The Radius of Neutron Stars

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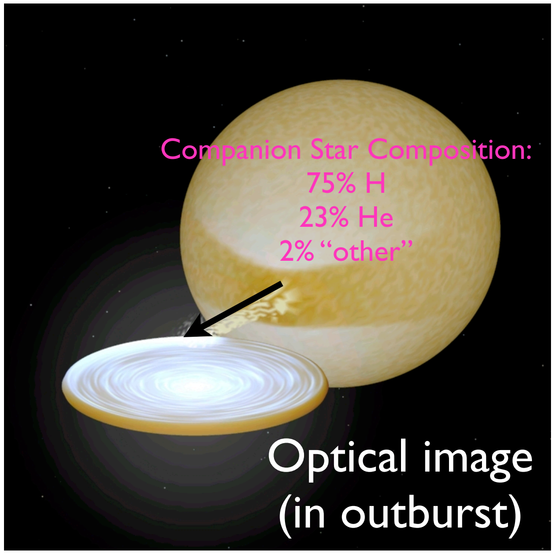
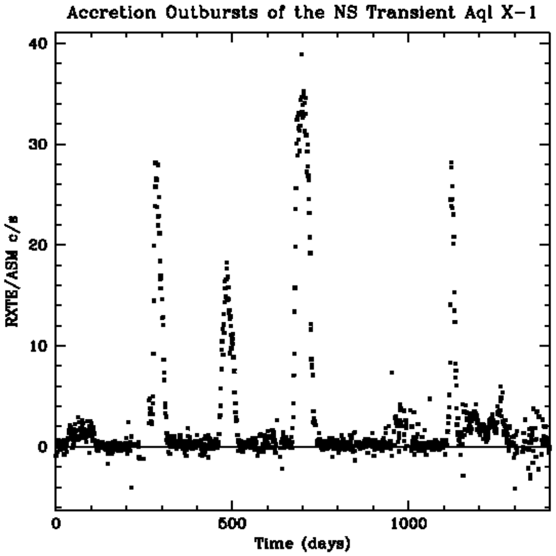
Quiescent Low Mass X-ray Binaries (qLMXB)

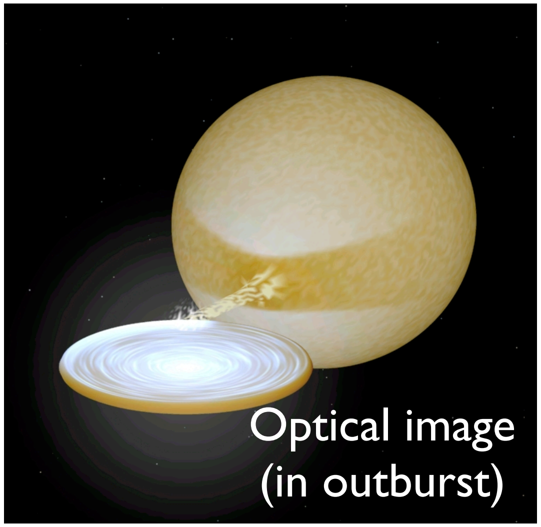
Outburst



Companion Star Composition:
75% H
23% He
2% “other”

Optical image
(in outburst)



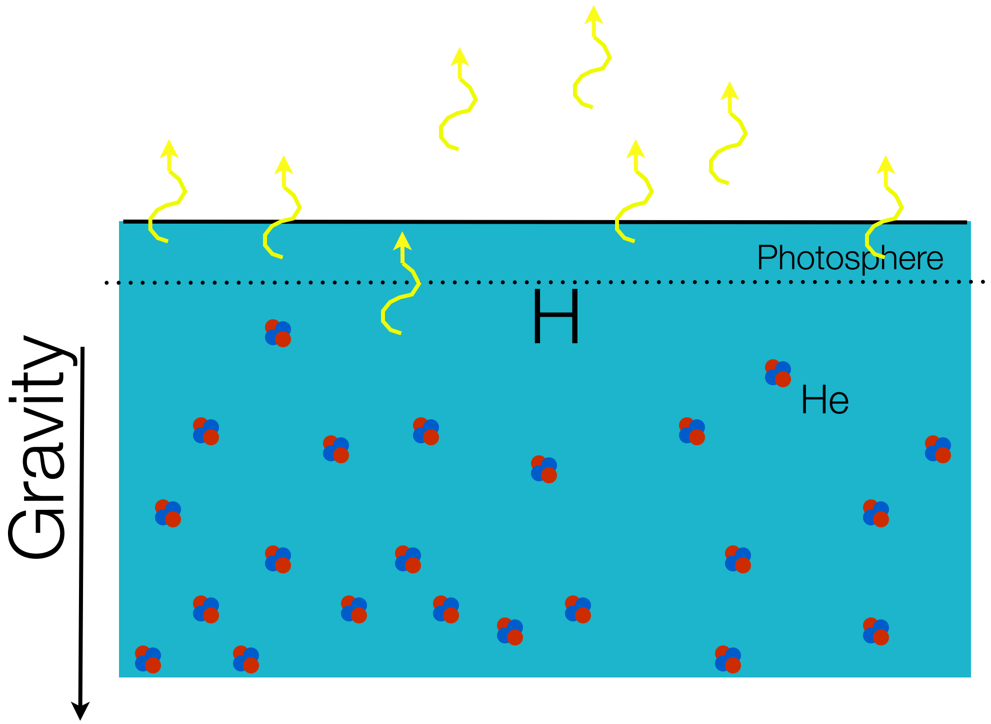


Optical image
(in outburst)

Quiescence

qLMXBs, in this scenario, have pure Hydrogen atmospheres

- When accretion stops, the He (and heavier elements, gravitationally settle on a timescale of ~10s of seconds (like rocks in water), leaving the photosphere to be pure Hydrogen (Alcock & Illarionov 1980, Bildsten et al 1992).



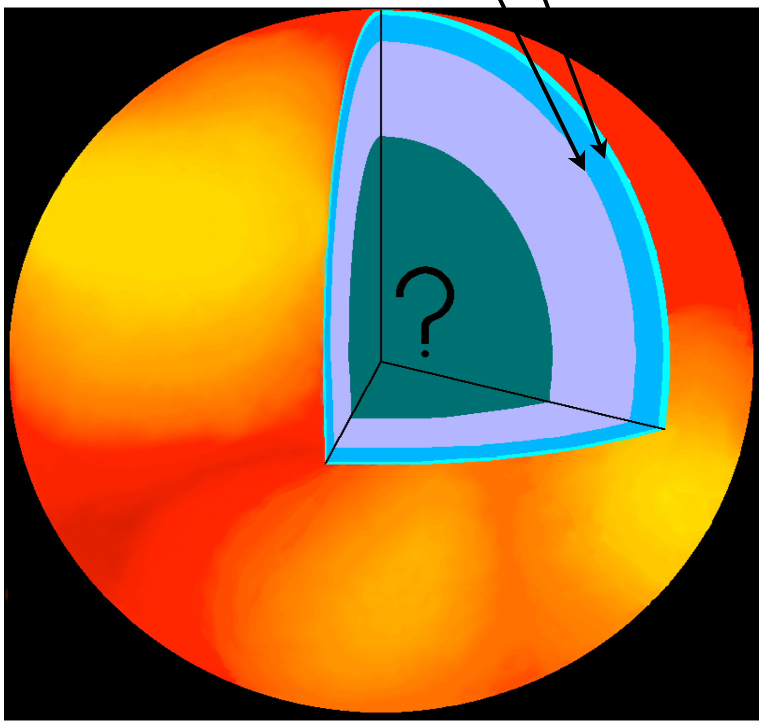
Deep Crustal Heating

Non-Equilibrium Processes in the Outer Crust
Beginning with ^{56}Fe (Haensel &Zdunik 1990, 2003)

ρ (g cm^{-3})	Reaction	Δρ/ρ	Q (Mev/np)
$1.5 \cdot 10^{9}$	$^{56}\text{Fe} \Rightarrow {}^{56}\text{Cr} - 2e- + 2\nu_e$	0.08	0.01
$1.1 \cdot 10^{10}$	$^{56}\text{Cr} \Rightarrow {}^{56}\text{Ti} - 2e- + 2\nu_e$	0.09	0.01
$7.8 \cdot 10^{10}$	$^{56}\text{Ti} \Rightarrow {}^{56}\text{Ca} - 2e- + 2\nu_e$	0.10	0.01
$2.5 \cdot 10^{10}$	$^{56}\text{Ca} \Rightarrow {}^{56}\text{Ar} - 2e- + 2\nu_e$	0.11	0.01
$6.1 \cdot 10^{10}$	$^{56}\text{Ar} \Rightarrow {}^{52}\text{S} + 4n - 2e- + 2\nu_e$	0.12	0.01

