The (theoretical) properties of very hot compound nuclei

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Motivation

- One would expect nuclei to grow bigger as they become hotter. How much bigger?

- How does the change in size affect statistical properties? What is the relationship between excitation energy, entropy and temperature?

- What is the limiting excitation energy or temperature above which the nucleus does not exist?

- In short, what can we expect when a nucleus gets hotter and hotter?

Finite temperature formalism

In the Hartree-Fock approximation, one uses the single-particle density operator

$$D = \frac{1}{Z_0} \exp\left(- \sum_i \alpha_i a_i^\dagger a_i \right)$$

with Z_0 the grand canonical partition function,

to define the thermodynamical potential

$$\Omega(D) = \mathrm{Tr}[DH] - T\, S(D) - \mu_n N(D) - \mu_p Z(D)$$

where the entropy is given by

$$S = Tr[D \ln D]$$

and *N(D)* and *Z(D)* are the neutron and proton number operators and μ_n and μ_p the associated chemical potentials.

The Hartree-Fock equations are obtained by minimizing the thermodynamical potential with respect to the wavefunctions, $|i\rangle = a_i^\dagger |0\rangle$

The single-particle occupations that result are

$$n_i = \mathrm{Tr}\left[D a_i^\dagger a_i\right] = \frac{1}{1 + \exp[(e_i - \mu)/T]}$$

Bound states only

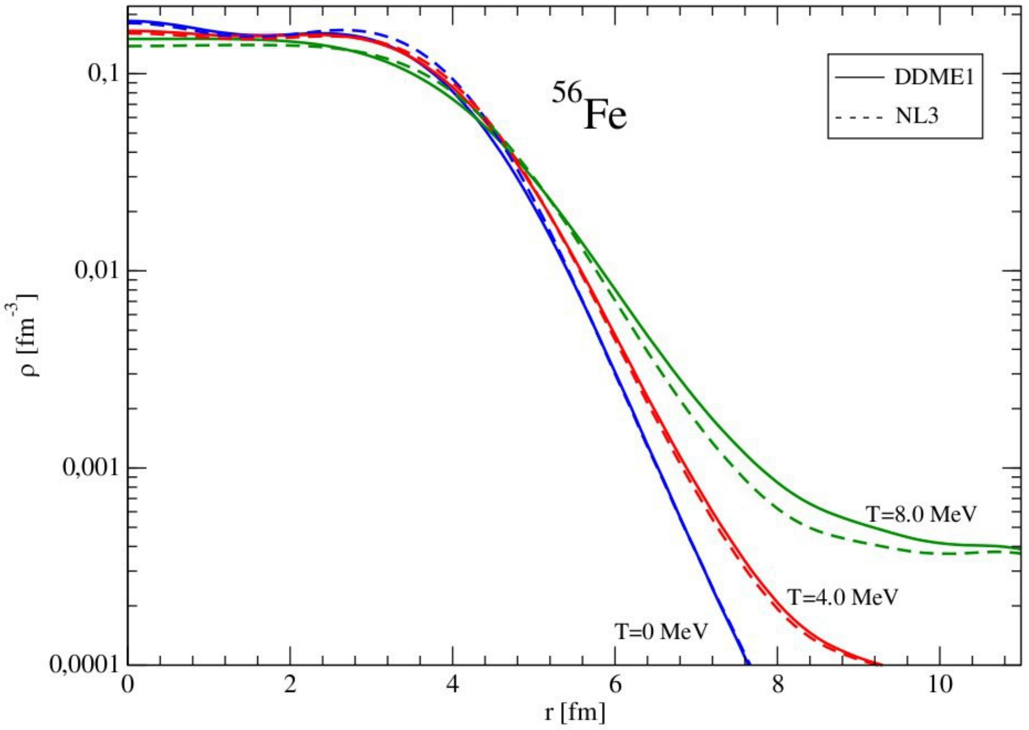
At low temperature, only bound single particle states have non-neglible occupations.

At higher temperatures, we can artificially restrict the occupations so that only bound single-particle states are occupied.

Two calculations:

- Hartree RMF;
- NL3 and DD-ME1 parameter sets;
- Harmonic oscillator basis;
- 30 major shells.

Anomalous behavior at 8 MeV, but artificially restricted anyway.



NL3 – G.A. Lalazissis, J. König and P. Ring, Phys.Rev. C 55, 540 (1997).

DD-ME1 – T. Niksic, D. Vretenar, P. Finelli, and P.Ring, Phys. Rev. C 66, 024306 (2002).

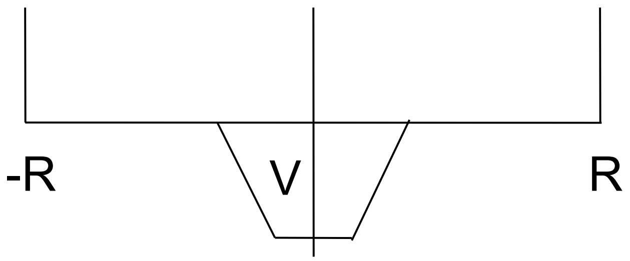
Continuum states

Consider scattering from a potential V(x) in a 1-D box [-R,R] with the condition that the wave function is zero on the edges of the box.

