Summary

Nusym13 is an exciting, educating and stimulating workshop

to find your L

Asy-soft

Asymmetry

Some random thoughts about $E_{sym}(\rho)$ at the end of Nusym13

that may be biased, inaccurate, misleading, incomplete, ignorant and …….. provocative, ……

Bao-An Li

Texas A&M University-Commerce

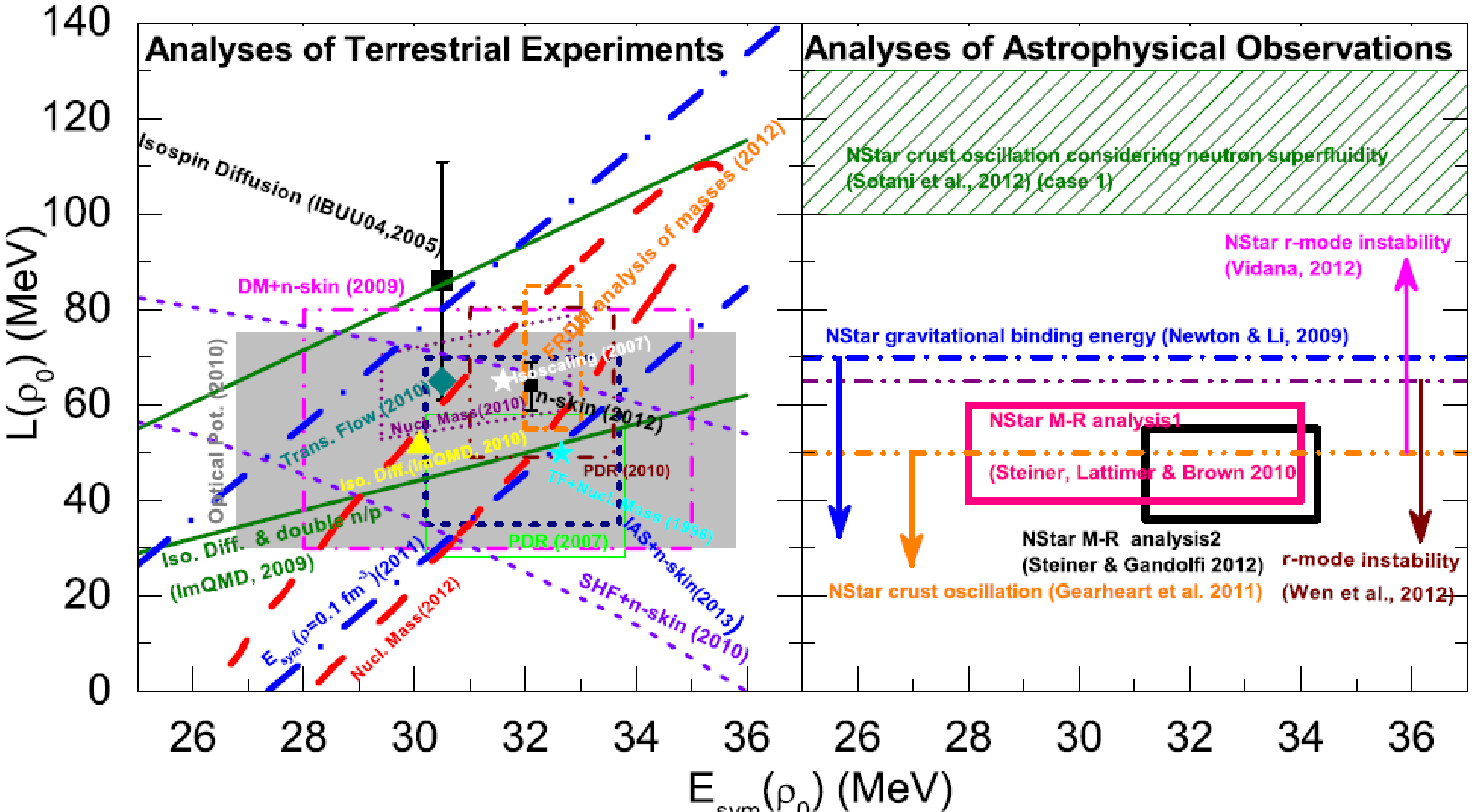
- What we may agree on about Esym and L at ρ_0 and its implications
- What have we learned about the EOS of low-density clustered matter and why it is physically unnecessary and ambiguous to define $E_{sym}(\rho)$ for such matter
- Why the $E_{sym}(\rho)$ especially at supra-saturation density is still so uncertain and what should we do to make progress
- What I may have been completely wrong about the $E_{sym}(\rho)$

NSF

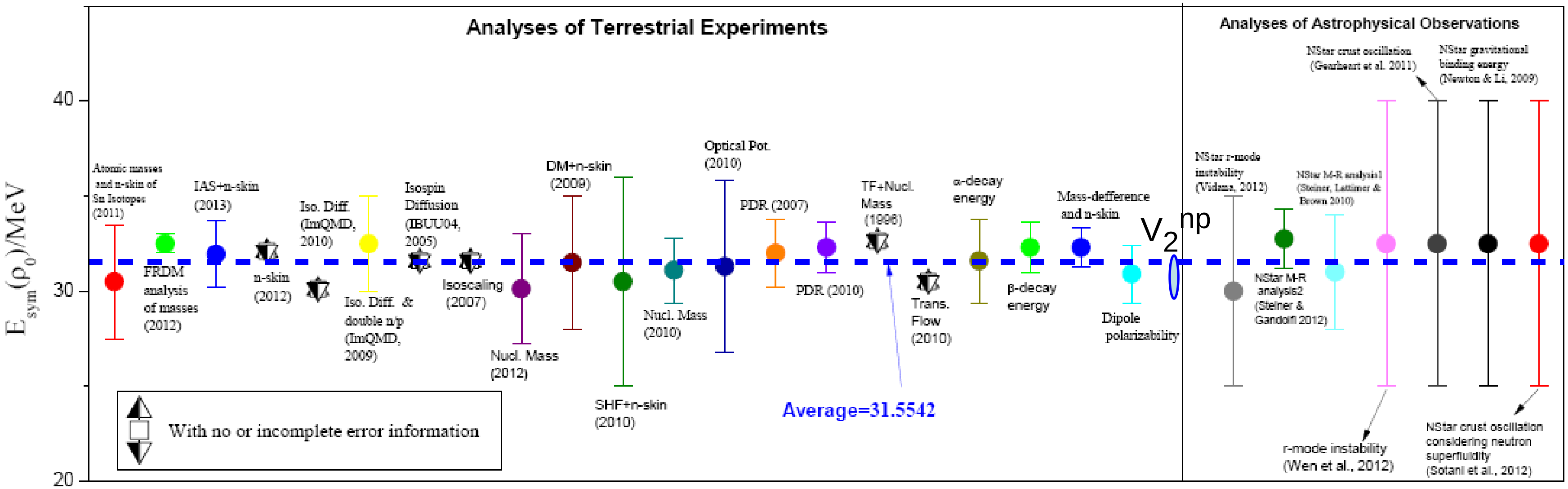
NASA

国家自然科学基金委员会
National Natural Science Foundation of China

Thanks to the hard work of many of you, your postdocs and students as well as supports of your funding agencies



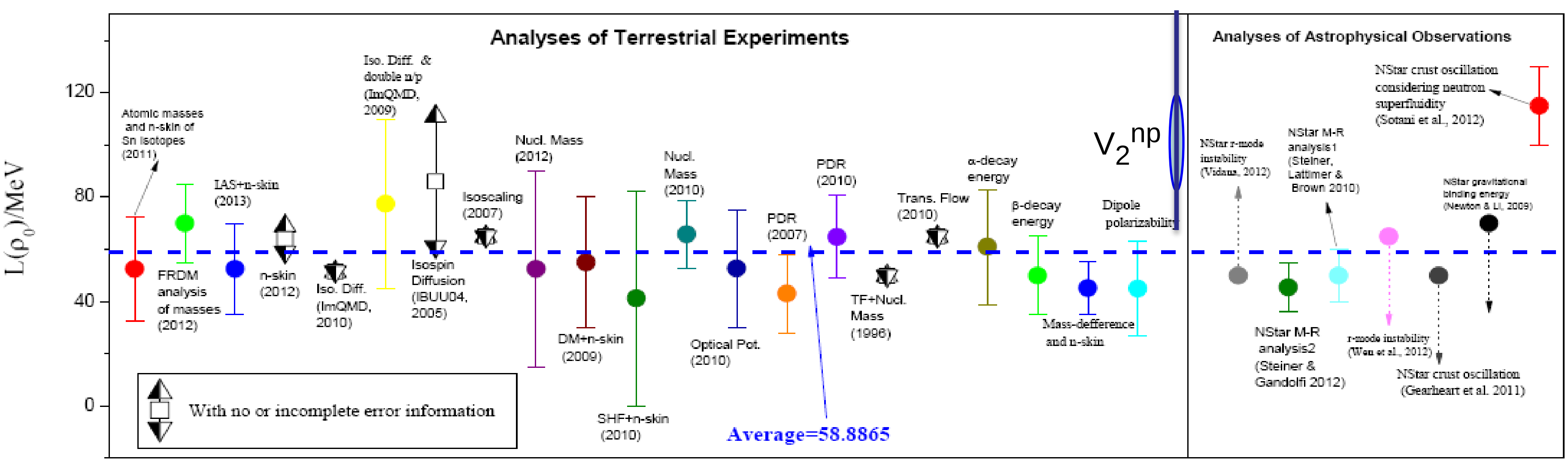
Nusym13 constraints on $E_{sym}(\rho_0)$ and L based on 29 analyses of some data



	Esym	L
average of the means	31.55415	58.88646
standard deviation	0.915867	16.52645

Currently impossible to estimate a physically meaningful error bar

Alex Brown: “K_{sym} is still a random number”

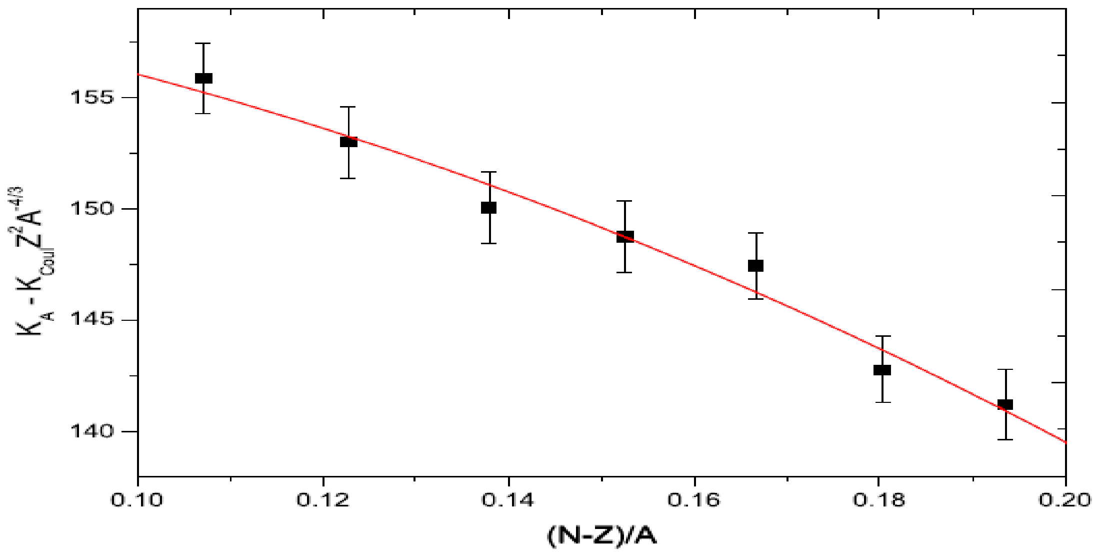


Umesh Garg: $K_\tau = -555 \pm 75$ MeV

$$K_{\rm sat}(\delta) = K_0 + K_{{\rm sat},2}\delta^2 + K_{{\rm sat},4}\delta^4 + O(\delta^6)$$

$$K_{{\rm sat},2} = K_{\rm sym} - 6L - \frac{J_0}{K_0}L,$$

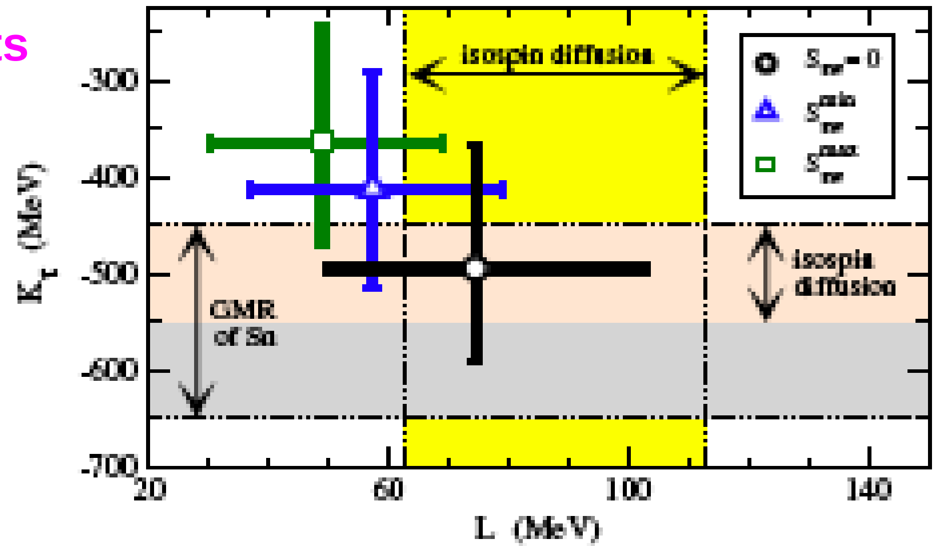
$$J_0 = 27\rho_0^3 \frac{d^3 E_0(\rho)}{d\rho^3}|_{\rho=\rho_0},$$



The large uncertainty in L and Ksym explain why it is so hard to pink down K_τ

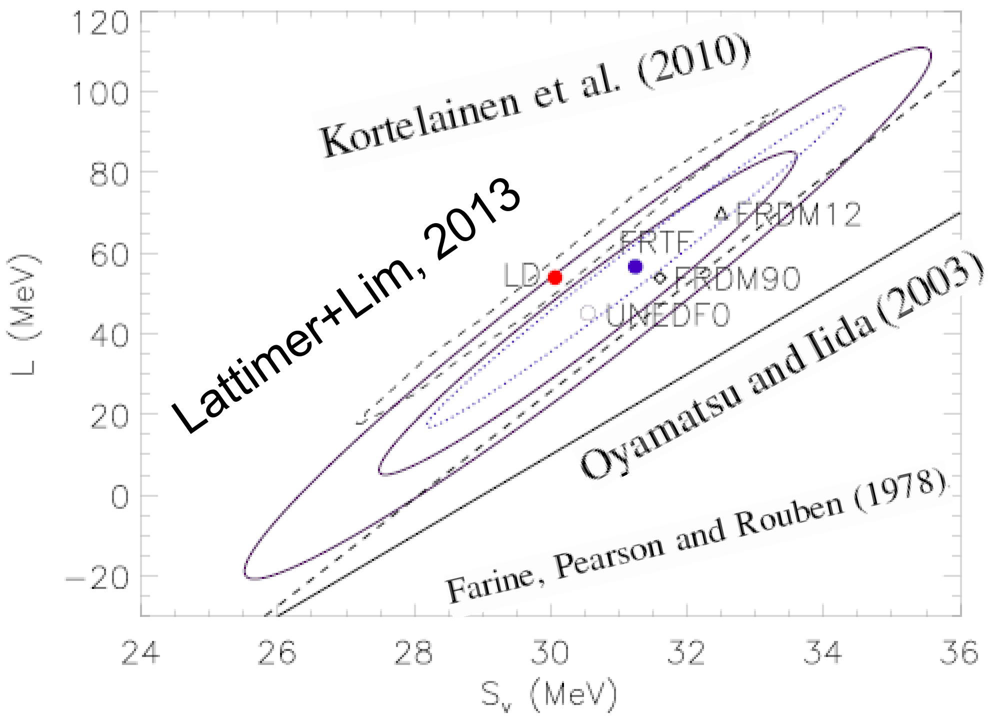
Many ongoing and planned experiments studying various modes to give better constraints on K_0, L and K_{sym}