Non-Equilibrium Processes in the Inner Crust

ρ (g cm^{-3})	Reaction	X_n	Q (Mev/np)
$9.1 \cdot 10^{11}$	$^{52}\text{S} \Rightarrow {}^{46}\text{Si} + 6n - 2e- + 2\nu_e$	0.07	0.09
$1.1 \cdot 10^{12}$	$^{46}\text{Si} \Rightarrow {}^{40}\text{Mg} + 6n - 2e- + 2\nu_e$	0.07	0.09
$1.5 \cdot 10^{12}$	$^{40}\text{Mg} \Rightarrow {}^{34}\text{Ne} + 6n - 2e- + 2\nu_e$		
	$^{34}\text{Ne} + {}^{34}\text{Ne} \Rightarrow {}^{68}\text{Ca}$	0.29	0.47
$1.8 \cdot 10^{12}$	$^{68}\text{Ca} \Rightarrow {}^{62}\text{Ar} + 6n - 2e- + 2\nu_e$	0.39	0.05
$2.1 \cdot 10^{12}$	$^{62}\text{Ar} \Rightarrow {}^{56}\text{S} + 6n - 2e- + 2\nu_e$	0.45	0.05
$2.6 \cdot 10^{12}$	$^{56}\text{S} \Rightarrow {}^{50}\text{Si} + 6n - 2e- + 2\nu_e$	0.50	0.06
$3.3 \cdot 10^{12}$	$^{50}\text{Si} \Rightarrow {}^{44}\text{Mg} + 6n - 2e- + 2\nu_e$	0.55	0.07
$4.4 \cdot 10^{12}$	$^{44}\text{Mg} \Rightarrow {}^{36}\text{Ne} + 6n - 2e- + 2\nu_e$		
	$^{36}\text{Ne} + {}^{36}\text{Ne} \Rightarrow {}^{72}\text{Ca}$		
	$^{68}\text{Ca} \Rightarrow {}^{62}\text{Ar} + 6n - 2e- + 2\nu_e$	0.61	0.28
$5.8 \cdot 10^{12}$	$^{62}\text{Ar} \Rightarrow {}^{60}\text{S} + 6n - 2e- + 2\nu_e$	0.70	0.02
$7.0 \cdot 10^{12}$	$^{60}\text{S} \Rightarrow {}^{54}\text{Si} + 6n - 2e- + 2\nu_e$	0.73	0.02
$9.0 \cdot 10^{12}$	$^{54}\text{Si} \Rightarrow {}^{48}\text{Mg} + 6n - 2e- + 2\nu_e$	0.76	0.03
$1.1 \cdot 10^{13}$	$^{48}\text{Mg} + {}^{48}\text{Mg} \Rightarrow {}^{96}\text{Cr}$	0.79	0.11
$1.1 \cdot 10^{13}$	$^{96}\text{Cr} \Rightarrow {}^{88}\text{Ti} + 8n - 2e- + 2\nu_e$	0.80	0.01



Begins Here

Ends Here

1.47 Mev per np

Brown, Bildsten & RR (1998)

Emergent Spectrum of a Neutron Star Hydrogen Atmosphere

- **H atmosphere calculated Spectra are ab initio radiative transfer calculations using the Eddington equations.**

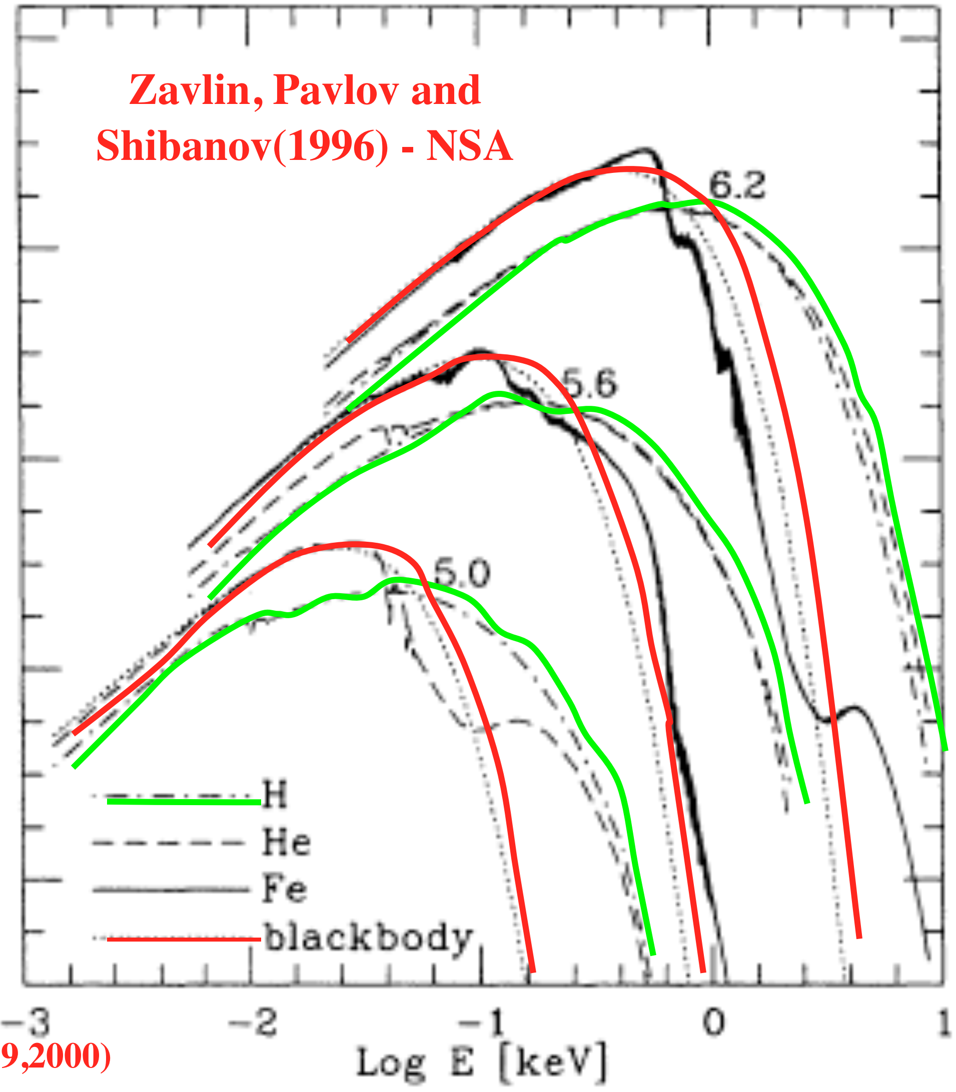
- **Rajagopal and Romani (1996); Zavlin et al (1996); Pons et al (2002; Heinke et al (2006) -- NSATMOS; Gaensicke, Braje & Romani (2001); Haakonsen et al (2012)**

All comparisons show consistency within ~few % (e.g. Webb et al 2007, Haakonsen 2012).

"Vetted": X-ray spectra of Zavlin, Heinke together have been used in several dozen works.

$$F = 4\pi T_{eff,\infty}^{4}\left(\frac{R_\infty}{D}\right)^2$$

$$R_\infty = \frac{R}{\sqrt{1-\frac{2GM}{c^2R}}}$$



Zavlin, Pavlov and Shibanov(1996) - NSA

RR et al (1999,2000)

Instruments for measurements of qLMXBs

Chandra X-ray Observatory

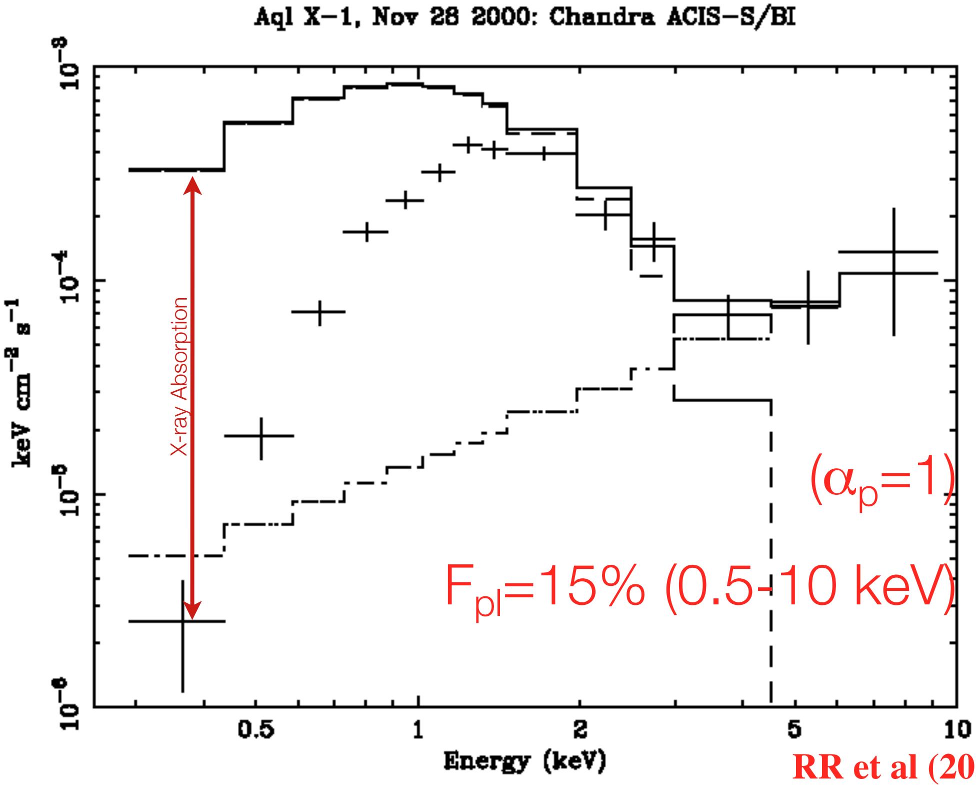
- Launched 1999 (NASA)
  - 1” resolution

XMM/Newton

- Launched 1999 (ESA)
- 6” resolution
- ~4x area of Chandra.