For a positive energy continuum state, this implies that

$$2kR + \delta(E) = n\pi$$

where δ(E) is the phase shift due to scattering from the potential.



The density of continuum single-particle states is

$$\rho(E) = \frac{dn}{dE} = \frac{2R}{\pi}\frac{dk}{dE} + \frac{1}{\pi}\frac{d\delta}{dE}$$

so that the thermodynamic potential can be written as the sum of three contributions

$$\Omega = T\sum_i \ln(1-n_i)$$

$$\to T\sum_{i\varepsilon b}\ln(1-n_i) + \frac{T}{\pi}\int_0^\infty \ln(1-n(E))\frac{d\delta}{dE}dE + 2R\frac{T}{\pi}\int_0^\infty \ln(1-n(E))\frac{dk}{dE}dE$$

Continuum states II

The first term in the thermodynamical potential is the contribution of the bound states,

$$\Omega = T\sum_{i\varepsilon b}\ln\left(1-n_i\right)+\frac{T}{\pi}\int_0^{\infty}\ln\left(1-n\left(E\right)\right)\frac{d\delta}{dE}dE+2R\frac{T}{\pi}\int_0^{\infty}\ln\left(1-n\left(E\right)\right)\frac{dk}{dE}dE$$

The second term furnishes the contribution of resonances – the phase shift near a resonance can be written as

$$\delta\left(E\right)\approx\delta_0+\mathrm{atan}\left(\frac{\Gamma/2}{E_R-E}\right)$$

and its derivative as

$$\frac{d\delta}{dE}\approx\frac{\Gamma/2}{\left(E-E_R\right)^2+\Gamma^2/4}\approx\pi\delta\left(E-E_R\right)$$

which contributes to the thermodynamic potential as

$$\Omega_R = T\,\ln\left(1-n_R\right)$$

The last term - the continuum contribution – diverges and is the same when V=0.

We can extract the finite contribution due to the potential by taking the difference,

$$\Delta\Omega\left(T,\mu\right)=\Omega\left(T,\mu,V\right)-\Omega\left(T,\mu,V=0\right)$$

Two solutions

In 3-D, including the Coulomb interaction, this is

$$\Delta\Omega(T,\mu) = \Omega(T,\mu,V+V_C) - \Omega(T,\mu,V=V_C)$$

For given values of the chemical potentials, the Hartree-Fock equations have two solutions:

- one corresponding to a nucleus + gas, with nucleon density ρ_{NG};

- another corresponding to the gas, with nucleon density ρ_G.

However, the solutions are unstable due to the Coulomb repulsion of the gas particles on themselves. To remedy this, only the Coulomb repulsion from the particles in the nucleus, $\rho_{NG} - \rho_G$, are included.

The thermodynamic potential is given by

$$\Delta\Omega(T,\mu,\rho_{NG}-\rho_G) = \Omega(T,\mu,\rho_{NG},V_N) - \Omega(T,\mu,\rho_G,V_N) + \frac{1}{2}\int \left[\rho_{NG,p}(\vec{r}) - \rho_{G,p}(\vec{r})\right] \frac{e^2}{|\vec{r}-\vec{r}'|} \left[\rho_{NG,p}(\vec{r}') - \rho_{G,p}(\vec{r}')\right] d^3r\, d^3r' + E_{CX}$$

B. Bonche, S. Levit, and D. Vautherin, Nucl. Phys. **A427** (1984) 278, 296; **A436** (1985) 265.

Nuclear densities

Two self-consistent calculations are performed – for the nucleus + gas and the gas.

The chemical potentials are such that Z and A correspond to $\rho_{NG} - \rho_G$.

RMF calculations w/pairing

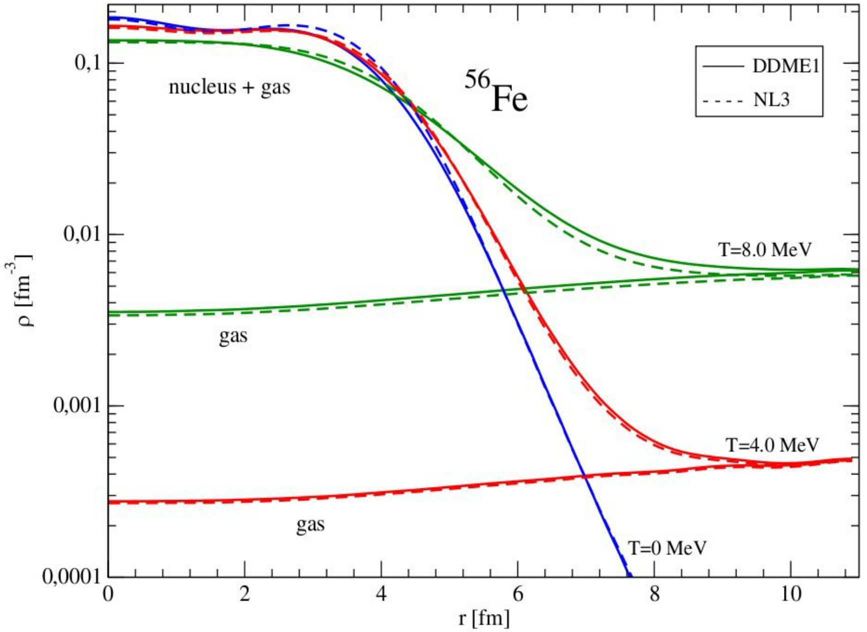
- Harmonic oscillator basis – 30 major shells;

