Cluster Production in pBUU
- Past and Future

Pawel Danielewicz

National Superconducting Cyclotron Laboratory
Michigan State University

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Boltzmann Equation Model (BEM/pBUU)

Degrees of freedom ($\mathcal{X}$):
nucleons, deuterons, tritons, helions ($A \leq 3$), $\Delta$, $N^*$, pions

Fundamentals:

- Relativistic Landau theory (Chin/Baym)

  *Energy functional* ($\epsilon$)

- Real-time Green’s function theory

  *Production/absorption rates* ($\mathcal{K}^<$, $\mathcal{K}^>$)

\[
\frac{\partial f}{\partial t} + \frac{\partial \epsilon}{\partial \mathbf{p}} \frac{\partial f}{\partial \mathbf{r}} - \frac{\partial \epsilon}{\partial \mathbf{r}} \frac{\partial f}{\partial \mathbf{p}} = \mathcal{K}^< (1 \mp f) - \mathcal{K}^> f
\]

production absorption rate
Single-Particle Energies & Functional

\[
\frac{\partial f}{\partial t} + \frac{\partial \epsilon}{\partial p} \frac{\partial f}{\partial r} - \frac{\partial \epsilon}{\partial r} \frac{\partial f}{\partial p} = \mathcal{K}^< (1 \mp f) - \mathcal{K}^> f
\]

The single-particle energies \( \epsilon \) are given in terms of the net energy functional \( E\{f\} \) by,

\[
\epsilon(p) = \frac{\delta E}{\delta f(p)}
\]

In the local cm, the mean potential is

\[
U_{opt} = \epsilon - \epsilon_{kin}
\]

and \( \epsilon_{kin} = \sqrt{p^2 + m^2} \)
Energy Functional

The functional:

\[ E = E_{vol} + E_{gr} + E_{iso} + E_{Coul} \]

where

\[ E_{gr} = \frac{a_{gr}}{\rho_0} \int d\mathbf{r} (\nabla \rho)^2 \]

For covariant volume term, ptcle velocities parameterized in local frame:

\[ v^*(p, \rho) = \frac{p}{\sqrt{p^2 + m^2} / \left(1 + c \frac{\rho}{\rho_0} \frac{1}{(1+\lambda p^2/m^2)^2} \right)^2} \]

precluding a supraluminal behavior, with \( \rho \) - baryon density. The 1-ptcle energies are then

\[ \epsilon(p, \rho) = m + \int_0^p dp' v^* + \Delta \epsilon(\rho) \]

Parameters in the velocity varied to yield different optical potentials characterized by values of effective mass,

\[ m^* = p_F / v_F. \]
Potential from p-scattering (Hama et al. PRC41(90)2737) & parameterizations

Ground-state densities from electron scattering and from functional minimization. From \( E(f) = \min \):

\[
0 = \epsilon \left( p^F(\rho) \right) - 2 a_{gr} \nabla^2 \left( \frac{\rho}{\rho_0} \right) - \mu
\]

\( \Rightarrow \) Thomas-Fermi eq.
Many-Body Theory

Transport eq. for nucleons follows from the eq. of motion for the 1-ptcle Green’s function (KB eq.). Transport eq. for deuterons ($A = 2$) from the eq. for 2-ptcle Green’s function??

Wigner function in second quantization

$$f(p; R, T) = \int dr \, e^{-i p r} \langle \hat{\psi}^\dagger (R - r/2, T) \hat{\psi}_H (R + r/2, T) \rangle$$

where $\langle \cdot \rangle \equiv \langle \psi | \cdot | \psi \rangle$ and $| \psi \rangle$ describes the initial state.

