

# Towards a model-independent constraint of the high-density dependence of the symmetry energy

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# Symmetry Energy

## EoS of Asymmetric Nuclear Matter

$$E(\rho, \beta) = E(\rho, \beta=0) + S(\rho)\beta^2 \quad \beta = \frac{\rho_n - \rho_p}{\rho}$$

$$S(\rho) = S(\rho_0) + \frac{L_{sym}}{3} \frac{\rho - \rho_0}{\rho_0} + \frac{K_{sym}}{18} \frac{(\rho - \rho_0)^2}{\rho_0^2}$$

## Theoretical estimates of L and K

B.A. Li *et al.* *Int.J.Mod.Phys. E7*, 147 (1998)

Force	Paris	SKM*	SI'	SIII	DHF (b)	DHF (e)
$L$	68.8	45.78	35.34	9.91	132	138
$K_{sym}$	37.56	-155.9	-259.1	-393.7	466	276

## Experiment

isospin diffusion/neutron skin thickness

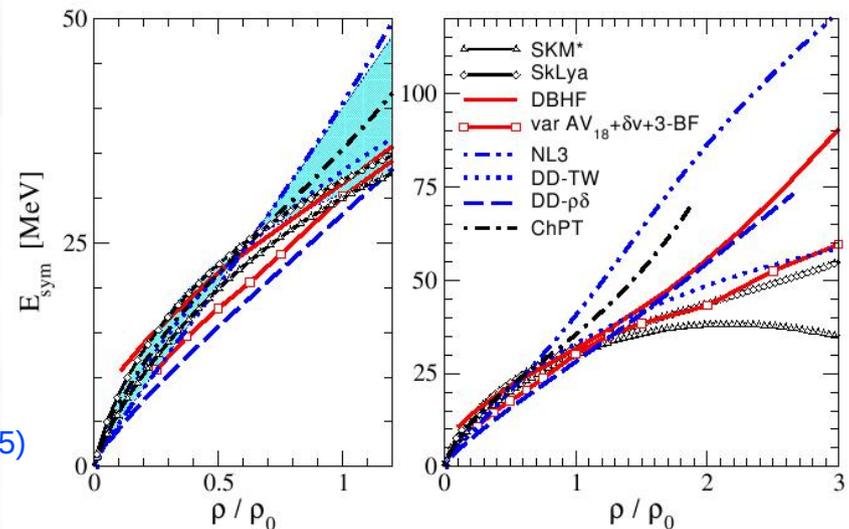
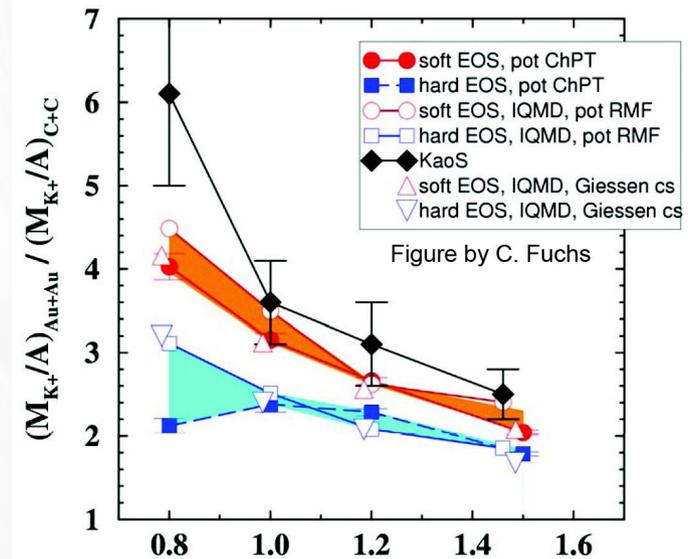
of Pb:  $L_{sym} \approx 65$  MeV B.A. Li *et al.* *PRC* 72,064611 (2005)

giant monopole resonances:

$K_{sym}$  -566 +/- 1350 MeV; 34 +/- 159 MeV

S. Shlomo *et al.* *PRC* 47, 529 (1993)

C. Fuchs *et al.* *PRL* 86, 1974 (2001)



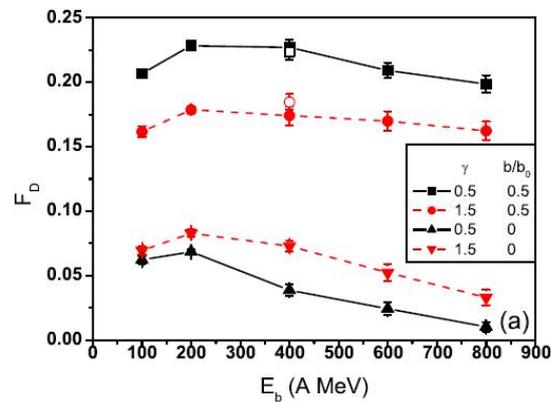
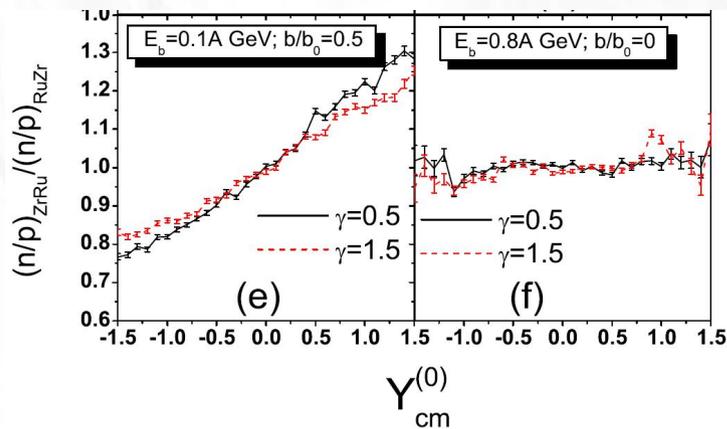
See also: M.B. Tsang *et al.* *PRC* 86, 015803 (2012)

# Observables

symmetry potential has opposite sign for neutrons (repulsive) and protons (attractive)