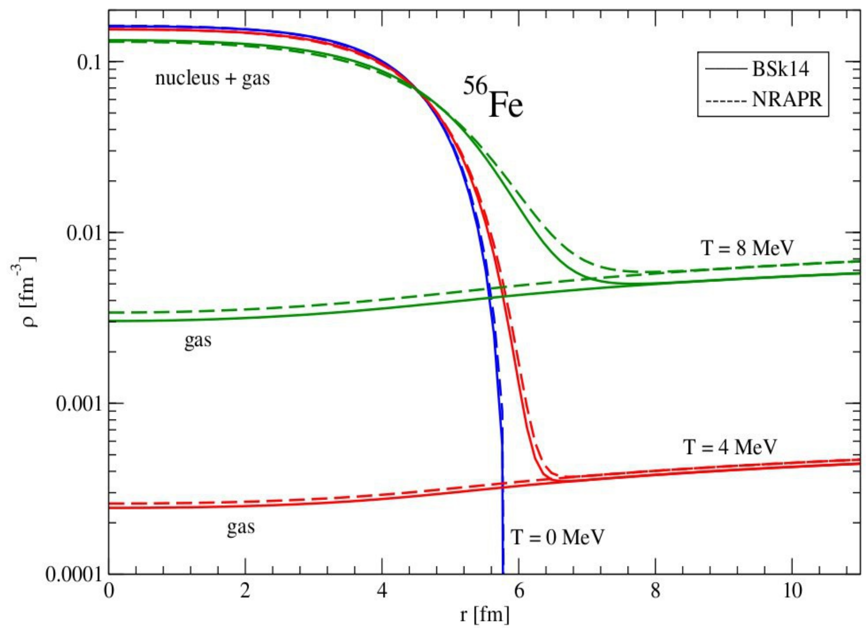
Skyrme Thomas-Fermi calculations

- Regular grid in a 1-D box;
- BSk14 and NPAPR parameter sets

BSK14 - S. Goriely, M. Samyn, J. Pearson, Phys. Rev. C 75, 064312 (2007).

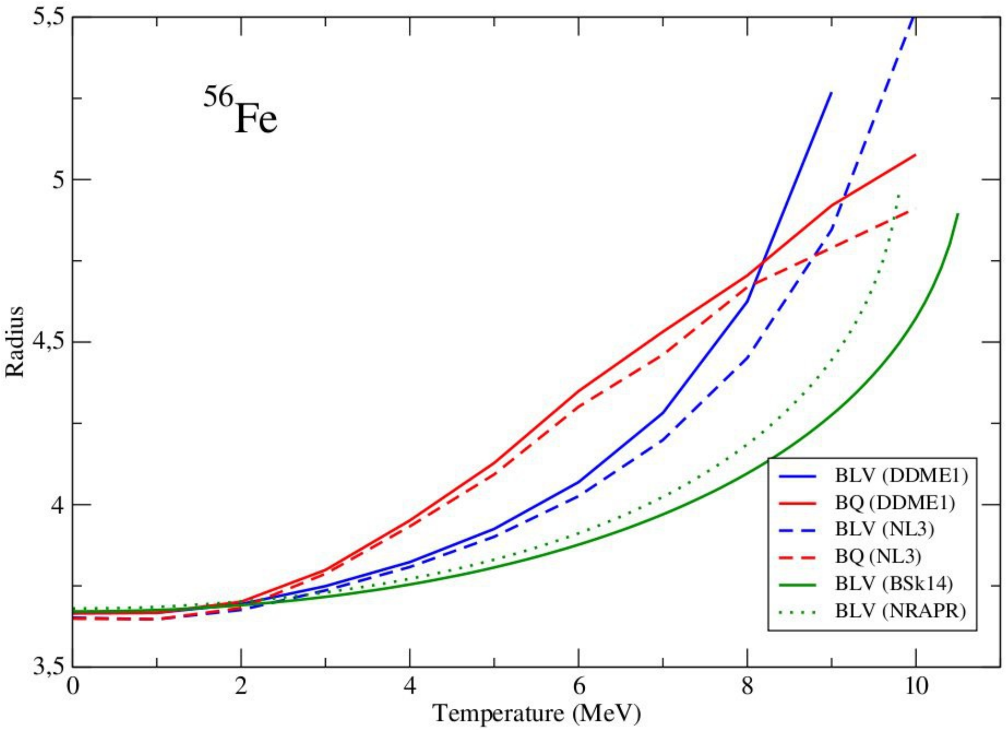
NPAPR - A. W. Steiner, M. Prakash, J. M. Lattimer, P. J. Ellis, Phys. Rep. 411, 325 (2005)





Rms radii

- Radii including only bound states increase fastest but saturate.
- Thomas-Fermi radii increase the slowest.
- The BLV radii diverge between 9 and 11 MeV, depending on the interaction.
- Without Coulomb, the BLV radii diverge at about 12 MeV.