Every photon is time tagged (~1 sec), with its energy measured (E/deltaE = 10) with full resolution imaging.

Aql X-1 with Chandra -- Field Source



X-ray Absorption

(α_p=1)

F_{pl}=15% (0.5-10 keV)

RR et al (2001b)

R_∞ (d/5 kpc)	$kT_{eff,\infty}$	N_H (1e20 cm^{-2})
13^{+5}_{-4} km	135^{+18}_{-12} eV	35^{+8}_{-7}

The LMXB Factories: Globular Clusters

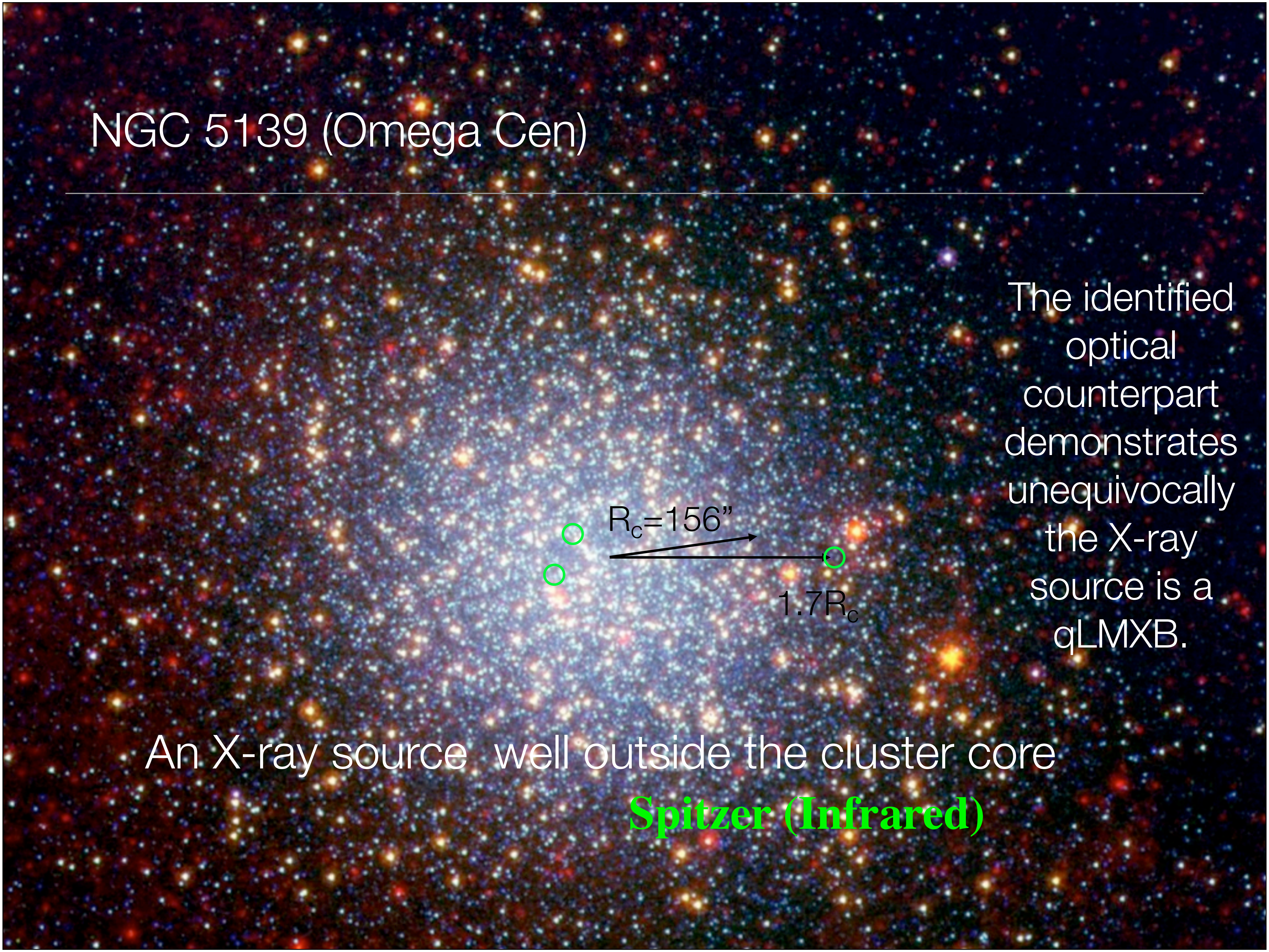
- GCs : overproduce LMXBs by 1000x vs. field stars
- Many have accurate distances measured.

NGC	D (kpc)	+/-(%)
104	5.13	4
288	9.77	3
362	10.0	3
4590	11.22	3
5904	8.28	3
7099	9.46	2
6025	7.73	2
6341	8.79	3
6752	4.61	2

Carretta et al (2000)

qLMXBs can be identified by their soft X-ray spectra, and confirmed with optical counterparts.

NGC 5139 (Omega Cen)



The identified optical counterpart demonstrates unequivocally the X-ray source is a qLMXB.

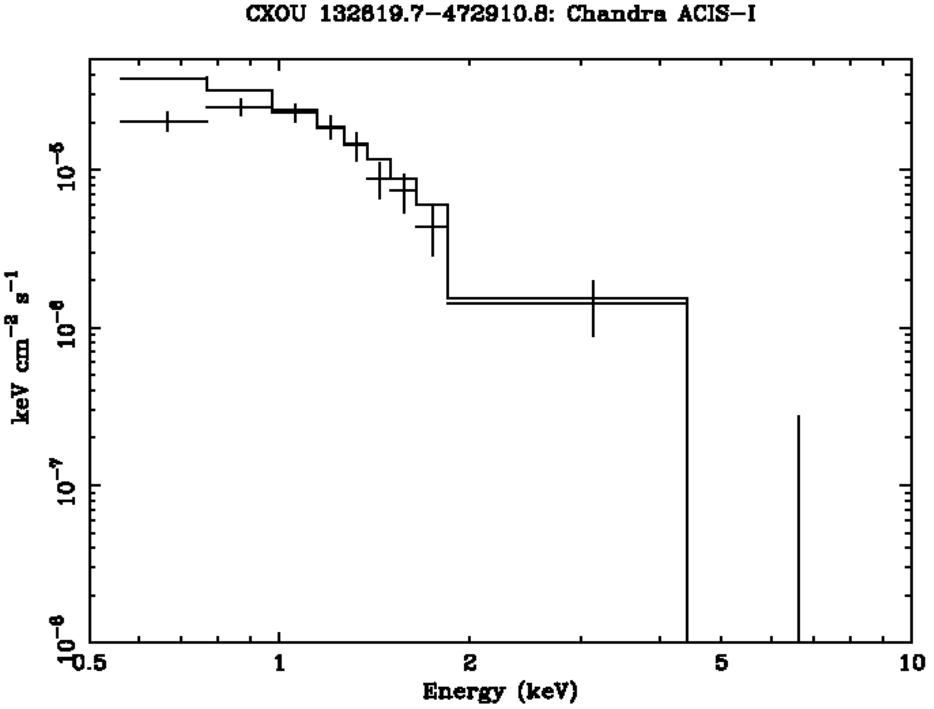
An X-ray source well outside the cluster core

Spitzer (Infrared)

NGC 5139 (Omega Cen)

X-ray Spectrum is inconsistent with any other type of known GC source (pulsars, CVs, coronal sources).

Full confirmation as LMXB requires Hubble photometry (which only exists for this 1 of our 5 sources).