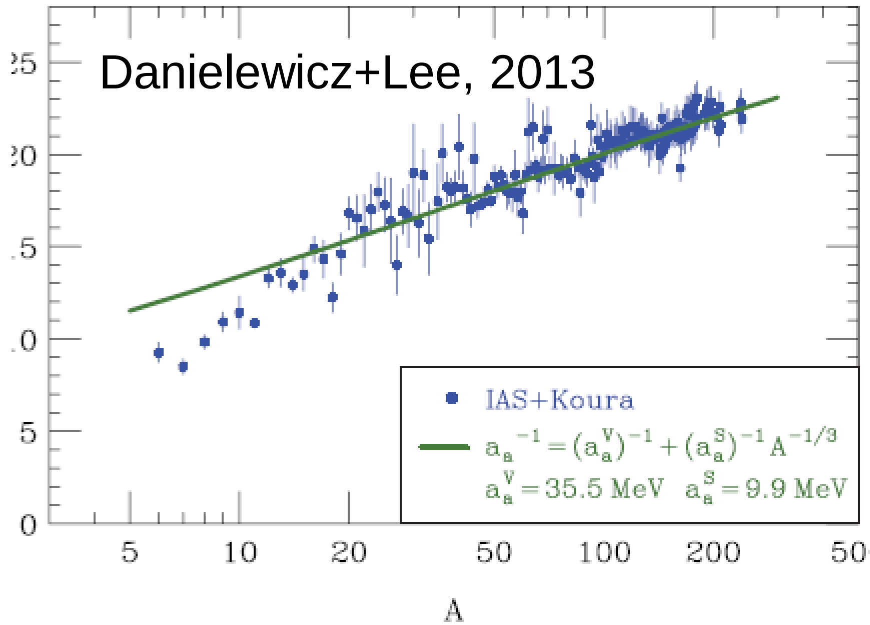
Existing estimates are consistent



M. Centelles *et al.*, Phys. Rev. Lett. **102**, 122502 (2009)

The most accurate and abundant data available for either global or nucleus-by-by nucleus analysis of Esym and L at ρ_0 are the atomic masses: detailed statistical significance analysis for the L-Esym correlation possible,





leading to so far the most accurate extraction (FRDM12):

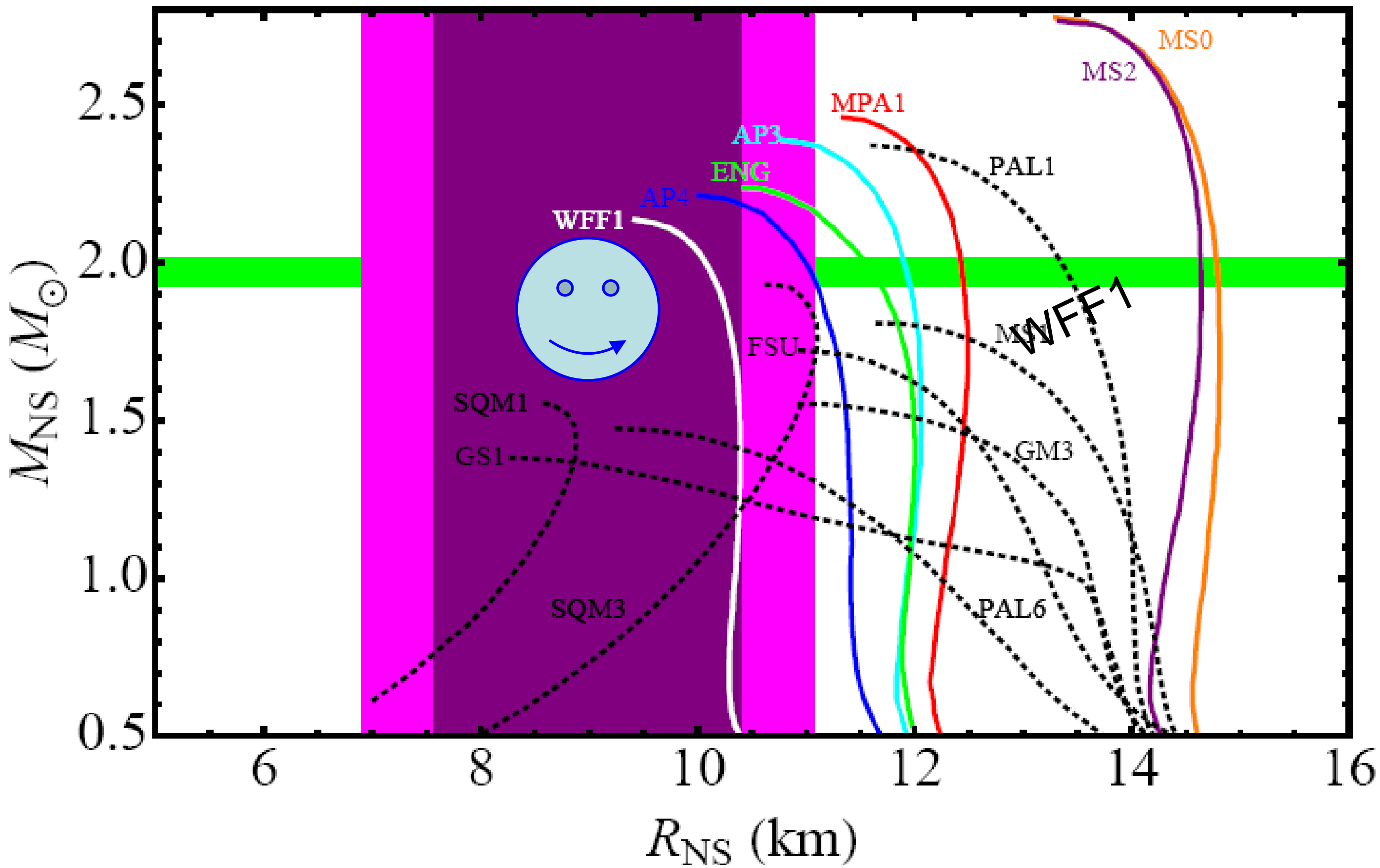
J=32.5±0.5 MeV and L=70±15

Peter Möller, William D. Myers, Hiroyuki Sagawa, and Satoshi Yoshida

Mostly consistent conclusions, necessary to use L-Esym correlations from other observables to pin down the Esym and L more accurately

Rutledge+Guillot: $R_{\rm NS} = 9.1^{+1.3}_{-1.5}\,{\rm km}$ (90%-confidence)

Independent of the masses of neutron stars

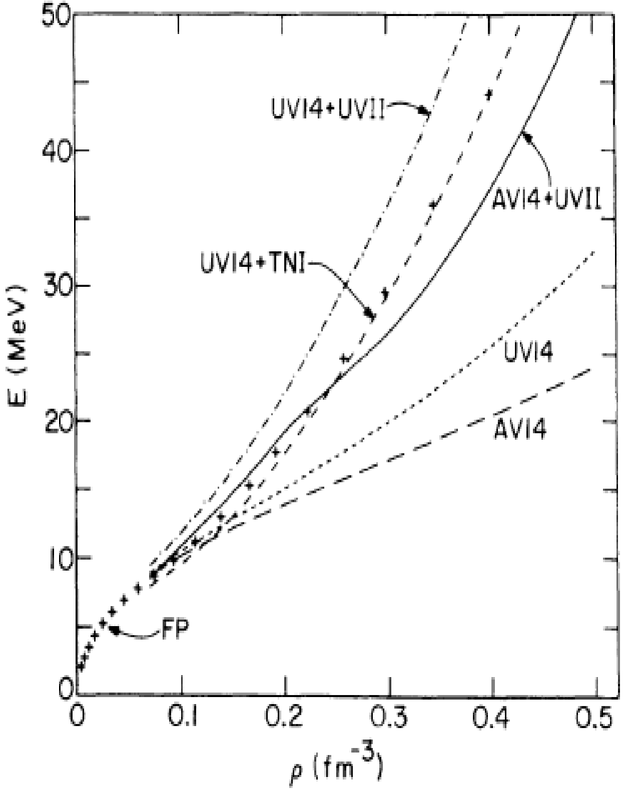


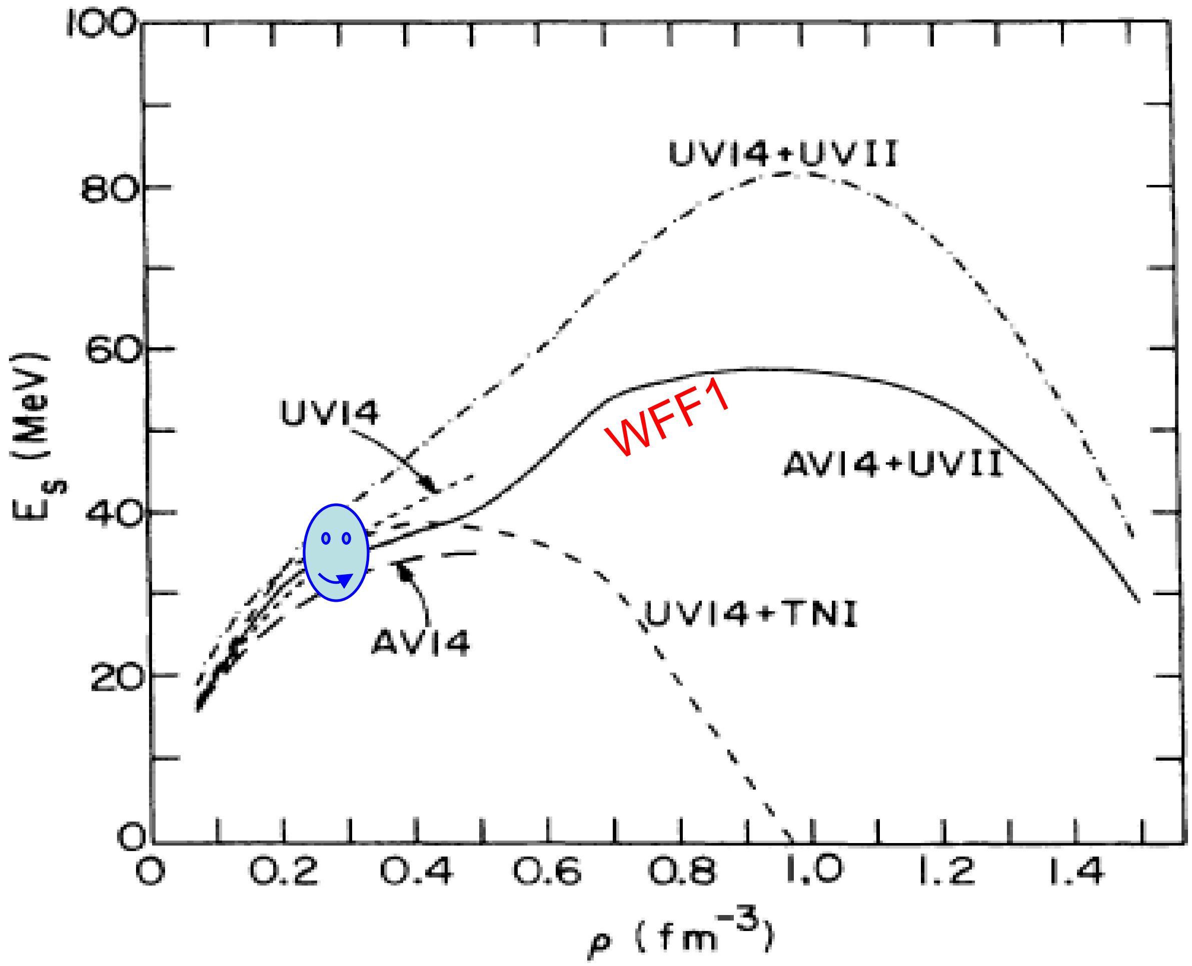
WFF1 has a soft EOS: K_0=209 MeV, E_{sym} ≈26 MeV, L ≈ 60 MeV (estimates)

WFF: Wiringa, Fiks and Fabrocini (1988), Phys. Rev. C 38, 1010

The L is not so different from those studies giving significantly larger radii, is the high density Esym rather than L more important here?

WFF: Wiringa, Fiks and Fabrocini (1988), Phys. Rev. C 38, 1010





WFF1 has a rather soft Esym in the density range of 2-3rho_0 (according to a study by Lattimer and Prakash, R_{ns} is most sensitive to the Esym in this region)

Can the symmetry energy become negative at high densities?