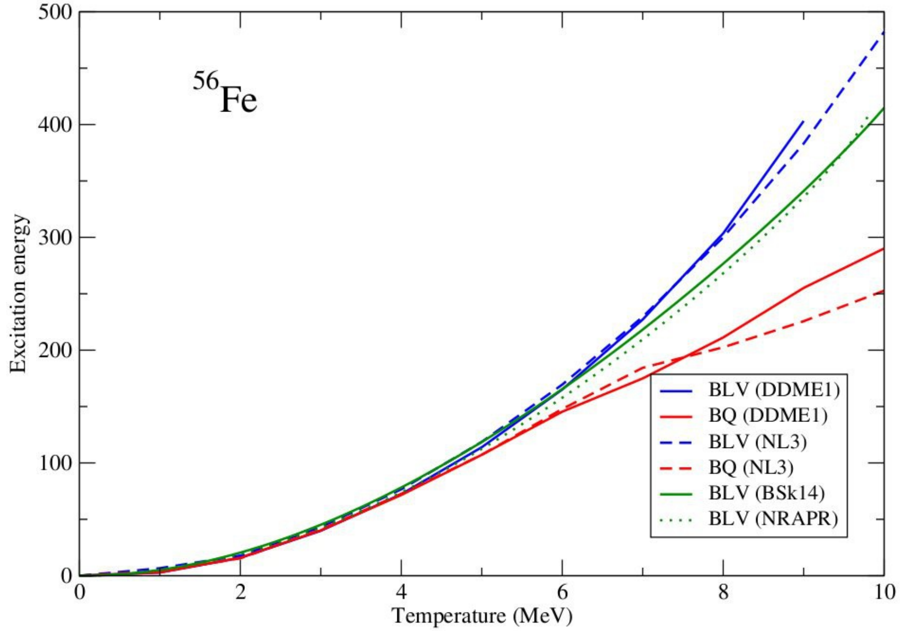
The BLV matter radii are well fit at T< 6 MeV by

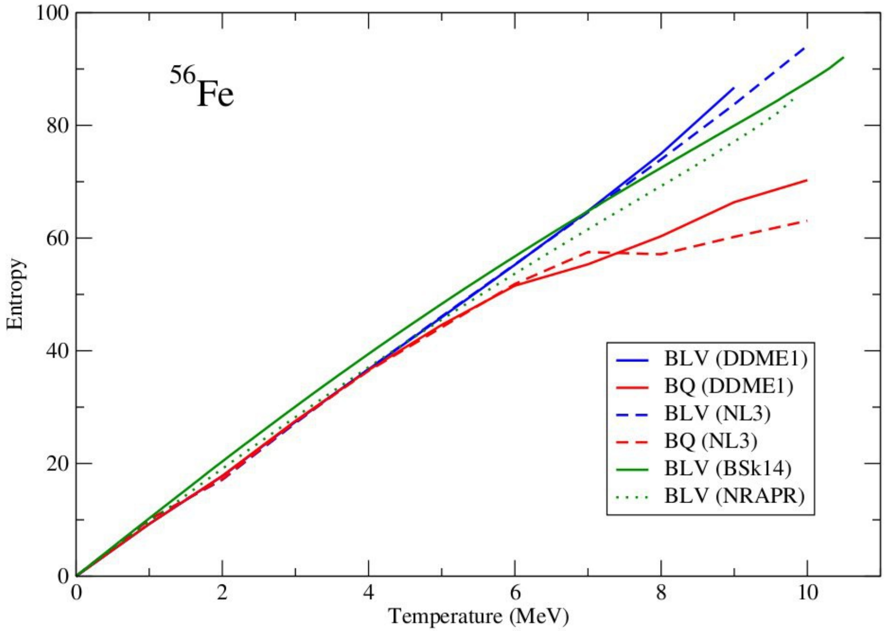
$$\left\langle r_m^2 \right\rangle = r_{m0}^2 A^{2/3} \left(1 + c_m T^2\right)$$

with

$$r_{m0} = 0.95 \pm 0.05 \text{ fm} \qquad c_m = 0.005 \pm 0.001 \text{ MeV}^{-2}$$