Evolution driven by a Hamiltonian. Interaction Hamiltonian:

$$\hat{H}^1 = \frac{1}{2} \int dx \, dy \, \hat{\psi}^\dagger (x) \hat{\psi}^\dagger (y) v(x - y) \hat{\psi} (y) \hat{\psi} (x),$$
Evolution Contour

\[ \langle \hat{O}_H(t_1) \rangle = \langle T^a \left[ \exp \left( -i \int_{t_1}^{t_0} dt' \, \hat{H}_I^1(t') \right) \right] \hat{O}_I(t_1) \]
\[ \quad \times T^c \left[ \exp \left( -i \int_{t_0}^{t_1} dt' \, \hat{H}_I^1(t') \right) \right] \rangle \]
\[ = \langle T \left[ \exp \left( -i \int_{t_0}^{t_1} dt' \, \hat{H}_I^1(t') \right) \right] \hat{O}_I(t_1) \rangle , \]

Expectation value expanded perturbatively in terms of \( V \) and noninteracting 1-ptcle Green’s functions on the contour

\[ iG_0(x, t, x', t') = \langle T \left[ \hat{\psi}_I(x, t) \hat{\psi}_I^\dagger(x', t') \right] \rangle \]
Single-Particle Evolution

Wigner function corresponds to a particular case of the Green’s function on contour:

\[ f(p; R, T) = \int dr e^{-ipr} (\mp i)G^<(R + r/2, T, R - r/2, T) \]

If we find an equation for \( G \), this will also be an equation for \( f \).

Dyson eq. from perturbation expansion:

\[ G = G_0 + G_0 \Sigma G \]

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Outcome of Evolution

Formal solution of the Dyson eq:

\[ \mp i G^<(x, t; x', t') = \int dx_1 \, dt_1 \, dx'_1 \, dt'_1 \, G^+(x, t; x_1, t_1) \]
\[ \times (\mp i) \Sigma^< (x_1, t_1; x'_1, t'_1) \, G^- (x, t; x_1, t_1) \]

and

\[ \mp i \Sigma^< (x, t; x', t') = \langle \hat{j}^\dagger (x', t') \hat{j}(x, t) \rangle_{irred} \]

where the source \( j \) is

\[ \hat{j}(x, t) = [\hat{\psi}(x, t), \hat{H}^1] \]
Quasiparticle Limit

Under slow spatial and temporal changes in the system, the Green’s function expressible in terms of the Wigner function $f$ and 1-particle energy $\epsilon_p$

$$\mp iG^<(x, t; x', t') \approx \int dp \ f\left(p; \frac{x + x'}{2}, \frac{t + t'}{2}\right) e^{i(p(x-x') - \epsilon_p(t-t'))}$$

Then also Boltzmann eq:

$$\frac{\partial f}{\partial t} + \frac{\partial \epsilon_p}{\partial p} \frac{\partial f}{\partial r} - \frac{\partial \epsilon_p}{\partial r} \frac{\partial f}{\partial p} = -i\Sigma^<(1 - f) - i\Sigma^> f$$

$$\mp i\Sigma^< : \quad \rightarrow \quad x \times \rightarrow$$
2-Particle Green’s Function

Transport eq. for deuterons \((A = 2)\) from the eq. for 2-ptcle Green’s function??

\[
iG^<_2 = \langle \hat{\psi}^\dagger(x'_1, t') \hat{\psi}^\dagger(x'_2, t') \hat{\psi}(x_2, t) \hat{\psi}(x_1, t) \rangle
\]

For the contour function:

\[
G_2 = G + G \times v \times G_2
\]

where \(G\) – irreducible part of \(G_2\) (w/o two 1-ptcle lines connected by the potential \(v\); anything else OK)

In terms of retarded Green’s function \(G^<_2\):

\[
iG^<_2 = (1 + v \times G^+_2) \times iG^<_2 \times (1 + v \times G^<_2)
\]
Deuteron Quasiparticle Limit

In the limit of slow spatial and temporal changes, deuteron contribution to the 2-ptcle Green’s function:

\[ iG_2^\leq = \langle \hat{\psi}^\dagger(x'_1 t') \hat{\psi}^\dagger(x'_2 t') \hat{\psi}(x_2 t) \hat{\psi}(x_1 t) \rangle \]

\[ \simeq \int dp \phi_d^*(r') \phi_d(r) e^{ip\left(\frac{r_1+r_2}{2} - \frac{r'_1+r'_2}{2}\right)} e^{-i\epsilon_d(t-t')} + \cdots, \]

where \( R = \frac{1}{4}(x_1 + x_2 + x'_1 + x'_2) \), \( r = x_1 - x_2 \)
\( \phi_d \) and \( f_d \) – internal wave function and cm Wigner function
\( \cdots\equiv \text{continuum} \)

