NuSym2013, FRIB/NSCL, East Lansing, USA, July 22-26, 2013

Probing the nuclear symmetry energy with HIC:

an overview of different transport models used in China

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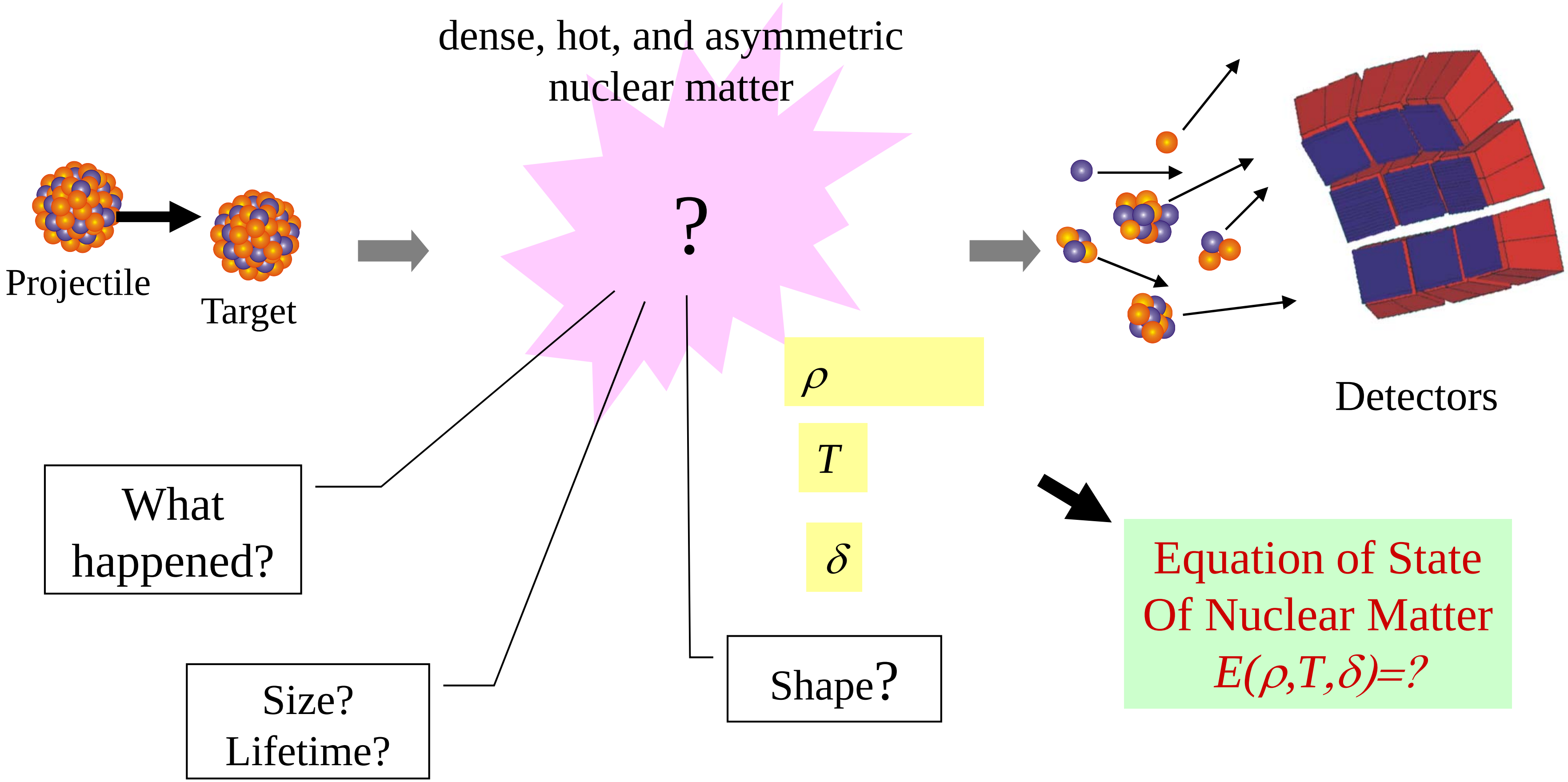
Outline

1. Introduction

2. Different transport models used in China

3. Observables for symmetry energy

Nuclear dynamics at intermediate and high energies by IQMD and IBL



- From compound nuclei ($\rho \approx \rho_0$, T≈1-2 MeV,)

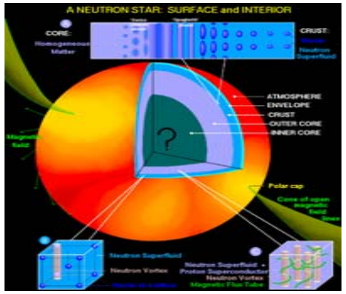
➡ hot nuclei($\rho \approx \rho_0$,T>5 MeV),

➡ highly excited nuclei ($\rho \approx 3\rho_0$,T>5 MeV)

➡ asymmetrical highly excited nuclei

($\rho \approx$ 2-3ρ_0,T>5 MeV, **$\delta>0$**)

- Physical indications ➡ IEOS ➡



$\rho \neq \rho_0$, $T > 0$, $\delta > 0$

$E(\rho, T, \delta) = ?$

G. J. Kunde et al., Phys. Rev. Lett. 77, 2897 (1996)

MSU, 1996-1998

- ^{112,124}Sn(40MeV/nucl.)+^{112,124}Sn
  isospin effects in multifragmentation

- ^{58}Fe(45-105 MeV/nucl.)+^{58}Fe,
  ^{58}Ni(45-105 MeV/nucl.)+^{58}Ni,

disappearance of isospin effects in multifragmentation

- Physical indications and challenges

$\rho \neq \rho_0$, $T > 0$, $\underline{\delta > 0}$

$E(\rho, T, \underline{\delta}) = ?$

Important to production of RIB &Neutron Stars !!!

