Pions in pBUU

Pawel Danielewicz
National Superconducting Cyclotron Laboratory
Michigan State University

Transport 2017: International Workshop on Transport Simulations for Heavy Ion Collisions under Controlled Conditions
FRIB-MSU, East Lansing, Michigan, March 27 - 30, 2017
**Interest: π as Probe of High-ρ Symmetry Energy**

B-A Li PRL88(02)192701: $S(\rho > \rho_0) \Rightarrow n/p_{\rho > \rho_0} \Rightarrow \pi^-/\pi^+$

Pions originate from high $\rho$
Simulations of Heavy-Ion Collisions

Separation of time and distance scales:

Short scales reduced to negligible extent with outcomes of events treated probabilistically

Long scales treated explicitly and deterministically

Cut-off scales: $t \sim 1 \text{ fm}/c$, $r \lesssim 1 \text{ fm}$

Primarily binary collision processes

Equation of state: if there is an optical potential affecting a particle, that particle impacts the interaction parts of thermodynamic functions.

Low-$E$ pion production: $N + N \leftrightarrow N + \Delta$, $\Delta \leftrightarrow N + \pi$
\( \Delta \text{ in } \pi-N \text{ Interactions} \)

\( \pi-p \) scattering cross sections

\[
\sigma = \frac{\pi}{p^2} \frac{2J+1}{2s+1} \frac{\Gamma^2}{(E - m_\Delta c^2)^2 + \Gamma^2/4} \equiv \frac{\pi}{p^2} \frac{2J+1}{2s+1} \Gamma \mathcal{A}_\Delta(E)
\]

\( J = 3/2, \ m_\Delta = 1232 \text{ MeV}/c^2, \ \Gamma(p) \propto p^3, \ \mathcal{A}_\Delta \) - spectral funct
Inelastic $NN$ Interactions

Decomposition of inelastic $NN$ cross section

Weil et al EPJA48(12)111
Production and Absorption: Detailed Balance

Time reversal symmetry: same magnitude of mtx element for forward & backward process, 
\[ |M_{NN\rightarrow N\Delta}| = |M_{N\Delta\rightarrow NN}|. \]

\[
\frac{dN_{\Delta}}{dt} \propto \int dp \, dm_{\Delta} \, \delta(p_{N} + p_{\Delta} - p_{N} - p_{N}) \\
\times \delta(\epsilon_{N} + \epsilon_{\Delta} - \epsilon_{N} - \epsilon_{N}) \\
\times |M_{NN\rightarrow N\Delta}|^{2} (f_{N} f_{N} - f_{N} f_{\Delta}) \, A_{\Delta}
\]

in equilibrium: \( f = e^{(\mu - \epsilon)/T} \)

\[
\sigma \, \nu \propto \int dp \, dm_{\Delta} \, \delta(E - E) \, |M|^{2} \, A_{\Delta}
\]

Detailed-balance relation: \( \sigma_{NN\rightarrow N\Delta} \rightleftharpoons \sigma_{N\Delta\rightarrow NN} \)

Relation nontrivial for \( \Delta \) due to mass spread.

Balance violated: no thermal distribution, no law of mass action.
Production and Absorption: Detailed Balance

Time reversal symmetry: same magnitude of mtx element for forward & backward process,
\[ |M_{NN \rightarrow N\Delta}| = |M_{N\Delta \rightarrow NN}|. \]

\[
\frac{dN_{\Delta}}{dt} \propto \int dp \, dm_{\Delta} \, \delta(p_N + p_{\Delta} - p_N - p_N) \\
\times \delta(\epsilon_N + \epsilon_{\Delta} - \epsilon_N - \epsilon_N) \\
\times |M_{NN \rightarrow N\Delta}|^2 (f_N f_{N\Delta} - f_N f_{\Delta}) \, A_{\Delta}
\]

in equilibrium: \( f = e^{(\mu - \epsilon)/T} \)

\[
\sigma \nu \propto \int dp \, dm_{\Delta} \, \delta(E - E) \, |M|^2 \, A_{\Delta}
\]

Detailed-balance relation: \( \sigma_{NN \rightarrow N\Delta} \leftrightarrow \sigma_{N\Delta \rightarrow NN} \)

Relation nontrivial for \( \Delta \) due to mass spread.