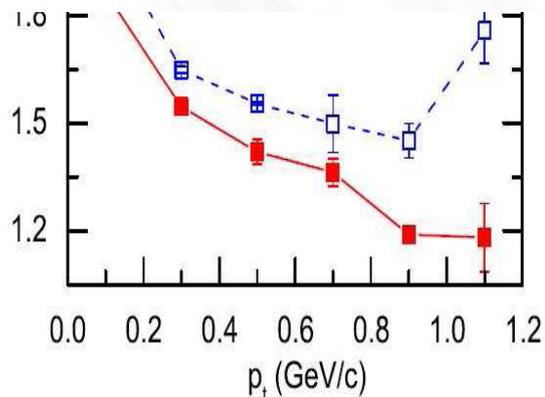
slope of double neutron to proton ratio (n/p)AB/(p/n)BA

Li, Li, Stoecker PRC 73, 051601 (2006)



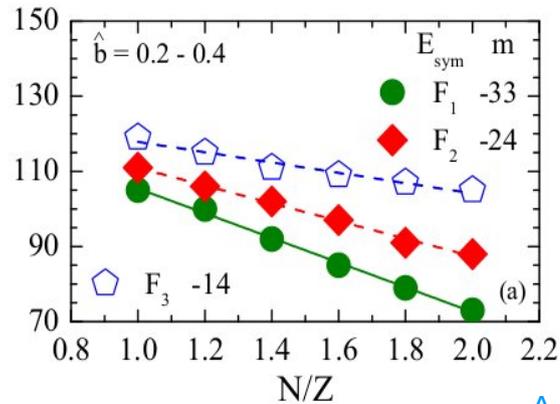
$$S(u) = S_0 u^\gamma \quad u = \rho / \rho_0$$

neutron/proton ratio at midrapidity



Yong, Li, Chen PLB 650, 344 (2007)

balance energy of transverse flow



$$\langle p_x^{dir} \rangle = \frac{1}{N} \sum_1^N \text{sign}(y_i) p_x(i)$$

A. Sood, PRC84, 014611 (2011)

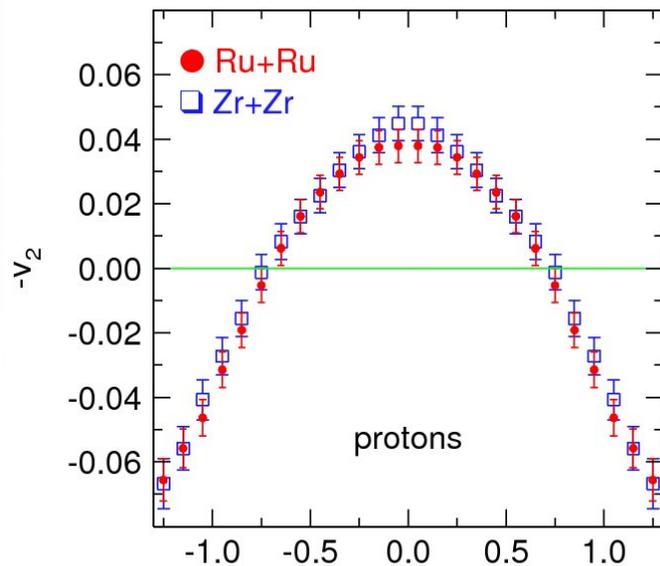
# Motivation: FOPI/FOPI-LAND

**FOPI Collaboration:** p and charged mass fragments (0.15-1.5 AGeV)

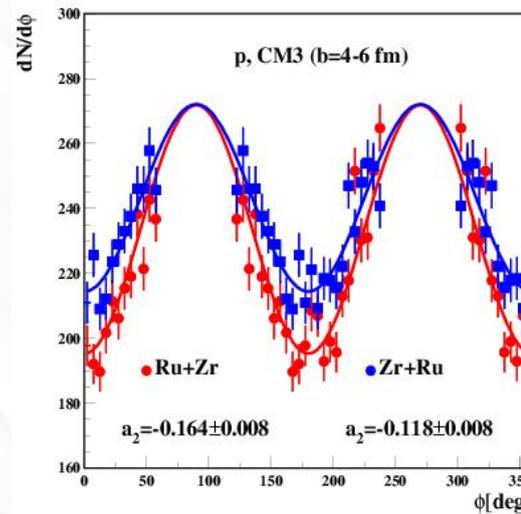
- isospin tracing [Rami et al., PRL 84,1120 \(2000\); Rami et al., NPA 646, 367 \(1999\)](#)
- stopping [Hong et al., PRC 66, 034901 \(2002\);](#)
- elliptic flow [A. Andronic et al. NPA 679,765 \(2001\)](#)

**FOPI LAND:** n, p and light mass fragments – elliptic flow

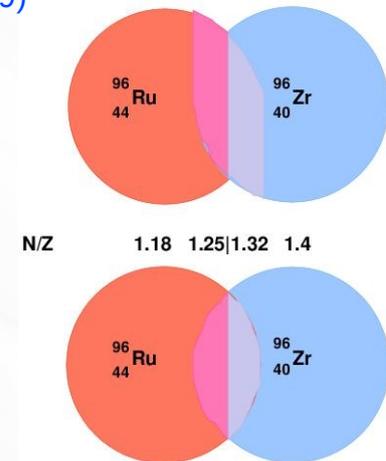
[Y. Leifels et al. PRL 71, 963 \(1993\); D. Lambrecht Z.Phys. 350,115 \(1994\)](#)



[W. Reisdorf, et al. NPA 876, 1 \(2012\)](#)



[M. Petrovici et al. \(FOPI\) preliminary/unpublished](#)



# Transport Model

## Quantum Molecular Dynamics (QMD):

Monte Carlo cascade + Mean field + Pauli-blocking+ in medium cross section

all 4\* resonances below 2 GeV - 10  $\Delta^*$  and 11  $N^*$

## baryon-baryon collisions:

all elastic channels

inelastic channels  $NN \rightarrow NN^*$ ,  $NN \rightarrow N\Delta$ ,  $NN \rightarrow \Delta N^*$ ,  $NN \rightarrow \Delta\Delta^*$ ,  $NR \rightarrow NR'$

**pion-absorption  $\Leftrightarrow$  resonance-decay channels:**  $\Delta \leftrightarrow N\pi$ ,  $\Delta^* \leftrightarrow \Delta\pi$ ,  $N^* \leftrightarrow N\pi$

**meson production/absorption:**  $\eta(547)$ ,  $\rho(770)$ ,  $\omega(782)$ ,  $\eta'(958)$ ,  $f_0(980)$ ,  $a_0(980)$ ,  $\Phi(1020)$