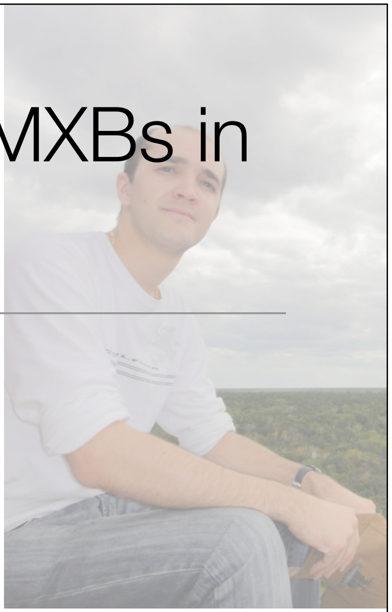


CXOU 132619.7−472910.8: Chandra ACIS−I

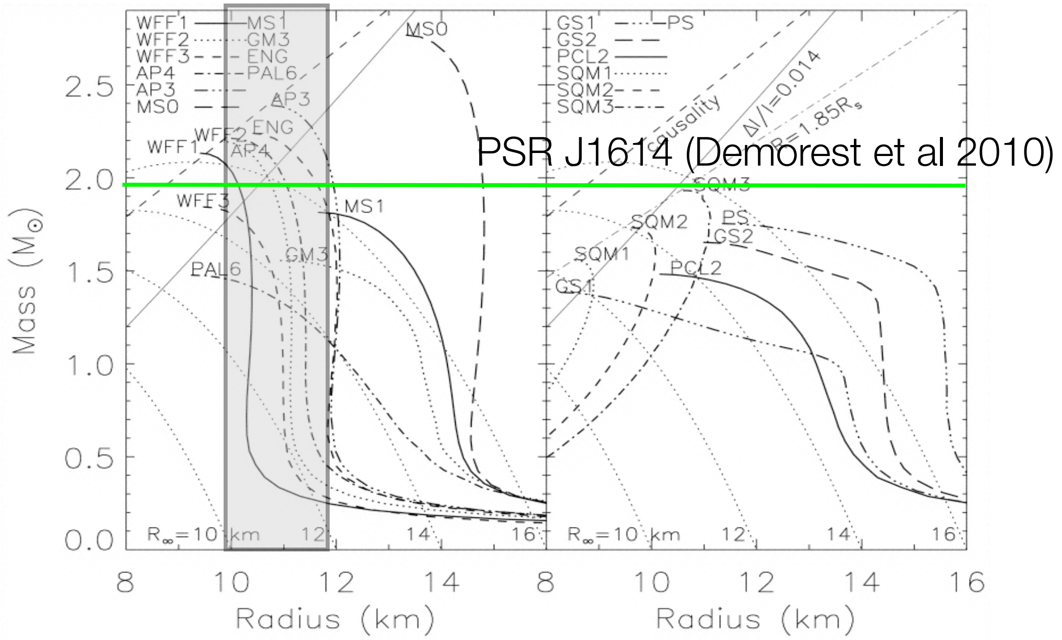
R_∞ (d/5 kpc)	$kT_{eff,\infty}$	N_H (1e20 cm^{-2})
14.3 ± 2.1 km	66^{+4}_{-5} eV	(9)

RR et al (2002)

Measuring the Radius of Neutron Stars from qLMXBs in Globular Clusters

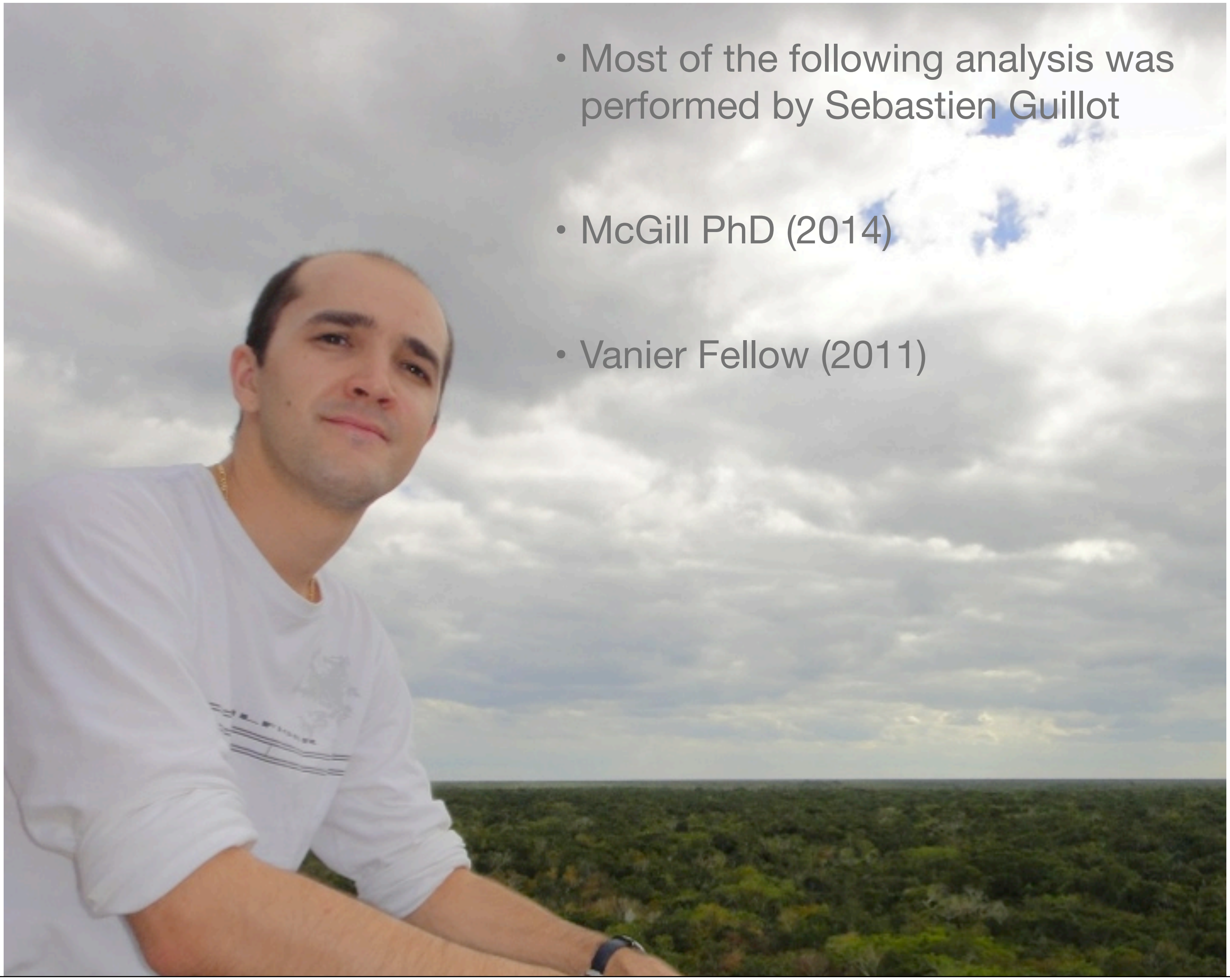


- The 1.97(4) solar mass neutron star favors hadronic dEOSs over quark and phase-transition dEOSs. These have the property of a quasi-constant neutron star radius.
- Analysis goal: Using all suitable qLMXB X-ray data sets of targets (there are five) provide the most reliable neutron star radius measurement possible.
- Assume the radius of neutron stars is **quasi-constant** (a constant, at astrophysically important masses, within measurement error).
- Perform a Markoff-Chain-Monte-Carlo (MCMC) and include all known uncertainties and use conservative assumptions.



PSR J1614 (Demorest et al 2010)

Measuring the Radius of Neutron Stars from qLMXBs in Globular Clusters



- Most of the following analysis was performed by Sebastien Guillot
- McGill PhD (2014)
- Vanier Fellow (2011)

Assumptions -- the systematic uncertainties.

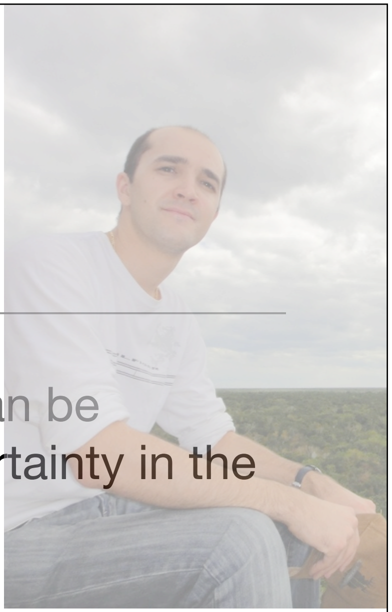
- **H atmosphere neutron stars.** (expected from a Hydrogen companion LMXB; can be proven through optical observations with Hubble, only done in one case, Omega Cen).

- **Low B-field (<10^{10} G) neutron stars.** (this is true for ‘standard’ LMXBs as a class, but difficult to prove on a case-by-case basis).

- **Emitting isotropically.** (comes naturally when powered by a hot core).

These assumptions reflect the best knowledge of these systems astronomy has in 2013.
If you don’t like these assumptions: “We find the assumptions not strongly supported and therefore ignore this result.”

Accounted-for Uncertainties

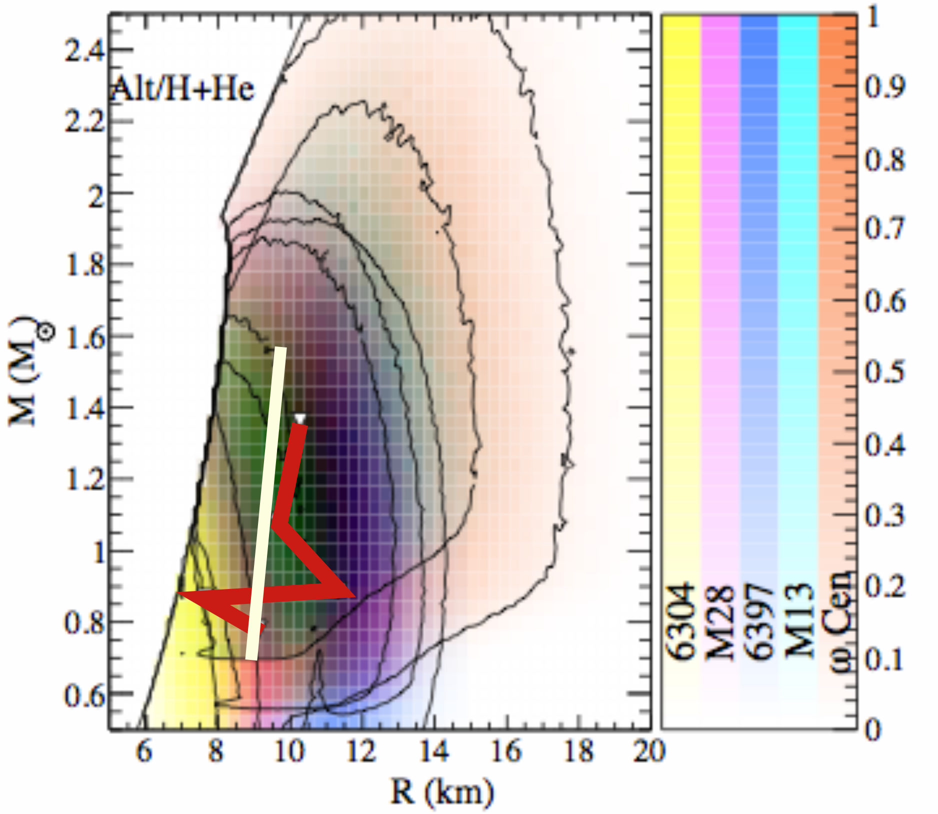


- In all previous works using qLMXB, the distance uncertainty -- which can be 2%-10% for each source -- has been neglected. Reflected in the uncertainty in the measured radius.
- X-ray absorption (due to the Hydrogen column density) is sometimes held fixed at radio-measured values, but is known to be systematically uncertain by x2, unless measured in the X-ray band. Reflected in the uncertainty in the measured radius.
- In some field sources (but no globular cluster sources) excess emission at high energies, not due to a H atmosphere, has been detected. Reflected in the uncertainty in the measured radius.
- Calibration uncertainty is included as a 3% intensity uncertainty.
- There are no remaining known quantified uncertainties.