Improved isospin dependent quantum molecular dynamics model

Quantum molecular dynamics model (QMD)

The QMD model represents the many body state of the system and thus contains correlation effects to all orders. In QMD, nucleon i is represented by a Gaussian form of wave function.

$$\psi_i(\boldsymbol{r}, t) = \frac{1}{(2\pi L)^{3/4}} e^{-[\boldsymbol{r}-\boldsymbol{r}_i(t)]^2/4L} e^{i\boldsymbol{p}_i(t)\cdot\boldsymbol{r}/\hbar}$$

After performing Wigner transformations, the density distribution of nucleon i is:

$$f_i(\boldsymbol{r},\boldsymbol{p},t) = \frac{1}{(\pi\hbar)^3} \exp\left[-\frac{[\boldsymbol{r}-\boldsymbol{r}_i(t)]^2}{2L} - \frac{[\boldsymbol{p}-\boldsymbol{p}_i(t)]^2\cdot 2L}{\hbar^2}\right]$$

From QMD model to IQMD model (1997,1998)

- mean field (corresponds to interactions)

$$U(\rho, \tau_z) = U^{loc} + U^{Yuk} + U^{Coul} + U^{Sym} + U^{MDI}$$

U^{loc} : density dependent potential

UYuk: Yukawa (surface) potential

U^{Coul}: Coulomb energy

U^{Sym}: symmetry energy

U^{MD}: momentum dependent interaction

- two-body collisions
- pauli blocking
- initialization
- coalescence model

IQMD model for studying isospin dependence of collective flow, PRC58(1998)2283

PHYSICAL REVIEW C VOLUME 58, NUMBER 4 OCTOBER 1998

Analysis of isospin dependence of nuclear collective flow in an isospin-dependent quantum molecular dynamics model

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(Received 24 April 1998)

Within the framework of an isospin-dependent quantum molecular dynamics model in which the initial neutron and proton densities are sampled according to the densities calculated from the Skyrme-Hartree-Fock method and the initial Fermi momenta of neutrons and protons are calculated from the Fermi gas model, we study systematically the transverse collective flow of different fragment types at an energy of 55 MeV/nucleon and the balance energy in the reactions ^{58}Fe+^{58}Fe and ^{58}Ni+^{58}Ni. The results from the present calculations indicate that the neutron-rich system (^{58}Fe+^{58}Fe) displays stronger negative deflection and has a higher balance energy, which are qualitatively in agreement with the experimental data. Furthermore, the effects of the isospin-dependent symmetry energy and nucleon-nucleon cross sections on collective flow are studied. [S0556-2813(98)07010-1]

IQMD model for studying isospin dependence of multifragmentation, **PRC60(1999)064604**

PHYSICAL REVIEW C, VOLUME 60, 064604

Isospin dependence of nuclear multifragmentation in $^{112}Sn+^{112}Sn$ and $^{124}Sn+^{124}Sn$ collisions at 40 MeV/nucleon

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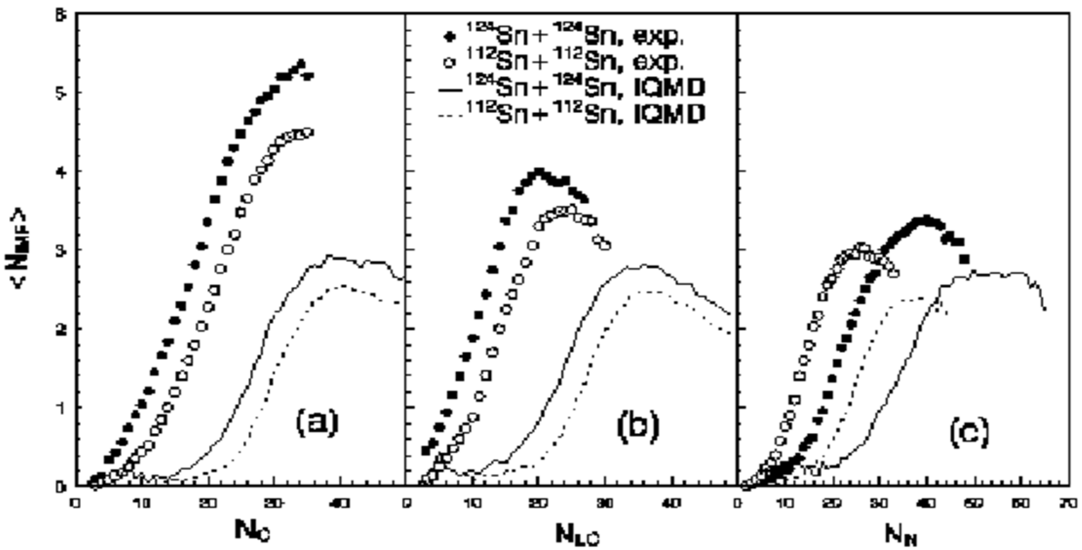
(Received 17 May 1999; published 9 November 1999)

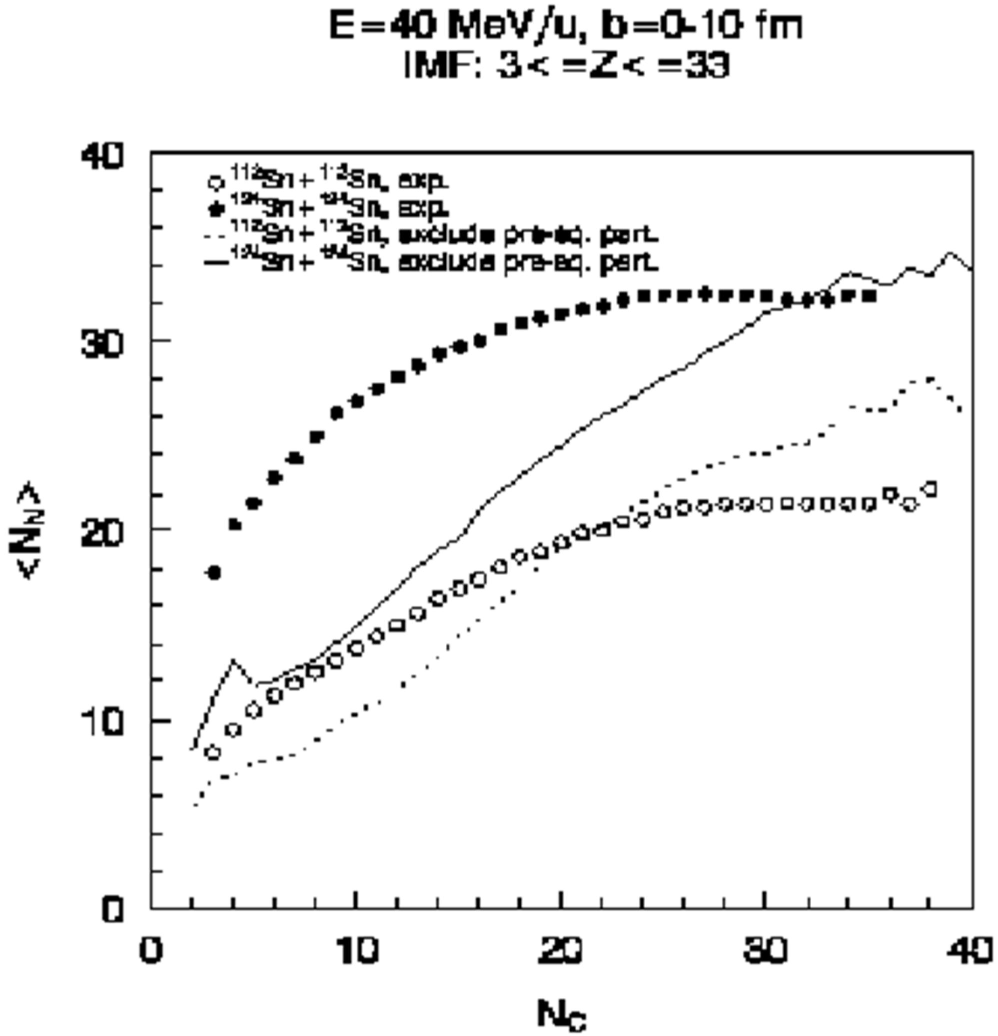
Within the framework of an isospin-dependent quantum molecular dynamics model, the multifragmentation in reactions of $^{112}Sn+^{112}Sn$ and $^{124}Sn+^{124}Sn$ at 40 MeV/nucleon is investigated. The calculated results are in good qualitative agreement with the experimental data which indicated that there were significantly different scalings of the mean number of intermediate mass fragments with the number of neutron and charged particles between the two reaction systems. Meanwhile, it is shown that the preequilibrium emission may affect strongly these scalings. [S0556-2813(99)05911-7]

