Handling of clusters in transport models

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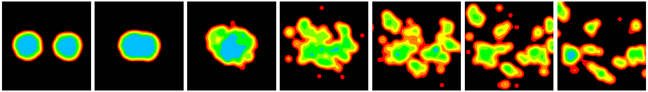
NuSYM13, July 22 – 26, 2013.

- Why clusters are important (for NuSYM)
- Why clusters are difficult, and how to solve
- Effects of clusters



An event of central collision of Xe + Sn at 50 MeV/nucleon (AMD calculation)

Clusters and NuSYM

1. ^{4}He clusters are symmetric ($N = Z = 2$). However, after emitting many ^{4}He clusters, the rest of the system is more neutron rich than the initial system.

$$(N_{\text{tot}}, Z_{\text{tot}}) - n_\alpha {}^4\text{He} = (N_{\text{tot}} - 2n_\alpha,\ Z_{\text{tot}} - 2n_\alpha)$$

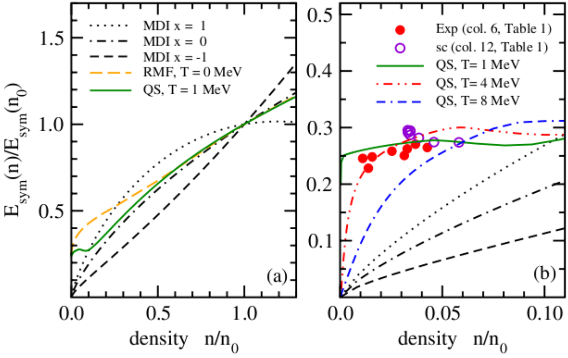
2. In dilute nuclear matter, it is known that clusters play essential roles.
3. Clusters (e.g. ^{3}H / ^{3}He) are measured by experiments to probe NuSYM.
4. Clusters are a major part of the disintegrating system in heavy-ion collisions, so collision dynamics may be influenced by the existence of clusters.
   - Four uncorrelated nucleons at $T = 10$ MeV: $\langle E \rangle = \frac{3}{2}T \times 4 = 60$ MeV
   - An α cluster at $T = 10$ MeV: $\langle E \rangle = \frac{3}{2}T \times 1 - 28.3\,\text{MeV} = -13.3$ MeV

Natowitz et al., PRL104 (2010) 202501.

Exp.: ^{64}Zn + ^{92}Mo, ^{197}Au at 35 MeV/u

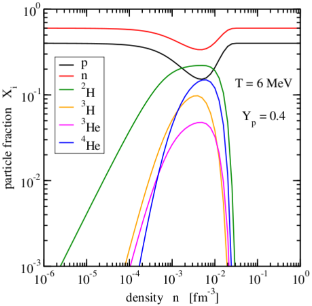
Composition of nucleons and clusters

⇒ Temperature, Density, Symmetry energy



EOS with chemical equilibrium

- Shen EOS
- Lattimer-Swesty EOS
- Ishizuka et al.,
- Botvina & Mishustin,
- Horowitz & Schwenk,
- Typel et al.,



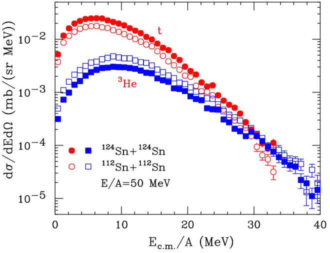
In dilute matter, clusters are important.

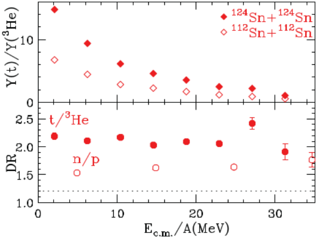
Generalized RMF by Typel

Clusters measured for NuSYM

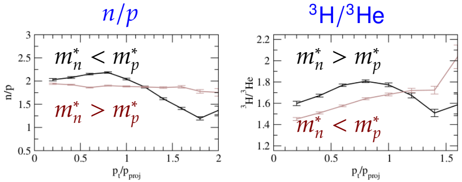
MSU data for Sn+Sn at 50 MeV/u

Liu et al., PRC86(2012)024605.





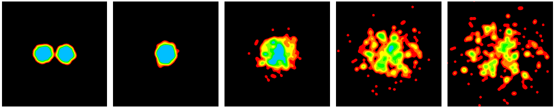
SAMURAI-TPC at RIKEN RIBF will measure light charged particles including ^{3}H and ^{3}He as well as pions.