The major innovation of Guillot et al (2013) is statistical.

- All work to date, in combining spectral fits, fit each source individually, then combined the best fit M and R afterwards, with error regions.
- Guillot et al (2013) required R to be the same for all sources.
- This “quasi-constant Radius” should be thought of as a simplified parametric model which can be compared to realistic EoSs.
- The result is an improvement in S/N over previous work which (for example) would use 5 sources independently, (approximately) as if we had 25 sources.
- A simplified explanation......

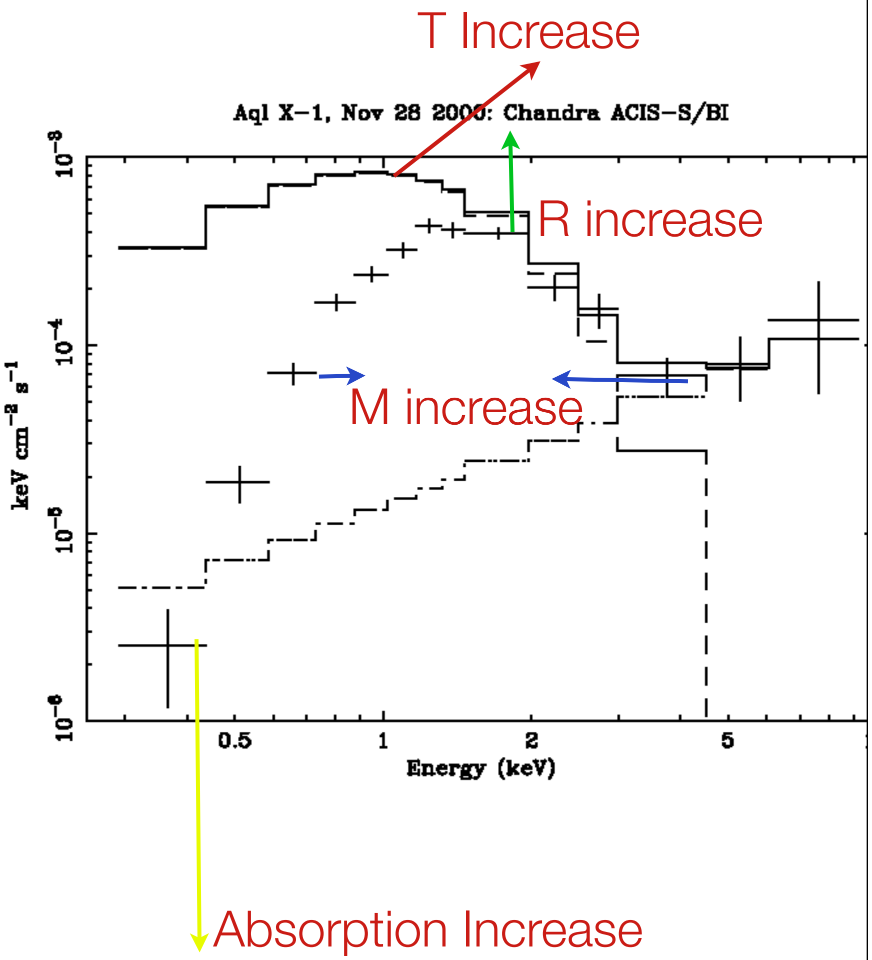
“Alt/H+He” (LS13)



- Draw your best Neutron Star M-R relationship in your head. Ready?

All previous EoS work treated measurements independently.

$$\frac{\partial \chi^2}{\partial(p1)\partial(p2)} = \begin{array}{ccccccccc}
\multicolumn{4}{c}{\overline{\text{Source 1}}} & \multicolumn{4}{c}{\overline{\text{Source 2}}} & \\
R_1 & M_1 & T_1 & N_{H,1} & R_2 & M_2 & T_2 & N_{H,2} & \\
\frac{1}{\sigma^2_{R_1}} & a_{1,2} & a_{1,3} & a_{1,4} & 0 & 0 & 0 & 0 & R_1 \\
a_{2,1} & \frac{1}{\sigma^2_{M_1}} & a_{2,3} & a_{2,4} & 0 & 0 & 0 & 0 & M_1 \\
a_{3,1} & a_{3,2} & \frac{1}{\sigma^2_{T_1}} & a_{3,4} & 0 & 0 & 0 & 0 & T_1 \\
a_{4,1} & a_{4,2} & a_{4,3} & \frac{1}{\sigma^2_{N_{H1}}} & 0 & 0 & 0 & 0 & N_{H,1} \\
0 & 0 & 0 & 0 & \frac{1}{\sigma^2_{R_2}} & a_{5,6} & a_{5,7} & a_{5,8} & R_2 \\
0 & 0 & 0 & 0 & a_{6,5} & \frac{1}{\sigma^2_{M_2}} & a_{6,7} & a_{6,8} & M_2 \\
0 & 0 & 0 & 0 & a_{7,5} & a_{7,6} & \frac{1}{\sigma^2_{T_2}} & a_{7,8} & T_2 \\
0 & 0 & 0 & 0 & a_{8,5} & a_{8,6} & a_{8,7} & \frac{1}{\sigma^2_{N_{H2}}} & N_{H,2}
\end{array}$$



“Joint Fits” - the major difference from previous work

$$\frac{\partial\chi^2}{\partial(p1)\partial(p2)} = \begin{array}{cccccccc|c}
\multicolumn{4}{c}{\text{Source 1}} & \multicolumn{4}{c}{\text{Source 2}} & \\
\mathrm{R} & \mathrm{M}_1 & \mathrm{T}_1 & \mathrm{N}_{\mathrm{H},1} & \mathrm{R} & \mathrm{M}_2 & \mathrm{T}_2 & \mathrm{N}_{\mathrm{H},2} & \\
\hline
\boxed{\frac{1}{\sigma_R^2}} & a_{1,2} & a_{1,3} & a_{1,4} & \boxed{\frac{1}{\sigma_R^2}} & a_{1,6} & a_{1,7} & a_{1,8} & \mathrm{R}_1 \\
a_{2,1} & \frac{1}{\sigma_{M_1}^2} & a_{2,3} & a_{2,4} & a_{2,5} & \frac{1}{\sigma_{M_1,M_2}^2} & a_{2,7} & a_{2,8} & \mathrm{M}_1 \\
a_{3,1} & a_{3,2} & \frac{1}{\sigma_{T_1}^2} & a_{3,4} & a_{3,5} & a_{3,6} & \frac{1}{\sigma_{T_1,T_2}^2} & a_{3,8} & \mathrm{T}_1 \\
a_{4,1} & a_{4,2} & a_{4,3} & \frac{1}{\sigma_{N_{H1}}^2} & a_{4,5} & a_{4,6} & a_{4,7} & \frac{1}{\sigma_{N_{H1},N_{H2}}^2} & \mathrm{N}_{\mathrm{H},1} \\
\boxed{\frac{1}{\sigma_R^2}} & a_{5,2} & a_{5,3} & a_{5,4} & \boxed{\frac{1}{\sigma_R^2}} & a_{5,6} & a_{5,7} & a_{5,8} & \mathrm{R}_2 \\
a_{6,1} & \frac{1}{\sigma_{M_2,M_1}^2} & a_{6,3} & a_{6,4} & a_{6,5} & \frac{1}{\sigma_{M_2}^2} & a_{6,7} & a_{6,8} & \mathrm{M}_2 \\
a_{7,1} & a_{7,2} & \frac{1}{\sigma_{T_2,T_1}^2} & a_{7,4} & a_{7,5} & a_{7,6} & \frac{1}{\sigma_{T_2}^2} & a_{7,8} & \mathrm{T}_2 \\
a_{8,1} & a_{8,2} & a_{8,3} & \frac{1}{\sigma_{N_{H2},N_{H1}}^2} & a_{8,5} & a_{8,6} & a_{8,7} & \frac{1}{\sigma_{N_{H2}}^2} & \mathrm{N}_{\mathrm{H},2}
\end{array}$$