Averaged number of IMF $\langle N_{IMF} \rangle$ as a function of N_C, N_{LC}, and N_N (4π analyzing)



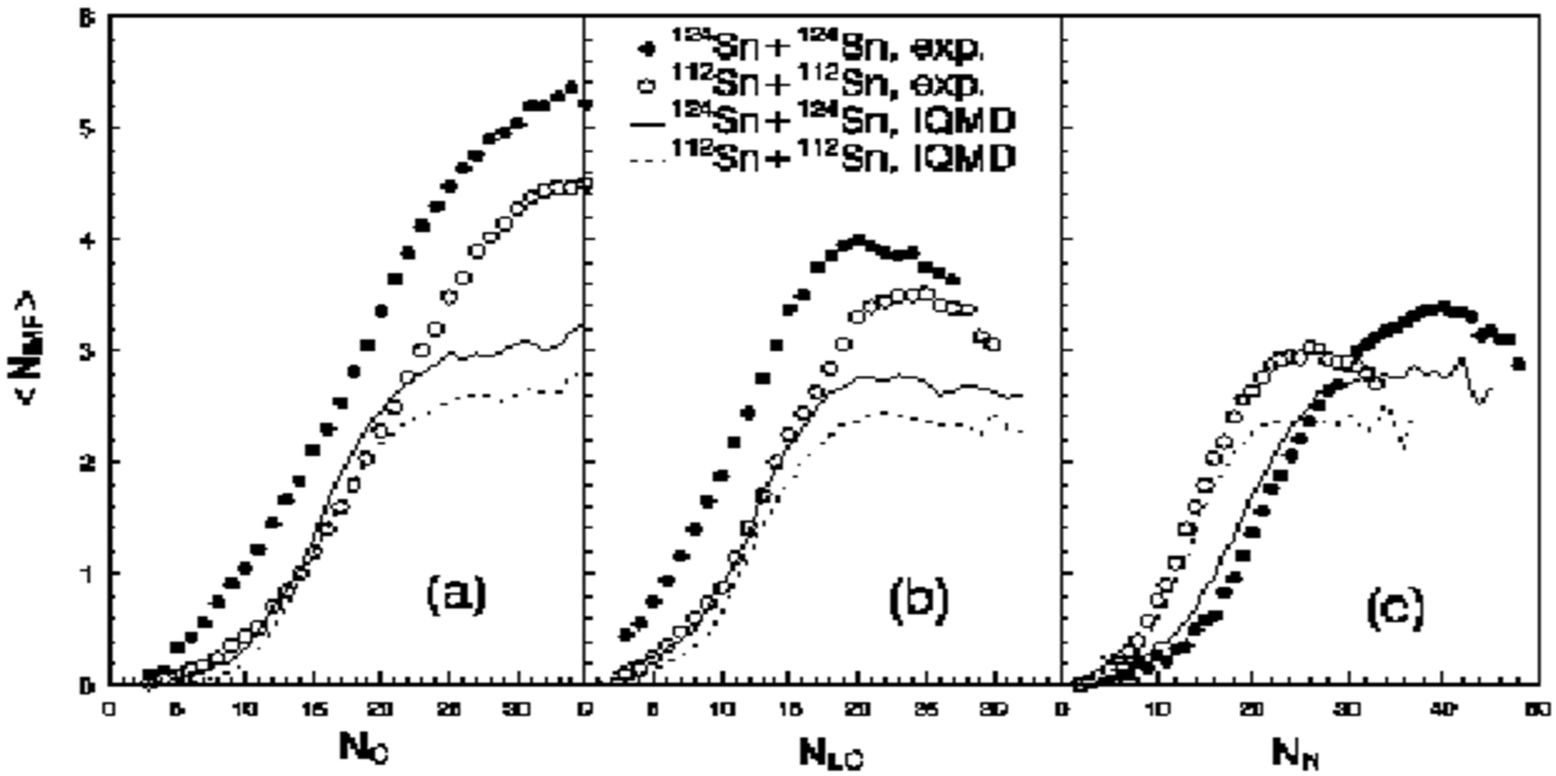
Exp data, G. J. Kunde et al., Phys. Rev. Lett. 77, 2897 (1996)

Average *n* multiplicity $\langle N_N \rangle$, as a function of charged-particle multiplicity N_C



Exp data, G. J. Kunde et al., Phys. Rev. Lett. 77, 2897 (1996)

Averaged number of IMF $\langle N_{IMF} \rangle$ as a function of N_C, N_{LC}, and N_N (4π — pre equilibrium emissions)



Exp data, G. J. Kunde et al., Phys. Rev. Lett. 77, 2897 (1996)

IQMD model for studying isospin dependence of radial flow, PLB459(1999)21

ELSEVIER

22 July 1999

PHYSICS LETTERS B

Physics Letters B 459 (1999) 21–26

Isospin dependence of radial flow in heavy-ion collisions at intermediate energies