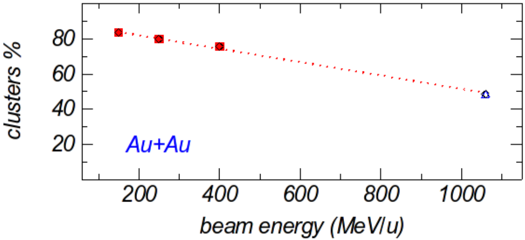
Au + Au at 400 MeV/u, central.

Di Toro et al., J. Phys. G 37(2010) 083101.

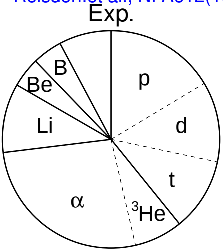
Large fraction of clusters in head-on collisions

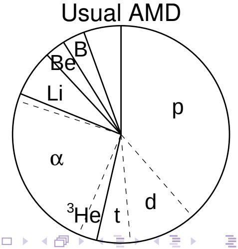


^{197}Au + ^{197}Au at 150 MeV/u



Reisdorf et al., NPA612(1997)493.





- Clusters (and fragments) are always the important part of the system.
- The actual proton multiplicity is much smaller than the prediction by usual dynamical models.

Clusters and NuSYM

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$$(N_{\text{tot}}, Z_{\text{tot}}) - n_\alpha {}^4\text{He} = (N_{\text{tot}} - 2n_\alpha,\ Z_{\text{tot}} - 2n_\alpha)$$

2. In dilute nuclear matter, it is known that clusters play essential roles.
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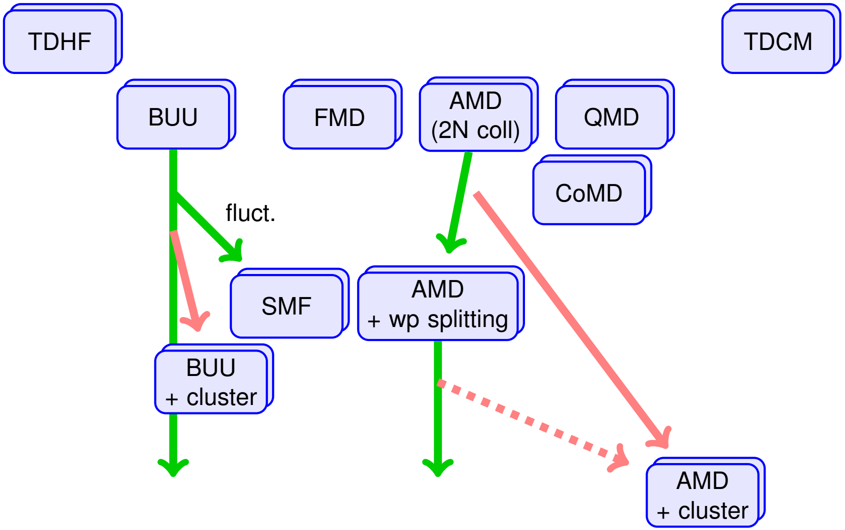
Various Microscopic Approaches



(In)stability of a formed cluster in free space

Mean field approaches (TDHF or Vlasov/BUU)

$$\phi_d(\mathbf{r}_1-\mathbf{r}_2)\,\Psi(\tfrac{\mathbf{r}_1+\mathbf{r}_2}{2}) \xrightarrow{\text{exact } e^{-iHt}} \phi_d(\mathbf{r}_1-\mathbf{r}_2)\,\Psi(\tfrac{\mathbf{r}_1+\mathbf{r}_2}{2},t)$$

$$\wr\wr \qquad\qquad\qquad\qquad \not\approx$$

$$\psi_p(\mathbf{r}_1)\,\psi_n(\mathbf{r}_2) \xrightarrow{\text{TDHF}} \psi_p(\mathbf{r}_1,t)\,\psi_n(\mathbf{r}_2,t)$$

Deuteron probability disappears due to the spurious coupling to the center-of-mass motion.

Wave-packet molecular dynamics approaches (e.g. AMD)

$$e^{-\nu\mathbf{r}_1^2+i\mathbf{P}\cdot\mathbf{r}_1}\;e^{-\nu\mathbf{r}_2^2+i\mathbf{P}\cdot\mathbf{r}_2} \xrightarrow{\text{MD}} e^{-\nu(\mathbf{r}_1-\mathbf{v}t)^2+i\mathbf{P}\cdot\mathbf{r}_1}\;e^{-\nu(\mathbf{r}_2-\mathbf{v}t)^2+i\mathbf{P}\cdot\mathbf{r}_2}$$

No problem once a cluster is formed, but the cluster-forming probability is the problem.

BUU with clusters

Danielewicz et al., NPA 533 (1991) 712.