## applied to study:

- dilepton emission in HIC: [K.Shekter, PRC 68, 014904 \(2003\)](#); [D. Cozma, PLB640,170 \(2006\)](#); [E.Santini PRC78,03410](#)
- EoS of symmetric nuclear matter: [C. Fuchs, PRL 86, 1974 \(2001\)](#); [Z.Wang NPA 645, 177 \(1999\)](#) (2008)
- In-medium effects and HIC dynamics: [C. Fuchs, NPA 626,987 \(1997\)](#); [U. Maheswari NPA 628,669 \(1998\)](#)

# Isospin dependence of EoS

a) **momentum dependent** – generalization of the Gogny interaction:

Das, Das Gupta, Gale, Li PRC67, 034611 (2003)

$$U(\rho, \beta, p, \tau, x) = A_u(x) \frac{\rho_{\tau'}}{\rho_0} + A_l(x) \frac{\rho_{\tau}}{\rho_0} + B(\rho/\rho_0)^{\sigma} (1 - x\beta^2) - 8\tau x \frac{B}{\sigma + 1} \frac{\rho^{\sigma-1}}{\rho_0^{\sigma}} \beta \rho_{\tau'}$$

$$+ \frac{2C_{\tau\tau}}{\rho_0} \int d^3p' \frac{f_{\tau}(\vec{r}, \vec{p}')}{1 + (\vec{p} - \vec{p}')^2/\Lambda^2} + \frac{2C_{\tau\tau'}}{\rho_0} \int d^3p' \frac{f_{\tau'}(\vec{r}, \vec{p}')}{1 + (\vec{p} - \vec{p}')^2/\Lambda^2}$$

b) **momentum dependent** – power law

add references:

$$U_{sym}(\rho, \beta) = \begin{cases} S_0(\rho/\rho_0)^{\gamma} - \text{linear, stiff} \\ a + (18.5 - a)(\rho/\rho_0)^{\gamma} - \text{soft, supersoft} \end{cases}$$

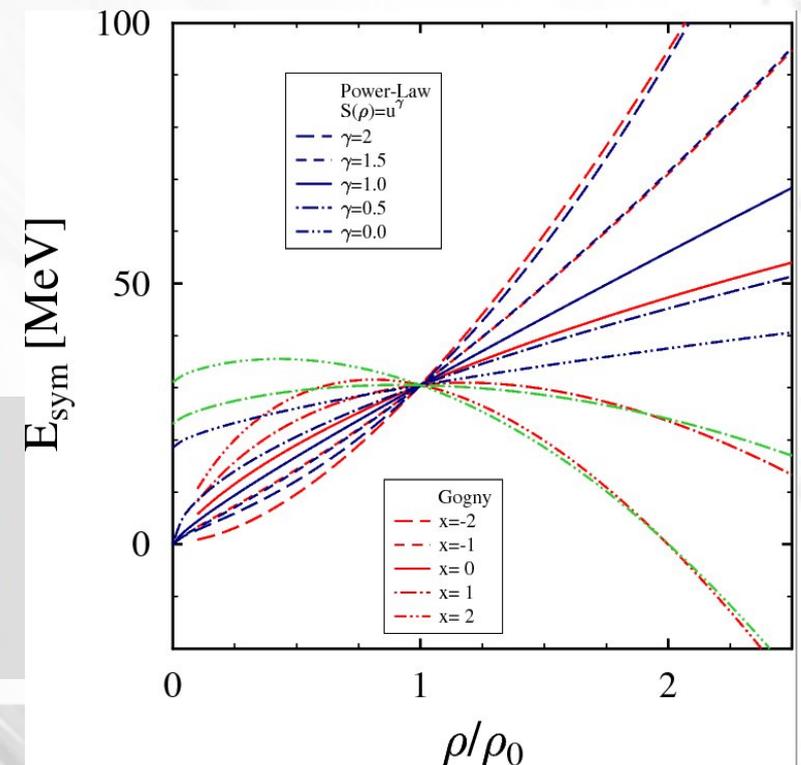
**nucleons and resonances**  
propagate in an **isospin**  
**dependent mean field**

$$U_{asym}(n^*) = U_{asym}(\Delta^0) = U_{asym}^n$$

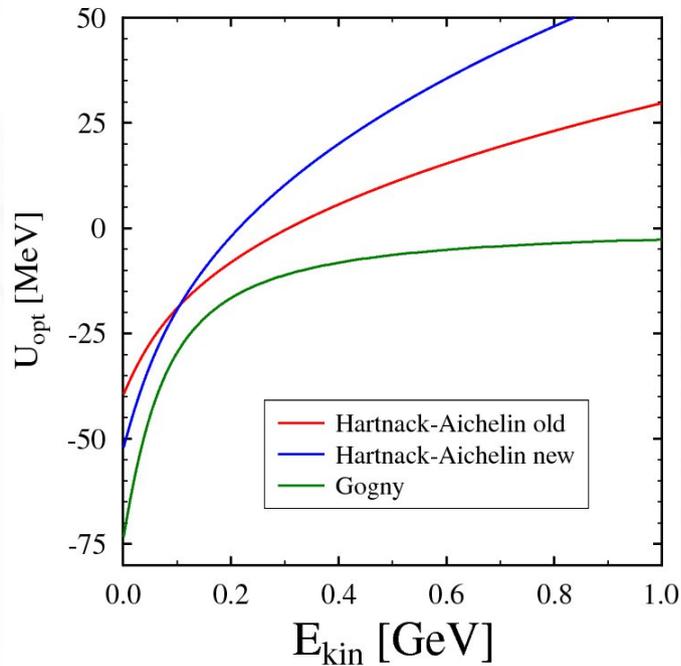
$$U_{asym}(p^*) = U_{asym}(\Delta^+) = U_{asym}^p$$