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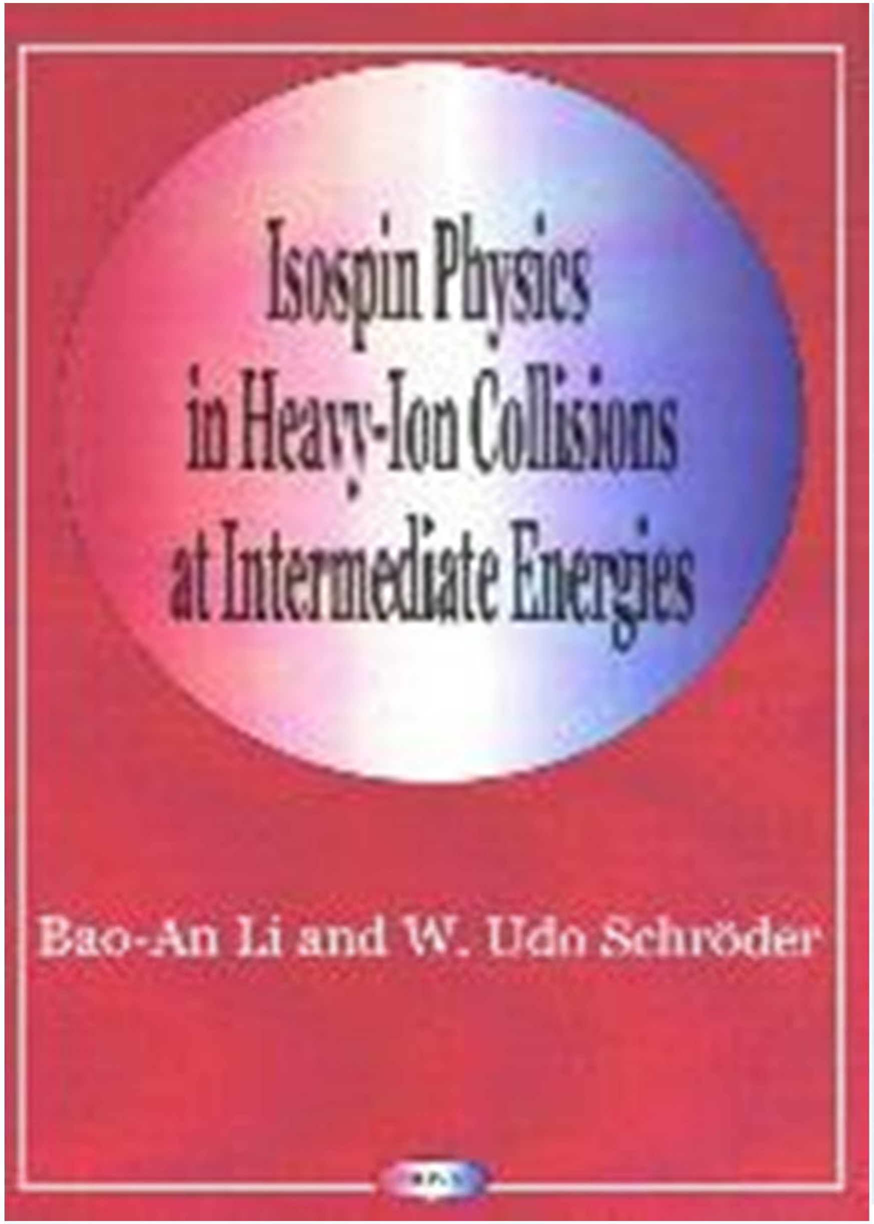
Received 30 March 1999; received in revised form 19 May 1999
Editor: W. Haxton

Abstract

Within the framework of an isospin-dependent quantum molecular dynamics model, the radial flows from the central collisions of ^{124}Ba + ^{124}Ba and ^{124}Sn + ^{124}Sn are studied at different incident energies. It is found that the more neutron-rich system exhibits smaller radial flow and it is shown that the radial flow is sensitive to the nucleon-nucleon cross sections and independent of the nuclear symmetry energy. It suggests that the experimental measurement of the radial flow for reaction systems with different ratios of neutron to proton provide a novel recipe for determining the isospin dependent in-medium nucleon-nucleon cross sections.

PACS: 25.70.Pq; 02.70.Ns; 24.10.Lx



Chapter 10:Isospin-Dependent Quantum Molecular Dynamics Model and Its Applications in Heavy Ion Collisions, F. S. Zhang, L. W. Chen, et al.

Physics Letters B 319 (1993) 35–40
North-Holland

PHYSICS LETTERS B

Boltzmann–Langevin equation, dynamical instal and multifragmentation

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Received 30 April 1993, revised manuscript received 15 October 1993
Editor: G.F. Bertsch

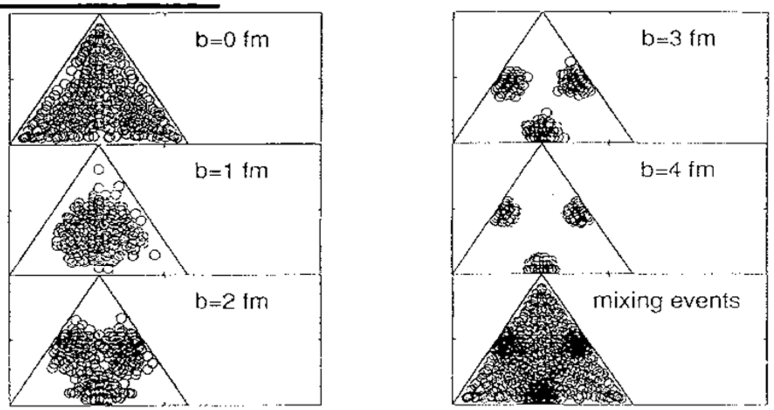

FIG. 8. The correlations between the three largest fragments of an event by a charge-Dalitz plot for the ^{40}Ca+^{40}Ca system at 90 MeV/nucleon with the events of impact parameters 0, 1, 2, 3, 4 fm and the mixing events with different impact parameters except $b = 0$ fm. The number of events is the same as in Fig. 5.

BUU → BLE
for studying nuclear multifragmentation

PHYSICAL REVIEW C VOLUME 51, NUMBER 6 JUNE 1995

Analysis of multifragmentation in a Boltzmann-Langevin approach

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(Received 31 October 1994)

By using the Boltzmann-Langevin equation, which incorporates dynamical fluctuations beyond usual transport theories, we simulate the ^{40}Ca+^{40}Ca reaction system at different beam energies 20, 60, and 90 MeV/nucleon for different impact parameters. Dynamical fluctuations become larger and larger with increasing bombarding energy and the system can reach densities corresponding to the unstable region of the nuclear matter equation of state at energies above 60 MeV/nucleon. By coupling the Boltzmann-Langevin equation with a coalescence model in the late stages of the reaction, we obtain the distribution of the intermediate mass fragments in each event. From the correlation analysis of these fragments, we recover some trends of recent multifragmentation data. A critical behavior analysis is also provided.