Yes, it happens when the tensor force due to rho exchange in the T=0 channel dominates

Making the EOS of symmetric matter increases faster than the EOS for pure n-matter

Pandharipande V R and Garde V K 1972 *Phys. Lett.* B 39 608

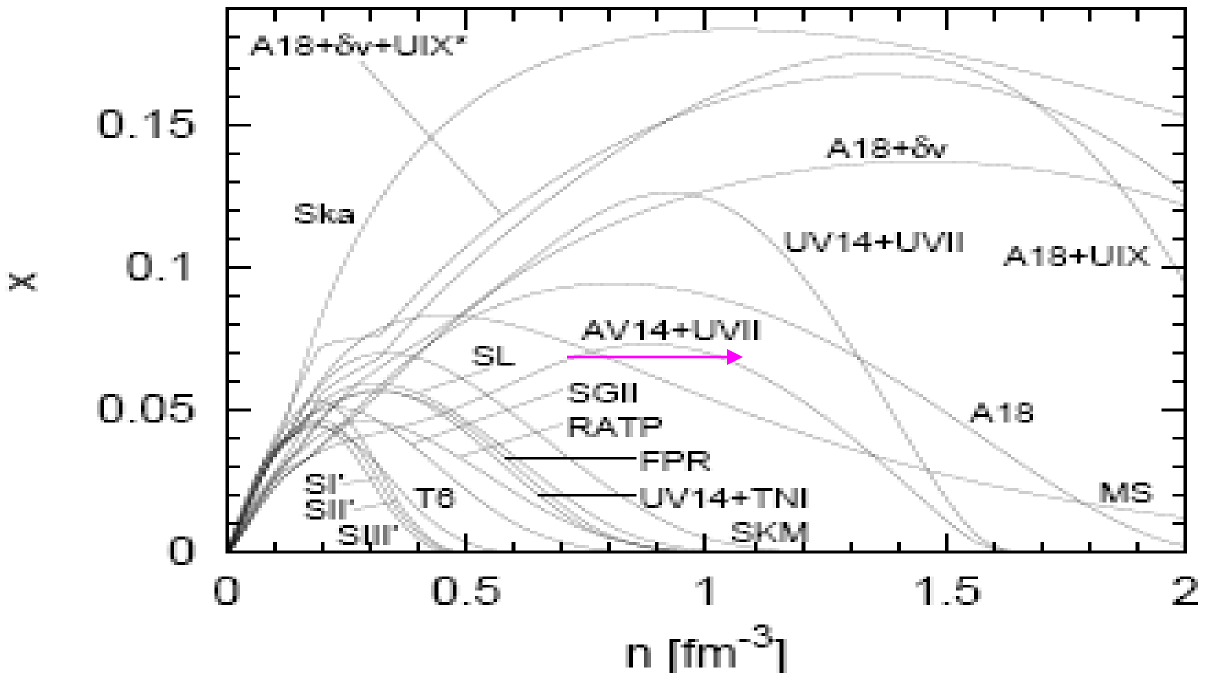
Wiringa R B, Fiks V and Fabrocini A 1988 *Phys. Rev.* C 38 1010

Kutschera M 1994 *Phys. Lett.* B 340 1

Example: proton fractions with interactions/models leading to negative symmetry energy

M. Kutschera et al., Acta Physica Polonica B37 (2006)

$$x = 0.048[E_{sym}(\rho)/E_{sym}(\rho_0)]^3(\rho/\rho_0)(1-2x)^3$$



Some questions about $E_{sym}(\rho)$

Why is it so hard to determine it?

Why L and $E_{sym}(\rho_0)$ are correlated?

Some hints and possible answers at the mean-field level

Relationship between the symmetry energy and the mean-field potentials

Lane potential $U_{n/p} = U_0 \pm U_{sym}\delta$

Both U_0 (ρ,k) and $U_{sym}(\rho,k)$ are density and momentum dependent

Symmetry energy $$E_{sym}(\rho) = \frac{1}{6}\frac{\partial(\overset{\text{kinetic}}{t} + \overset{\text{isoscalar}}{U_0})}{\partial k}|_{k_F} k_F + \frac{1}{2}\overset{\text{isovector}}{U_{sym}}(\rho, k_F)$$

Isoscarlar effective mass $$m^*/m = [1 + \frac{m}{k_F}\frac{\partial U_0}{\partial k}|_{k_F}]^{-1}$$

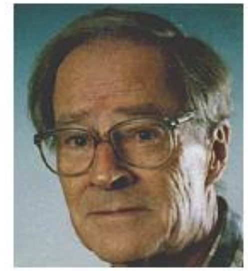
$$E_{sym}(\rho) = \frac{1}{3}\boxed{\frac{\hbar^2 k_F^2}{2m^*}} + \frac{1}{2}U_{sym}(\rho, k_F)$$

Using K-matrix theory, the conclusion is independent of the interaction

K. A. Brueckner and J. Dabrowski, Phys. Rev. **134**, B722 (1964); J. Dabrowski and P. Haensel, Phys. Lett. B **42**, 163 (1972); Phys. Rev. C **7**, 916 (1973); Can. J. Phys. **52**, 1768 (1974).

Hugenholtz-Van Hove theorem

Powerful tool to check thermodynamical consistency



N. M. Hugenholtz

L. Van Hove

Microphysics governing the $E_{sym}(\rho)$ and $L(\rho)$ according to the Hugenholtz-Van Hove (HVH) theorem

$$E_{sym}(\rho) = \frac{1}{3}\frac{\hbar^2 k^2}{2m_0^*}|_{k_F} + \frac{1}{2}U_{sym,1}(\rho, k_F),$$

$$L(\rho) = \frac{2}{3}\frac{\hbar^2 k^2}{2m_0^*}|_{k_F} - \frac{1}{6}\left(\frac{\hbar^2 k^3}{m_0^{*2}}\frac{\partial m_0^*}{\partial k}\right)|_{k_F} + \frac{3}{2}U_{sym,1}(\rho, k_F) + \frac{\partial U_{sym,1}}{\partial k}|_{k_F} \cdot k_F + 3U_{sym,2}(\rho, k_F),$$

$$U_{n/p}(k,\rho,\delta) = U_0(k,\rho) \pm U_{sym1}(k,\rho)\bullet\delta + U_{sym2}(k,\rho)\bullet\delta^2 + o(\delta^3)$$

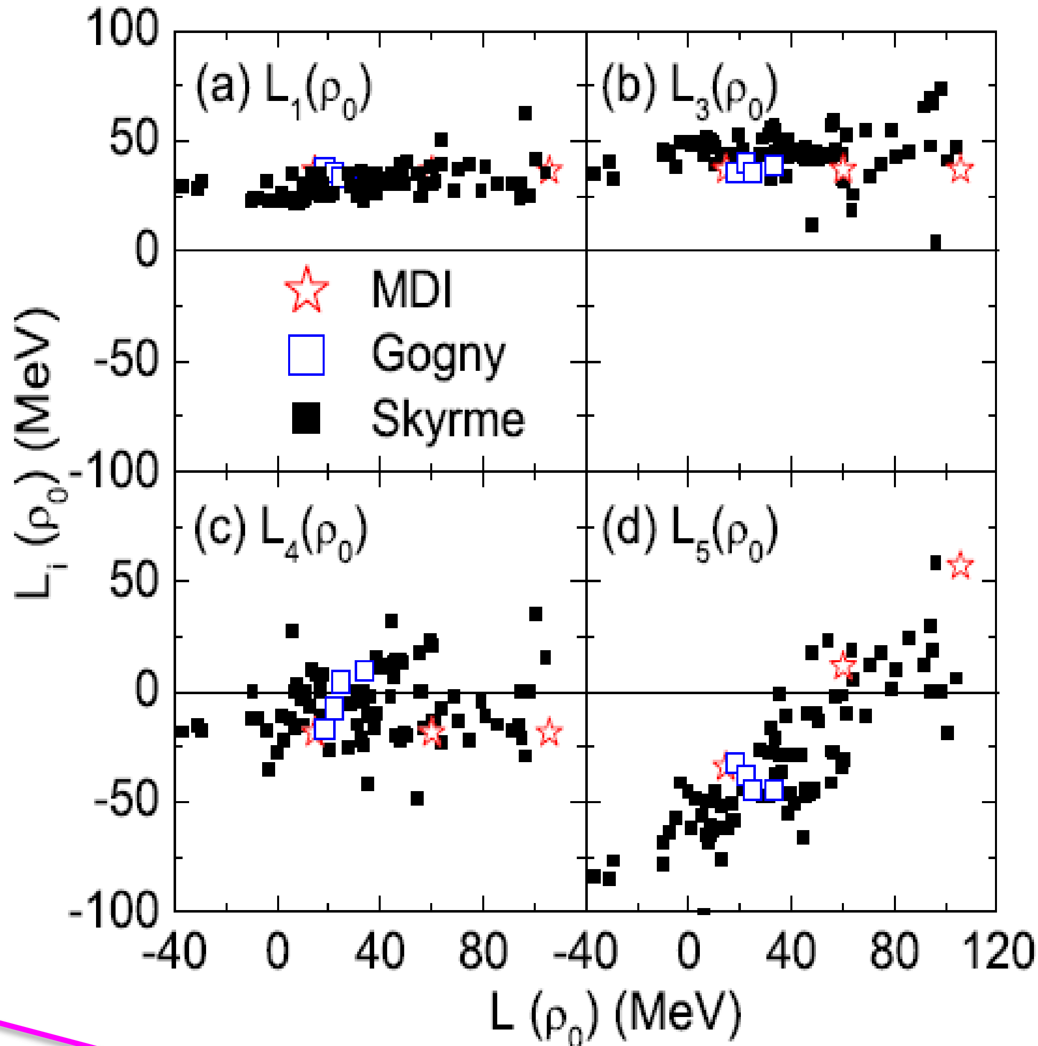
C. Xu, B.A. Li, L.W. Chen and C.M. Ko, NPA 865, 1 (2011)

R. Chen et al., PRC 85, 024305 (2012).

Rong Chen, Bao-Jun Cai, Lie-Wen Chen, Bao-An Li, Xiao-Hua Li, Chang Xu

PRC 85, 024305 (2012).

$$
\begin{aligned}
L_1(\rho) &= \frac{2}{3}\frac{\hbar^2 k_F^2}{2m_0^*(\rho,k_F)} \\
L_2(\rho) &= -\frac{1}{6}\frac{\hbar^2 k_F^3}{m_0^{*2}(\rho,k_F)}\frac{\partial m_0^*(\rho,k)}{\partial k}|_{k_F} \\
L_3(\rho) &= \frac{3}{2}U_{sym,1}(\rho,k_F) \\
L_4(\rho) &= \frac{\partial U_{sym,1}(\rho,k)}{\partial k}|_{k_F}\cdot k_F \\
L_5(\rho) &= 3U_{sym,2}(\rho,k_F).
\end{aligned}
$$



Need FRIB to determine

$$U_{n/p}(k,\rho,\delta) = U_0(k,\rho) \pm U_{sym1}(k,\rho)\bullet\delta + U_{sym2}(k,\rho)\bullet\delta^2 + o(\delta^3)$$

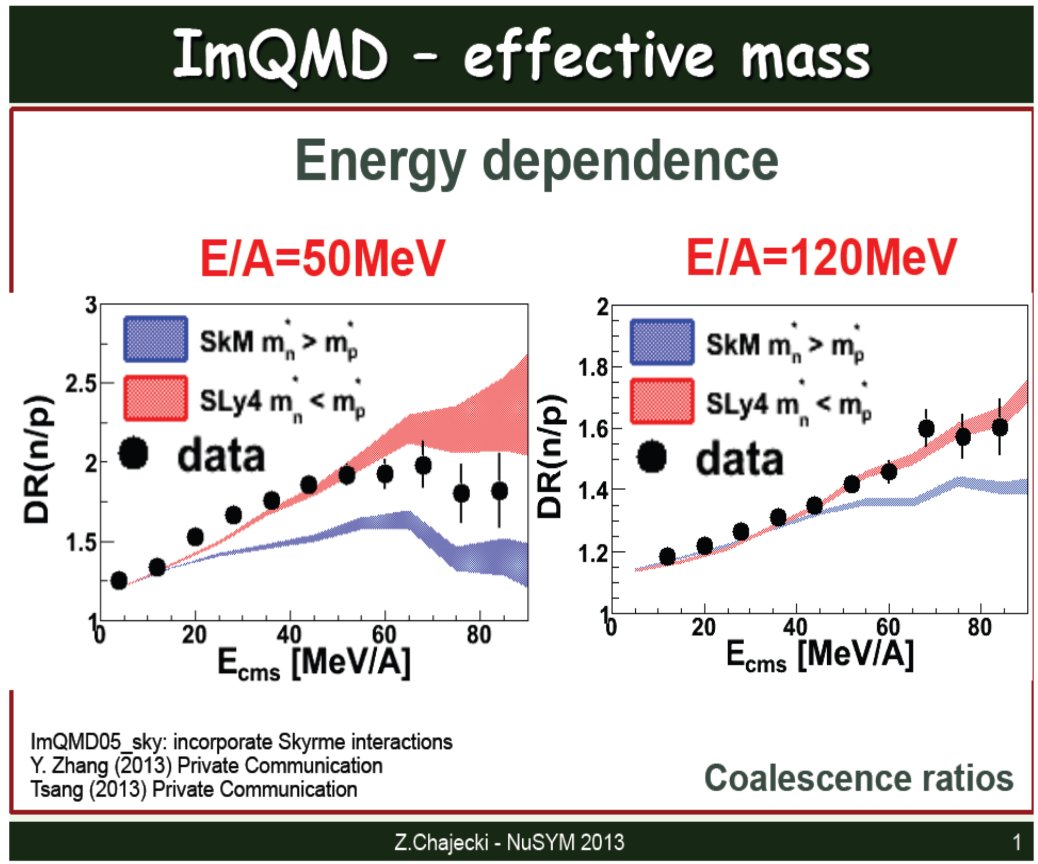
A major issue near saturation density

$$\frac{m_n^* - m_p^*}{m} = -2\delta \frac{m}{\hbar^2 k_F} \frac{dU_{\rm sym}}{dk}\bigg|_{k_F} \bigg/ \left(1 + 2\frac{m}{\hbar^2 k_F}\frac{dU_0}{dk}\bigg|_{k_F}\right)$$

Possible experimental tests:

Esym and the effective
mass splitting are NOT
2 independent issues/quantities!

The effective mass splitting is an
Important part of the L and mainly
responsible for its uncertainty!



