RECENT PROGRESS IN HIGH-PRECISION CHIRAL NUCLEAR FORCES

R. Machleidt
University of Idaho
OUTLINE

- Current status & current issues
- How to address the open issues?
- Consistent interactions up to N4LO
- Keeping the error budget low
- Conclusions
CURRENT STATUS
Motivation for the chiral EFT approach

- **QCD at low energy is strong.**
- Quarks and gluons are confined into colorless hadrons.
- Nuclear forces are residual forces (similar to van der Waals forces)
- Separation of scales
• **Calls for an EFT:**
  
  soft scale: $Q \approx m_\pi$, hard scale: $\Lambda_\chi \approx m_\rho$; pions and nucleons are relevant d.o.f.

• **Low-momentum expansion:**
  
  $(Q/\Lambda_\chi)^\nu$ with $\nu$ bounded from below.

• **Most general Lagrangian consistent with all symmetries of low-energy QCD, particularly, chiral symmetry** which is spontaneously broken.

• **Weakly interacting Goldstone bosons = pions.**

• **$n$-$n$ and $n$-$N$ perturbatively**

• **NN has bound states:**

  (i) **NN potential perturbatively**

  (ii) **apply nonpert. in LS equation.**

  *(Weinberg)*
<table>
<thead>
<tr>
<th>Order</th>
<th>2N Force</th>
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<tr>
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<td><img src="image22" alt="N^5LO Diagram" /></td>
<td><img src="image23" alt="N^5LO Diagram" /></td>
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</tr>
</tbody>
</table>
WHAT HAVE WE ACHIEVED WITH THOSE FORCES?

- There has been some success (ground state of 10B, drip lines, nuclear matter saturation, orbit evolution, etc.), but some persistent problems remain.
- In the few-body sector: Ay puzzle, N-d break-up, ...
N-d A_y calculations by Witala et al.

- 2N (N3LO) force only
- 2N (N3LO) + 3N (N3LO) forces

Graph shows comparison of different force models with experimental data from TUNL nd and calculations involving chiral N^3LO + 3NF N^3LO (ππ+D+E) and chiral N^3LO + 3NF N^3LO (ππ+2π1π+D+E) forces.
CURRENT STATUS AND OPEN ISSUES

- Current status: 2NFs and 3NFs up to N3LO are applied in nuclear few- and many-body systems.
- In general, quite a bit of success, but some persistent problems remain.
- In the few-body sector: Ay puzzle, N-d break-up, ...
- Light nuclei: Spectra not perfect.
SPECTRA OF SOME OXYGEN ISOTOPES

From Roth
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- The radii of nuclei
Radii and Binding Energies in Oxygen Isotopes: A Challenge for Nuclear Forces

V. Lapoux, V. Somà, C. Barbieri, H. Hergert, J. D. Holt, and S. R. Stroberg

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Department of Physics, University of Surrey, Guildford GU2 7XH, United Kingdom
National Superconducting Cyclotron Laboratory and Department of Physics and Astronomy, Michigan State University, East Lansing, Michigan 48824, USA
TRIUMF, 4004 Wesbrook Mall, Vancouver, British Columbia, Canada V6T 2A3
(Received 29 April 2016; published 27 July 2016)

FIG. 1. Oxygen binding energies. Results from SCGF (DGF and GGF) and IMSRG calculations with EM and NNLO_{sat} are displayed along with experimental data.

FIG. 5. Matter radii from our analysis and given in Table I, compared to calculations with EM [27–29] and NNLO_{sat} [36]. Bands span results from GGF and MR-IMSRG schemes.
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- Overbinding of intermediate-mass nuclei
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Overbinding of intermediate-mass nuclei

Oxygen

Calcium

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From Hergert et al., PRC 90, 041302 (2014).
Overbinding of intermediate-mass nuclei

**Oxygen**

![Graph showing binding energies of Oxygen isotopes.]

**FIG. 1.** Oxygen binding energies. Results from SCGF (DGF and GGF) and IMSRG calculations with EM and NNLO$_{sat}$ are displayed along with experimental data.

**Tin**

![Graph showing binding energies of Tin isotopes.]