PLB319(1993)35
PRC51(1995)3201

BLE → IBLE for studying isospin effects in nuclear multifragmentation, NPA807(2008)71

Available online at www.sciencedirect.com
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Nuclear Physics A 807 (2008) 71–78
NUCLEAR PHYSICS A
www.elsevier.com/locate/nuclphysa

Fragmentation cross sections of ^{20}Ne collisions with different targets at 600 MeV/nucleon

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Received 5 February 2008; received in revised form 28 March 2008; accepted 30 March 2008

Available online 3 April 2008

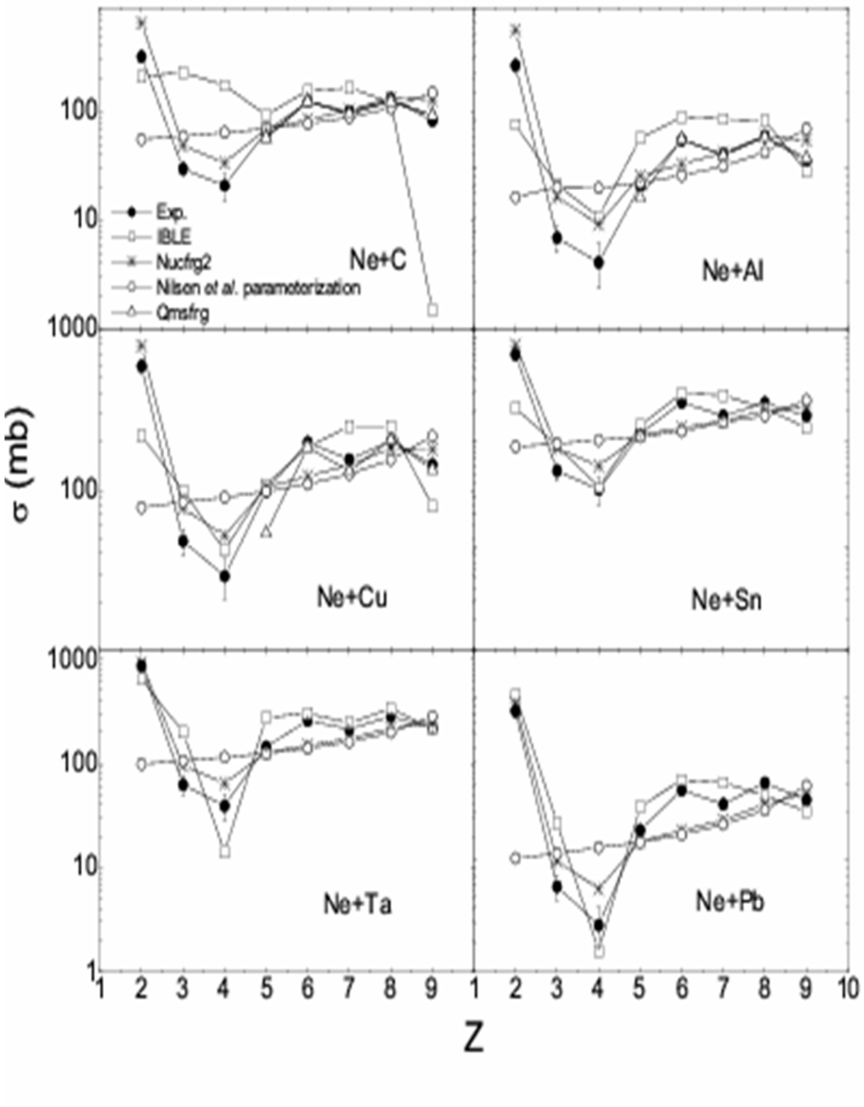
Abstract

Within the framework of the isospin-dependent Boltzmann–Langevin equation, the production cross sections of fragments are calculated for reactions of Ne collisions with C, Al, Cu, Sn, Ta, and Pb targets at 600 MeV/nucleon. It is found that the production cross sections for fragments $Z = 2$ to 9 are qualitatively reproduced by the present calculations except for C target. The enhancement of even-Z fragments (C, O) cross sections shown in the experimental data is not well reproduced except for Ta target, however the observed suppression of the F fragment cross sections is described very well. The suppression of F production is discussed in terms of isotopic distribution of fragments. This is the first time to use the isospin-dependent Boltzmann–Langevin equation model to calculate the fragmentation cross sections for these reaction systems.

PACS: 25.70.-z; 25.70.Pq; 98.70.Sa

Keywords: Low and intermediate energy heavy-ion reactions; Multifragment emission; Cosmic rays

Fig. 1. Comparison of calculated production cross sections with available data [20] (solid circles). Open squares denote calculated results by the IBLE model, crosses, open circles and open triangles represent the results calculated by Nucfrg2 model [7], Nilsen et al. parametrization [8] and Qmsfrg model [21] respectively.