Coupled equations for $f_n(\mathbf{r},\mathbf{p},t)$, $f_p(\mathbf{r},\mathbf{p},t)$, $f_d(\mathbf{r},\mathbf{p},t)$, $f_t(\mathbf{r},\mathbf{p},t)$, $f_h(\mathbf{r},\mathbf{p},t)$ are solved by the test particle method.

$$\frac{\partial f_n}{\partial t} + \mathbf{v}\cdot\frac{\partial f_n}{\partial \mathbf{r}} - \frac{\partial U_n}{\partial \mathbf{r}}\cdot\frac{\partial f_n}{\partial \mathbf{p}} = I_n^{\text{coll}}[f_n, f_p, f_d, f_t, f_h]$$

$$\frac{\partial f_p}{\partial t} + \mathbf{v}\cdot\frac{\partial f_p}{\partial \mathbf{r}} - \frac{\partial U_p}{\partial \mathbf{r}}\cdot\frac{\partial f_p}{\partial \mathbf{p}} = I_p^{\text{coll}}[f_n, f_p, f_d, f_t, f_h]$$

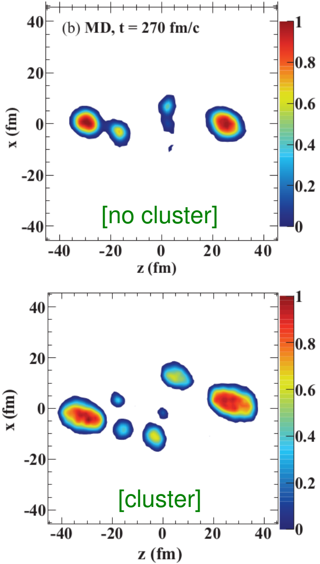
$$\frac{\partial f_d}{\partial t} + \mathbf{v}\cdot\frac{\partial f_d}{\partial \mathbf{r}} - \frac{\partial U_d}{\partial \mathbf{r}}\cdot\frac{\partial f_d}{\partial \mathbf{p}} = I_d^{\text{coll}}[f_n, f_p, f_d, f_t, f_h]$$

$$\frac{\partial f_t}{\partial t} + \mathbf{v}\cdot\frac{\partial f_t}{\partial \mathbf{r}} - \frac{\partial U_t}{\partial \mathbf{r}}\cdot\frac{\partial f_t}{\partial \mathbf{p}} = I_t^{\text{coll}}[f_n, f_p, f_d, f_t, f_h]$$

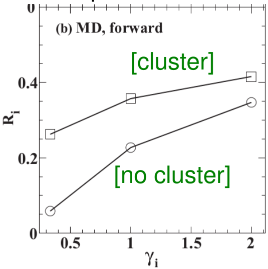
$$\frac{\partial f_h}{\partial t} + \mathbf{v}\cdot\frac{\partial f_h}{\partial \mathbf{r}} - \frac{\partial U_h}{\partial \mathbf{r}}\cdot\frac{\partial f_h}{\partial \mathbf{p}} = I_h^{\text{coll}}[f_n, f_p, f_d, f_t, f_h]$$

Effects of clusters on isospin diffusion (BUU)

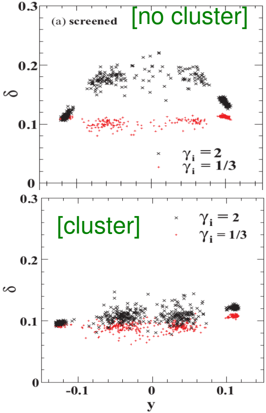
Coupland, Lynch, Tsang, Danielewicz, Zhang, PRC 84 (2011) 054603.



Isospin Diffusion

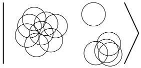


γ_i: density-dep. of E_{sym}



Distribution of fragments ($A > 2$)

in isospin asymmetry and rapidity



AMD wave function

$$|\Phi(Z)\rangle = \det_{ij}\Big[\exp\Big\{-\nu\Big(\mathbf{r}_j - \frac{\mathbf{Z}_i}{\sqrt{\nu}}\Big)^2\Big\}\chi_{\alpha_i}(j)\Big]$$

$$\mathbf{Z}_i = \sqrt{\nu}\mathbf{D}_i + \frac{i}{2\hbar\sqrt{\nu}}\mathbf{K}_i$$

ν : Width parameter = (2.5 fm)$^{-2}$

χ_{α_i} : Spin-isospin states = $p\uparrow, p\downarrow, n\uparrow, n\downarrow$

Time-dependent variational principle

$$\delta\int_{t_1}^{t_2}\frac{\langle\Phi(Z)|(i\hbar\frac{d}{dt}-H)|\Phi(Z)\rangle}{\langle\Phi(Z)|\Phi(Z)\rangle}dt = 0, \qquad \delta Z(t_1) = \delta Z(t_2) = 0$$

