. \]

Stochastic Mean-Field (SMF) model:
Fluctuations are projected on the coordinate space by agitating the spacial density profile

Some applications ……

- **Fragmentation studies in central and semi-peripheral collisions**

- **Isospin effects at Fermi energies**

- **Small amplitude dynamics (collective modes) and low-energy reaction dynamics**

![Fragmentation studies graph](image1)

![Isospin effects graph](image2)

![Small amplitude dynamics graph](image3)