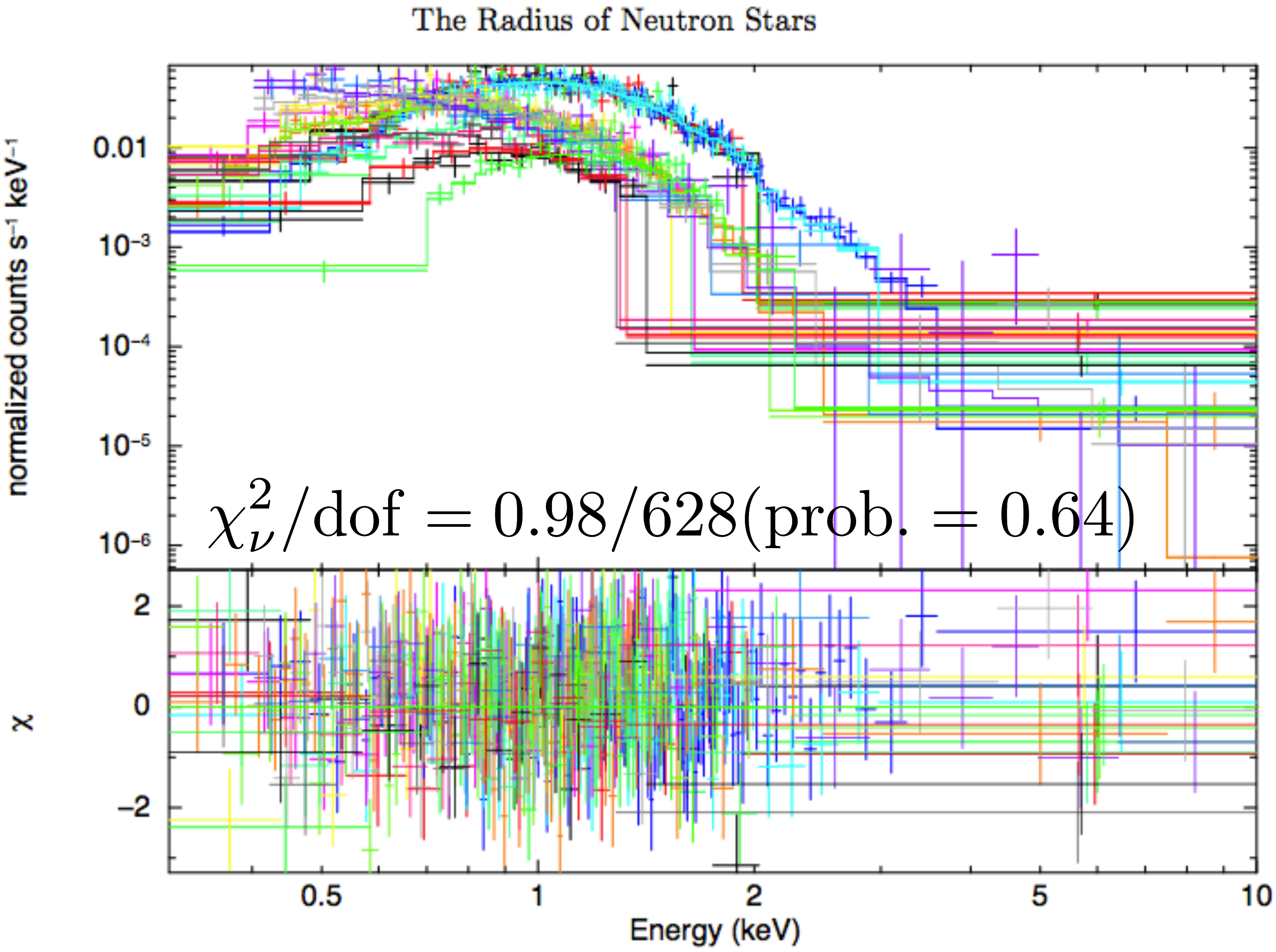
Every parameter (M, R, T, N_H) of all five sources affects every other parameter of every source

$$\frac{S}{N} \propto \frac{N^2(\#\text{ of Source Matrix Elements})}{N(\#\text{ of sources})} = N$$

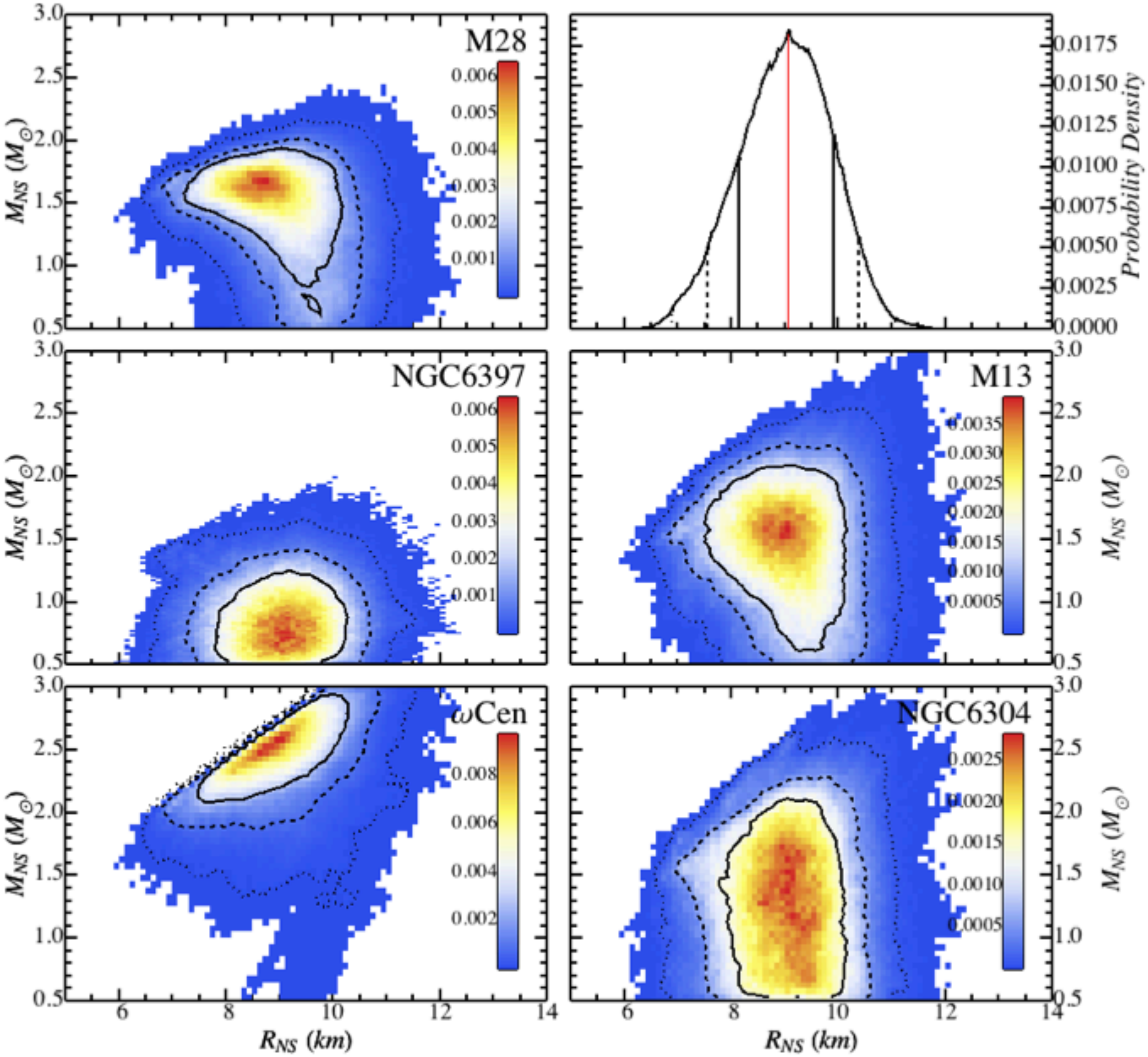
In comparison to using the sources “independently”, its as if we have 25 sources, instead of 5 sources.

Best H atmosphere (+ PL) spectral fit of all 5 qLMXBs

- This model is a statistically acceptable fit to the X-ray spectral data. This is an a posteriori confirmation that the data are consistent with our assumptions.
- After finding the best fit a MCMC method was used to find the uncertainty regions for all parameters - -- the Radius, Mass, Temperature, absorption, distance, and power-law normalization.