$$U_{asym}(\Delta^{++}) = 2U_{asym}^p - U_{asym}^n$$

$$U_{asym}(\Delta^-) = 2U_{asym}^n - U_{asym}^p$$



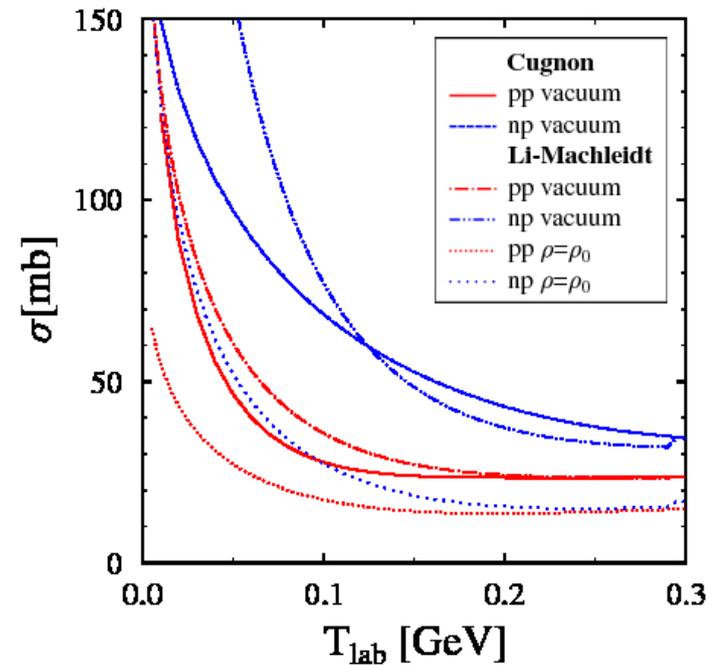
# Optical potential/NN Cross-Sections



Das, Das Gupta, Gale, Li PRC 67, 034611 (2003)  
Hartnack and Aichelin, PRC 49, 2801 (1994)

$$V_{opt}^{(HA)}(p-p') = V_0 + \delta \ln^2[(p-p')^2 \epsilon + 1]$$

$$V_0 = -54 \text{ MeV}; \delta = 1.58 \text{ MeV}; \epsilon = 500 \text{ GeV}^{-2}$$



Li, Machleidt PRC 48, 1702 (1993)  
Li, Machleidt PRC 49, 566 (1994)

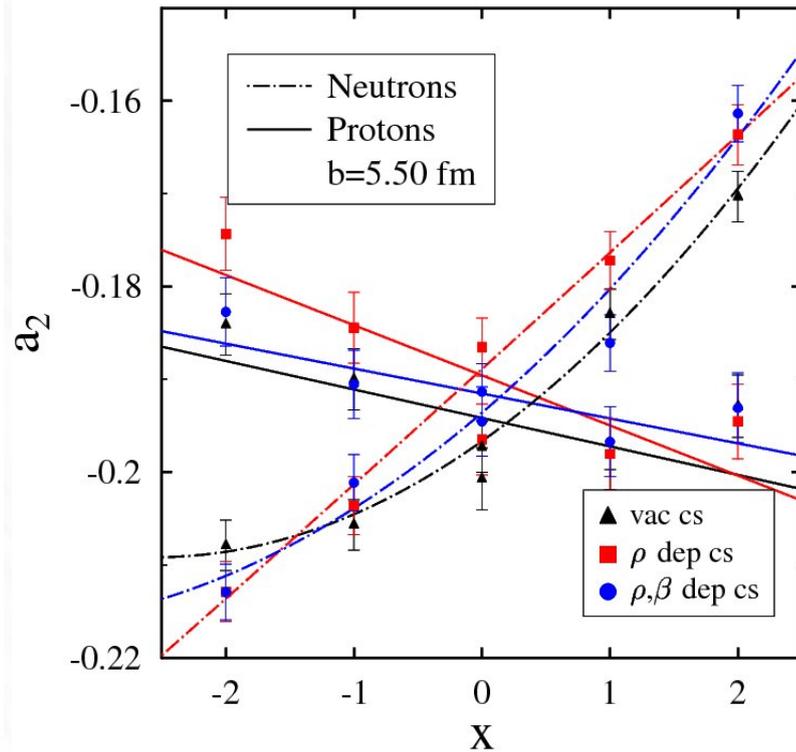
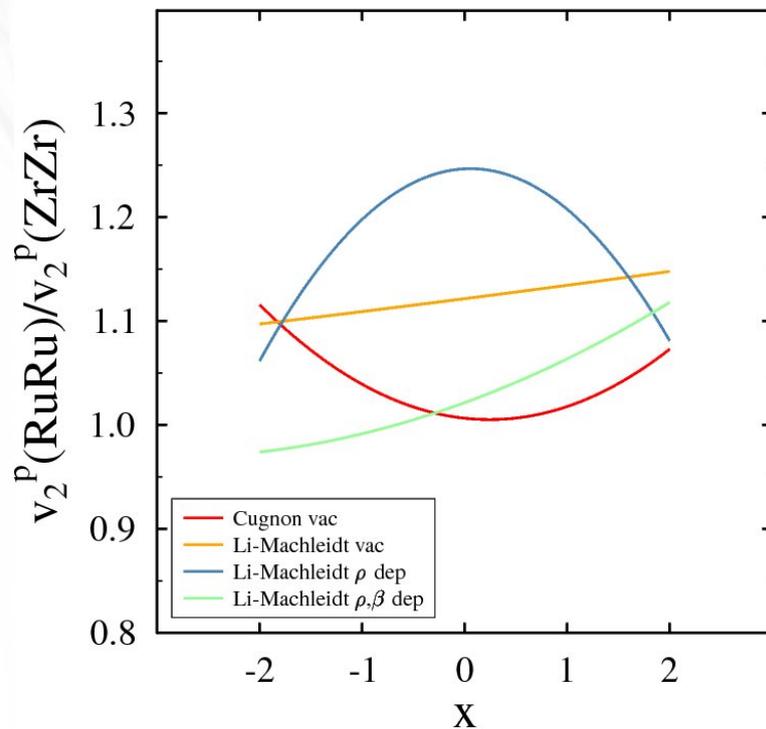
isospin asymmetry dependence of NNCS

$$\sigma_{NN}(\rho, \beta) = \sigma_{NN}(\rho, \beta = 0) \frac{m_1(\rho, \beta) m_2(\rho, \beta)}{m_1(\rho, \beta = 0) m_2(\rho, \beta = 0)}$$