Excitation energy and entropy





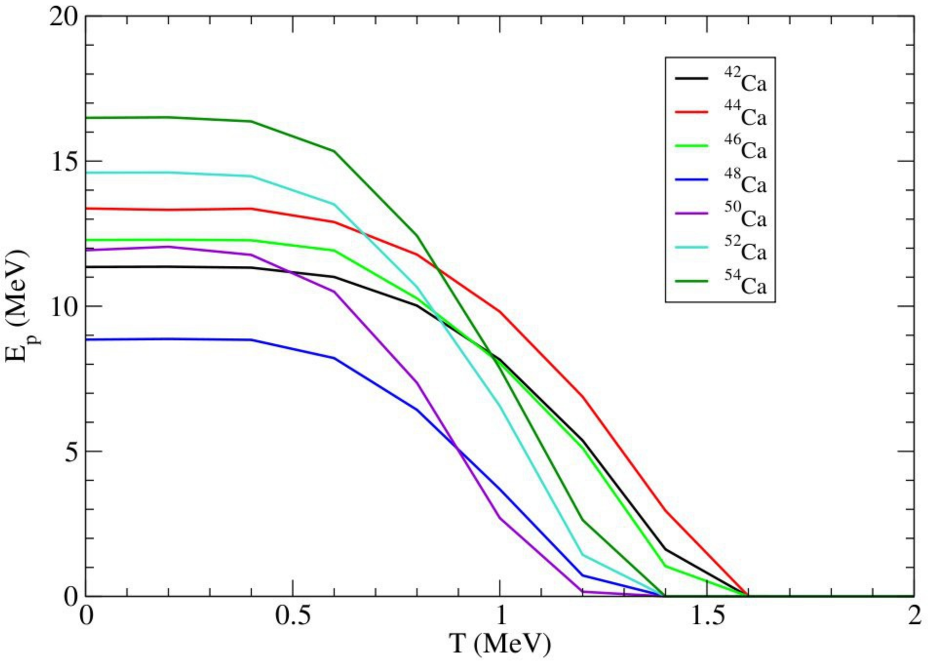
On the scale shown here,

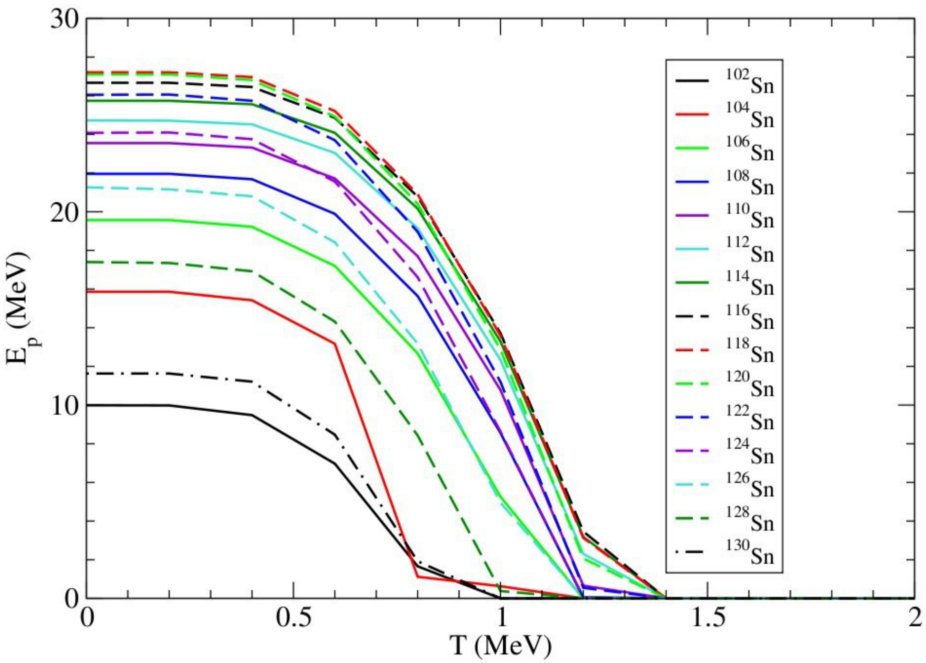
- the excitation energy appears to vary quadratically and the entropy linearly with the temperature, in all cases, up to about 5 MeV (Fermi gas behavior);

- above 5 MeV, the calculations including only bound states begin to show saturation effects;

- Pairing and shell effects enter at low temperatures.

Pairing





The RMF calculations were performed using an extended BCS approximation and a relativistic zero-range pairing interaction. (BVC and D. Hirata, Phys. Rev. C62 (2000) 054310.)

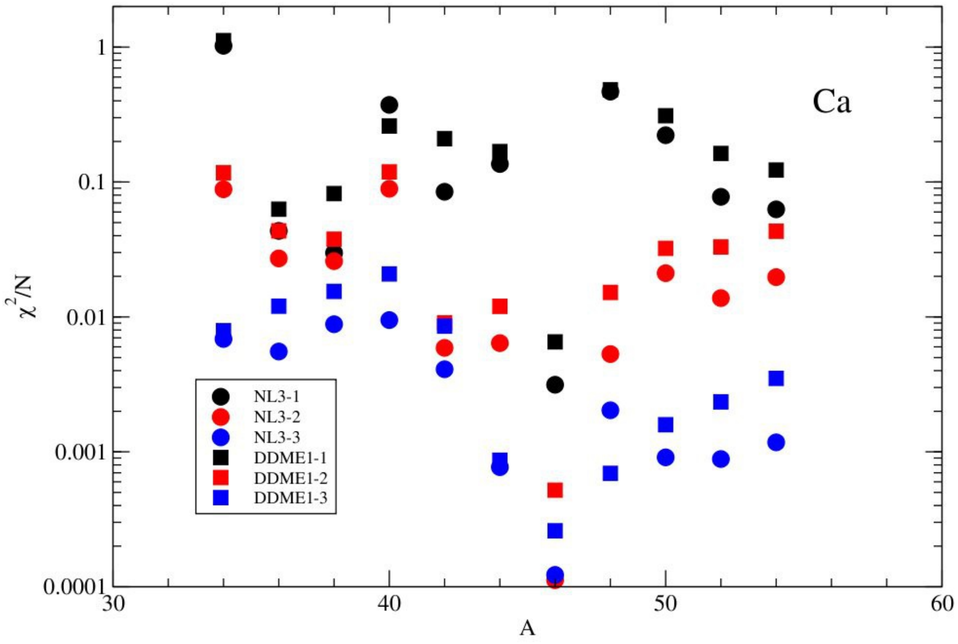
The pairing energy has the typical energy dependence and decreases to zero below T=1.5 MeV.