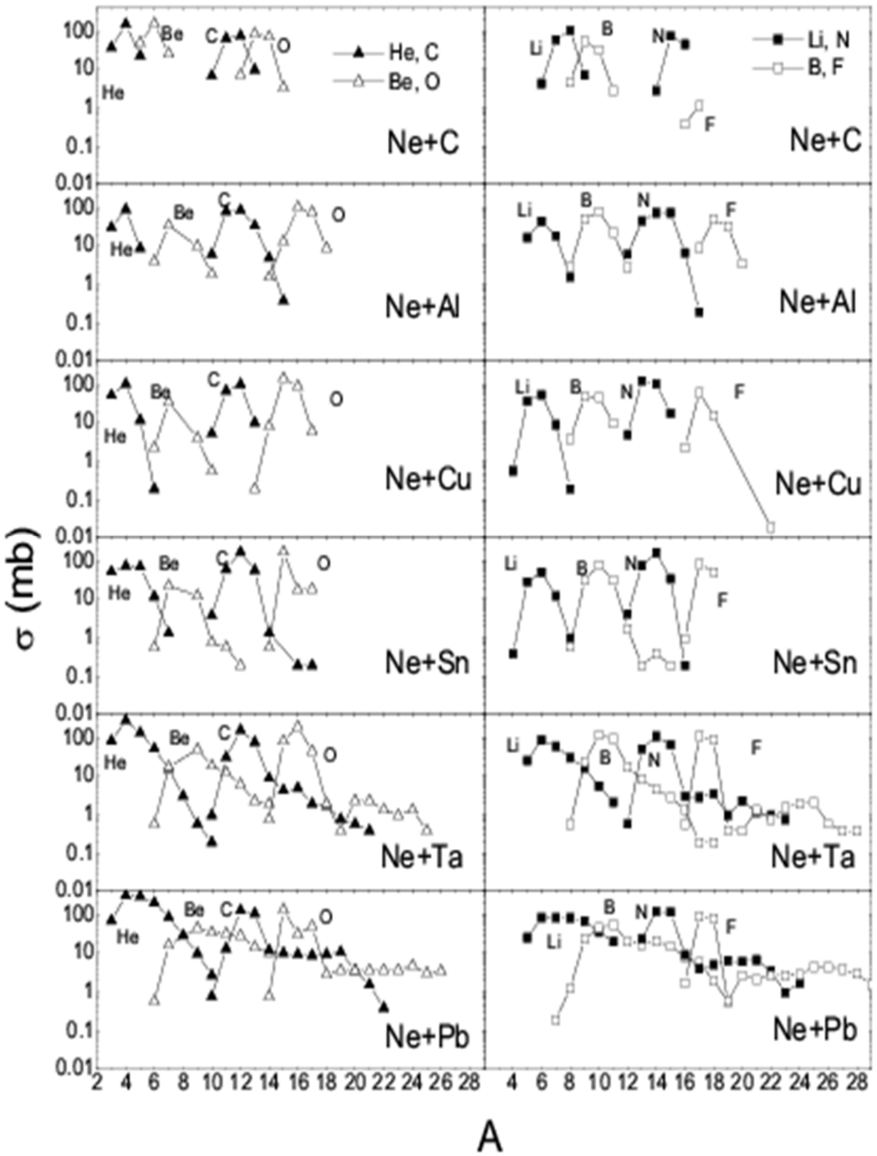
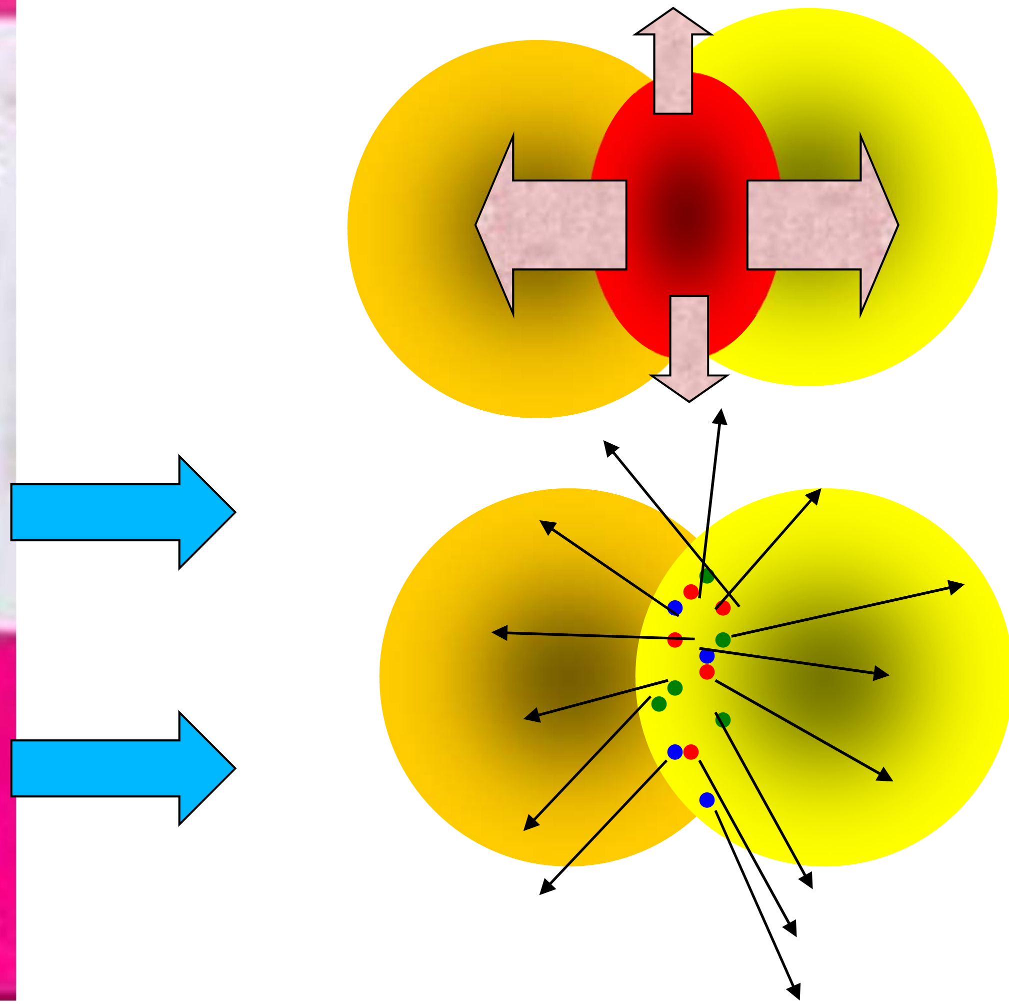


Fig. 2. Calculated production cross sections as a function of mass. Solid (open) triangles represent the calculation for fragments He and C (Be and O), and solid (open) squares represent the calculation for fragments Li and N (B and F).