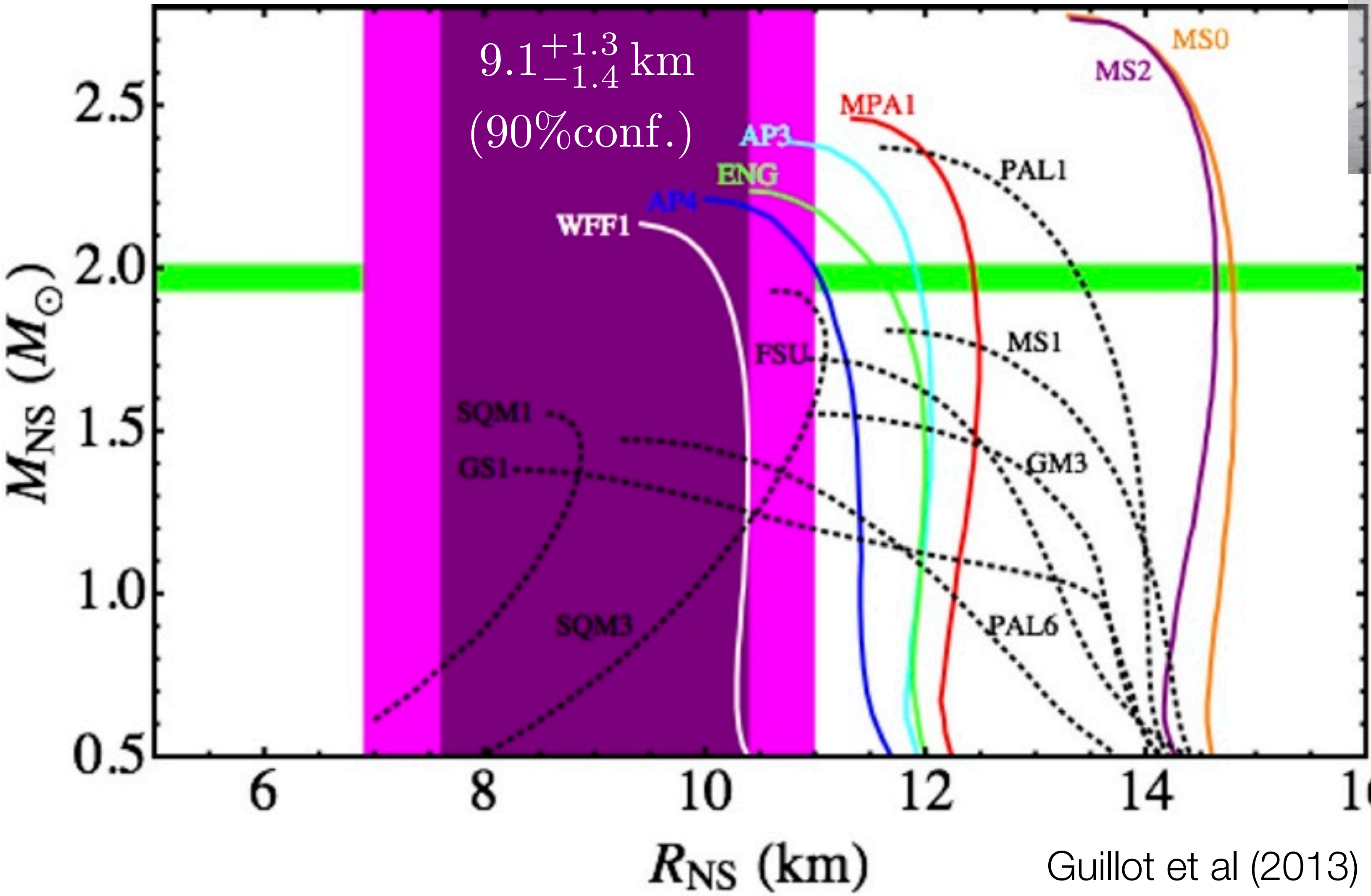


$$\chi^2_\nu/\mathrm{dof} = 0.98/628(\mathrm{prob.} = 0.64)$$


M28
NGC6397
M13
ωCen
NGC6304
M_{NS} ($M_\odot$)
R_{NS} (km)
Probability Density
0.5
1.0
1.5
2.0
2.5
3.0
6
8
10
12
14
0.0000
0.0025
0.0050
0.0075
0.0100
0.0125
0.0150
0.0175
0.001
0.002
0.003
0.004
0.005
0.006
0.008
0.0005
0.0010
0.0015
0.0020
0.0025
0.0030
0.0035

The Neutron Star Radius

<11 km (99% conf).



$9.1^{+1.3}_{-1.4}$ km (90%conf.)

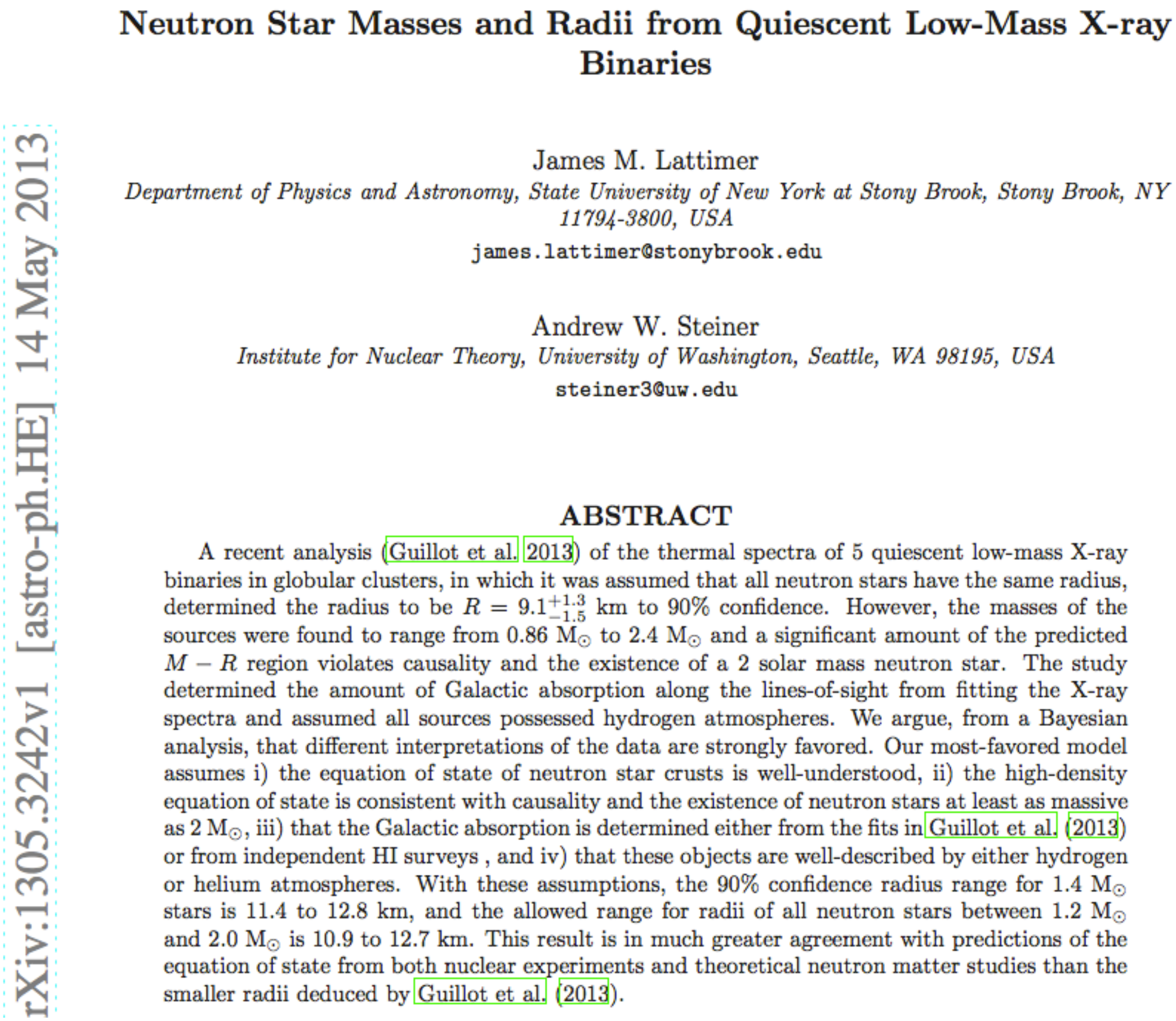
Guillot et al (2013)

M-R by J. Lattimer

WFF1= Wiring, Fiks and Fabrocini (1988)

Contains uncertainties from:
Distance
All spectral parameters
Calibration

LS13 (submitted)



Neutron Star Masses and Radii from Quiescent Low-Mass X-ray Binaries

rXiv:1305.3242v1 [astro-ph.HE] 14 May 2013

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ABSTRACT

A recent analysis (Guillot et al. 2013) of the thermal spectra of 5 quiescent low-mass X-ray binaries in globular clusters, in which it was assumed that all neutron stars have the same radius, determined the radius to be $R = 9.1^{+1.3}_{-1.5}$ km to 90% confidence. However, the masses of the sources were found to range from 0.86 $M_\odot$ to 2.4 $M_\odot$ and a significant amount of the predicted $M - R$ region violates causality and the existence of a 2 solar mass neutron star. The study determined the amount of Galactic absorption along the lines-of-sight from fitting the X-ray spectra and assumed all sources possessed hydrogen atmospheres. We argue, from a Bayesian analysis, that different interpretations of the data are strongly favored. Our most-favored model assumes i) the equation of state of neutron star crusts is well-understood, ii) the high-density equation of state is consistent with causality and the existence of neutron stars at least as massive as 2 $M_\odot$, iii) that the Galactic absorption is determined either from the fits in Guillot et al. (2013) or from independent HI surveys , and iv) that these objects are well-described by either hydrogen or helium atmospheres. With these assumptions, the 90% confidence radius range for 1.4 $M_\odot$ stars is 11.4 to 12.8 km, and the allowed range for radii of all neutron stars between 1.2 $M_\odot$ and 2.0 $M_\odot$ is 10.9 to 12.7 km. This result is in much greater agreement with predictions of the equation of state from both nuclear experiments and theoretical neutron matter studies than the smaller radii deduced by Guillot et al. (2013).