# Elliptic Flow

$$\frac{dN}{d\phi} \sim 1 + 2v_1 \cos\phi + 2v_2 \cos 2\phi$$

$$a_2 = 2v_2$$



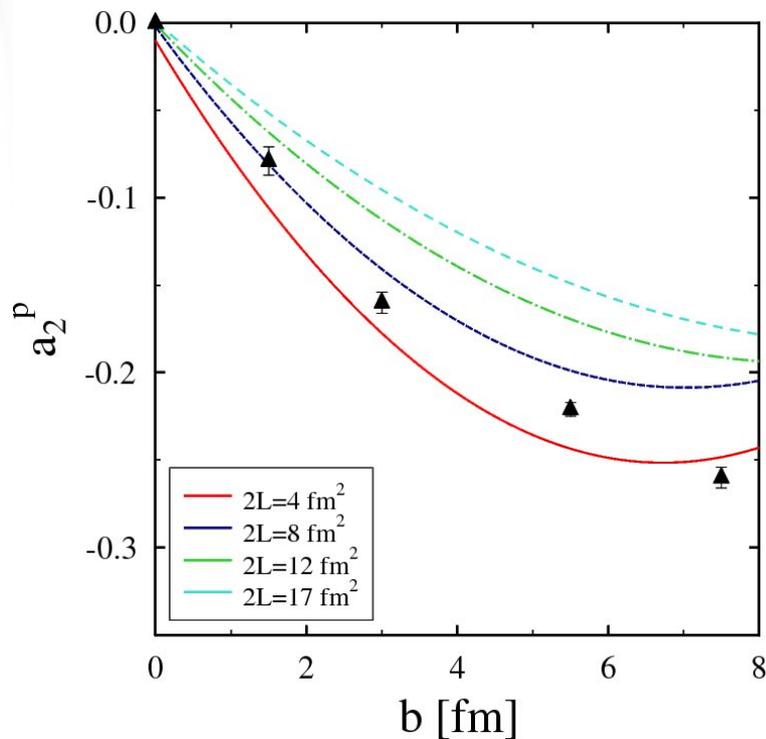
# Extra model parameters

Width of the wave packet:  $L$

$$\Psi(\vec{r}, \vec{p}, t) \sim \exp(-(\vec{x} - \vec{r})^2/L) \exp(i\vec{x}\vec{p})$$

$$f_i(\vec{r}, \vec{p}, t) \sim \exp(-(\vec{r} - \vec{r}_i)^2/2L) \exp(-(\vec{p} - \vec{p}_i)^2 L/2)$$

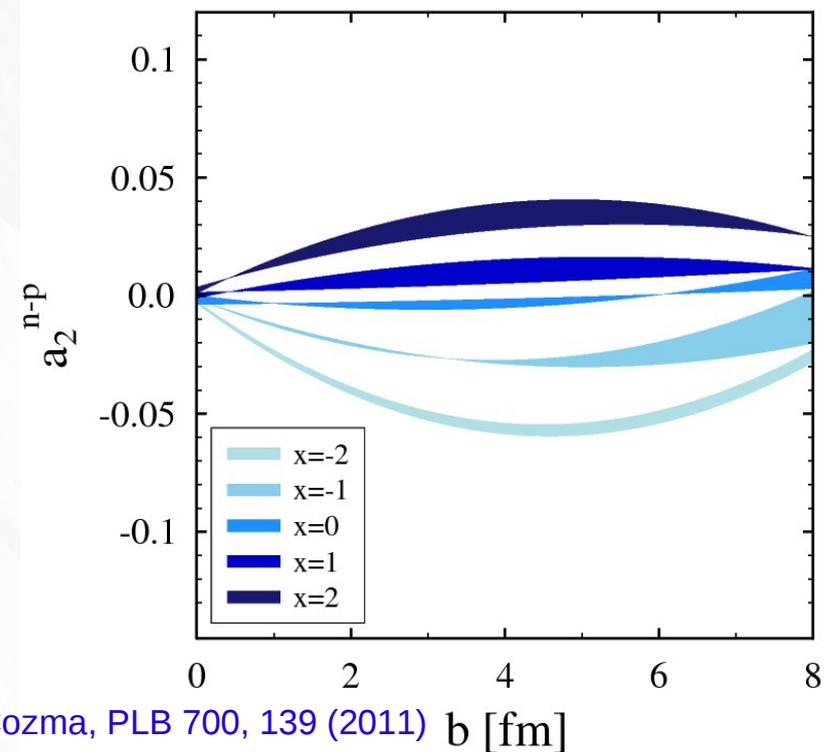
stability of heavy nuclei:  $L=8 \text{ fm}^2$   
 C. Hartnack et al. Eur.Phys.J.A1,151 (1998)



Compressibility modulus:  $K$

$$E(\rho) = E(\rho_0) + \frac{K}{18} \frac{(\rho - \rho_0)^2}{\rho_0^2}$$

$K=210 \text{ MeV (soft) / 380 MeV (stiff)}$



M.D. Cozma, PLB 700, 139 (2011)

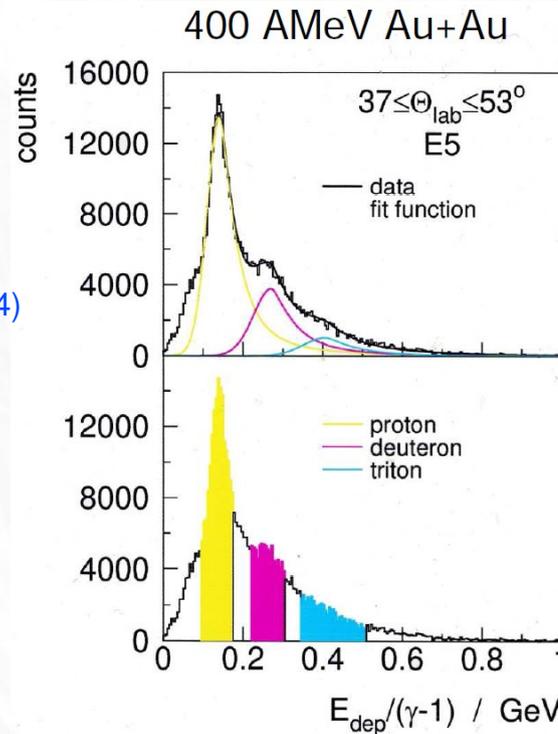
Experimental data (FOPI): A. Andronic et al. Nucl. Phys. A 679, 765 (2001)