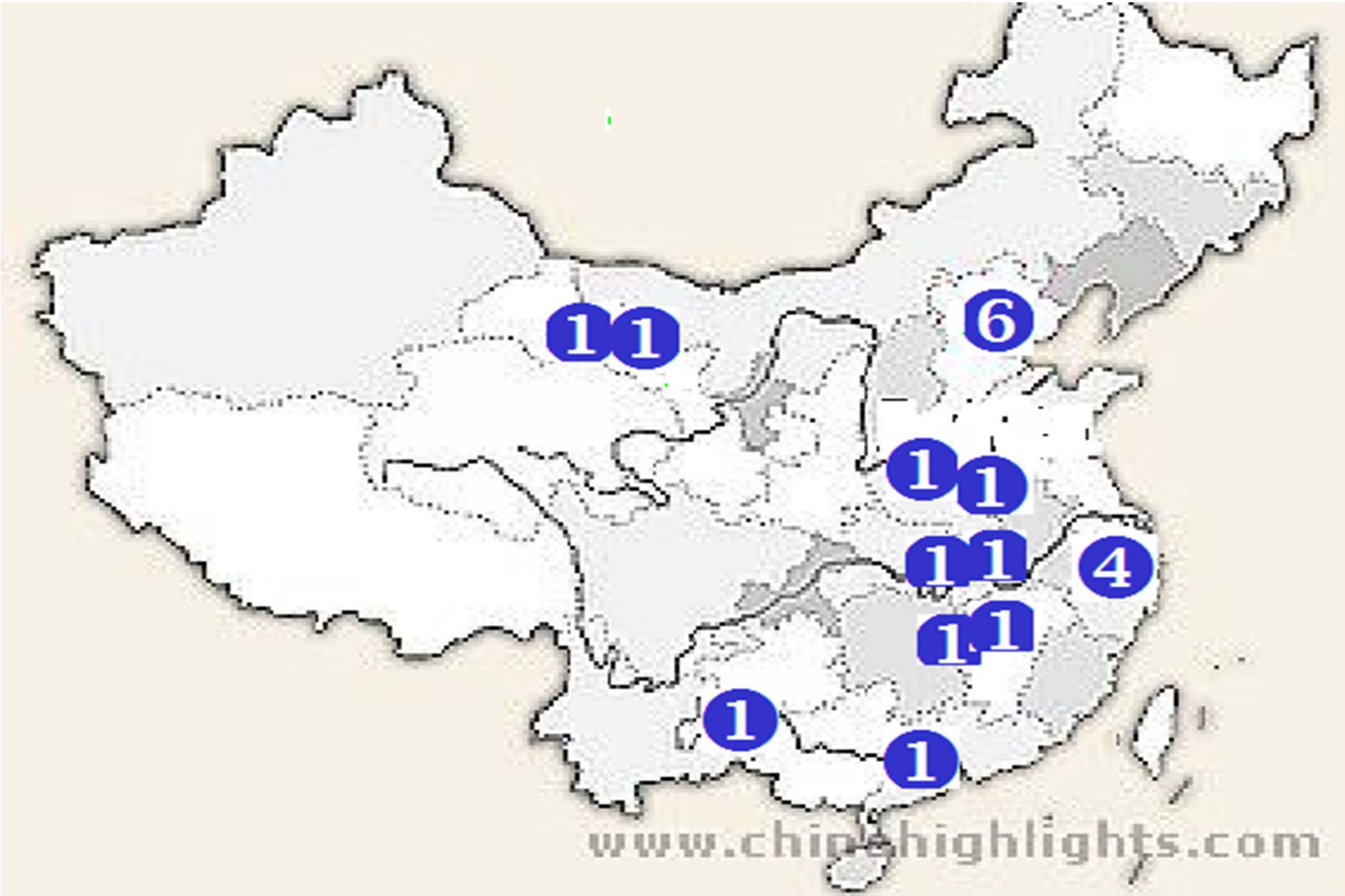
Nuclear Multifragmentation,

F. S. Zhang and L. X. Ge,

Science Press, Beijing, 1998



Transport models in heavy ion collisions in China, 2011.9(20)

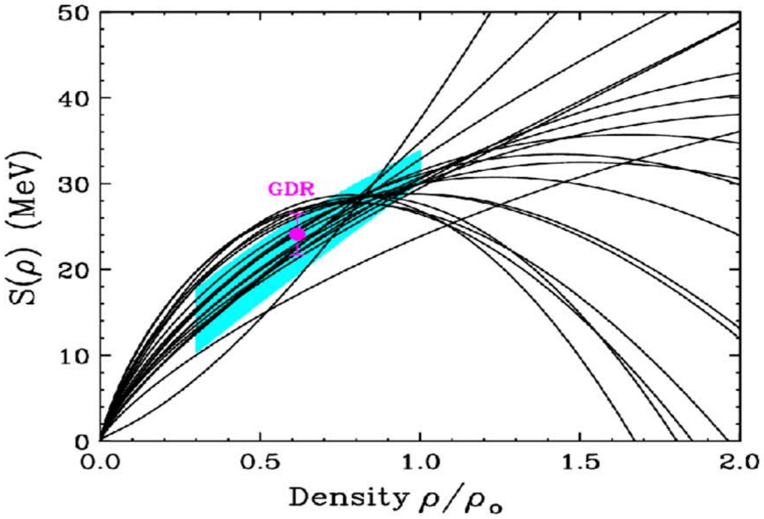


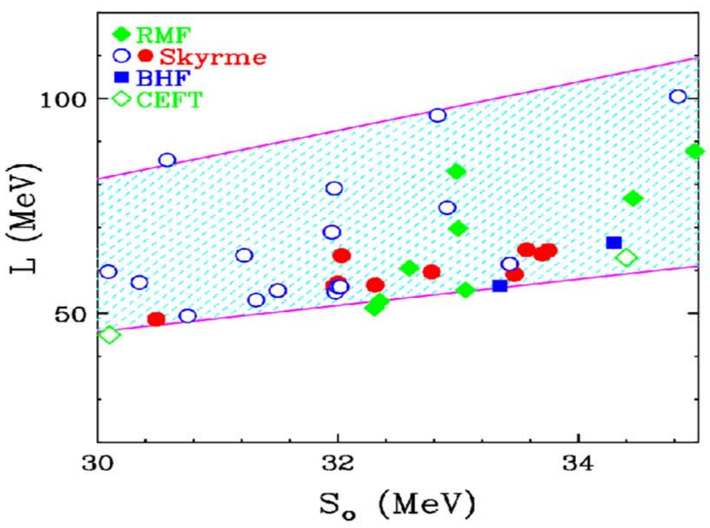
2011.9 Transport models in HIC in China (Huzhou)

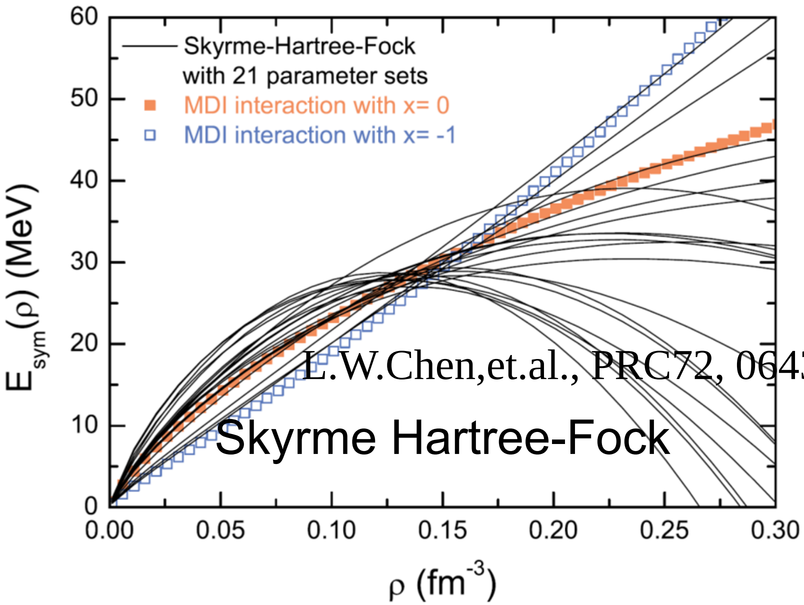
1. CIAE(YZ Zhuo, ZX LI, XZ Wu, YX Zhang, K Zhao, XH Lu, …)
2. ITP(EG Zhao, SG Zhou, …)
3. IHEP(B Liu, …)
4. IMP(JQ Li, W Zuo, ZQ Feng, GC Yong, …)
5. SINAP(YG Ma, DQ Fang…)
6. PKU(FR Xu, YX Liu,…)
7. TsinghuaU(ZG Xiao, …)
8. BNU(JB Bao, FS Zhang, …)
9. NJU(ZZ Ren, C Xu,…)
10. USTC(Q Wang, …)
11. SJTU(LW Chen, ..)
12. FudanU(ZH Li, BR Wei, …)
13. SZU(N Wang,…)
14. GXNU(N Wang, M Liu, L O, …)
15. HNNU(CW Ma, …)
16. HUTC(CW Shen, QF Li, …)
17. TSTC(YZ Xing, …)
18. AYTC(JL Tian, …)
19. SHIT (WJ Guo, …)
20. JNU(BA Bian, …)