From Hergert
CURRENT STATUS AND OPEN ISSUES

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- In general, quite a bit of success, but some persistent problems remain.
- In the few-body sector: Ay puzzle, N-d break-up, ...
- Light nuclei: Spectra not perfect.
- The radii of nuclei
- Overbinding of intermediate-mass nuclei
- Convergence of the chiral expansion in the many-body system
Because of the problems just pointed out, improvement of current nuclear forces is called for.

- How?
- Revisit the lower orders
Chiral nuclear forces

Status A.D. 2010
<table>
<thead>
<tr>
<th>Order</th>
<th>(Q/\Lambda_{\chi})^n</th>
<th>2N Force</th>
<th>3N Force</th>
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<tbody>
<tr>
<td>LO</td>
<td>(Q/\Lambda_{\chi})^0</td>
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<td>NLO</td>
<td>(Q/\Lambda_{\chi})^2</td>
<td><img src="image" alt="Diagram" /></td>
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<td>NNLO</td>
<td>(Q/\Lambda_{\chi})^3</td>
<td><img src="image" alt="Diagram" /></td>
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<td>N^3LO</td>
<td>(Q/\Lambda_{\chi})^4</td>
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<tr>
<td>N^4LO</td>
<td>(Q/\Lambda_{\chi})^5</td>
<td><img src="image" alt="Diagram" /></td>
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<td>(Q/\Lambda_{\chi})^6</td>
<td><img src="image" alt="Diagram" /></td>
<td><img src="image" alt="Diagram" /></td>
</tr>
</tbody>
</table>

**NNLO revisited:**
- Ekstroem et al., 2013+
- Carlsson et al., 2016

**NNLO**
- \(\text{opt} \)
- \(\text{sat} \)
- \(\text{sep} \)
- \(\text{sim} \)

**NNLO/N3LO revisited:**
- Piarulli et al., 2015+

**Local potentials.**
BECAUSE OF THE PROBLEMS JUST POINTED OUT, IMPROVEMENT OF CURRENT NUCLEAR FORCES IS CALLED FOR.

- How?
- Revisit the lower orders
- Move on to higher orders
Status A.D. 2010
1-loop graphs: 5 topologies

Krebs et al. (2012, 2013)
1-loop graphs: 5 topologies

Krebs et al. (2012, 2013)
1-loop graphs: 5 topologies

2PE              2PE-1PE              Ring        Contact-1PE        Contact-2PE

Krebs et al. (2012, 2013)
1-loop graphs: 5 topologies

Krebs et al. (2012, 2013)

3NF contacts at N4LO
Girlanda, Kievsky, Viviani, PRC 84, 014001 (2011)

\[ k_i = p_i - p_i' \text{ and } Q_i = p_i + p_i', \] 
\[ p_i \text{ and } p_i' \text{ being the initial and final momenta of nucleon } i, \] 
\[ the potential in momentum space is found to be \]

\[
V = \sum_{i \neq j \neq k} \left[-E_1 k_i^2 - E_2 k_i^2 \tau_i \cdot \tau_j - E_3 k_i^2 \sigma_i \cdot \sigma_j - E_4 k_i^2 \sigma_i \cdot \sigma_j \tau_i \cdot \tau_j - E_5 (3k_i \cdot \sigma_i k_i \cdot \sigma_j - k_i^2) \tau_i \cdot \tau_j - E_6 (3k_i \cdot \sigma_i k_i \cdot \sigma_j - k_i^2) \tau_i \cdot \tau_j - \frac{i}{2} E_7 k_i \times (Q_i - Q_j) \cdot (\sigma_i + \sigma_j) + \frac{i}{2} E_8 k_i \times (Q_i - Q_j) \cdot (\sigma_i + \sigma_j) \tau_j \cdot \tau_k - E_9 k_i \cdot \sigma_i k_j \cdot \sigma_j - E_{10} k_i \cdot \sigma_i k_j \cdot \sigma_j \tau_i \cdot \tau_j\right].
\] (15)
All possible 20 isospin-spin-momentum/position structures occur in the 3NF at N4LO!