# FOPI-LAND

first detection of neutron  
squeeze-out

Y. Leifels et al. PRL 71, 963 (1993)

D. Lambrecht et al., ZPA 350, 115 (1994)



$$E_{\text{sym}} = 22(\rho/\rho_0)^y + 12(\rho/\rho_0)^{2/3} \text{ MeV}$$

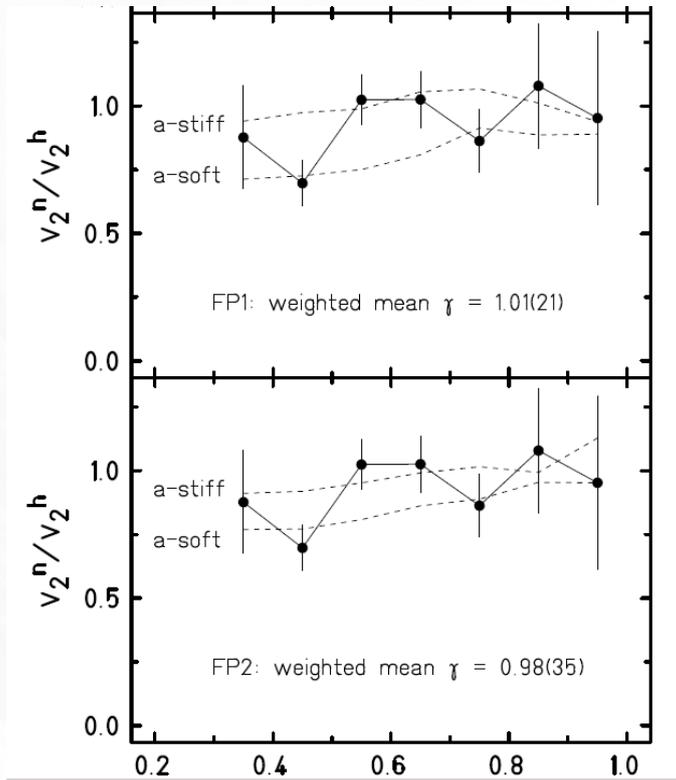
$$v_2^n/v_2^h \quad \gamma = 1.01 \pm 0.21 / 0.98 \pm 0.35$$

$$5.5 < b < 7.5 \quad \gamma = 0.58 \pm 0.27 / 0.35 \pm 0.44$$

$$v_2^n/v_2^p \quad \gamma = 0.99 \pm 0.28 / 0.85 \pm 0.47$$

$$\gamma = 0.9 \pm 0.4$$

clustering algorithm – good Z=1 reproduction



P. Russotto et al. PLB 697, 471 (2011)

UrQMD model (upgrades):

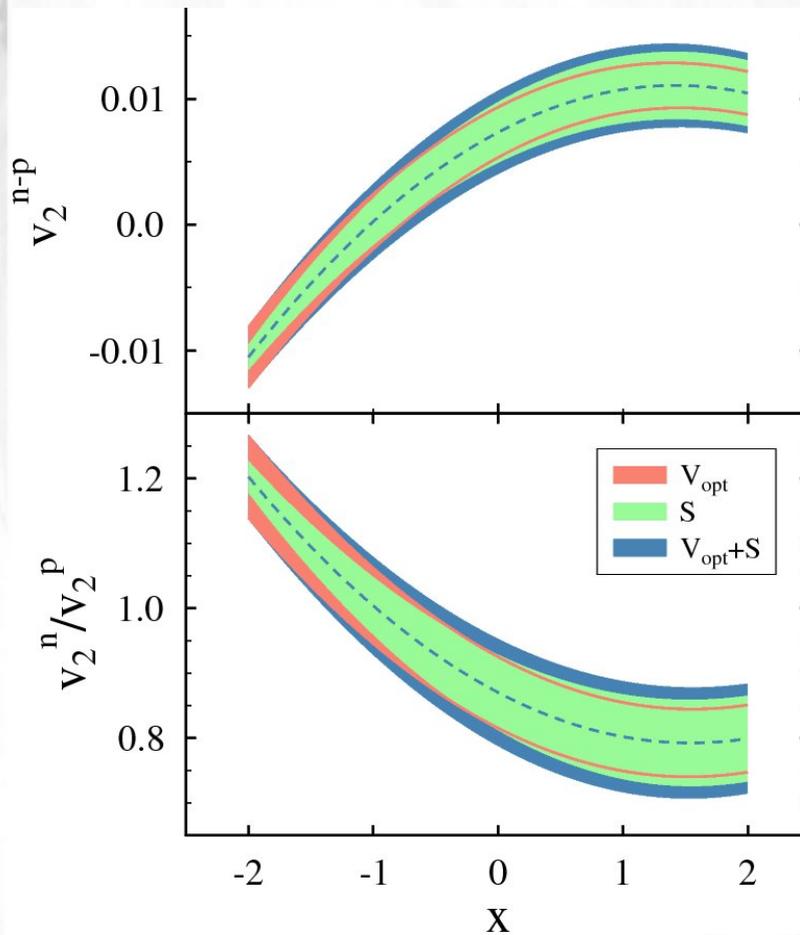
Q. Li et al. PRC 83, 044617 (2011)

Y. Wang et al., nucl-th 1305.4730 (2013)

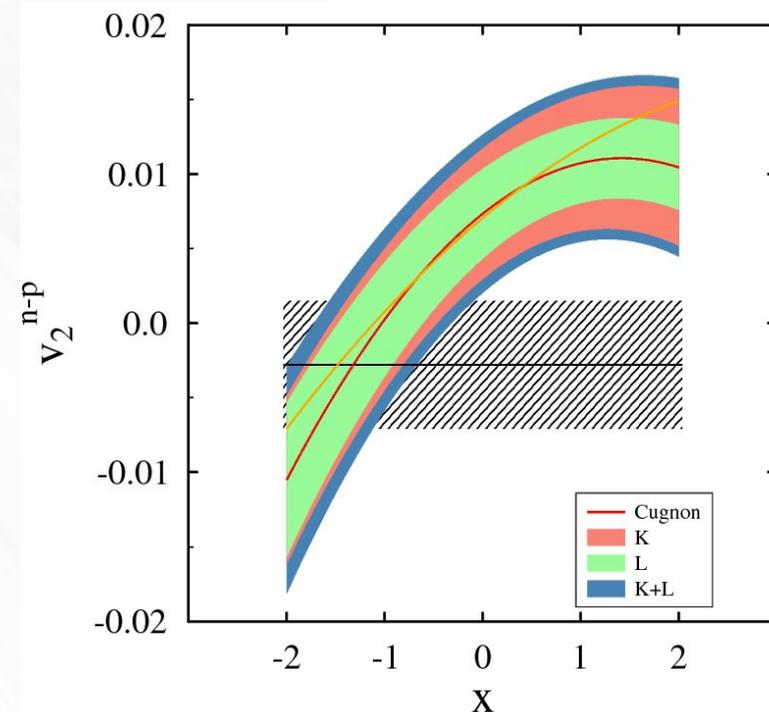
# Model dependence & FOPI-LAND

optical potential & symmetry energy

wave function width (L) and compressibility modulus (K)