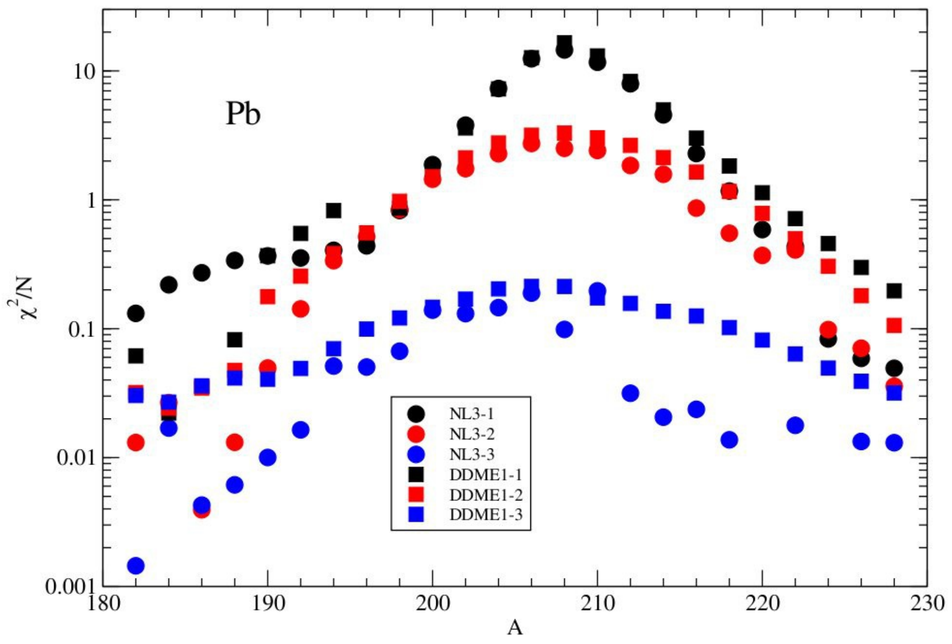
Shell effects

The RMF calculations also showed the effects of shell closures. To get an idea of their importance we looked at

$$\chi^2 = \sum_{T_{min}}^{T_{max}} \left(E_{RMF}(T) - E_0 - c_0 T^2 \right)^2$$

where T_{max} = 6 MeV and T_{min} was varied between 1 and 3 MeV.





Liquid-drop model fit to the energy

- Due to the effects of pairing and shell closures, ground state energies cannot be used a a reference for the functional dependence of the excitation energy at high energy.
- Both the constant and temperature dependent terms must be fit.
- We take

$$E = c_1 A + c_2 A^{2/3} + c_4 A d^2 + c_5 A^{1/3} + c_6 \frac{Z(Z-1)}{A^{1/3}} + (c_7 A + c_8 A^{2/3} + c_9 A d^2) T^2$$

where

$$d = \frac{1}{(1 + c_3 A^{-1/3})} \frac{N-Z}{A}$$

Fits were performed using

- RMF and Skyrme T- F calculations
- 180 nuclei with 8 ≤ Z ≤ 82 and 12 ≤ A ≤ 250
- temperatures in the range 2 MeV ≤ T ≤ 6 MeV.

Liquid-drop model fit to the energy II

The parametrization:

$$E = c_1 A + c_2 A^{2/3} + c_4 A d^2 + c_5 A^{1/3} + c_6 \frac{Z(Z-1)}{A^{1/3}} + (c_7 A + c_8 A^{2/3} + c_9 A d^2) T^2$$

where

$$d = \frac{1}{(1 + c_3 A^{-1/3})} \frac{N-Z}{A}$$

Modelos	c_1	c_2	c_3	c_4	c_5	c_6	c_7	c_8	c_9	χ^2/N
BSk14	-14.71	11.73	0.655	27.29	6.94	0.664	0.064	0.077	-0.091	3.5
NRAPR	-14.35	9.98	0.718	28.24	9.17	0.649	0.057	0.087	-0.095	3.6
DD-ME1	-15.83	21.16	1.042	32.79	-7.79	0.675	0.062	0.093	-0.112	7.0
NL3	-15.27	17.68	1.145	32.47	-1.88	0.650	0.059	0.090	-0.085	6.6
G.S.	-15.8	18.3	0.0	23.7	0.0	0.714	0.0625	0.139	0.0	$--$

- Skyrme volume and surface terms are smaller than those in G.S. fit;
- Symmetry energy term c_4 is higher because of c_3 dependence not in G.S. Fit;
- Temperature-dependent volume and surface terms – c_7 and c_8 – smaller.