Symmetry (isovector) potential and its major uncertainties

- Isospin-dependence of NN correlations and the tensor force

Experimental indication: BNL (p,p'pp) and (p,p'pn) and JLab (e,e'pp) and (e,e'pn) experiments,

A. Tang et al., PRL 90, 042302 (2003); E. Piasetzky et al., PRL97, 162504 (2006);

B. R. Subedi et al. Science 320,1476 (2008)………………

Theories: MANY papers,

R. Schiavilla et al., PRL 98, 132501 (2007); M. Alvioli et al., PRL 100, 162503 (2008);

L. Frankfurt, M. Sargsian and M. Strikman; T. Neff, H. Feldmeier; W. Dickhoff, Wiringa, ………………

Within an interacting Fermi gas model, schematically,

Structure of the nucleus, M.A. Preston and R.K. Bhaduri (1975)

NN correlation functions

$$U_{sym}(k_F,\rho) = \frac{1}{4}\rho \int [V_{T1}(r_{ij}) f^{T1}(r_{ij}) - V_{T0}(r_{ij}) f^{T0}(r_{ij})] d^3 r_{ij}$$

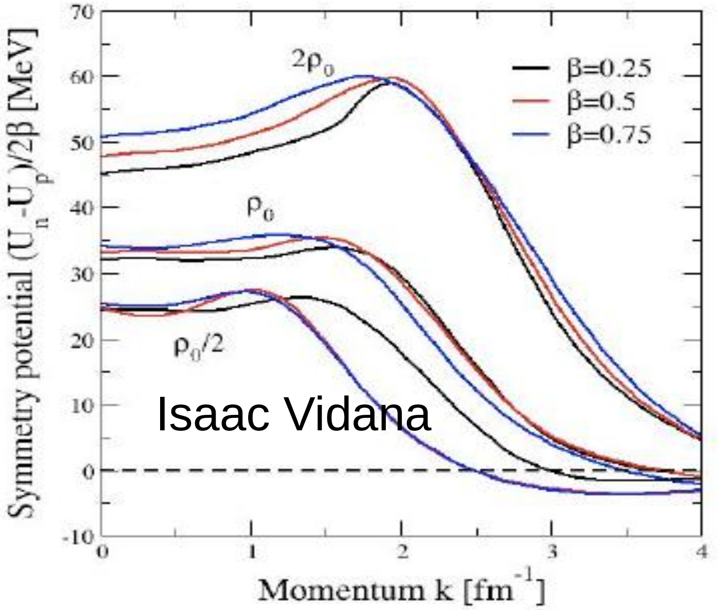
- Spin-isospin dependence of 3-body forces
- Short-range tensor force due to rho meson exchange

$$V_{T0} = V'_{np} \quad \text{(n-p pair in the T=0 state)}$$

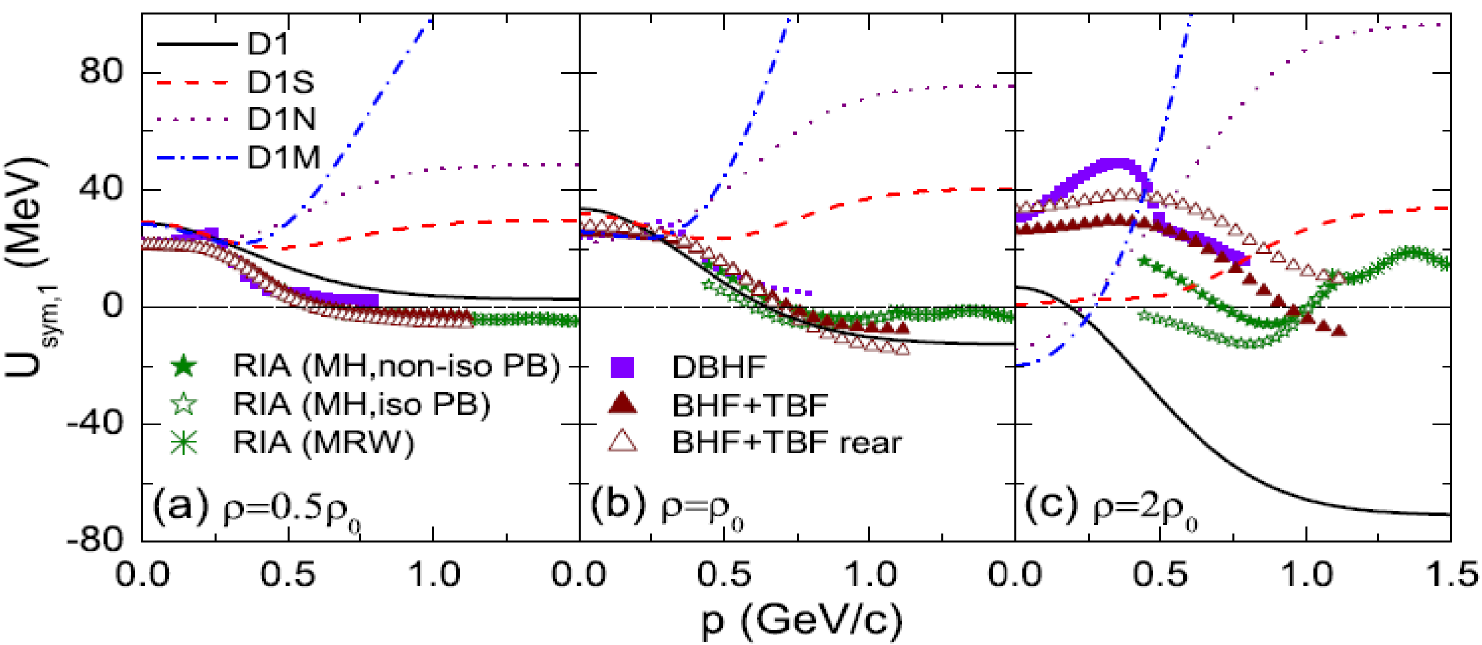
$$V_{T1} = V_{nn} = V_{pp} = V_{np} \quad \text{(charge independence in the T=1 state)}$$

$U_{sym,1}$ (ρ,p) in several models

$$\frac{m_n^* - m_p^*}{m} = -2\delta \frac{m}{\hbar^2 k_F} \frac{dU_{sym}}{dk}\bigg|_{k_F} \Bigg/ \left(1 + 2\frac{m}{\hbar^2 k_F} \frac{dU_0}{dk}\bigg|_{k_F}\right)$$



R. Chen et al., PRC 85, 024305 (2012).



Symmetry potential near saturation density from global nucleon optical potentials

Systematics based on world data accumulated since 1969:

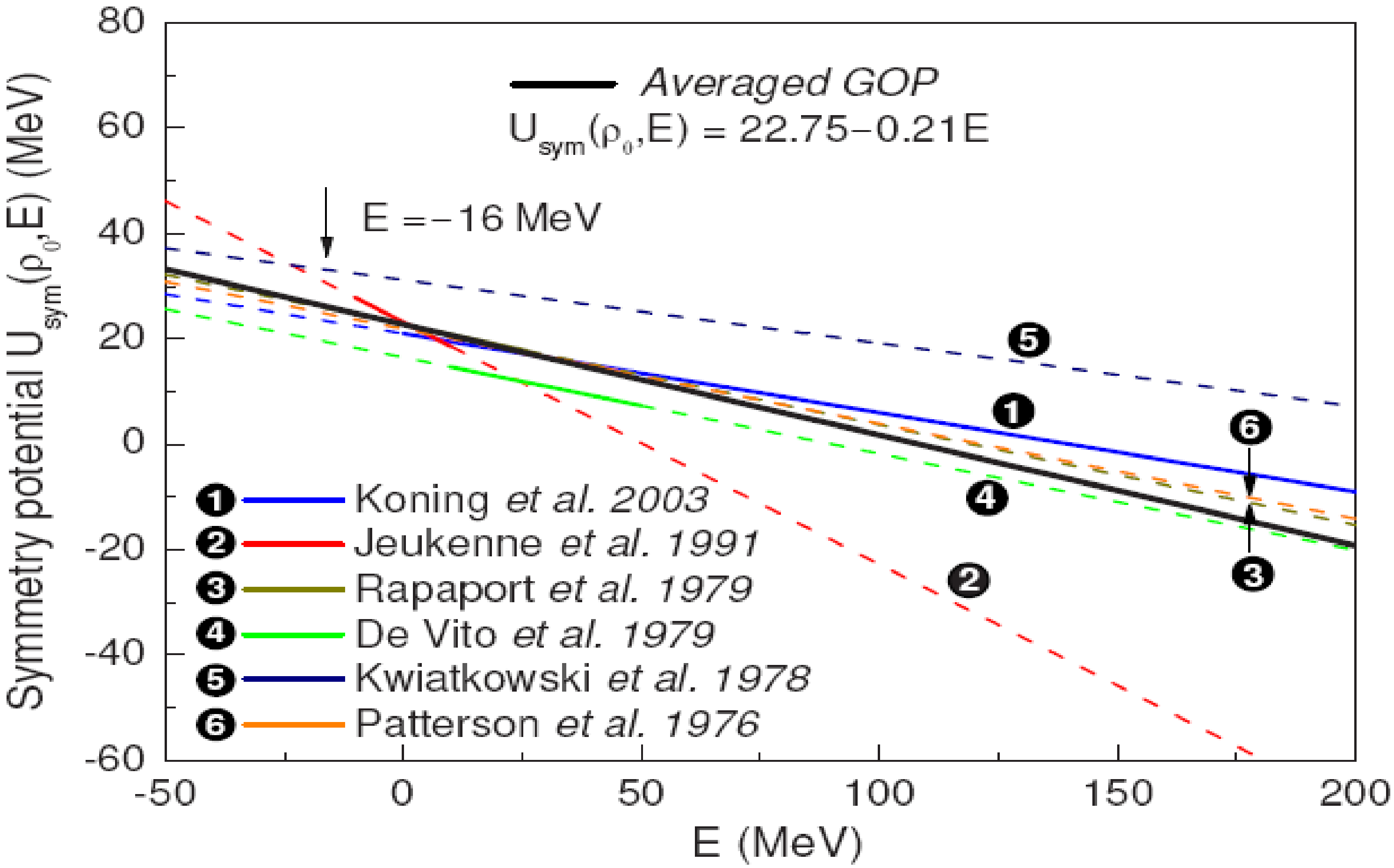
(1) Single particle energy levels from pick-up and stripping reaction

(2) Neutron and proton scattering on the same target at about the same energy

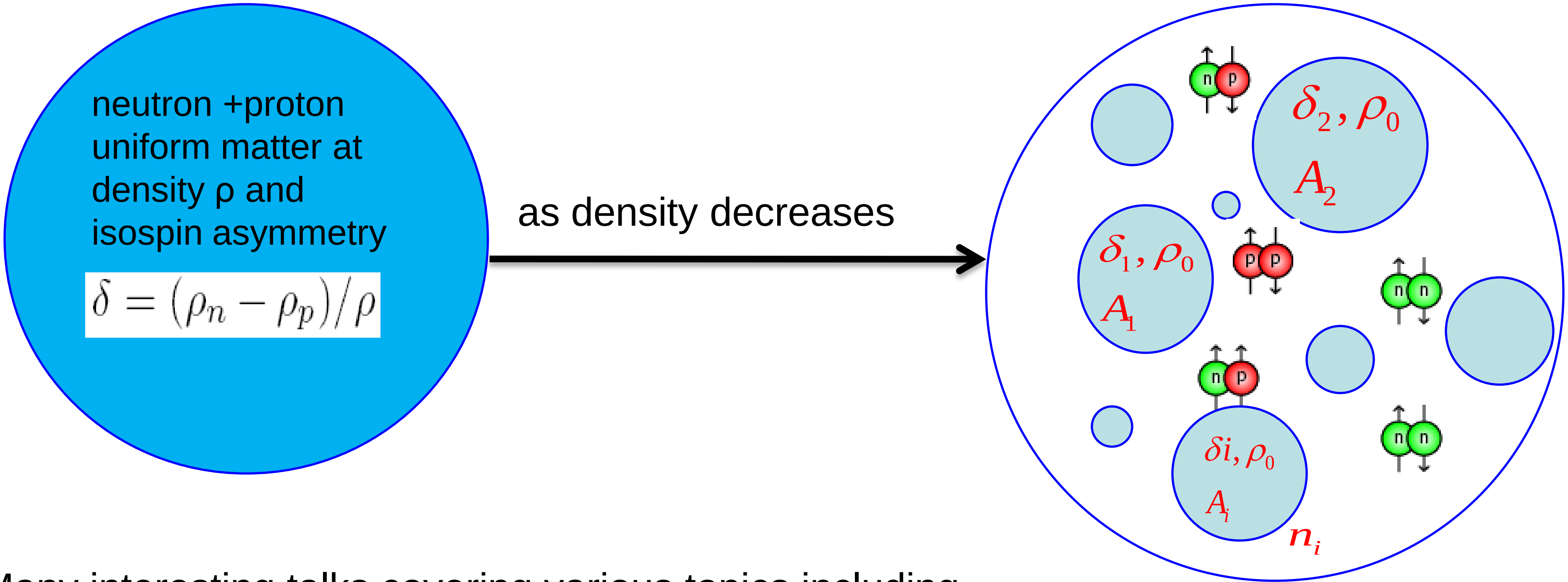
(3) Proton scattering on isotopes of the same element

(4) (p,n) charge exchange reactions

Chang Xu, Bao-An Li, Lie-Wen Chen Phys.Rev.C82:054607,2010



Some basic issues on low density, hot neutron-rich matter



Many interesting talks covering various topics including

$$\delta = \sum_i n_i \frac{A_i}{A} \delta_i \qquad \langle \rho \rangle = \frac{A}{V} \leq \rho_0$$

At finite Temperature T

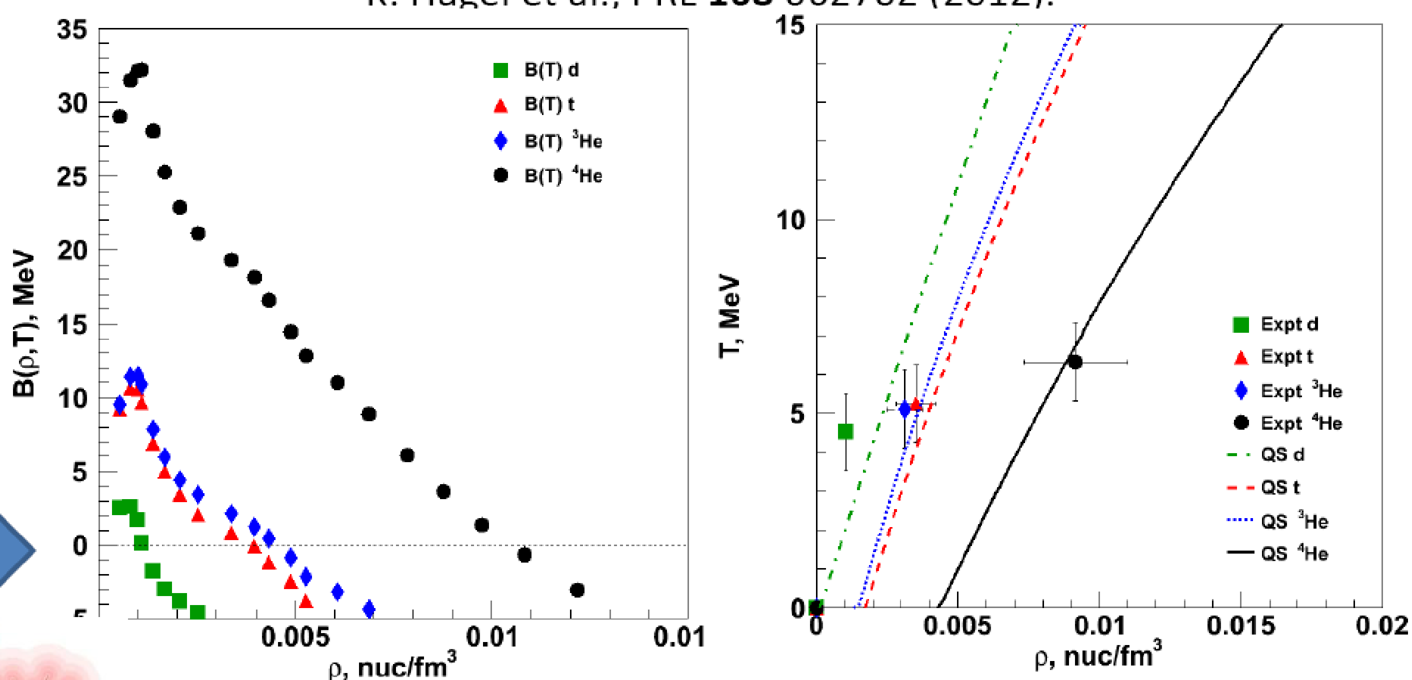
- What is the EOS of clustered neutron-rich matter with pairing and its astrophysical ramifications
- In-medium properties of finite nuclei, Mott points, isospin dependence of the Caloric curve…
- Symmetry energy of hot nuclei and the meaning of isoscaling coefficients
- The origin of the Wigner term or linear symmetry energy