$K=190 \div 280$  MeV  
 $L=2.5 \div 7.0$  fm<sup>2</sup>



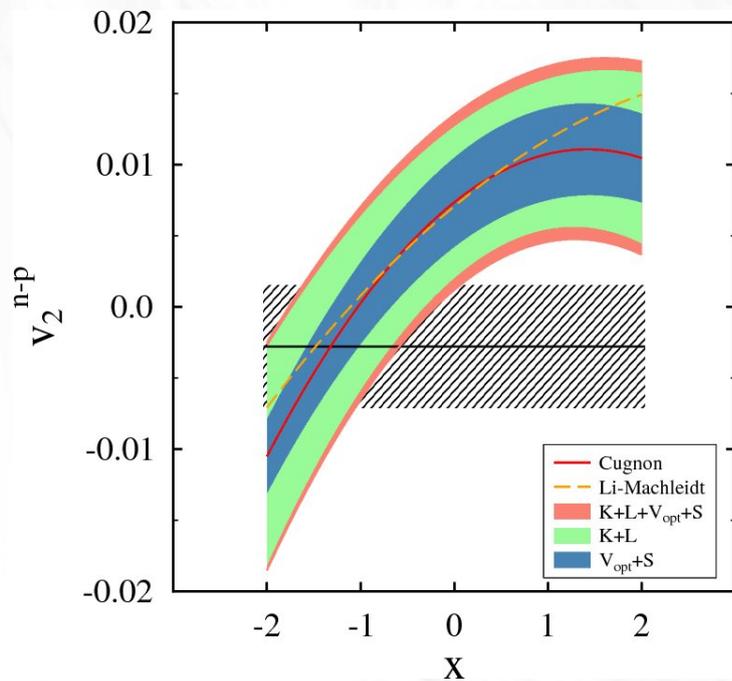
M.D. Cozma et al. ArXiv:1305.5417 [nucl-th]

see also L.Zhang et.al Eur.Phys.J. A48, 30 (2012)

# Constraints on Asy-EoS

**flow difference:**

$$x = -1.50^{+1.75}_{-1.00}$$

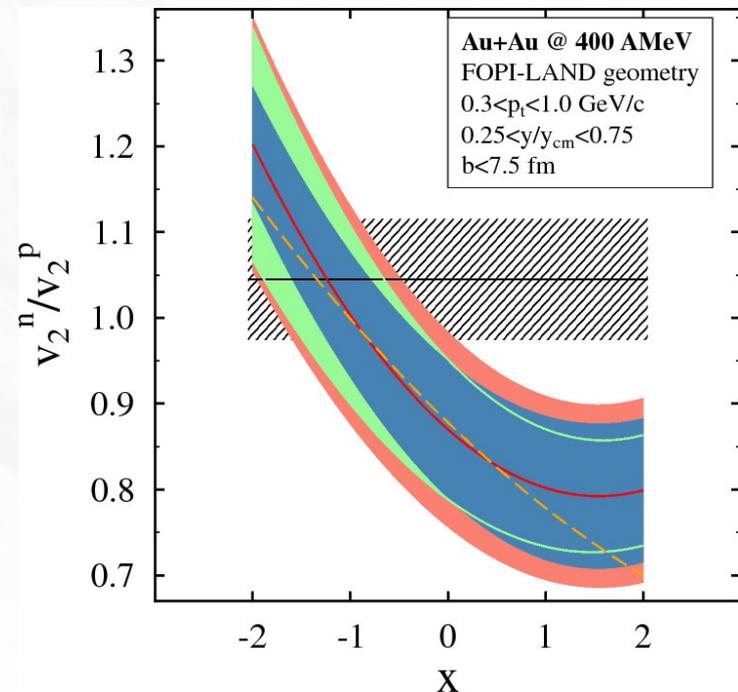


$$L_{\text{sym}} = 129^{+46}_{-80} \text{ MeV}$$

$$K_{\text{sym}} = 272^{+291}_{-508} \text{ MeV}$$

**flow ratio:**

$$x = -1.25^{+1.25}_{-1.00}$$



$$L_{\text{sym}} = 118^{+45}_{-57} \text{ MeV}$$

$$K_{\text{sym}} = 199^{+291}_{-362} \text{ MeV}$$

# Density dependence of SE

## studies

presented in

M.D. Cozma PLB 700, 139 (2011)

P. Russotto et al. PLB 697, 471 (2011)

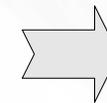
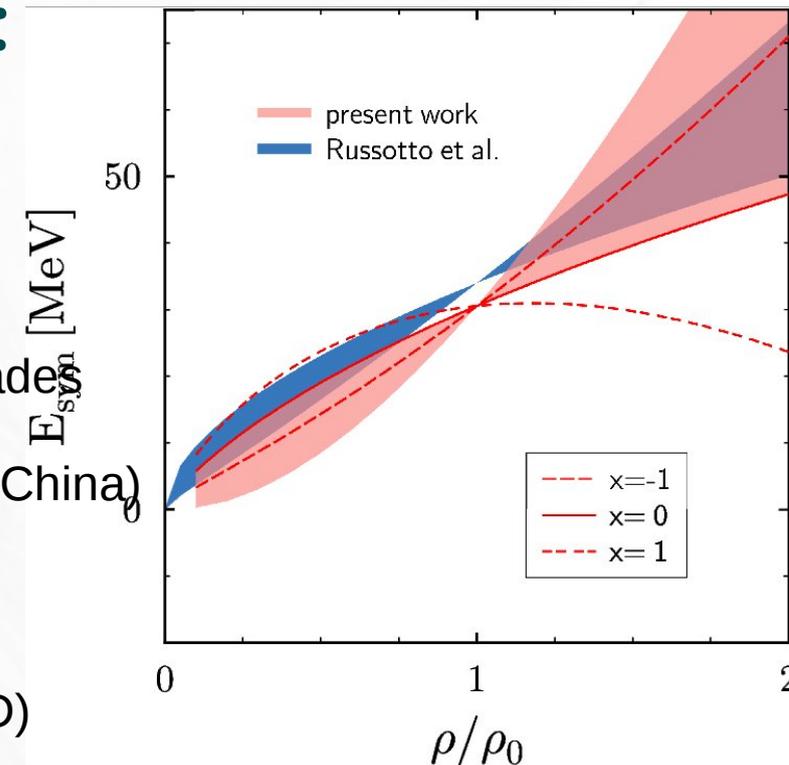
M.D. Cozma et al. ArXiv:1305.5417 [nucl-th]