<table>
<thead>
<tr>
<th>Generators $\mathcal{G}$ in momentum space</th>
<th>Generators $\tilde{\mathcal{G}}$ in coordinate space</th>
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</thead>
<tbody>
<tr>
<td>$G_1 = 1$</td>
<td>$\tilde{G}_1 = 1$</td>
</tr>
<tr>
<td>$G_2 = \tau_1 \cdot \tau_3$</td>
<td>$\tilde{G}_2 = \tau_1 \cdot \tau_3$</td>
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<tr>
<td>$G_3 = \bar{\sigma}_1 \cdot \bar{\sigma}_3$</td>
<td>$\tilde{G}_3 = \bar{\sigma}_1 \cdot \bar{\sigma}_3$</td>
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<tr>
<td>$G_4 = \tau_1 \cdot \tau_3 \bar{\sigma}_1 \cdot \bar{\sigma}_3$</td>
<td>$\tilde{G}_4 = \tau_1 \cdot \tau_3 \bar{\sigma}_1 \cdot \bar{\sigma}_3$</td>
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<tr>
<td>$G_5 = \tau_2 \cdot \tau_3 \bar{\sigma}_1 \cdot \bar{\sigma}_2$</td>
<td>$\tilde{G}_5 = \tau_2 \cdot \tau_3 \bar{\sigma}_1 \cdot \bar{\sigma}_2$</td>
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<tr>
<td>$G_6 = \tau_1 \cdot (\tau_2 \times \tau_3) \bar{\sigma}_1 \cdot (\bar{\sigma}_2 \times \bar{\sigma}_3)$</td>
<td>$\tilde{G}_6 = \tau_1 \cdot (\tau_2 \times \tau_3) \bar{\sigma}_1 \cdot (\bar{\sigma}_2 \times \bar{\sigma}_3)$</td>
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<tr>
<td>$G_7 = \tau_1 \cdot (\tau_2 \times \tau_3) \bar{\sigma}_2 \cdot (\bar{q}_1 \times \bar{q}_3)$</td>
<td>$\tilde{G}<em>7 = \tau_1 \cdot (\tau_2 \times \tau_3) \bar{\sigma}<em>2 \cdot (\bar{r}</em>{12} \times \bar{r}</em>{23})$</td>
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<tr>
<td>$G_8 = \bar{q}_1 \cdot \bar{\sigma}_1 \bar{q}_1 \cdot \bar{\sigma}_3$</td>
<td>$\tilde{G}<em>8 = \hat{r}</em>{23} \cdot \bar{\sigma}<em>1 \hat{r}</em>{23} \cdot \bar{\sigma}_3$</td>
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<tr>
<td>$G_9 = \bar{q}_1 \cdot \bar{\sigma}_3 \bar{q}_3 \cdot \bar{\sigma}_1$</td>
<td>$\tilde{G}<em>9 = \hat{r}</em>{23} \cdot \bar{\sigma}<em>3 \hat{r}</em>{12} \cdot \bar{\sigma}_1$</td>
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<tr>
<td>$G_{10} = \bar{q}_1 \cdot \bar{\sigma}_1 \bar{q}_3 \cdot \bar{\sigma}_3$</td>
<td>$\tilde{G}<em>{10} = \hat{r}</em>{23} \cdot \bar{\sigma}<em>1 \hat{r}</em>{12} \cdot \bar{\sigma}_3$</td>
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<tr>
<td>$G_{11} = \tau_2 \cdot \tau_3 \bar{q}_1 \cdot \bar{\sigma}_1 \bar{q}_1 \cdot \bar{\sigma}_2$</td>
<td>$\tilde{G}<em>{11} = \tau_2 \cdot \tau_3 \hat{r}</em>{23} \cdot \bar{\sigma}<em>1 \hat{r}</em>{23} \cdot \bar{\sigma}_2$</td>
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<td>$G_{12} = \tau_2 \cdot \tau_3 \bar{q}_1 \cdot \bar{\sigma}_1 \bar{q}_3 \cdot \bar{\sigma}_2$</td>
<td>$\tilde{G}<em>{12} = \tau_2 \cdot \tau_3 \hat{r}</em>{23} \cdot \bar{\sigma}<em>1 \hat{r}</em>{12} \cdot \bar{\sigma}_2$</td>