Joe Natowitz

Density Dependent Binding Energies of Light Clusters – The Mott Point

- Successful QSM Model of Roepke et al. Incorporates In-Medium modification of Cluster Binding Energies . Clusters become unbound wrt medium at the Mott Point
- Mott points determined in reactions of 47AMeV ^{40}Ar and ^{64}Zn projectiles with $^{112, 124}$Sn are in close agreement with the theoretical estimates

K. Hagel et al., PRL **108** 062702 (2012).



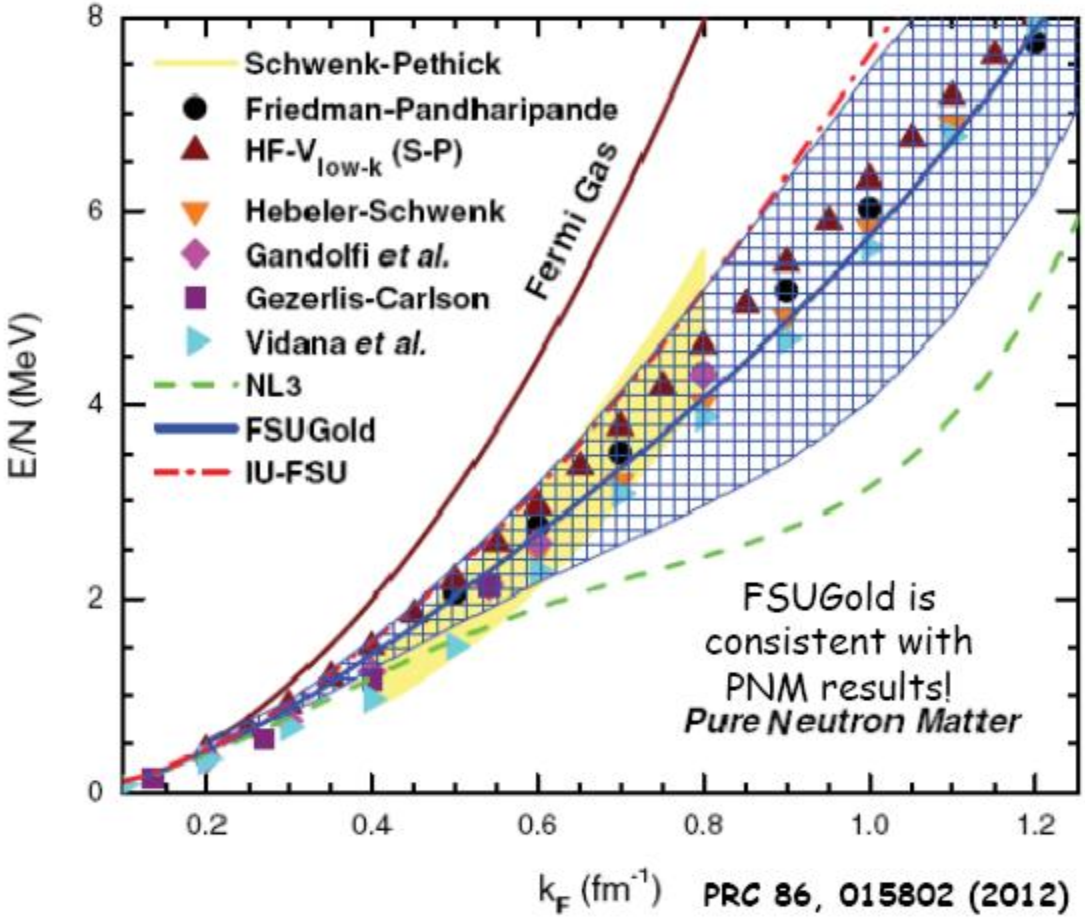
The Mott Point

CYCLOTRON INSTITUTE
Texas A&M University

Comments on the EOS of pure neutron matter and its role in constraining the $E_{sym}(\rho)$=EOS(pnm)-EOS(snm)

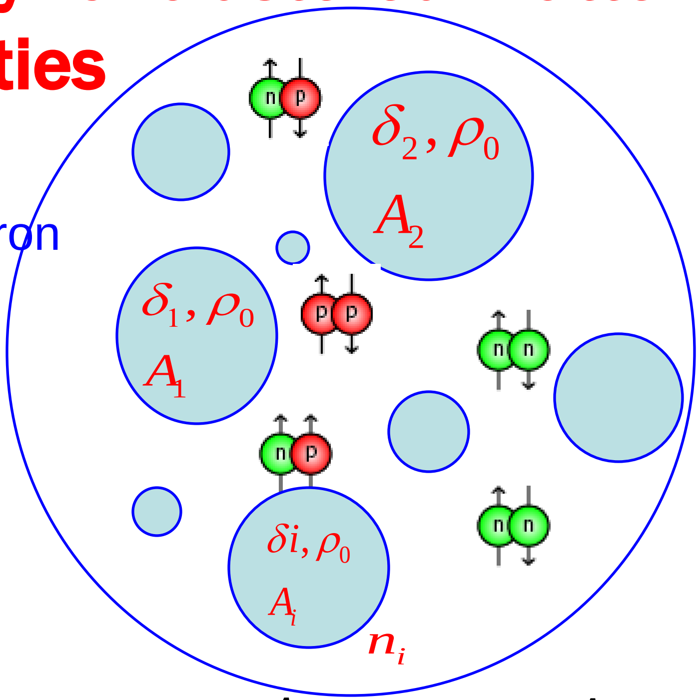
- Impressive progress made in calculating the EOS_{PNM} providing a theoretical boundary condition to calibrate the EOS of asymmetric matter
- It does constrain the $E_{sym}(\rho)$ around saturation point assuming the EOS of symmetric matter is well understood
- It does NOT constrain the $E_{sym}(\rho)$ away from ρ_0 where it is harder to calculate the EOS of SNM due to the tensor force, etc

EOS_{PNM} provides a theoretical boundary condition to calibrate the EOS of asymmetric matter



Comments on the "symmetry energy" of clustered matter at very low densities

To my best knowledge, for all practical purposes of calculating the EOS for supernovae simulation and neutron star properties, it is **Unnecessary** to define a "Esym" for the clustered matter, what is needed are in-medium properties of hot nuclei and their Esym



It is physically ambiguous to define and talk about the "Esym" for clustered matter

$$\delta = \sum_i n_i \frac{A_i}{A} \delta_i \qquad \langle \rho \rangle = \frac{A}{V} \leq \rho_0$$

For clustered matter there is no more n $\leftrightarrow$ p invariance because of the Coulmb term in the binding energies of nuclei, interactions among them and the asymmetry between proton and neutron driplines, the EOS can have odd terms in δ and it does NOT minimize at δ=0

$$E(\overline{\rho},\delta) = E_0(\overline{\rho},\delta=0) + E_{s1}(\overline{\rho})\times\delta + E_{s2}(\overline{\rho})\times\delta^2 + E_{s3}(\overline{\rho})\times\delta^3 + o(\delta^4)$$

Promising Probes of the $E_{sym}(\rho)$ in Nuclear Reactions

At sub-saturation densities

- Global nucleon optical potentials from n/p-nucleus and (p,n) reactions
- **Thickness of n-skin in ^{208}Pb measured using various approaches and sizes of n-skins of unstable nuclei from total reaction cross sections**
- **n/p ratio of FAST, pre-equilibrium nucleons**
- **Isospin fractionation and isoscaling in nuclear multifragmentation**
- **Isospin diffusion/transport**
- **Neutron-proton differential flow**
- **Neutron-proton correlation functions at low relative momenta**
- **t/^{3}He ratio and their differential flow**

Towards supra-saturation densities

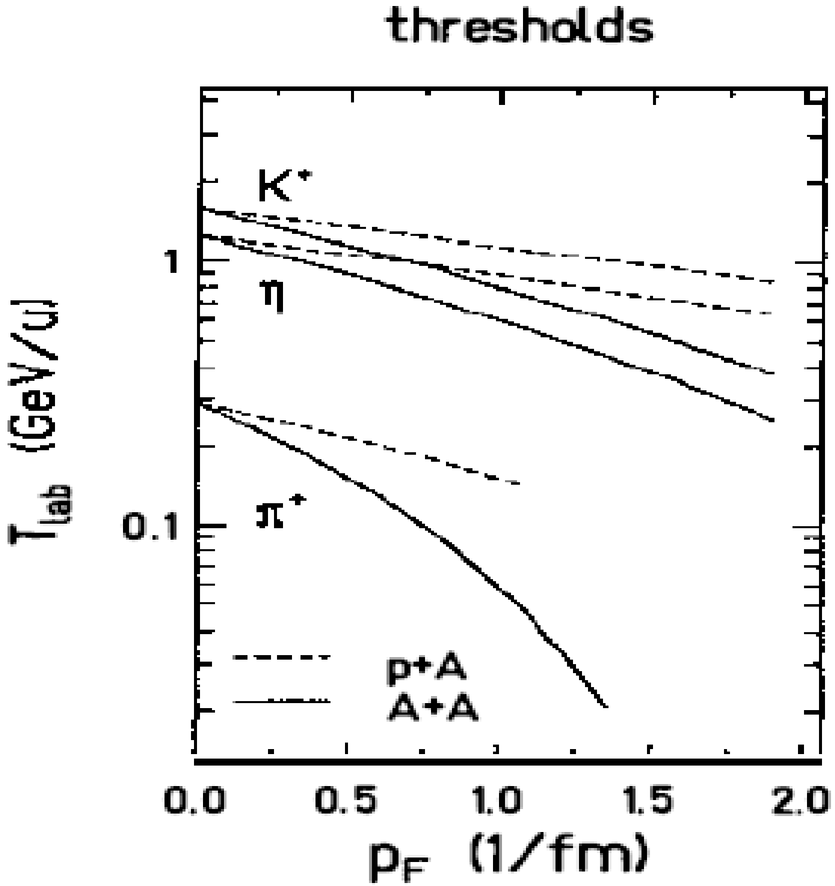
- **π^-/π^+ ratio, K^+/K^0 ?**
- **Neutron-proton differential transverse flow**
- **n/p ratio of squeezed-out nucleons perpendicular to the reaction plane**
- **Nucleon elliptical flow at high transverse momentum**
- **t-^{3}He differential and difference transverse flow**

(1) Correlations of multi-observable are important
(2) Detecting neutrons simultaneously with charged particles is critical

B.A. Li, L.W. Chen and C.M. Ko, *Physics Reports 464, 113 (2008)*

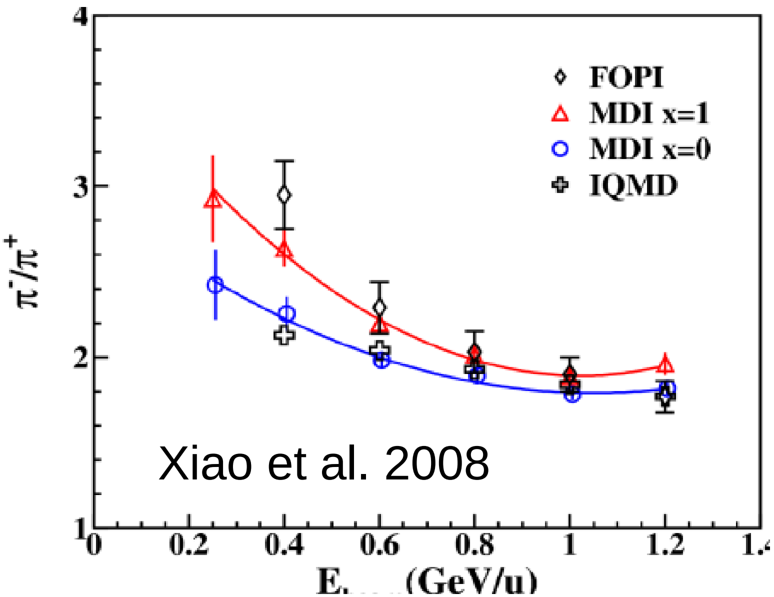
Probing the symmetry energy at supra-saturation densities

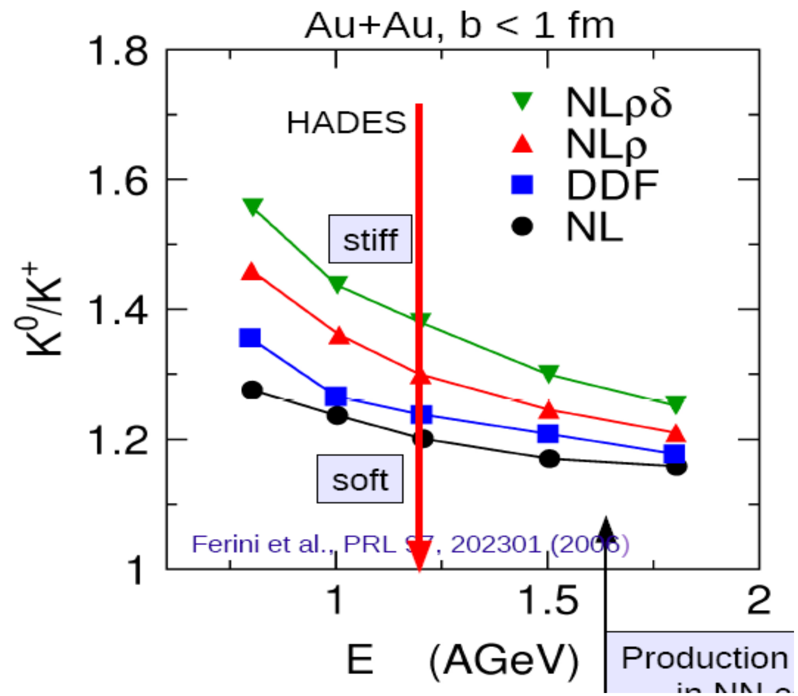
- <u>π^-/π^+</u>, K^+/K^0, η
- **Neutron-proton differential or relative flows**
- **Neutrino flux of supernova explosions (Luke Roberts)**
- **Strength and frequency of gravitational waves (Will Newton)**