Equation of motion for the wave packet centroids Z

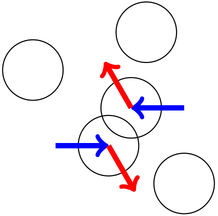
$$\frac{d}{dt}\mathbf{Z}_i = \{\mathbf{Z}_i, \mathcal{H}\}_{\rm PB} \qquad \text{or} \qquad i\hbar\sum_{j=1}^{A}\sum_{\tau=x,y,z}C_{i\sigma,j\tau}\frac{dZ_{j\tau}}{dt} = \frac{\partial\mathcal{H}}{\partial Z_{i\sigma}}$$

$$\mathcal{H} = \frac{\langle\Phi(Z)|H|\Phi(Z)\rangle}{\langle\Phi(Z)|\Phi(Z)\rangle} + \text{(c.m. correction)},$$

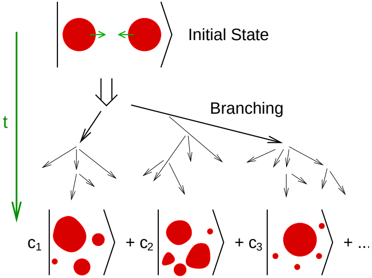
H: Effective interaction (e.g. Skyrme force)

Stochastic two-nucleon collisions

- Cross section $\frac{d\sigma_{NN}}{d\Omega}(E,\theta)$ in nuclear medium.
- Pauli blocking for the final state.
  (Almost automatic in AMD)



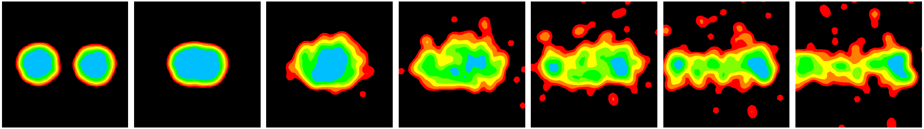
$$W_{i\to f} = \frac{2\pi}{\hbar}|\langle\Psi_f|V|\Psi_i\rangle|^2\delta(E_f - E_i)$$



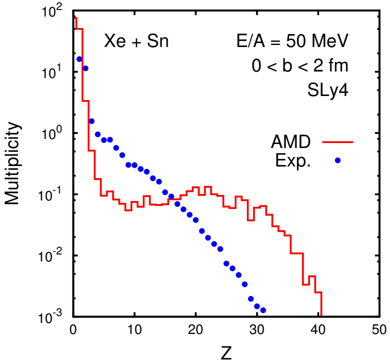
Stochastic equation of motion

$$\frac{d}{dt}\mathbf{Z}_i = \{\mathbf{Z}_i, \mathcal{H}\}_{\mathrm{PB}} + \text{(NN collisions)}$$

Results of AMD with Two-Nucleon Collisions



Xe + Sn central collisions at 50 MeV/u



- AMD with NN collisions
- INDRA data, Hudan et al., PRC 67 (2003)

	AMD	INDRA
$M(p)$	40.2	8.4
$M(\alpha)$	2.5	10.1

Clusters have to be handled in a special way

Two-nucleon collision:

$$W_{i\to f} = \tfrac{2\pi}{\hbar}|\langle\Psi_f|V|\Psi_i\rangle|^2\delta(E_f - E_i)$$

$$\sum_f |\Psi_f\rangle\langle\Psi_f| = 1$$

What is a suitable complete basis set for the final states of a two-nucleon scattering?

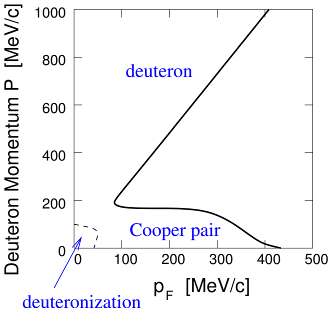
- A usual choice is to change only the two.

$$\sum_{k_1,k_2}\Big|\varphi_{k_1}(1)\varphi_{k_2}(2)\Psi(3,4,\ldots)\Big\rangle\Big\langle\varphi_{k_1}(1)\varphi_{k_2}(2)\Psi(3,4,\ldots)\Big|$$

- If a deuteron will propagate in medium, a more suitable basis will include

$$\Big|\varphi_{k_1}(1)\psi_{\mathrm{d}}(2,3)\Psi(4,\ldots)\Big\rangle\Big\langle\varphi_{k_1}(1)\psi_{\mathrm{d}}(2,3)\Psi(4,\ldots)\Big| + \cdots$$

Deuteron pole in medium ($T = 0$)



Danielewicz and Bertsch, NPA 533 (1991) 712.