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<td>$G_{13} = \tau_2 \cdot \tau_3 \bar{q}_3 \cdot \bar{\sigma}_1 \bar{q}_1 \cdot \bar{\sigma}_2$</td>
<td>$\tilde{G}<em>{13} = \tau_2 \cdot \tau_3 \hat{r}</em>{12} \cdot \bar{\sigma}<em>1 \hat{r}</em>{23} \cdot \bar{\sigma}_2$</td>
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<tr>
<td>$G_{14} = \tau_2 \cdot \tau_3 \bar{q}_3 \cdot \bar{\sigma}_1 \bar{q}_3 \cdot \bar{\sigma}_2$</td>
<td>$\tilde{G}<em>{14} = \tau_2 \cdot \tau_3 \hat{r}</em>{12} \cdot \bar{\sigma}<em>1 \hat{r}</em>{12} \cdot \bar{\sigma}_2$</td>
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<td>$G_{15} = \tau_1 \cdot \tau_3 \bar{q}_2 \cdot \bar{\sigma}_1 \bar{q}_2 \cdot \bar{\sigma}_3$</td>
<td>$\tilde{G}<em>{15} = \tau_1 \cdot \tau_3 \hat{r}</em>{13} \cdot \bar{\sigma}<em>1 \hat{r}</em>{13} \cdot \bar{\sigma}_3$</td>
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<td>$G_{16} = \tau_2 \cdot \tau_3 \bar{q}_3 \cdot \bar{\sigma}_2 \bar{q}_3 \cdot \bar{\sigma}_3$</td>
<td>$\tilde{G}<em>{16} = \tau_2 \cdot \tau_3 \hat{r}</em>{12} \cdot \bar{\sigma}<em>2 \hat{r}</em>{12} \cdot \bar{\sigma}_3$</td>
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<td>$G_{17} = \tau_1 \cdot \tau_3 \bar{q}_1 \cdot \bar{\sigma}_3 \bar{q}_1 \cdot \bar{\sigma}_3$</td>
<td>$\tilde{G}<em>{17} = \tau_1 \cdot \tau_3 \hat{r}</em>{23} \cdot \bar{\sigma}<em>1 \hat{r}</em>{23} \cdot \bar{\sigma}_3$</td>
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<td>$G_{18} = \tau_1 \cdot (\tau_2 \times \tau_3) \bar{\sigma}_1 \cdot \bar{\sigma}_3 \bar{\sigma}_2 \cdot (\bar{q}_1 \times \bar{q}_3)$</td>
<td>$\tilde{G}_{18} = \tau_1 \cdot (\tau_2 \times \tau_3) \bar{\sigma}_1 \cdot \bar{\sigma}<em>3 \bar{\sigma}<em>2 \cdot (\hat{r}</em>{12} \times \hat{r}</em>{23})$</td>
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<td>$G_{19} = \tau_1 \cdot (\tau_2 \times \tau_3) \bar{\sigma}_3 \cdot \bar{q}_1 \bar{q}_1 \cdot (\bar{\sigma}_1 \times \bar{\sigma}_2)$</td>
<td>$\tilde{G}<em>{19} = \tau_1 \cdot (\tau_2 \times \tau_3) \bar{\sigma}<em>3 \cdot \hat{r}</em>{23} \hat{r}</em>{23} \cdot (\bar{\sigma}_1 \times \bar{\sigma}_2)$</td>
</tr>
<tr>
<td>$G_{20} = \tau_1 \cdot (\tau_2 \times \tau_3) \bar{\sigma}_1 \cdot \bar{q}_1 \bar{\sigma}_3 \cdot \bar{q}_3 \bar{\sigma}_2 \cdot (\bar{q}_1 \times \bar{q}_3)$</td>
<td>$\tilde{G}<em>{20} = \tau_1 \cdot (\tau_2 \times \tau_3) \bar{\sigma}<em>1 \cdot \hat{r}</em>{23} \bar{\sigma}<em>3 \cdot \hat{r}</em>{12} \bar{\sigma}<em>2 \cdot (\hat{r}</em>{12} \times \hat{r}</em>{23})$</td>
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</tbody>
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N4LO 2NF Contributions
Entem, Kaiser, Machleidt, Nosyk,
PRC 91, 014002 (2015)
N5LO 2NF Contributions
Entem, Kaiser, Machleidt, Nosyk,
PRC 92, 064001 (2015)
From Entem, Kaiser, Machleidt, Nosyk, PRC 91, 014002 (2015)
Converged at N4LO