Transport eq from integral quantum eq of motion:

\[ \frac{\partial f_d}{\partial T} + \frac{\partial \epsilon_d}{\partial p} \frac{\partial f_d}{\partial R} - \frac{\partial \epsilon_d}{\partial R} \frac{\partial f_d}{\partial p} = \mathcal{K}^\text{<} (1 + f_d) - \mathcal{K}^\text{>} f_d \]
Wave Equation

From Green's function eq, the equation for wavefunction:

\[
(\epsilon_d(P) - \epsilon_N(P/2 + p) - \epsilon_N(P/2 - p)) \phi_d(p) \\
- (1 - f_N(P/2 + p) - f_N(P/2 - p)) \int dp' \nu(p - p') \phi_d(p') = 0
\]

In zero-temperature matter, discrete states lacking over a vast range of momenta
Cluster Production & Absorption

?? Production & absorption rates: \[ \hat{\mathcal{K}} = \phi^* v \hat{\mathcal{G}} v \phi \]

Leading contribution

\[ \mathcal{K} = \int dr dr' \phi_d^* v \langle \hat{\psi}^\dagger(x_1, t') \hat{\psi}(x_1, t) \rangle \langle \hat{\psi}^\dagger(x_2, t') \hat{\psi}(x_2, t) \rangle v \phi_d \]

Leading-order in the quasiparticle expansion: neutron & proton come together and make a deuteron.

If system approximately uniform and stationary, the process not allowed by energy-momentum conservation.

Process possible in a mean field varying in space, but, in nuclear case, the high-energy production rate low – tested in Glauber model.
3-Nucleon Collisions

First correction to the pure 1-particle state, from a coupling to p-h excitations, yields a contribution to the d-production due to 3-nucleon collisions.

Still more nucleons involved in production of heavier clusters.
Deuteron Production

Detailed balance:

\[ |M_{npN \rightarrow Nd}|^2 = |M_{Nd \rightarrow Nnp}|^2 \propto d\sigma_{Nd \rightarrow Nnp} \]

Thus, production can be described in terms of breakup.

Problem: Breakup cross section only known over limited range of final states - Interpolation/extrapolation needed

Impulse approximation works at high incident energy

\[ |M_{Nd \rightarrow Npn}|_{IA}^2 = |M_{2}|^2 + |M_{1}|^2 + |M_{3}|^2 \]
Renormalized Impulse Approximation

Renormalization factor for squared matrix element to get breakup cross section right as a function of energy

\[ \frac{d\sigma^{Nd \rightarrow Nnp}}{d\Omega} \propto F \sigma_{NN} |\phi_d(p)|^2 \]
Single-Particle Spectra

\[ C + C \rightarrow p + X \; \text{0.8 GeV/nucleon} \]

\[ C + C \rightarrow d + X \; \text{0.8 GeV/nucleon} \]

Proton & Deuteron inclusive spectra

Histograms: calculations using

\[ |\mathcal{M}^{npN \rightarrow Nd}|^2 = |\mathcal{M}^{Nd \rightarrow npN}|^2 \propto d\sigma_{Nd \rightarrow npN} \]

and \( \langle f \rangle < 0.2 \) cut-off for deuterons
$A = 3$ Particles + Tests

$A=3$-ptcles from 4N collisions

Christiane Kuhrt: solving finite-$T$ Galitski-Feynman (GF) and modified (in-medium) Alt-Grassberger-Sandhas eqs

solid lines: finite-$T$ GF for cross-sections and existence

dashed lines: free cross sections + $\langle f \rangle$ cut-off

symbols: INDRA data $^{129}$Xe + $^{119}$Sn at 50 MeV/nucleon

Clusters in pBUU
Cluster Yields and Entropy

Compression in central reactions accompanied by heating. Is the matter heated as much as expected for shock compression??