- **Independently developed** transport codes and upgrades  
 QMD – Tuebingen/ upgraded in Bucharest  
 UrQMD – Frankfurt/ upgraded Q.Li et al. Huzhou (China)

- **different parametrizations** of the symmetry energy  
 - momentum dependent (QMD) /  
 momentum independent (UrQMD)

- inclusion of **in-medium effects**  
 - in medium NN cross-section (both QMD and UrQMD)

- **thorough study** of various model parameters:  
 - width of nucleon wave-function (L)  
 - compressibility modulus of nuclear matter (K)  
 - impact of optical potential



similar result

$$x = -1.0 \pm 1.0$$

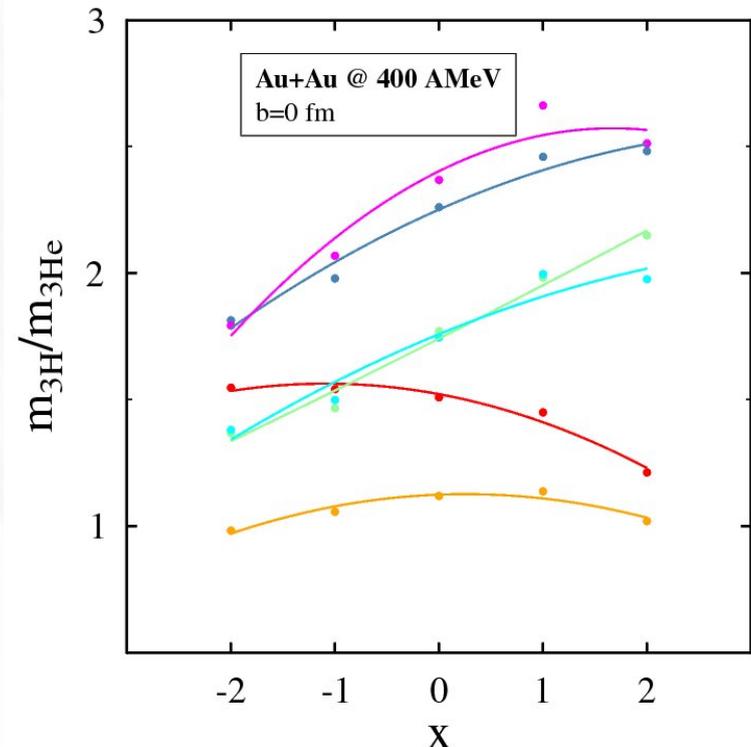
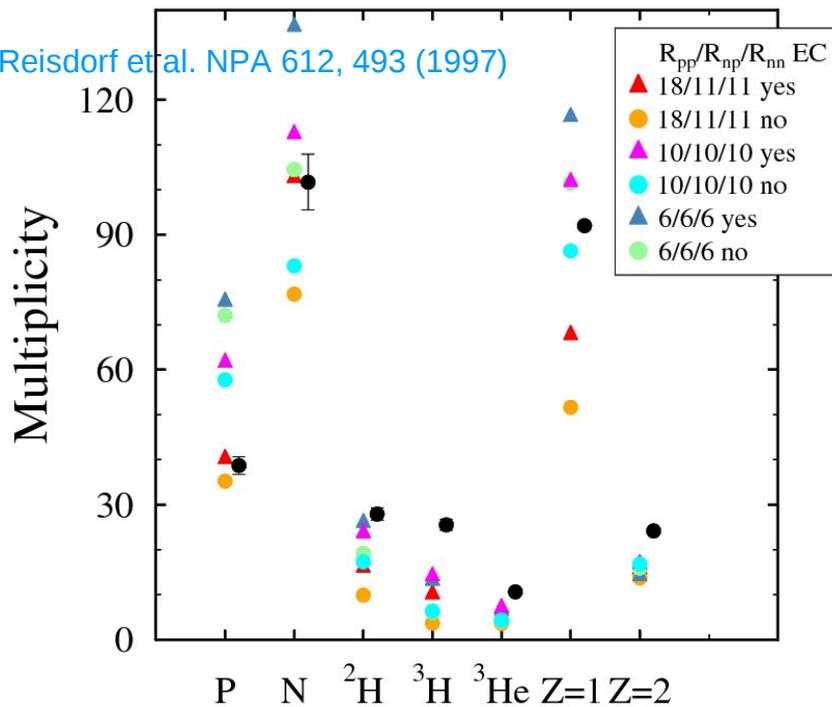
$$L_{\text{sym}} = 106 \pm 46 \text{ MeV}$$

$$K_{\text{sym}} = 127 \pm 290 \text{ MeV}$$

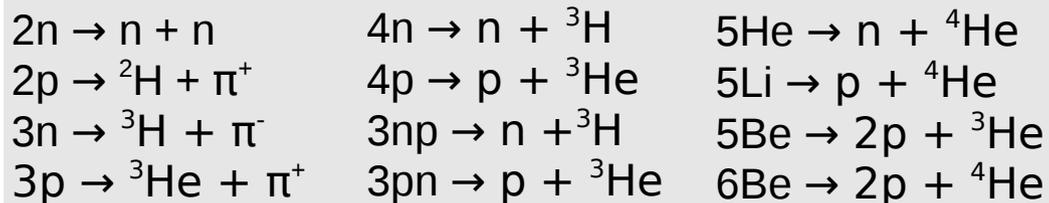
# A note on light mass fragments

- phase space coalescence model: Y.Zhang et al. PRC 85, 051602R (2012)
- modest goal: describe FOPI neutron, proton, deuteron, Z=1 multiplicities

W. Reisdorf et al. NPA 612, 493 (1997)



-prescription for spurious clusters (guided by exp. fraction of free neutrons/protons)



on the topic see also

A. Le Fevre et al., talk at ASYEOS 2012  
G-C. Yong et al. PRC 80, 044608 (2009)

# Summary & Outlook

-**Elliptic flow observables** – sensitive to the density dependence of symmetry energy

-**Thorough study** of model dependence

-**Similar extracted constraints** of the density dependence of symmetry energy

## OUTLOOK:

- **reduce model dependence** by describing a larger set of observables
- **more accurate experimental** data (ASYEOS, NEULAND) will also help push the constraints towards higher density regions
- extend the analysis to **light mass fragments**
- **pion ratios** – a test of our understanding of hadronic interactions: constraints from elliptic flow observables may serve as a benchmark