More recent calculation by a quantum statistical approach by G. Röpke, NPA 867 (2011) 66.

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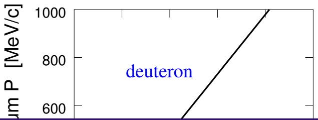
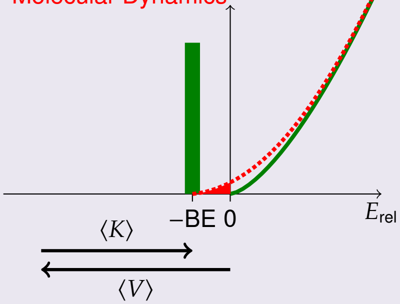


Density of states for p-n system

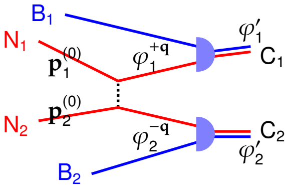


Cluster Formation Cross Section

Similar to Danielewicz et al., NPA533 (1991) 712.

$N_1 + B_1 \; + \; N_2 + B_2 \;\rightarrow C_1 + C_2$

- N_1, N_2 : Colliding nucleons
- B_1, B_2 : Spectator nucleons/clusters
- C_1, C_2 : N, $(2N)$, $(3N)$, $(4N)$ (up to α cluster)



$$v_{\rm NN}\, d\sigma(\text{NBNB} \rightarrow \text{CC})$$

$$= |\langle\varphi_1'|\varphi_1^{+\mathbf{q}}\rangle|^2 \; |\langle\varphi_2'|\varphi_2^{-\mathbf{q}}\rangle|^2 \; |M|^2\, \delta(\mathcal{H}-E)\, p_{\rm rel}^2 dp_{\rm rel} d\Omega$$

$$\left(v_{\rm NN}\, d\sigma_{\rm NN} = |M|^2\, \delta(\mathcal{H}-E)\, p_{\rm rel}^2 dp_{\rm rel} d\Omega \right)$$

$$\frac{d\sigma}{d\Omega} = F_{\rm kin}\, |\langle\varphi_1'|\varphi_1^{+\mathbf{q}}\rangle|^2 \; |\langle\varphi_2'|\varphi_2^{-\mathbf{q}}\rangle|^2 \left(\frac{d\sigma}{d\Omega}\right)_{\rm NN\rightarrow NN}$$

$$\mathbf{p}_{\rm rel} = \tfrac{1}{2}(\mathbf{p}_1 - \mathbf{p}_2) = p_{\rm rel}\hat{\mathbf{\Omega}}$$

$$\mathbf{q} = \mathbf{p}_1 - \mathbf{p}_1^{(0)} = \mathbf{p}_2^{(0)} - \mathbf{p}_2$$

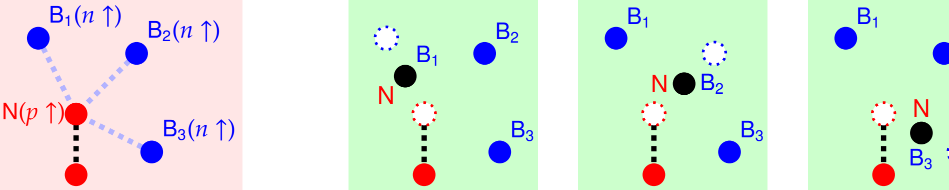
$$\varphi_1^{+\mathbf{q}} = \exp(+i\mathbf{q}\cdot\mathbf{r}_{N_1})\varphi_1^{(0)}$$

$$\varphi_2^{-\mathbf{q}} = \exp(-i\mathbf{q}\cdot\mathbf{r}_{N_2})\varphi_2^{(0)}$$

The cross section is given from the NN cross section.

Construction of Final States

Clusters (in the final states) are assumed to have $(0s)^N$ configuration.



$\lvert\Phi^{\mathbf{q}}\rangle$	$\lvert\Phi'_1\rangle$	$\lvert\Phi'_2\rangle$	$\lvert\Phi'_3\rangle$
After $\mathbf{p}^{(0)} \to \mathbf{p}^{(0)} + \mathbf{q}$	N + $\mathrm{B}_1 \to \mathrm{C}_1$	N + $\mathrm{B}_2 \to \mathrm{C}_2$	N + $\mathrm{B}_3 \to$ C

Final states are not orthogonal: $N_{ij} \equiv \langle\Phi'_i|\Phi'_j\rangle \neq \delta_{ij}$