Experimental measure of entropy: relative cluster yields

\[ E = T S - P V + \mu A \quad \Leftrightarrow \quad 3 A T / 2 \approx T S - A T + \mu A \]

as at freeze-out ideal gas and then

\[ \frac{S}{A} \approx \frac{5}{2} - \frac{\mu}{T} \]

In equilibrium

\[ \frac{N_d}{N_p} \propto \frac{\exp \left( \frac{2 \mu}{T} \right)}{\exp \left( \frac{\mu}{T} \right)} \quad \Rightarrow \quad \frac{S}{A} \approx 3.9 - \log \left( \frac{N_d}{N_p} \right) \]
Validity of Entropy Determination

Entropy per nucleon

Nb + Nb 650 MeV/nucleon

S/A

directly from dynamics

deuteron formula

number of ejected nucleons

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Collective Expansion

Is expansion viscous or isentropic?? Is pressure carrying out work producing a collective expansion of matter?

\[ \langle E_x \rangle = \frac{3}{2} T + \frac{m_x \langle v^2 \rangle}{2} = \frac{3}{2} T + A_x \frac{m_N \langle v^2 \rangle}{2} \]

In isentropic expansion, average kinetic energy should increase with fragment mass.

Energy increases linearly!

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Clusters in pBUU

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Head-On Au + Au (FOPI)

Rapidity Distribution

400 MeV/nucleon

proton, $b=1\,\text{fm}$

- $\text{MF}^N$π
- FOPI
$A = 3$ in Head-On Au + Au (FOPI)

Rapidity Distributions

$^{3}\text{H}, b=1\text{fm}$

$^{3}\text{He}, b=1\text{fm}$

400 MeV/nucleon
Semicentral Au + Au (FOPI)

Elliptic Flow

400 MeV/nucleon
Semicentral Au + Au (FOPI)

Elliptic Flow

400 MeV/nucleon
Future of Light-Cluster Production in Transport

Production rate for cluster of mass $A$:

$$
\mathcal{K}^<(p_A) = \int d\mathbf{p}'_1 \ldots d\mathbf{p}'_N' \, dp_1 \ldots dp_{N-1} \, |\mathcal{M}_{1'+\ldots+N'\rightarrow 1+\ldots+A}|^2
\times \delta(p'_1 + \ldots + p'_{N'} - p_1 - \ldots - p_{N-1} - p_A)
\times \delta(\epsilon_1' + \ldots + \epsilon'_{N'} - \epsilon_1 - \ldots - \epsilon_{N-1} - \epsilon_A)
\times f_1' \cdots f_{N'} (1 \pm f_1) \cdots (1 \pm f_{N-1})
$$

Determination and sampling of separate $|\mathcal{M}|^2$ for every possible process... Potential nightmare! E.g.

- $N + \Delta \leftrightarrow d + \pi$ (AGS)
- $d + d + N \leftrightarrow \alpha + N$ (etc.)

Any simplifications??
Simplified Matrix Elements

Batko, Randrup, Vetter
NPA536(92)786

$|\mathcal{M}|^2 \propto 1 \Rightarrow \text{Mini Fireball}$

??Too much dissipation??

Generalized coalescence:

$$|\mathcal{M}|^2 \propto \theta(p_0 - |\frac{p_A}{A} - p'_1|) \cdots \theta(p_0 - |\frac{p_A}{A} - p'_{N'}|)$$

Branching??  Automation needed!
Conclusions

- Real-time many-body theory provides fundamentals for production of clusters in transport theory
- Few-body collisions or rapidly changing mean-field conditions are needed to spur cluster production
- Detailed balance must be obeyed for thermodynamic consistency
- Breakup data yield production rates in collisions
- Clusters emphasize collective motion and provide information on phase-space densities and entropy
- Production description needs to be simplified in extending reach of theory.

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