From Entem, Kaiser, Machleidt, Nosyk, PRC 92, 064001 (2015)
The Map of the Chartered Waters Of the Forces

Status A.D. 2017

High-Precision Nucl. Forces
MSU, 03/29/2017

R. Machleidt
NOW THAT WE HAVE CHARTERED THE WATERS OF THE FORCES, HOW DO WE ADDRESS THE ISSUES?
Apply the forces of this map systematically, order by order.
And that first and above all requires High-quality NN potentials, Constructed consistently through all orders.
“HIGH QUALITY”, “CONSISTENTLY”, … WHAT DOES THAT MEAN?

- Use \( \pi\)-N LECs determined in \( \pi\)-N analysis with the highest possible precision: Roy-Steiner Analysis (Hoferichter et al., PRL 115, 192301 (2015)).
Matching Pion-Nucleon Roy-Steiner Equations to Chiral Perturbation Theory

Martin Hoferichter, 3, Jacobo Ruiz de Elvira,*, Bastian Kubis,⁴ and Ulf-G. Meißner⁴, ⁵

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²ExtreMe Matter Institute EMMI, GSI Helmholtzzentrum für Schwerionenforschung GmbH, D-64291 Darmstadt, Germany
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⁵Institut für Kernphysik, Institute for Advanced Simulation, Jülich Center for Hadron Physics, JARA-HPC, and JARA-FAME, Forschungszentrum Jülich, D-52425 Jülich, Germany

(Received 28 July 2015; published 4 November 2015)

We match the results for the subthreshold parameters of pion-nucleon scattering obtained from a solution of Roy-Steiner equations to chiral perturbation theory up to next-to-next-to-next-to-leading order, to extract the pertinent low-energy constants including a comprehensive analysis of systematic uncertainties and correlations. We study the convergence of the chiral series by investigating the chiral expansion of threshold parameters up to the same order and discuss the role of the Δ(1232) resonance in this context. Results for the low-energy constants are also presented in the counting scheme usually applied in chiral nuclear effective field theory, where they serve as crucial input to determine the long-range part of the nucleon-nucleon potential as well as three-nucleon forces.

* 2015 Klaus Erkelenz Prize Winners (University of Bonn, Germany)
Matching Pion-Nucleon Roy-Steiner Equations to Chiral Perturbation Theory

Martin Hoferichter,¹,²,³ Jacobo Ruiz de Elvira,⁴ Bastian Kubis,⁴ and Ulf-G. Meißner⁴,⁵

MAIN CHARACTERISTICS:

- Set of coupled partial-wave dispersion relations constraint by analyticity, unitarity, and crossing symmetry.
- Additional crucial constraint: High-accuracy π-N scattering lengths extracted from pionic atoms.
- Matching to π-N LECs done in the subthreshold region, which is best for nuclear forces.
- Comprehensive error analysis.
- Small errors.
## π-N LECs from Roy–Steiner Analysis
(Hoferichter et al., PRL 115, 192301 (2015))

TABLE II: The πN LECs as determined in the Roy-Steiner-equation analysis of πN scattering conducted in Ref. [35]. The given orders of the chiral expansion refer to the \(NN\) system. Note that the orders, at which the LECs are extracted from the πN system, are always lower by one order as compared of the \(NN\) system in which the LECs are applied. The \(c_i\), \(d_i\), and \(\bar{e}_i\) are the LECs of the second, third, and fourth order πN Lagrangian [26] and are in units of GeV\(^{-1}\), GeV\(^{-2}\), and GeV\(^{-3}\), respectively. The uncertainties in the last digits are given in parentheses after the values.