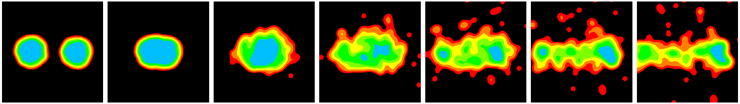
The probability of cluster formation with one of B's:

$$\hat{P} = \sum_{ij} |\Phi'_i\rangle N^{-1}_{ij} \langle\Phi'_j|, \qquad P = \langle\Phi^{\mathbf{q}}|\hat{P}|\Phi^{\mathbf{q}}\rangle \qquad \neq \sum_i |\langle\Phi'_i|\Phi^{\mathbf{q}}\rangle|^2$$

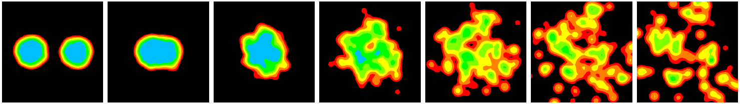
$$\begin{cases} P & \Rightarrow \text{Choose one of the candidates and make a cluster.} \\ 1 - P & \Rightarrow \text{Don't make a cluster (with any n}\uparrow\text{).} \end{cases}$$

Effect of Clusters on the Density Evolution

Without cluster correlations (AMD with NN collisions)

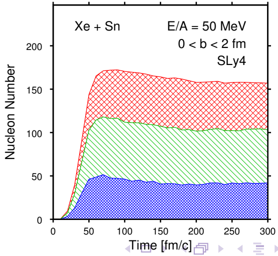


With cluster correlations



During the time evolution, clusters are . . .

- formed at NN collisions.
- propagated by AMD equation. (nothing special)
- broken by NN collisions. (nothing special)



Non-clustered

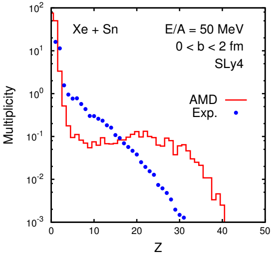
(2N)

(3N)

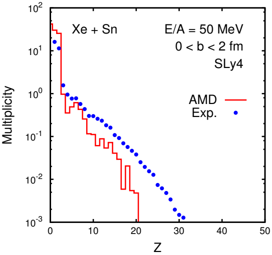
(4N)

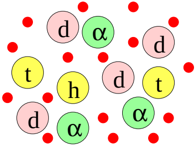
Effects of Cluster Correlations on Fragmentation

Usual NN collisions



With Clusters





Very strong tendency of turning into cluster gas.

	w/o C	with C	INDRA
$M(p)$	40.2	10.9	8.4
$M(\alpha)$	2.5	23.2	10.1
$Z_{\text{gas}}/Z_{\text{tot}}$	55%	78%	(40-50%)

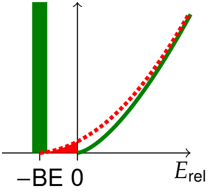
- Gas = $\sum$ (particles of $A \leq 4$)
- Liquid = $\sum$ (heavier fragments)

Cluster-Cluster Correlations

Relative motions between clusters should be treated quantum mechanically. In AMD,

- The binding energy a few clusters is reasonably correct,
- but the phase space of bound configuration is too small.

$$\text{e.g. } {}^7\text{Li} = \alpha + t - 2.5 \text{ MeV}$$

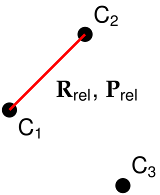


$$|\alpha + t\rangle \to |{}^7\text{Li}\rangle \text{ with probability} \approx |\langle {}^7\text{Li}|\alpha + t\rangle|^2$$

At every time step, **Clusters** C_1 **and** C_2 **are bound:** $\mathbf{P}_{\text{rel}} \to 0$,

- **if** C_j is the cluster closest to $\mathbf{C}_i$, $(i, j) = (1, 2)$ or $(2, 1)$,
- **and if** they are moderately separated, $|\mathbf{R}_{\text{rel}}| < R_{\text{max}}$,
- **and if** they are moving slowly away from each other, $|\mathbf{P}_{\text{rel}}| < P_{\text{max}}$ and $\mathbf{P}_{\text{rel}} \cdot \mathbf{R}_{\text{rel}} > 0$.

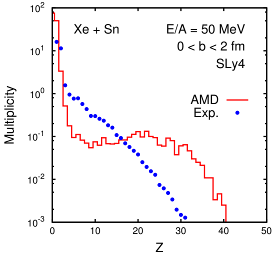
$$P_{\text{max}}^2/2\mu = 8 \text{ MeV}, \qquad R_{\text{max}} = 5 \text{ fm} \qquad \text{(adjustable)}$$



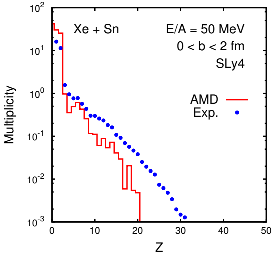
Energy is conserved by scaling the relative momentum between the C_1-C_2 pair and a third cluster C_3.