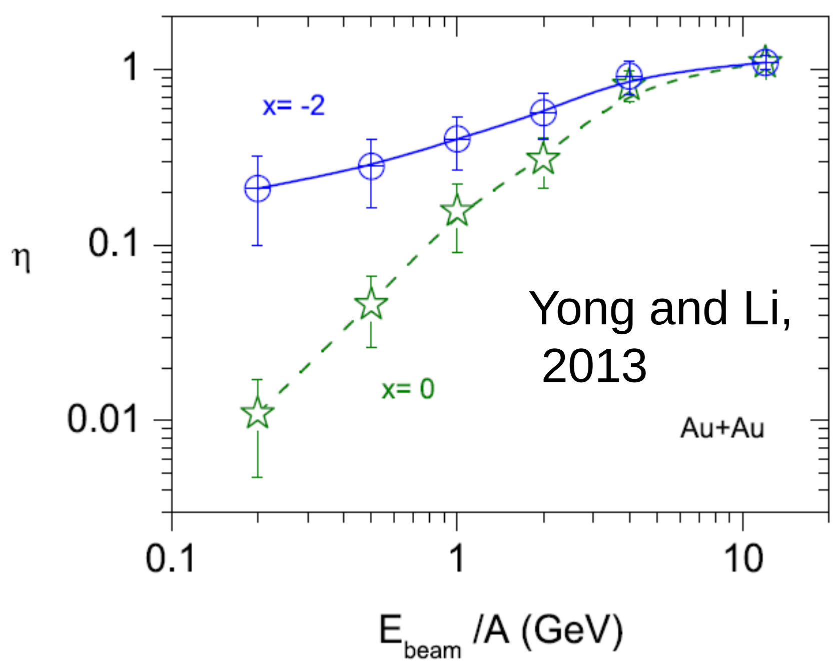


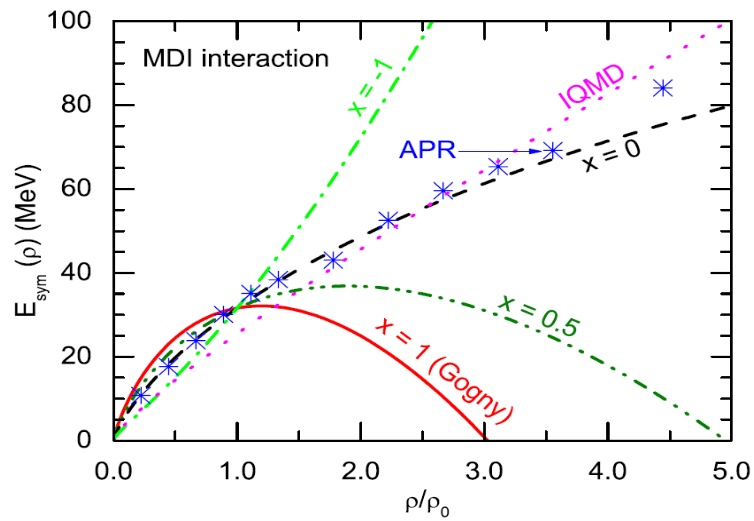
U. Mosel, Ann. Rev. Nucl. Part. Sci. 41, (1991) 29

Yvonne Leifels





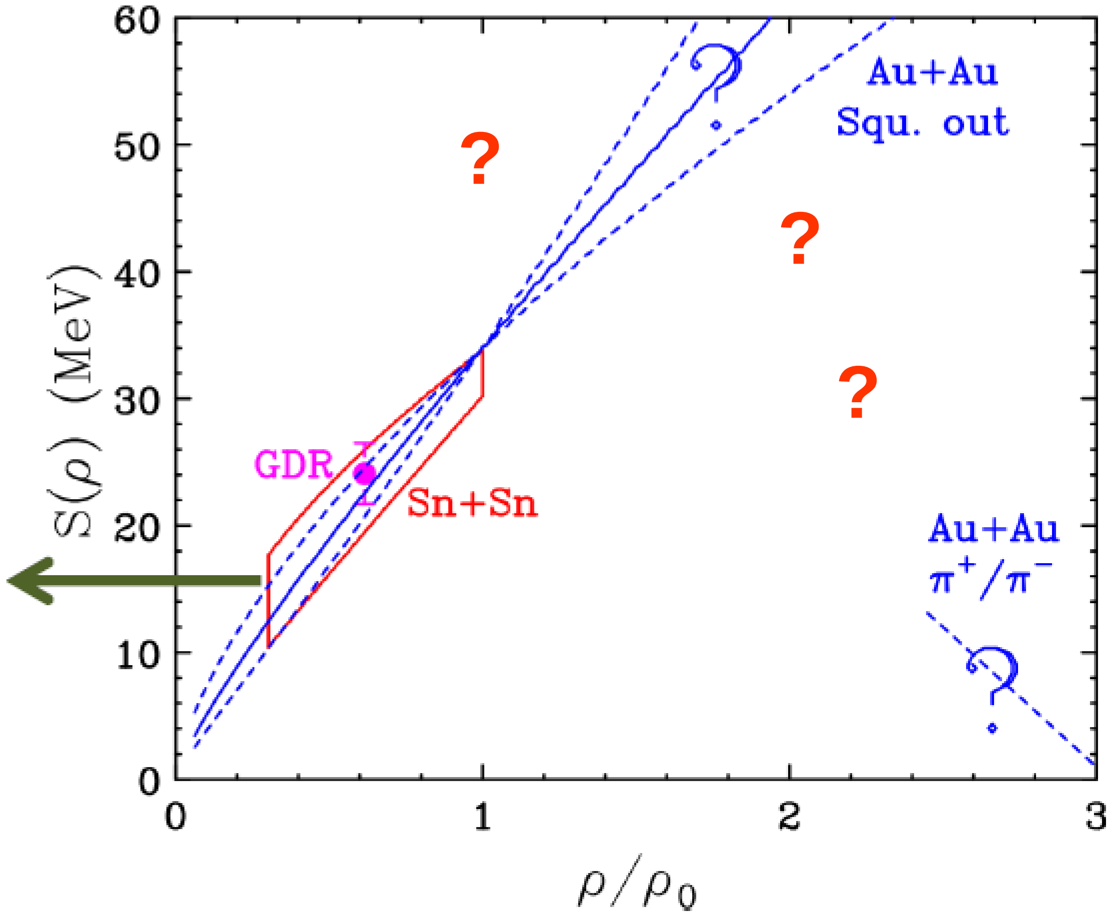




The high-density Esym situation is really a mess and Bao-An Li should be blamed^2

Besides the permanent possibility that some mistakes are in the code, What I may have been wrong about the physics of Esym?

Isospin dependence of tensor forced induced short-range NN correlations



Tensor force induced (1) high-momentum tail in single-particle momentum distribution and (2) isospin dependence of NN correlation

Theory of Nuclear matter
H.A. Bethe
Ann. Rev. Nucl. Part. Sci., 21, 93-244 (1971)

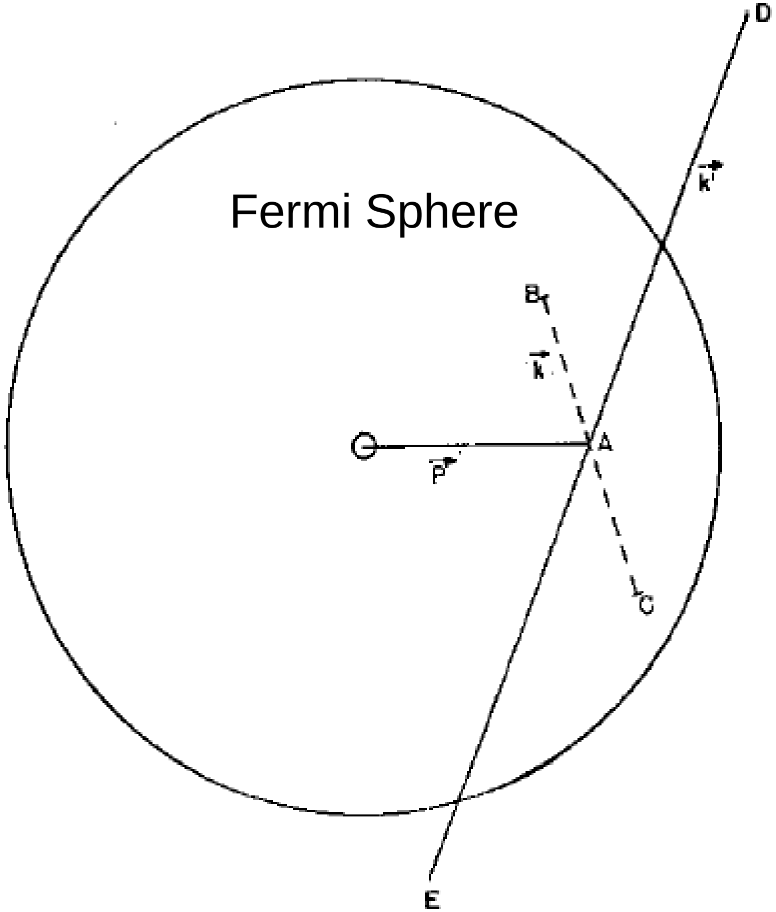
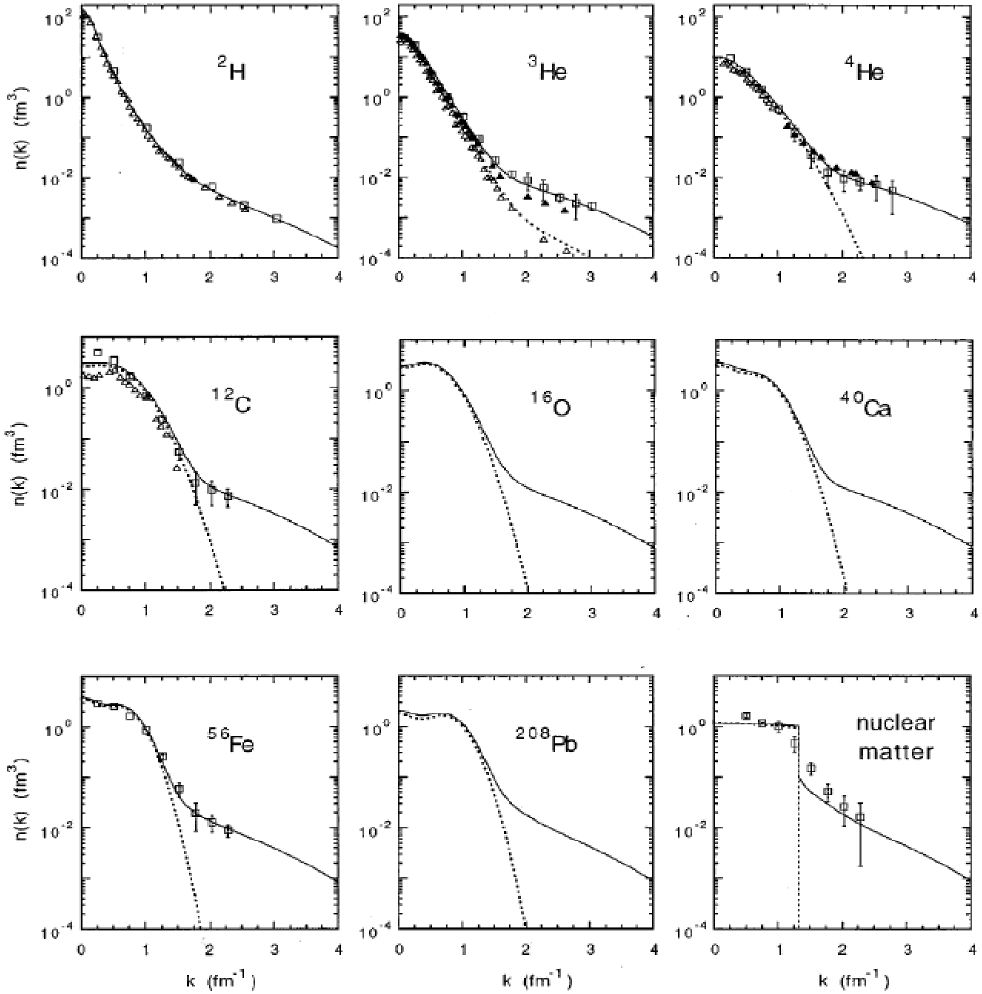
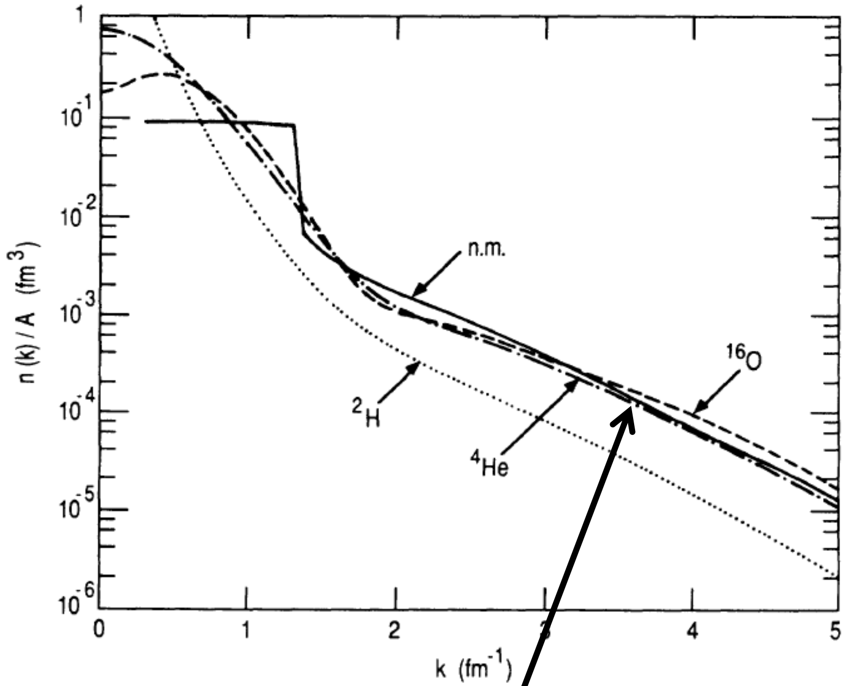


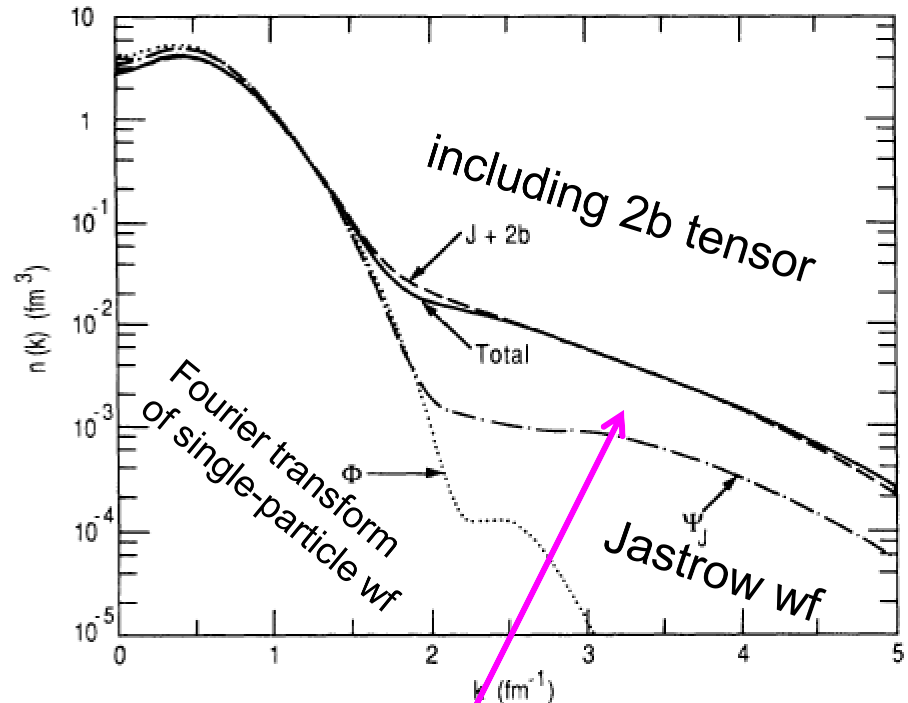
FIGURE 10. Two nucleons are initially in states B and C, having average momentum $\boldsymbol{P}$ and relative momentum $\boldsymbol{k}$. When they interact they are shifted to states D and E outside the Fermi sphere, with relative momentum $\boldsymbol{k}'$. If they are initially in a ^3S state and interact by tensor force, then they are in a $^3D_1$ state in DE.