<table>
<thead>
<tr>
<th>(c_1)</th>
<th>NNLO</th>
<th>N(^3)LO</th>
<th>N(^4)LO</th>
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</thead>
<tbody>
<tr>
<td>(-0.74(2))</td>
<td></td>
<td>(-1.07(2))</td>
<td>(-1.10(3))</td>
</tr>
<tr>
<td>(c_2)</td>
<td></td>
<td>3.20(3)</td>
<td>3.57(4)</td>
</tr>
<tr>
<td>(c_3)</td>
<td>(-3.61(5))</td>
<td>(-5.32(5))</td>
<td>(-5.54(6))</td>
</tr>
<tr>
<td>(c_4)</td>
<td>(2.44(3))</td>
<td>3.56(3)</td>
<td>4.17(4)</td>
</tr>
<tr>
<td>(d_1 + \bar{d}_2)</td>
<td></td>
<td>1.04(6)</td>
<td>6.18(8)</td>
</tr>
<tr>
<td>(\bar{d}_3)</td>
<td></td>
<td>(-0.48(2))</td>
<td>(-8.91(9))</td>
</tr>
<tr>
<td>(\bar{d}_5)</td>
<td></td>
<td>0.14(5)</td>
<td>0.86(5)</td>
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<tr>
<td>(d_14 - \bar{d}_{15})</td>
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<td>(-1.90(6))</td>
<td>(-12.18(12))</td>
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<tr>
<td>(\bar{e}_{14})</td>
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<td></td>
<td>1.18(4)</td>
</tr>
<tr>
<td>(\bar{e}_{17})</td>
<td></td>
<td></td>
<td>(-0.18(6))</td>
</tr>
</tbody>
</table>

**Very small errors!**
RECALL A TYPICAL PROBLEM FROM THE PAST . . .

- One had to assume that, e.g., \( c_3 \cong 3.4 - 6.0 \)
- Leading to a huge uncertainty for the 3NF contribution.
- Inconsistency with \( c_3 \) used in the NN interaction.
- **This is all over now!**
- Uncertainty of the NN interaction due to the uncertainty in \( c_i \)'s absolutely negligible.
- Uncertainty of the 3NF contribution due to the uncertainty in \( c_i \)'s: negligible as compared to truncation error.
"HIGH QUALITY", "CONSISTENTLY", ... WHAT DOES THAT MEANS?

- Use $\pi$-N LECs determined in $\pi$-N analysis with the highest possible precision: Roy-Steiner Analysis (Hoferichter et al., PRL 115, 192301 (2015)).

- NN potentials are fit to NN data (and not to phase shifts) using all NN data below pion production threshold published up to December 2016.
Reproduction of the NN Data

TABLE V: $\chi^2$/datum for the fit of the 2016 NN data base by $NN$ potentials at various orders of chiral EFT ($\Lambda = 500$ MeV in all cases).

<table>
<thead>
<tr>
<th>$T_{lab}$ bin (MeV)</th>
<th>No. of data</th>
<th>LO</th>
<th>NLO</th>
<th>NNLO</th>
<th>$N^3LO$</th>
<th>$N^4LO$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>proton-proton</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0–100</td>
<td>795</td>
<td>520</td>
<td>18.9</td>
<td>2.28</td>
<td>1.18</td>
<td></td>
</tr>
<tr>
<td>0–190</td>
<td>1206</td>
<td>430</td>
<td>43.6</td>
<td>4.64</td>
<td>1.69</td>
<td>1.12</td>
</tr>
<tr>
<td>0–290</td>
<td>2132</td>
<td>360</td>
<td>70.8</td>
<td>7.60</td>
<td>2.09</td>
<td>1.21</td>
</tr>
<tr>
<td>neutron-proton</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0–100</td>
<td>1180</td>
<td>114</td>
<td>7.2</td>
<td>1.38</td>
<td>0.93</td>
<td>0.94</td>
</tr>
<tr>
<td>0–190</td>
<td>1697</td>
<td>96</td>
<td>23.1</td>
<td>2.29</td>
<td>1.10</td>
<td>1.06</td>
</tr>
<tr>
<td>0–290</td>
<td>2721</td>
<td>94</td>
<td>36.7</td>
<td>5.28</td>
<td>1.27</td>
<td>1.10</td>
</tr>
<tr>
<td>$pp$ plus $np$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0–100</td>
<td>1975</td>
<td>283</td>
<td>11.9</td>
<td>1.74</td>
<td>1.03</td>
<td>1.00</td>
</tr>
<tr>
<td>0–190</td>
<td>2288</td>
<td>285</td>
<td>21.6</td>
<td>3.27</td>
<td>1.45</td>
<td>1.13</td>
</tr>
<tr>
<td>0–290</td>
<td>4853</td>
<td>206</td>
<td>51.5</td>
<td>6.30</td>
<td>1.63</td>
<td>1.15</td>
</tr>
</tbody>
</table>