Effects of Cluster and C-C Correlations on Fragmentation

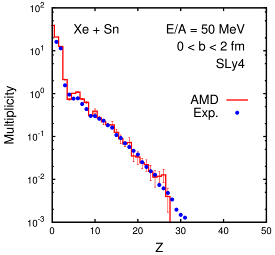
Usual NN collisions



With Clusters



With C & C-C

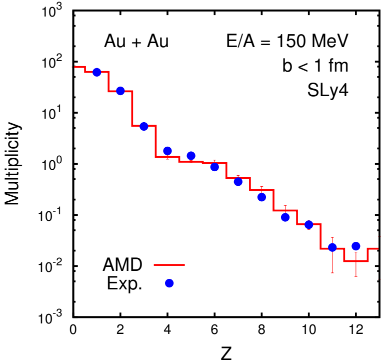


	w/o C	with C	C & C-C	INDRA
$M(p)$	40.2	10.9	10.8	8.4
$M(\alpha)$	2.5	23.2	10.7	10.1
$Z_{\mathrm{gas}}/Z_{\mathrm{tot}}$	55%	78%	43%	(40-50%)

- Gas = $\sum$ (particles of $A \leq 4$)
- Liquid = $\sum$ (heavier fragments)

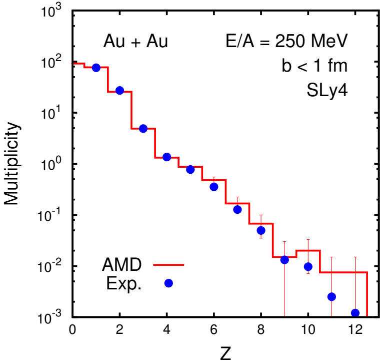
Au + Au Central Collisions at Higher Energies

E/A = 150 MeV



	with C & C-C	FOPI
$M(p)$	32.8	26.1
$M(\alpha)$	20.1	21.0
$Z_{\text{gas}}/Z_{\text{tot}}$	71%	73%

E/A = 250 MeV



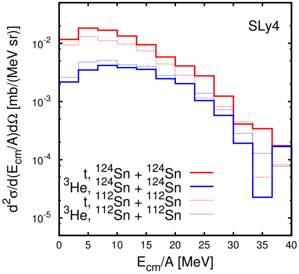
	with C & C-C	FOPI
$M(p)$	42.0	31.9
$M(\alpha)$	19.4	18.2
$Z_{\text{gas}}/Z_{\text{tot}}$	80%	83%

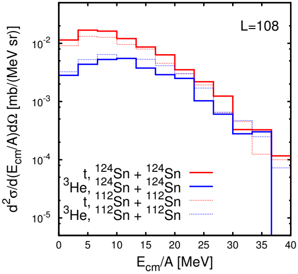
FOPI data: Reisdorf et al., NPA 612 (1997) 493.

Energy Spectra of Clusters

$^{124}Sn + ^{124}Sn$ and $^{112}Sn + ^{112}Sn$ central collisions at 50 MeV/nucleon

⇒ Energy spectra of tritons and ^{3}He emitted to transverse directions





SLy4 ($L = 46$ MeV)

$L = 108$ MeV

- Triton/^{3}He difference is consistent with the gas part of fractionation.

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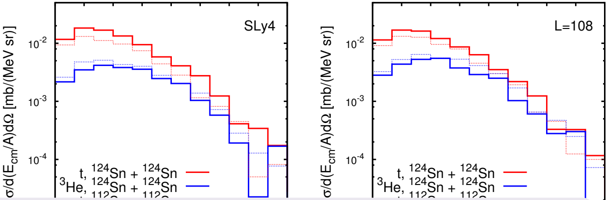

$Y(t)/Y(^3\text{He})$ ratio for ^{124}Sn + ^{124}Sn		
	$L = 46$	$L = 108$
$E/A < 10$ MeV	4.76	3.57
$E/A > 20$ MeV	2.25	1.65