Balance violated: no thermal distribution, no law of mass action!
\[ \Delta \leftrightarrow N + \pi \quad \Rightarrow \quad U_\Delta = U_N + U_\pi \]

‘Conservation’ of potential consistent with the quark perspective. Also greatly facilitates calculations of process kinematics as thresholds in kinetic energy stay put.

Ferini et al NPA762(05)147: \( U_\pi = 0 \) & \( U_\Delta = U_N \) employed in most models, including IBUU.

However, a strong isospin-dependent potential is needed to explain the existence of pionic atoms!

pBUU: \( U \) dependent on conserved quantities, density of baryon number and isospin - \( \pi \) end up with potentials that depend on isospin & symmetry energy.
\[ \Delta \leftrightarrow N + \pi \quad U_\Delta \equiv U_N + U_\pi \]

'Conservation' of potential consistent with the quark perspective. Also greatly facilitates calculations of process kinematics as thresholds in kinetic energy stay put.

Ferini et al NPA762(05)147: \( U_\pi = 0 \) & \( U_\Delta = U_N \) employed in most models, including IBUU.

However, a strong isospin-dependent potential is needed to explain the existence of pionic atoms!

pBUU: \( U \) dependent on conserved quantities, density of baryon number and isospin - \( \pi \) end up with potentials that depend on isospin & symmetry energy.
π vs Baryon Optical Potentials

\[ \Delta \leftrightarrow N + \pi \quad U_\Delta \equiv U_N + U_\pi \]

’Conservation’ of potential consistent with the quark perspective. Also greatly facilitates calculations of process kinematics as thresholds in kinetic energy stay put.

Ferini et al NPA762(05)147: \( U_\pi = 0 \) & \( U_\Delta = U_N \) employed in most models, including IBUU.

However, a strong isospin-dependent potential is needed to explain the existence of pionic atoms!

pBUU: \( U \) dependent on conserved quantities, density of baryon number and isospin - \( \pi \) end up with potentials that depend on isospin & symmetry energy.
$\pi$ vs Baryon Optical Potentials

$\Delta \leftrightarrow N + \pi \quad U_\Delta \equiv U_N + U_\pi$

’Conservation’ of potential consistent with the quark perspective. Also greatly facilitates calculations of process kinematics as thresholds in kinetic energy stay put.

Ferini et al NPA762(05)147: $U_\pi = 0$ & $U_\Delta = U_N$ employed in most models, including IBUU.

However, a strong isospin-dependent potential is needed to explain the existence of pionic atoms!

pBUU: $U$ dependent on conserved quantities, density of baryon number and isospin - $\pi$ end up with potentials that depend on isospin & symmetry energy
Symmetry-Energy Derived $\pi$ Potential

Jun Hong&PD PRC90(14)024605  Nucl density: Thomas-Fermi
Pions Probe System at High-$\rho$!

Song&Ko PRC91(15)014901

PD PRC51(95)716

$\pi$ test the maximal densities reached and collective motion then

π test the maximal densities reached and collective motion then
Pions as Probe of High-$\rho$ Symmetry Energy

B-A Li PRL88(02)192701: $S(\rho > \rho_0) \Rightarrow n/p_{\rho > \rho_0} \Rightarrow \pi^-/\pi^+$

Pions originate from high $\rho$
Interpretation of FOPI Data

Reisdorf et al NPA781(07)459

Transport IBUU04 Xiao et al PRL102(09)062502

Symmetry energy dropping with $\rho$, at $\rho > \rho_0$!?
Net $\pi$ Yields and $U(\rho, p)$ in pBUU