(Includes ct's In F-waves.)
Neutron-Proton Phase Shifts
Cutoff Variations

NNLO

N4LO
The Potentials are non-local and soft

TABLE VII: Two- and three-nucleon bound-state properties as predicted by NN potentials at various orders of chiral EFT ($\Lambda = 500$ MeV in all cases). (Deuteron: Binding energy $B_d$, asymptotic $S$ state $A_S$, asymptotic $D/S$ state $\eta$, structure radius $r_{stt}$, quadrupole moment $Q$, $D$-state probability $P_D$; the predicted $r_{stt}$ and $Q$ are without meson-exchange current contributions and relativistic corrections. Triton: Binding energy $B_t$.) $B_d$ is fitted, all other quantities are predictions.

<table>
<thead>
<tr>
<th></th>
<th>LO</th>
<th>NLO</th>
<th>NNLO</th>
<th>N³LO</th>
<th>N⁴LO</th>
<th>Empirical$^a$</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Deuteron</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$B_d$ (MeV)</td>
<td>2.224575</td>
<td>2.224575</td>
<td>2.224575</td>
<td>2.224575</td>
<td>2.224575</td>
<td>2.224575(9)</td>
</tr>
<tr>
<td>$A_S$ (fm$^{-1/2}$)</td>
<td>0.8526</td>
<td>0.8828</td>
<td>0.8844</td>
<td>0.8853</td>
<td>0.8852</td>
<td>0.8846(9)</td>
</tr>
<tr>
<td>$\eta$</td>
<td>0.0302</td>
<td>0.0262</td>
<td>0.0257</td>
<td>0.0257</td>
<td>0.0258</td>
<td>0.0256(4)</td>
</tr>
<tr>
<td>$r_{stt}$ (fm)</td>
<td>1.911</td>
<td>1.971</td>
<td>1.968</td>
<td>1.970</td>
<td>1.973</td>
<td>1.97507(78)</td>
</tr>
<tr>
<td>$Q$ (B$^2$)</td>
<td>3.018</td>
<td>3.079</td>
<td>3.079</td>
<td>3.079</td>
<td>3.079</td>
<td>3.079(4)</td>
</tr>
<tr>
<td>$P_D$ (%)</td>
<td>7.29</td>
<td>3.40</td>
<td>4.49</td>
<td>4.15</td>
<td>4.10</td>
<td>—</td>
</tr>
<tr>
<td><strong>Triton</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$B_t$ (MeV)</td>
<td>11.02</td>
<td>8.31</td>
<td>8.21</td>
<td>8.09</td>
<td>8.08</td>
<td>8.48</td>
</tr>
</tbody>
</table>
Concerning the *ab initio* explanation of intermediate and heavy nuclei we are faced with tough issues.

But, let’s not (yet) give up on the systematic use of chiral EFT.

This requires order-by-order calculations up to N4LO using consistent 2NF and 3NF (and 4NF).

For this purpose, we have constructed a family of NN potentials that keeps the error budget as low as possible: Essentially no uncertainties in the π-N LECs (Roy-Steiner!), Accurate fit to the 2016 NN data base (≈5000 data).

The NN potentials are relatively soft and require less 3NF as compared to some other chiral NN potentials that are floating around (like, locals, “semi-locals”).

Systematic calculations with different families of chiral interactions may hopefully give us clues for how to solve the remaining problems.
These days that's what passes for an optimist...

The end is not necessarily near!
But, one farther day,
The End
will come. Be patient.