LS13:
How it should be done.

- Download all X-ray data from the NASA Archive heasarc.gsfc.nasa.gov. All observations are freely available, as are all standard analysis tools. This is done 1000s of times every year by astronomers and results in 1000s of papers annually. That said, it is not idiot-proof.

- Extract X-ray photon spectra from each source. (Hereafter: "Data" means "X-ray photon spectra", and nothing else.)

- Perform a (for example) Chi-square minimization (or other figure of merit) comparison between the proposed photon spectral model and the X-ray photon spectral data. Is the chi-square "acceptable"? Is it "better" than alternative models?

LS 13:
What is actually done

- LS13 gives the impression that our group gave them photon spectral data. We did not. We provided numeric values for our best (M,R,T,NH) fits to each source, and their error regions -- which anyone could read from figures in our papers.
- LS13 uses an (unpublished) semi-analytic model for the spectrum (A minor issue: why not use the heavily vetted and widely distributed models NSATMOS (Heinke 2006) or NSA (Zavlin et al 1996)?)
- Normalizes this model against G13 best-fit values and uncertainties, and then compares a Bayseian likelihood of this best fit model to an extrapolated model using different assumptions applied to their analytic model.
- LS13 is not "data analysis" in any sense at all. This is "data modeling theory". It answers the question: "If the data look like our model, this is what the results would be.". Also: "If someone were to do our analysis with the data, and the data are described by our analytic model, then this is what the results would be." It does not say what the data are actually saying.

from LS13

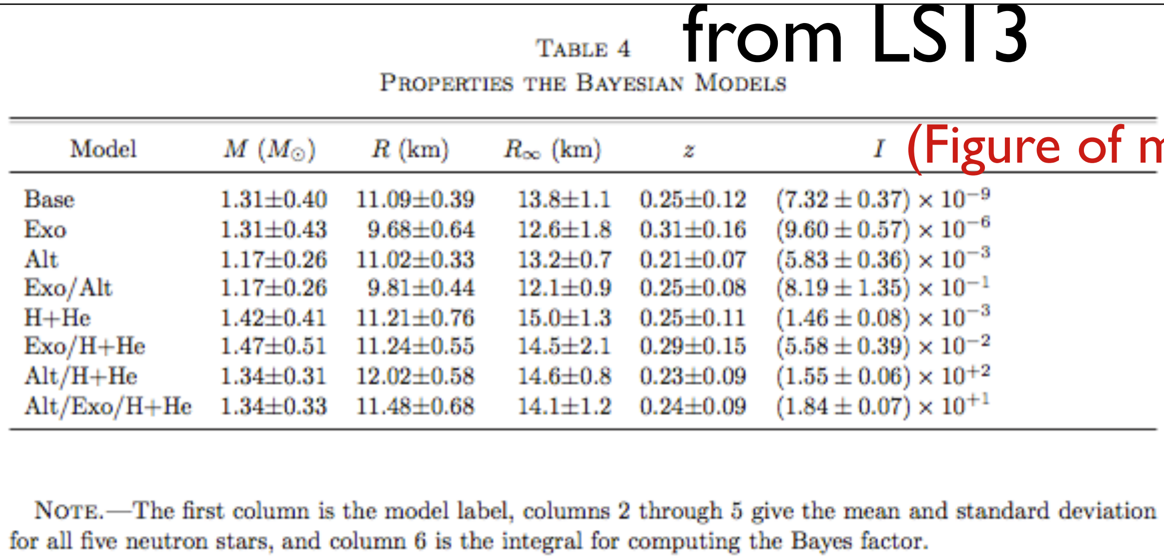

TABLE 4
PROPERTIES THE BAYESIAN MODELS

Model	M ($M_\odot$)	R (km)	R_∞ (km)	z	I
Base	1.31±0.40	11.09±0.39	13.8±1.1	0.25±0.12	$(7.32 \pm 0.37) \times 10^{-9}$
Exo	1.31±0.43	9.68±0.64	12.6±1.8	0.31±0.16	$(9.60 \pm 0.57) \times 10^{-6}$
Alt	1.17±0.26	11.02±0.33	13.2±0.7	0.21±0.07	$(5.83 \pm 0.36) \times 10^{-3}$
Exo/Alt	1.17±0.26	9.81±0.44	12.1±0.9	0.25±0.08	$(8.19 \pm 1.35) \times 10^{-1}$
H+He	1.42±0.41	11.21±0.76	15.0±1.3	0.25±0.11	$(1.46 \pm 0.08) \times 10^{-3}$
Exo/H+He	1.47±0.51	11.24±0.55	14.5±2.1	0.29±0.15	$(5.58 \pm 0.39) \times 10^{-2}$
Alt/H+He	1.34±0.31	12.02±0.58	14.6±0.8	0.23±0.09	$(1.55 \pm 0.06) \times 10^{+2}$
Alt/Exo/H+He	1.34±0.33	11.48±0.68	14.1±1.2	0.24±0.09	$(1.84 \pm 0.07) \times 10^{+1}$

NOTE.—The first column is the model label, columns 2 through 5 give the mean and standard deviation for all five neutron stars, and column 6 is the integral for computing the Bayes factor.

(Figure of merit)

- "I" is a "Bayes Integral" - their "goodness" statistic. LS13 claims it is from comparison with data. It is not. This is not a valid "data analysis" method.
- Which of your "Bayesian Preferred" models are consistent with the observed X-ray spectra for the five sources, and which are not? *This is answered in every data analysis paper ever written. It is not answered in LS13.*

Question for LS13

- Which of your "Bayesian Preferred" models are consistent with the observed X-ray spectra for the five sources, and which are not? In short, what are their "null hypothesis probabilities"? *This is answered in every data analysis paper ever written. It is not answered in LS13.*

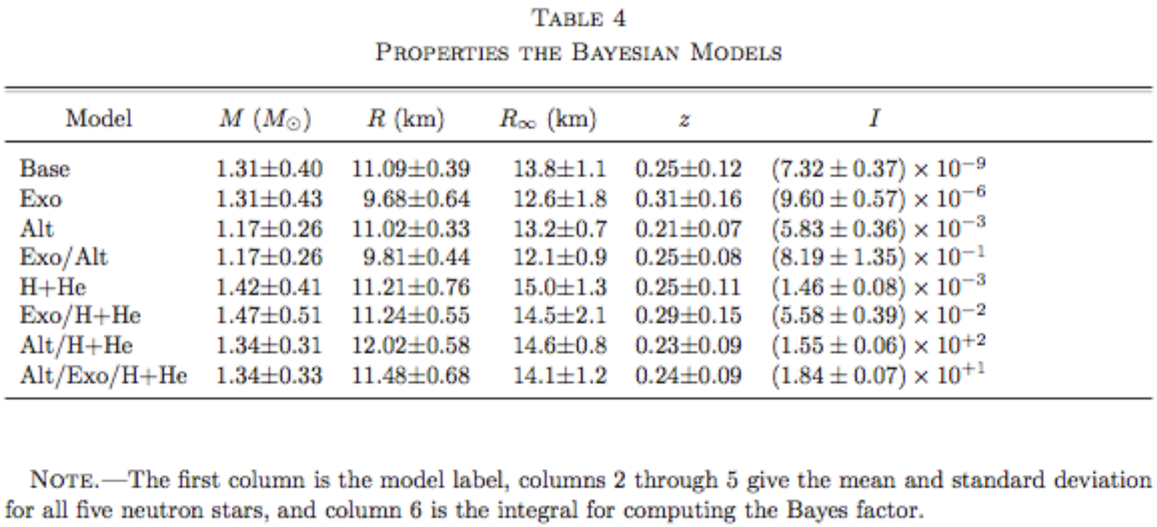

TABLE 4
PROPERTIES THE BAYESIAN MODELS

Model	M ($M_\odot$)	R (km)	R_∞ (km)	z	I
Base	1.31±0.40	11.09±0.39	13.8±1.1	0.25±0.12	$(7.32 \pm 0.37) \times 10^{-9}$
Exo	1.31±0.43	9.68±0.64	12.6±1.8	0.31±0.16	$(9.60 \pm 0.57) \times 10^{-6}$
Alt	1.17±0.26	11.02±0.33	13.2±0.7	0.21±0.07	$(5.83 \pm 0.36) \times 10^{-3}$
Exo/Alt	1.17±0.26	9.81±0.44	12.1±0.9	0.25±0.08	$(8.19 \pm 1.35) \times 10^{-1}$
H+He	1.42±0.41	11.21±0.76	15.0±1.3	0.25±0.11	$(1.46 \pm 0.08) \times 10^{-3}$
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NOTE.—The first column is the model label, columns 2 through 5 give the mean and standard deviation for all five neutron stars, and column 6 is the integral for computing the Bayes factor.

Partial List of Problems with LS13

- LS13 does not produce a statistical comparison between X-ray photon spectral data and their model. This is the only means by which any model can be tested. MAJOR
- LS13 assumes specific absorptions (NH values) and constrains them to be fixed. This is an inferior approach to leaving this a free parameter for the data fit (as done by G13). MINOR.
- Uncertainties in all parameters don't contain distance uncertainty, possibility of hard power-law contribution (G13 accounts for both). MINOR.