Reisdorf et al NPA781(07)459

$^{197}$Au+$^{197}$Au

$\pi^-$ FOPI $\pi^+$ FOPI

(a) mom-indep.MF (b) mom-dep.MF

Jun Hong & PD PRC90(14)024605, $\pi^-$ and $\pi^+$

?Imperfect Mom Dependence?? [No sensitivity to $\pi/\Delta$ rates] affects maximal densities reached
\[ \pi \text{ Yields Reproduced with Softened } U(p) \]

solid: softened \( U(p) \)

but then...
Inferior Description of Midrapidity Flow Anisotropy

solid: new $U(p)$, dashed: old $U(p)$

Jun Hong & PD PRC90(14)024605

$R_N \leftrightarrow$ elliptic flow

too weak with new $U(p)$
FOPI $\pi^-/\pi^+$ Reproduced by pBUU

... irrespectively of $U(\rho)$, right panel

Left panel: discrepancies in the literature - correlation vs anticorrelation of $S(\rho > \rho_0)$ with $\pi^-/\pi^+$. 
FOPI $\pi^-/\pi^+$ Reproduced by pBUU

\[ \text{...irrespective of } S_{\text{int}}(\rho) = S_0 \left( \frac{\rho}{\rho_0} \right)^\gamma : \]

?no hope?  

\textbf{Au+Au}
Original Idea Still Correct for High-$E$ $\pi$'s

\[ S_{\text{int}}(\rho) = S_0 \left(\frac{\rho}{\rho_0}\right)^\gamma \]
$n/p$ Ratio in pBUU at $\rho > \rho_0$

changes with the supranormal symmetry energy:

$$S_{\text{int}}(\rho) = S_0 (\rho/\rho_0)^\gamma$$
Why Differences for Net $\pi$ Ratios?

In pBUU isospin-driven $\pi^\pm$ optical potential

$\pi/\Delta$ rate sensitivities claimed in Larionov&Mosel NPA728(03)135; Prassa et al NPA789(07)311 and Song&Ko PRC91(15)014901. Virtually none there in pBUU!
Changing mo-dep of MF: either $v_2$ good or near-threshold $M_\pi$, but not both!

$$R_N = \frac{1 - v_2}{1 + v_2}$$
Tinkering with Incompressibility

Results so far for $K = 210$ MeV.

While elliptic flow is more sensitive to the momentum dependence of mean field, or $m^* / m$, the sensitivity to incompressibility $K$ is also there!

$K = 380$ MeV
Sensitivity of Elliptic Flow to $m^*/m$ and $K$

$K = 270 \text{ MeV}$
and changing $m^*/m$

$m^*/m = 0.7$
and changing $K$

Hysteresis in both cases due to competition between density and momentum dependence
Sensitivity of $M_\pi$ to Incompressibility $K$

$m^*/m = 0.75$ and changing $K$
Raising $K$ Allows to Describe Both $M_\pi$ and $\nu_2$!

Bands for $K = (240 - 300)$ MeV & optimal $m^*/m$

→ Constraints on EOS, at moderately supranormal densities, à la LeFèvre et al
Energy Per Nucleon

Symmetric Matter

![Graph showing energy per nucleon](image.png)
Pressure

Symmetric Matter

![Graph showing pressure vs density with various theoretical and experimental data points.](image)
Conclusions

- Detailed balance must be obeyed for thermodynamic consistency
- Uncertainties in the near-threshold $\pi$ production include $\pi$ & $\Delta$ optical potentials & in-medium rates
- Pions probe high-$\rho$ matter, net density, $n/p$-ratio, collective flow there! \dots $U(p)$ & $K$
- pBUU reproduces FOPI $\pi^-/\pi^+$, irrespectively of details in $U$ and $S$
- High-energy $\pi^+/\pi^-$ ratio more robust than ratio of net yields
- Efforts to reproduce simultaneously collective flow and pion yields lead to EOS constraints at moderately supranormal densities

Supported by National Science Foundation under Grant US PHY-1403906
Conclusions

- Detailed balance must be obeyed for thermodynamic consistency
- Uncertainties in the near-threshold $\pi$ production include $\pi$ & $\Delta$ optical potentials & in-medium rates
- Pions probe high-$\rho$ matter, net density, $n/p$-ratio, collective flow there! . . . $U(p)$ & $K$
- $p$BUU reproduces FOPI $\pi^-/\pi^+$, irrespectively of details in $U$ and $S$
- High-energy $\pi^+/\pi^-$ ratio more robust than ratio of net yields
- Efforts to reproduce simultaneously collective flow and pion yields lead to EOS constraints at moderately supranormal densities

Supported by National Science Foundation under Grant US PHY-1403906