Symmetry energy

The symmetry energy is found to be

$$E_{sym} \approx Ad^2 \left(30 - 0.1T^2\right) \text{ MeV}$$

It is about 10% below its ground state value at a temperature of 6 MeV.

Why does it decrease? The principal effect is the volume expansion.

With

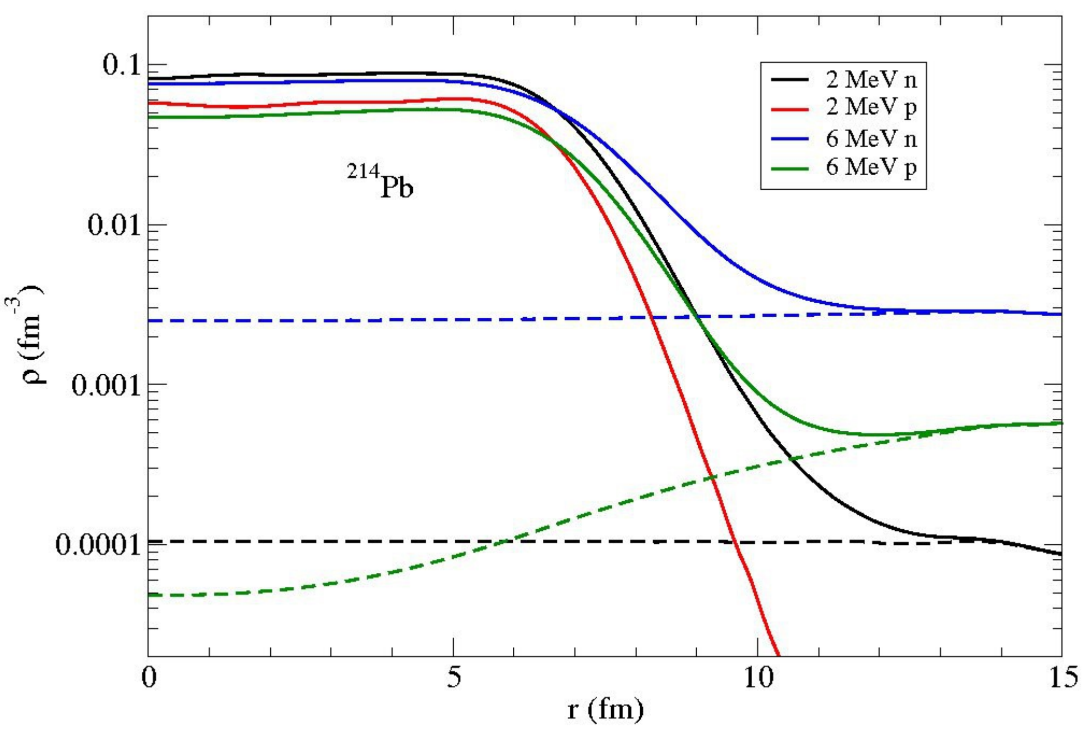
$$\left\langle r_m^2 \right\rangle = r_{m0}^2 A^{2/3} \left(1 + c_m T^2\right)$$

we have

$$E_{sym} \approx E_{sym,0} - c_m \tilde{L} T^2 / 2$$

where

$$\tilde{L} = 3\rho_0 \frac{dE_{sym}}{d\rho}\bigg|_{\rho_0}$$



Equilibrium

The compound nucleus is assumed to be in equilibrium. We estimate the local equilibration time in terms of the typical width of a shell-model state

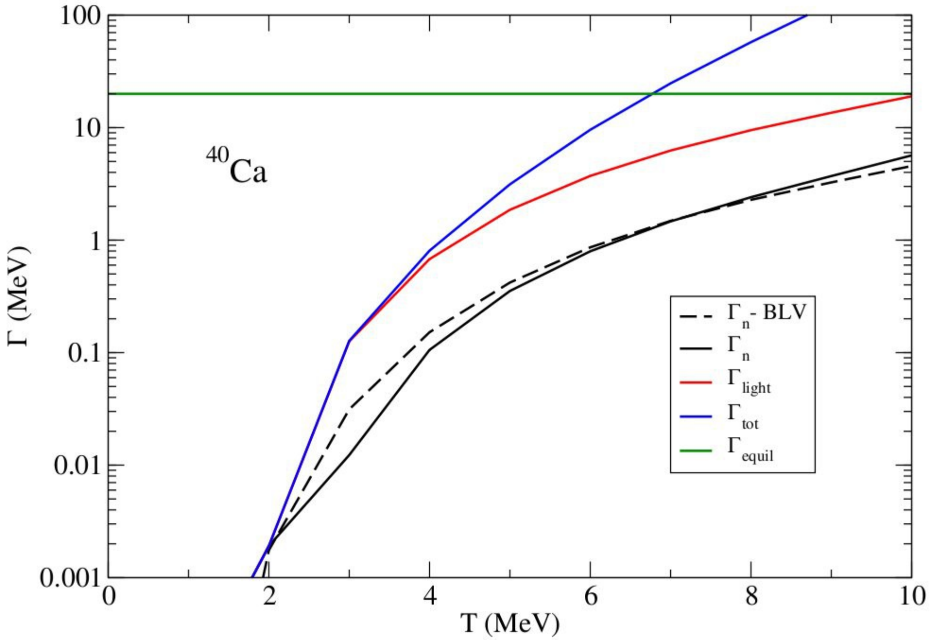
$$\hbar/\tau_{eq} = \Gamma_{eq} \approx 20\,\mathrm{MeV}$$