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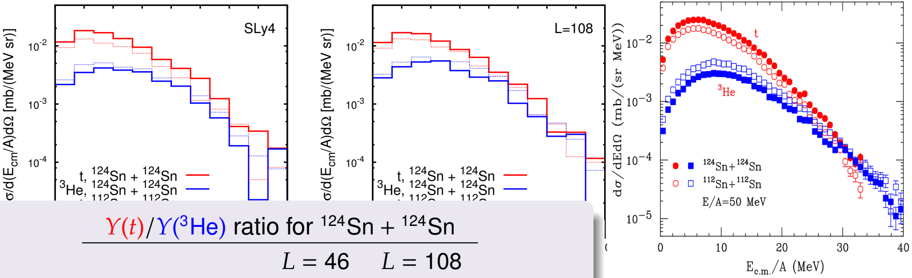

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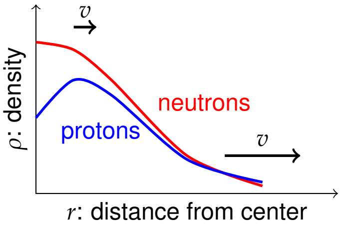
Data @NSCL/MSU
PRC86(2012)024605

- Triton/^{3}He difference is consistent with the gas part of fractionation.
- To reproduce data, there should be more low-energy tritons and less high-energy tritons, and low-energy ^{3}He particles (or protons) should be less. (known as ^{3}He puzzle)

Compression and Expansion Dynamics of Neutrons and Protons



Compared to the current AMD results (with SLy4), neutrons have to be more slowly expanding in order to explain the data of

- the triton spectrum
- the small yield of proton-rich nuclei
- the large energy of proton-rich nuclei

Momentum dependence of the symmetry potential (m_n^* v.s. m_p^*) and something more should be studied.

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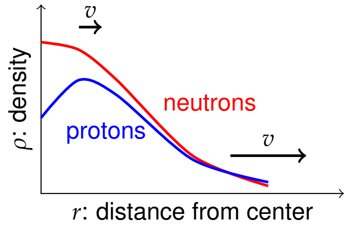

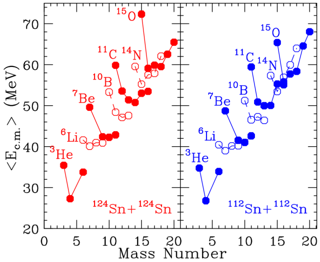

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Data @NSCL/MSU

Liu et al., PRC86(2012)024605

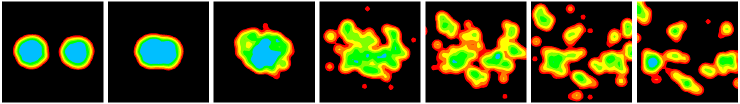
Kinetic energies of proton-rich fragments are anomalously large.

(Yields of low-energy proton-rich fragments are anomalously small.)

(generalized ^{3}He puzzle)

Summary

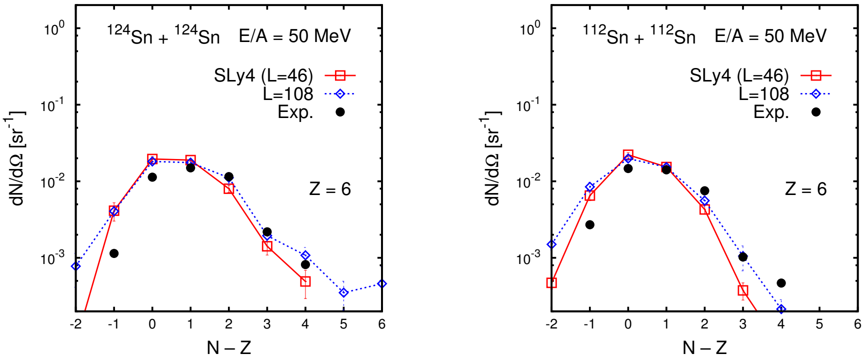
- Importance of clusters
  - as probes for nuclear symmetry energy
  - Cluster correlations are strong enough to affect the equation of state and the collision dynamics
- Handling of clusters in transport models
  - Treat clusters as particles (BUU), or specific quantum final states in two-nucleon collisions (AMD: clusters with $A = 2, 3, 4$)
  - Correlations between clusters are also important for fragment formation.
- Clusters (^{3}H and ^{3}He) in central collisions at 50 MeV/u
  - Dependence on symmetry energy, as expected from the behavior of gas part in fractionation
  - Need to understand the ^{3}He puzzle. Due to the momentum dependence of the symmetry potential, and/or something unknown?



An event of central collision of Xe + Sn at 50 MeV/nucleon (AMD calculation)

Fragment Isotope Distributions

MSU Data: T.X. Liu et al., PRC 014603 (2004).



$\langle N-Z\rangle$ = 0.75 and 0.99 $\langle N-Z\rangle$ = 0.36 and 0.53

- The average asymmetry and the width are sensitive to the symmetry energy.
- Compared to data, $Z \geq N$ fragments are overproduced.