S. Fantoni and V. R. Pandharipande, Nucl. Phys. A **427**, 473 (1984).
C. Ciofi degli Atti and S. Simula, Phys. Rev. C **53**, 1689 (1996).

Variational many-body calculations





Universal shape of high-momentum tail

→ due to short-range interaction of two nearby nucleons

→ scaling of weighted (e,e') inclusive xsections from light to heavy nuclei: the ratio of weighted xsection should be independent of the scattering variables

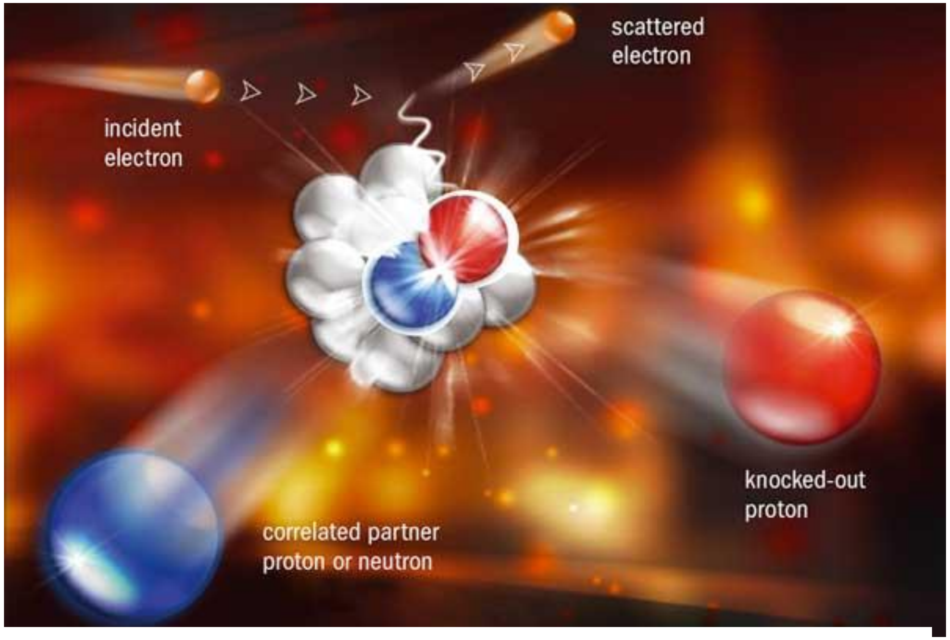
Tensor force dominance:

270 MeV/c < P < 600 MeV/c

3N correlations, repulsive core and nucleon resonances start playing a role at higher momentum

S.C. Pieper, R.B. Wiringa, and V.R. Pandharipande, Phys. Rev. C **46**, 1741 (1992).

Isospin-dependence of Short Range NN Correlations and Tensor Force



Two-nucleon knockout by a p or e

A.Tang et al, PRL 90, 042301 (2003)
R. Subedi et al. Science 320, 1475 (2008)

Triggered on nucleon pairs with zero total momentum

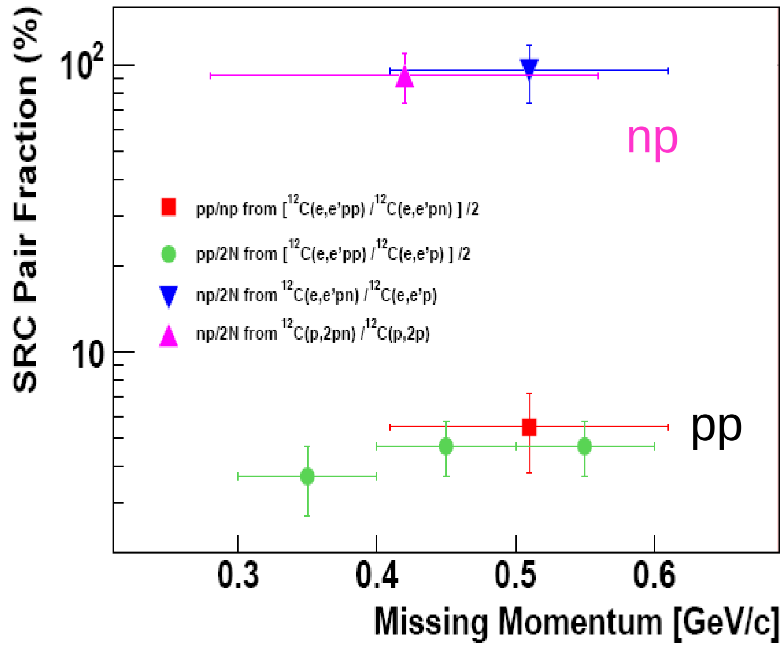


Figure 2: The fractions of correlated pair combinations in carbon as obtained from the (e,e'pp) and (e,e'pn) reactions, as well as from previous (p,2pn) data. The results and references are listed in Table 1.

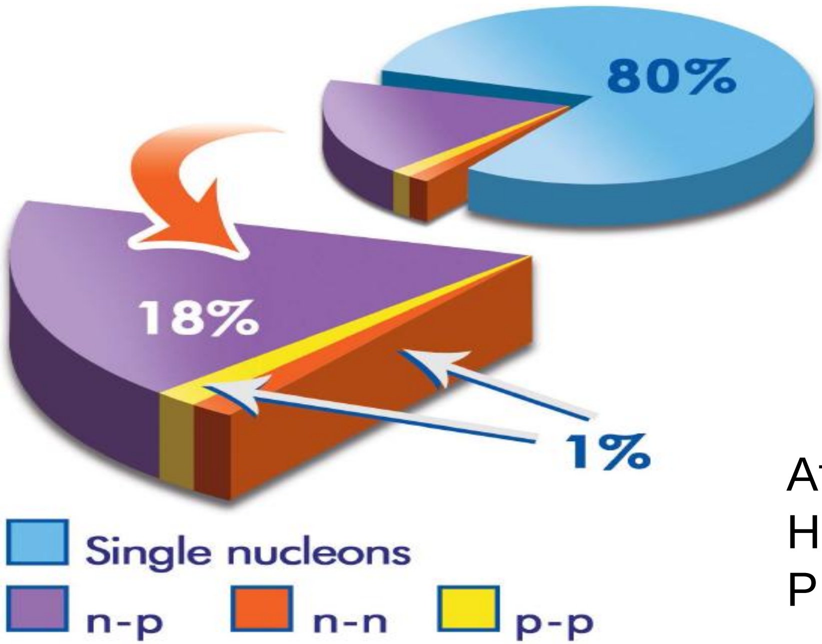


Figure 3: The average fraction of nucleons in the various initial state configurations of ^{12}C.

At finite total momentum, the effect is reduced,
H. Baghdasaryan *et al.* (CLAS)
PRL **105**, 222501 (2010)

$$r(A,{}^2\mathrm{H}) = \frac{\sigma_{eA}}{\sigma_{ed}}\frac{(\sigma_{ep}+\sigma_{en})}{(Z\sigma_{ep}+N\sigma_{en})},$$

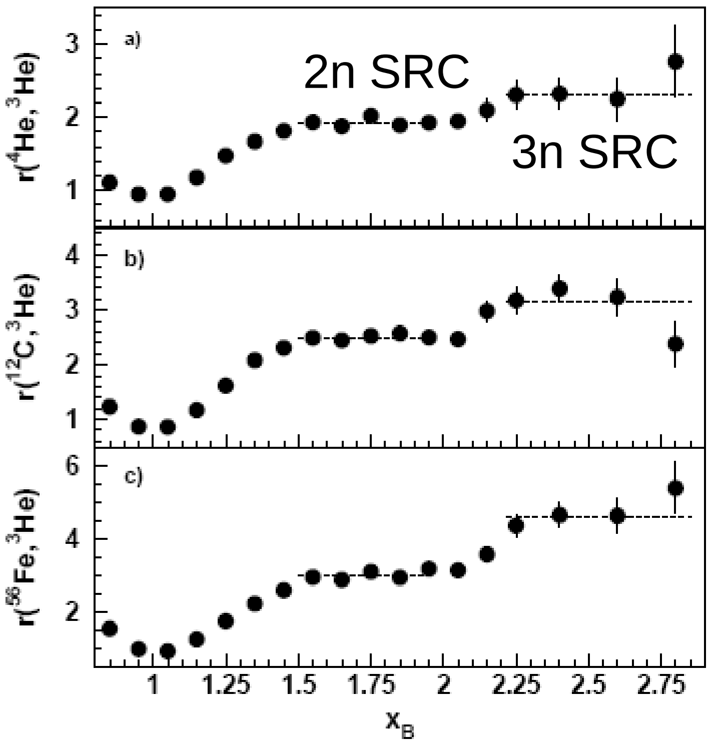


FIG. 1: Weighted cross section ratios of (a) ^{4}He, (b) ^{12}C and (c) ^{56}Fe to ^{3}He as a function of x_B for $Q^2 > 1.4$ GeV^2. The horizontal dashed lines indicate the NN and $3N$ scaling regions used to calculate the per-nucleon probabilities for 2- and $3N$ SRCs in nucleus A relative to ^{3}He.

K.S. Egiyan et al (CLAS), PRL96, 082501 (2006)

Table 1: Shown are the average ratios, $r(\mathrm{A}/^3\mathrm{He})$ for the scaling regions $1.5 < x_B < 2$ and $2.2 < x_B < 2.7$ along with the extracted absolute probabilities $a_{2N}(A)$ and $a_{3N}(A)$ that in a nucleus A a two- or three-nucleon short-range correlation is taking place at a given instant as extracted in Ref. [81]. $a_{2N}^{T=0}$ is the value of a_{2N} obtained if one assumed isoscalar dominance, e. that only np SRCs contribute to the distribution of high-momentum nucleons.

	$r(\mathrm{A}/^3\mathrm{He})$ $1.5 < x_B < 2$	$a_{2N}(A)$ [%]	$a_{2N}^{T=0}(A)$ [%]	$r(\mathrm{A}/^3\mathrm{He})$ $2.2 < x_B < 2.7$	$a_{3N}(A)$ [%]
^{3}He	1	8.0±1.6	8.0±1.6	1	0.18±0.06
^{4}He	1.93±0.03	15.4±3.2	13.5±2.8	2.33±0.13	0.42±0.14
^{12}C	2.49±0.15	19.8±4.4	17.4±3.9	3.18±0.27	0.56±0.21
Fe	2.98±0.18	23.9±5.3	20.3±4.5	4.63±0.33	0.83±0.27

Absolute probability per nucleon in the high momentum tail due to n-p short-range tensor force

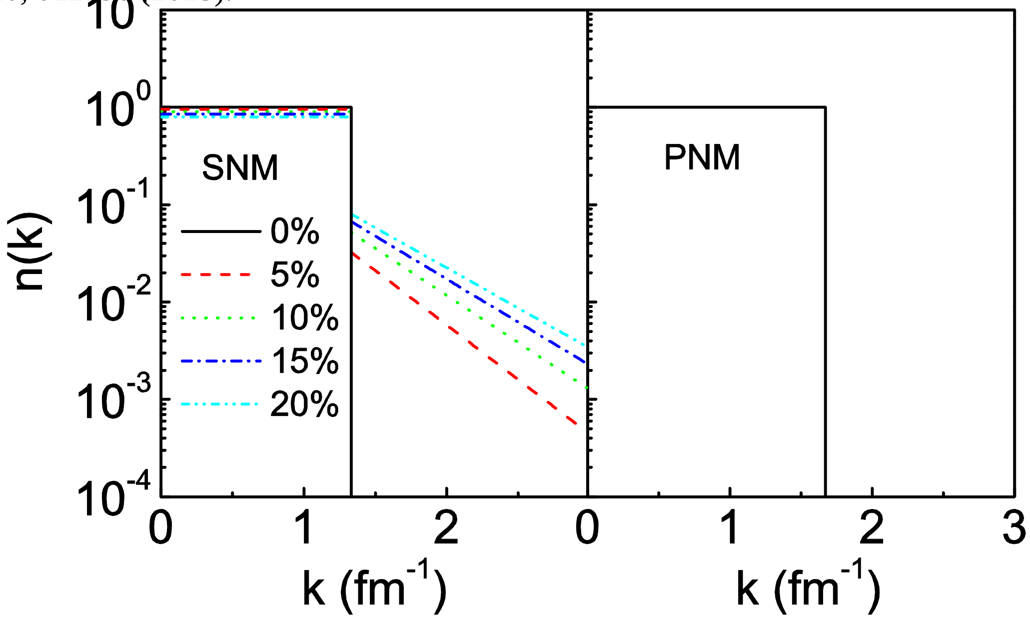
$$x_B = \frac{Q^2}{2M\nu}$$

Four-momentum transfer

Energy transfer

L. Frankfurt, M. Sargsian, and M. Strikman, Int. Jour. Mod. Phys. A **23**, 2991 (2008).
J. Arrington, D. W. Higinbotham, G. Rosner, M. Sargsian, arXiv:1104.1196, Prog. Part. Nucl. Phys (2011).

420, 012190 (2013).