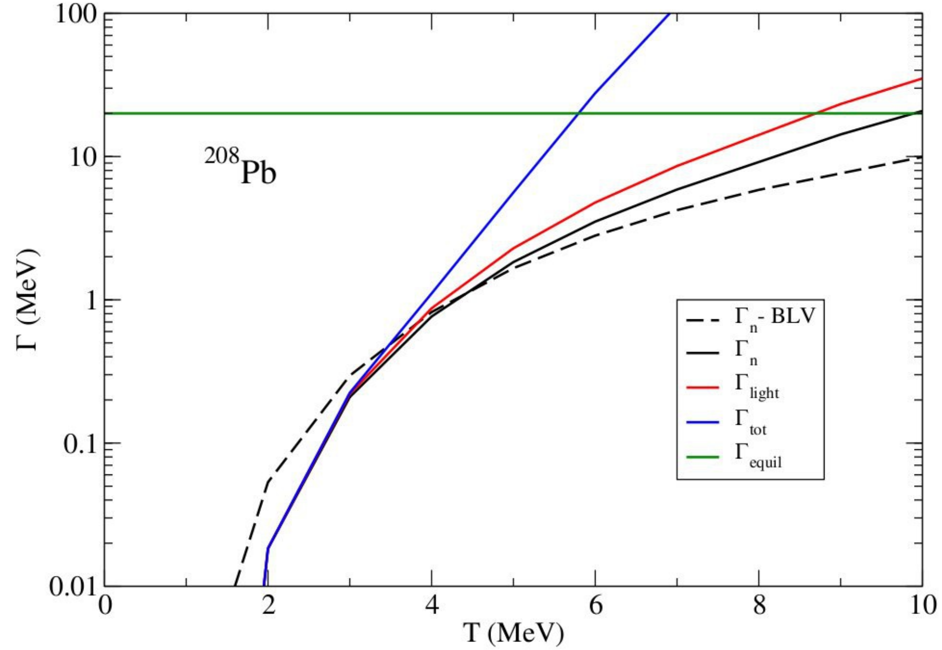
N. Frazier, B.A. Brown, V. Zelevinsky, Phys. Rev. C54 (1996) 1665.

The width of the BLV nucleus can be estimated in terms of the incident gas flux n(e) as well as in terms of its Weisskopf decay width.

$$\Gamma_{BLV} \approx \hbar \langle \sigma v n \rangle = \frac{g\mu}{\pi^2\hbar^2} \int e\,\sigma(e)\,n(e)\,de$$

$$\Gamma_W = \frac{g\mu}{\pi^2\hbar^2} \int e\,\sigma_{inv}(e)\,\frac{\rho_f(\varepsilon_0 - Q - e, 0)}{\rho_{cn}(\varepsilon_0, 0)}\,de$$





Summary

- As expected, a nucleus expands as it is heated. The nuclear radius grows approximately quadratically with the temperature and is about 10% larger than the ground state radius at 6 MeV.

- The excitation energy also grows approximately quadratically with the temperature, except at temperatures below about 2 MeV, where pairing and shell effects are important.

- The symmetry energy is temperature dependent and decreases by about 10% from the ground state value at a temperature of 6 MeV.

- The calculations suggest that nuclei are unstable due to Coulomb repulsion at temperatures above about 8 MeV.

- Decay times suggest that an equilibrated hot nucleus cannot exist at temperatures of more than 5 or 6 MeV.

Things to do

- Improve the description of the geometry

$$\left\langle r^2 \right\rangle \to \frac{1}{1+\exp\left[\left(r-R\right)/a\right]}$$

- Study fluctuations – in radius and deformation, at least

- Clusters – detailed balance?

$$\omega_{fn}\left(\varepsilon_0\right) = \prod_{l=1}^{k}\frac{1}{N_l!}\left(\frac{V}{\left(2\pi\hbar\right)^3}\right)^{n-1}\int\prod_{j=1}^{n}d^3p_j\,\delta\left(\sum_{j=1}^{n}\vec{p}_j\right)$$

$$\times\int\prod_{j=1}^{n}\left(\omega_{bj}\left(\varepsilon_j\right)d\varepsilon_j\right)\,\delta\left(\varepsilon_0-B_0-E_{c0}-\sum_{j=1}^{n}\left(\frac{p_j^2}{2m_j}+\varepsilon_j-B_j-E_{cj}\right)\right)$$