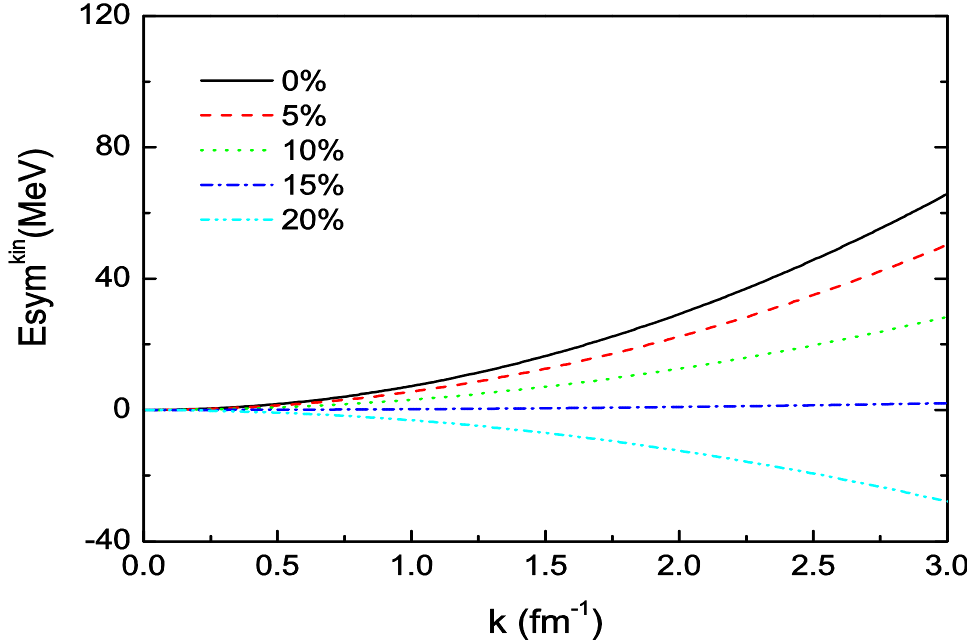


$$n(k) = a \ (k \le k_F)$$
$$= e^{bk} \ (k > k_F),$$

under the normalization condition [35] that

$$\frac{3}{k_F^3}\int_0^{\infty} n(k)k^2dk = 1.$$

Kinetic part of the symmetry energy can be negative



While the Fermi momentum for PNM
Is higher than that for SNM at the same density in the mean-field models,
if more than 15% nucleons are in the high-momentum tail of SNM due to the tensor force for n-p T=0 channel, the symmetry energy becomes negative

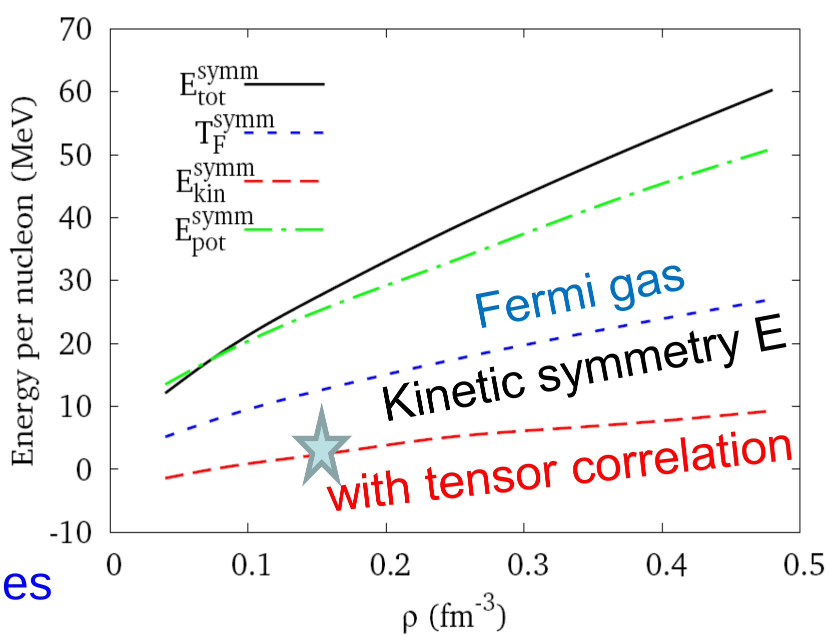
Chang Xu, Ang Li, Bao-An Li
J. of Phys: Conference Series, 420, 012190 (2013)

Confirmation by Microscopic Many-Body Theories

1. Nuclear symmetry energy and the role of the tensor force
Isaac Vidana, Artur Polls, Constanca Providencia, arXiv:1107.5412v1,
PRC84, 062801(R) (2011)
Brueckner--Hartree--Fock approach using the Argonne V18 potential plus the Urbana IX three-body force

2. High momentum components in the nuclear symmetry energy
Arianna Carbone, Artur Polls, Arnau Rios, arXiv:1111.0797v1
Euro. Phys. Lett. 97, 22001 (2012).
Self-Consistent Green's Function Approach with Argonne Av18, CDBonn, Nij1, N3LO interactions

3. Alessandro Lovato, Omar Benhar et al.,
extracted from results already published in
Phys. Rev. C83:054003,2011
Using Argonne V'$_6$ interaction
Fermi-Hyper-Netted-Chain (FHNC)
Single Operator Chains (SOC)



They all included the tensor force and many-body correlations using different techniques

Two Consequences of small kinetic contribution to the total Esym

(1) Effects of the symmetry POTENTIAL should be increased!

Can be zero! But ~ 25 MeV is currently used in all transport models

Heavy-ion reactions

probing

Kinetic

Potential

$$S(\rho)=\frac{C_{s,k}}{2}\left(\frac{\rho}{\rho_0}\right)^{2/3}+\frac{C_{s,p}}{2}\left(\frac{\rho}{\rho_0}\right)^{\gamma_i}$$

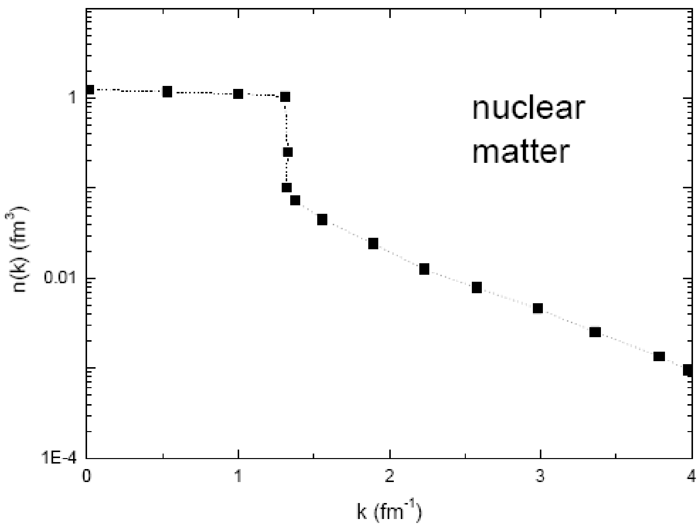
added in afterwards by hand using the Fermi gas model to fix the parameter Cs,p and gamma

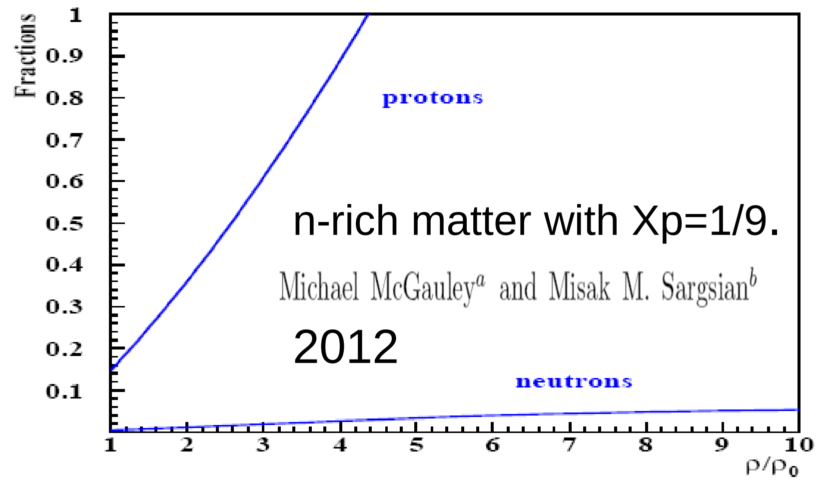
$C_{s,k}$=25 MeV

3bf

$$V(\rho,\delta)=\frac{A_1}{2\rho_0}\rho^2+\frac{A_2}{2\rho_0}\rho^2\delta^2+\frac{B}{\sigma+1}\frac{\rho^{\sigma+1}}{\rho_0^\sigma}(1-x\delta^2)$$
$$+\frac{1}{\rho_0}\sum_{\tau,\tau'}C_{\tau,\tau'}\int\int d^3p d^3p'\frac{f_\tau(\vec{r},\vec{p})f_{\tau'}(\vec{r},\vec{p}')}{1+(\vec{p}-\vec{p}')^2/\Lambda^2}$$

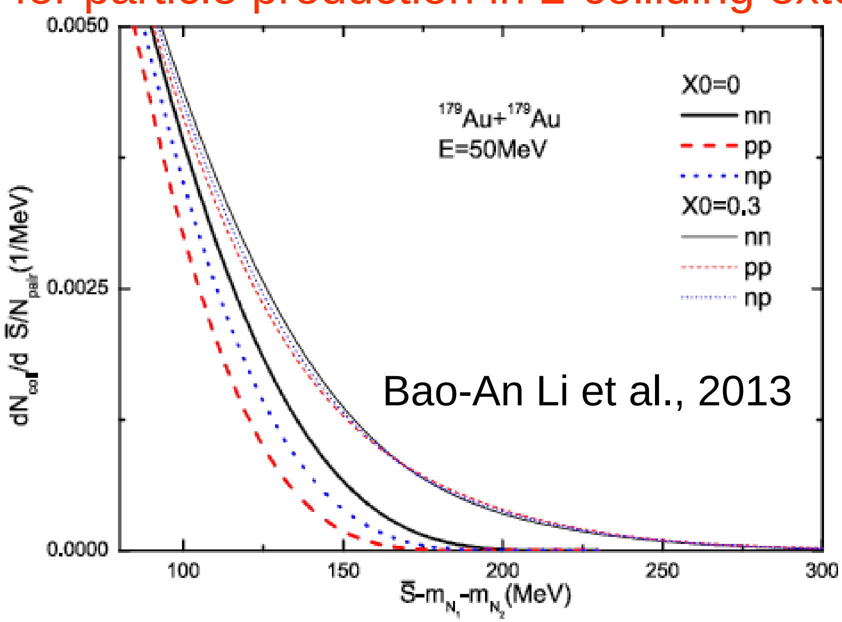
(2) Effects on sub-threshold pion ratio, etc

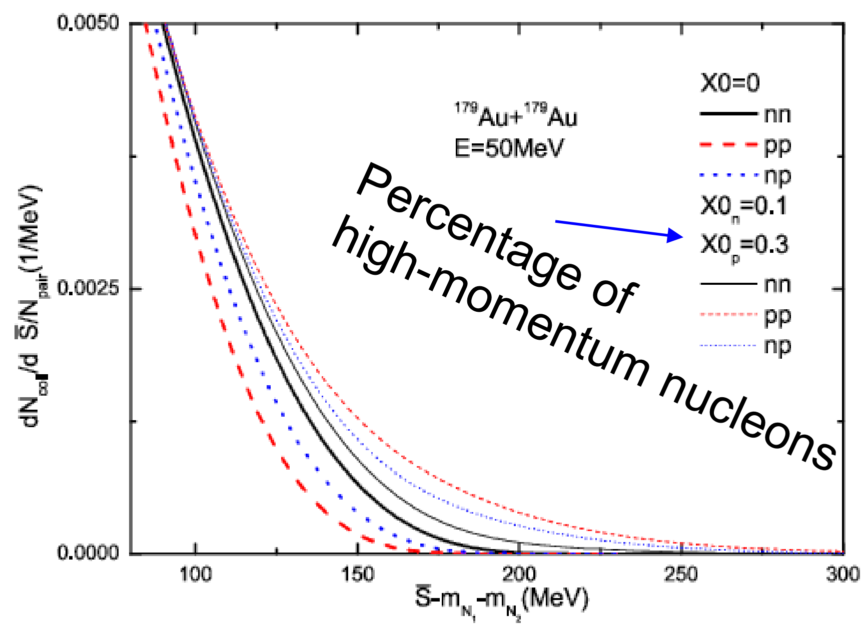


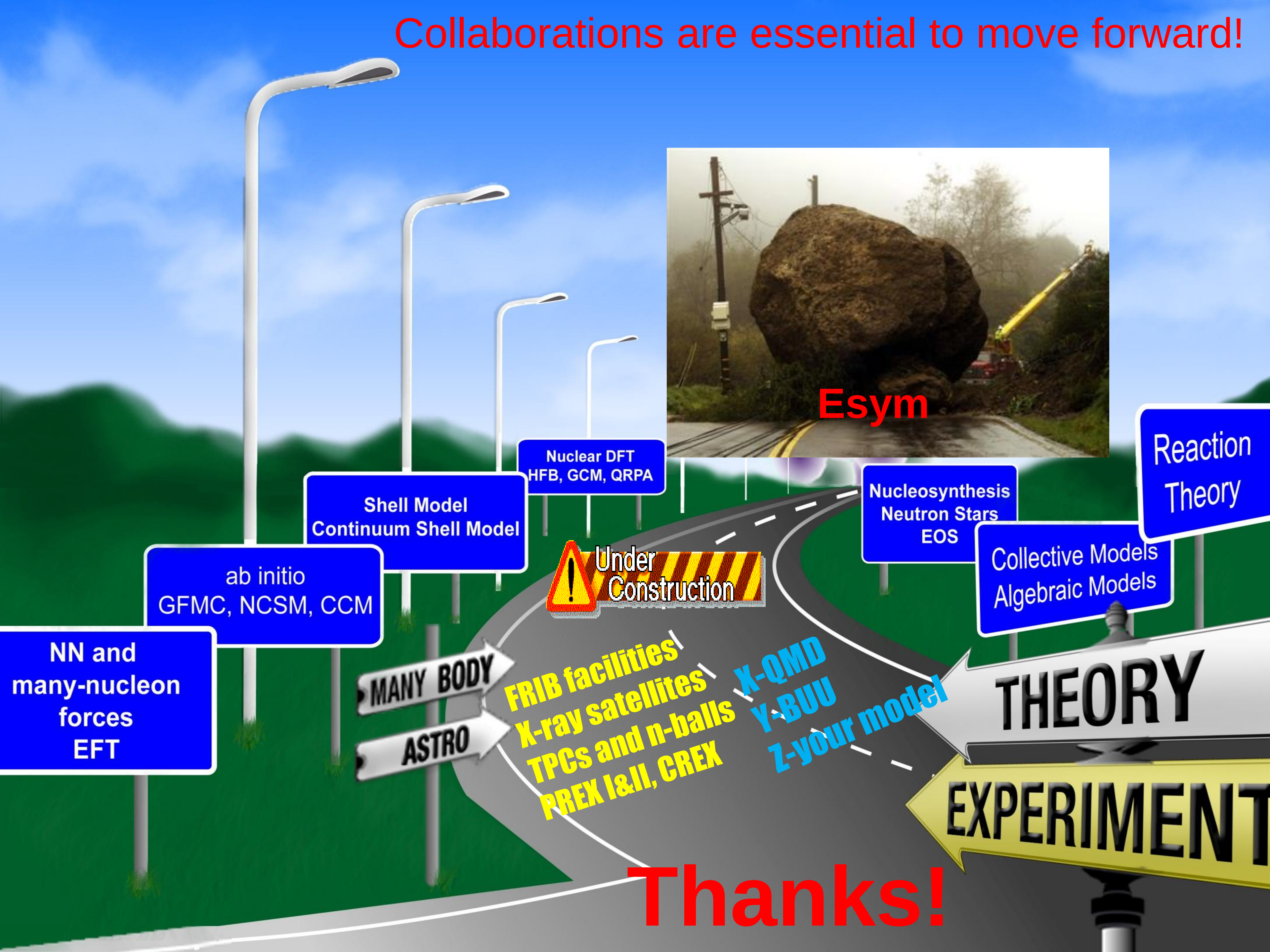


Fraction of high-momentum nucleons in neutron-rich matter

Distribution of the available energy for particle production in 2-colliding extended Fermi spheres






Collaborations are essential to move forward!
Esym
Reaction Theory
Nuclear DFT
HFB, GCM, QRPA
Shell Model
Continuum Shell Model
ab initio
GFMC, NCSM, CCM
NN and
many-nucleon
forces
EFT
Nucleosynthesis
Neutron Stars
EOS
Collective Models
Algebraic Models
Under Construction
MANY BODY
ASTRO
FRIB facilities
X-ray satellites
TPCs and n-balls
PREX I&II, CREX
X-QMD
Y-BUU
Z-your model
THEORY
EXPERIMENT
Thanks!