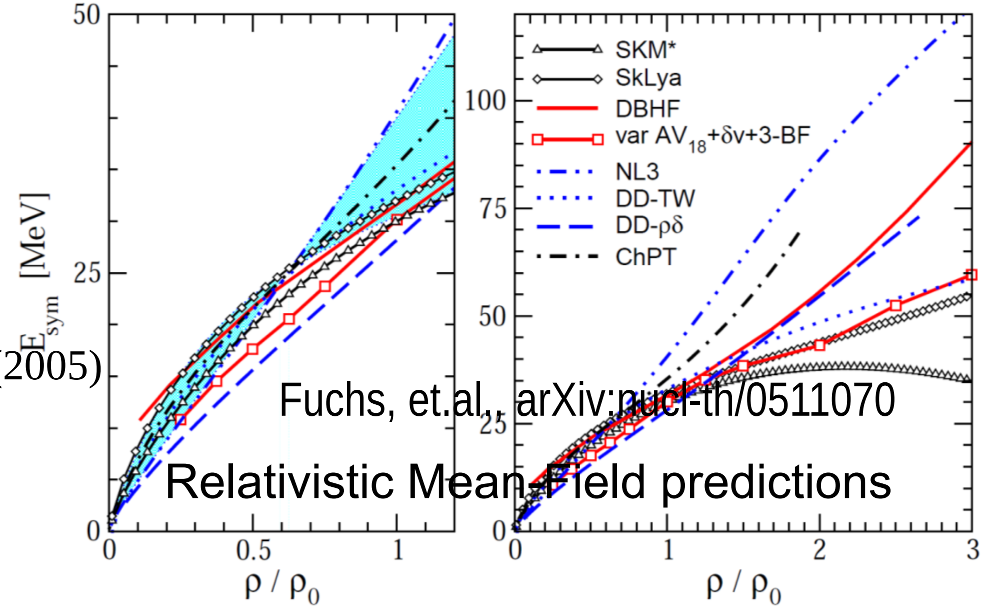
Nuclear Matter Symmetry Energy

Experimental work, M.B. Tsang *et al.* PRC86, 015803 (2012)









$\rho < \rho\_0$: confirm some of isospin-dependent effects, constrain the symmetry energy

$\rho > \rho\_0$: even the trend of the symmetry energy with increasing ρ is not constrained

Outline

1. Introduction

2. Different transport models used in China

3. Observables for symmetry energy

2 Different Transport Models Used in China

Quantum Molecular Dynamics like: solve N-body equation of motion

$$\Phi_i(\vec{r}) = \frac{1}{(2\pi\sigma_r^{\ 2})^{3/4}} \exp\left[-\frac{(\vec{r}-\vec{r}_i)2}{4\sigma_r^{\ 2}} + \frac{i}{\hbar}\vec{r}\cdot\vec{p}_i\right]$$

$$f(\vec{r},\vec{p}) = \sum_i \frac{1}{(\pi\hbar)^3} \exp\left[-\frac{(\vec{r}-\vec{r}_i)^2}{2\sigma_r^{\ 2}} - \frac{2\sigma_r^{\ 2}}{\hbar^2}(\vec{p}-\vec{p}_i)^2\right]$$

$$\dot{\vec{p}}_i = -\frac{\partial H}{\partial r_i}, \quad \dot{\vec{r}}_i = \frac{\partial H}{\partial p_i}$$

Two body collision: occurs between nucleons

Version: QMD, IQMD, ImQMD, LQMD, CoMD, UrQMD , ……
AMD, FMD

Boltzmann-like: f(r,p,t) one body phase space density

$$\left(\frac{\partial}{\partial t} + \frac{\vec{p}}{m}\cdot\nabla_r - \nabla_r U\left(\hat{f}\right)\cdot\nabla_p\right)\hat{f}(\vec{r},\vec{p},t) = K\left(\hat{f}\right) + \delta K(\vec{r},\vec{p},t)$$

$$f(\vec{r},\vec{p}) = \frac{1}{\tilde{N}}\sum \delta(\vec{r}-\vec{r}_i)\delta(\vec{p}-\vec{p}_i)$$

Two-body collision: occurs between test part.

Version: IBUU04, BLE,……

2.1Quantum Molecular Dynamics Like Models

Quantum Molecular Dynamics like: solve N-body equation of motion

$$\Phi_i(\vec{r}) = \frac{1}{(2\pi\sigma_r{}^2)^{3/4}} \exp\left[-\frac{(\vec{r}-\vec{r}_i)2}{4\sigma_r{}^2} + \frac{i}{\hbar}\vec{r}\cdot\vec{p}_i \right]$$

$$f(\vec{r},\vec{p}) = \sum_i \frac{1}{(\pi\hbar)^3} \exp\left[-\frac{(\vec{r}-\vec{r}_i)^2}{2\sigma_r{}^2} - \frac{2\sigma_r{}^2}{\hbar^2}(\vec{p}-\vec{p}_i)^2 \right]$$

$$\dot{\vec{p}}_i = -\frac{\partial H}{\partial r_i}, \quad \dot{\vec{r}}_i = \frac{\partial H}{\partial p_i}$$

Two body collision: occurs between nucleons

Version: QMD, IQMD, ImQMD, LQMD, UrQMD , ……
AMD, FMD

Quantum Molecular Dynamics model (J. Aichelin, Phys. Rep. 202 (1991) 233)
and Extension of the QMD model (isospin, low and high energies etc.)

Isospin QMD (IQMD) by Nantes group (Hartnack, Aichelin);
Ch. Hartnack, R.K. Puri, J. Aichelin, et al., EPJA 1 (1998) 151

IQMD extend by Feng-Shou Zhang, Lie-Wen Chen, et al., BNU, Beijing and Lanzhou); PRC 58(1998)2283, PLB 459(1999)21

IQMD in BNU

Lanzhou QMD (LQMD, IMP CAS, Lanzhou)
Z.-Q. Feng et al., NPA 750 (2005) 232, PLB 683 (2010) 140

LQMD in IMP

IQMD in SINAP, Shanghai Yu-Gang Ma et al
PRC 84(2001)034612, PRC 83(2011)064607

IQMD in SINAP

……

QMD (Fuchs et al., Tuebingen Uni, Germany);
Improved QMD (ImQMD) by Zhuxia Li, Ning Wang et al., CIAE, Beijing)
PRC 65(2002)064608

ImQMD in CIAE

……

Ultra-relativistic QMD (UrQMD) by Frankfurt group
S.A. Bass et al., Prog. Part. Nucl. Phys. 41 (1998) 255

UrQMD in Huzhou

Extend by Qing-Feng Li et al. Huzhou Teachers College, Huzhou
PRC 72(2005)034613

QMD

wave function

The N-body phase-space distribution function

$$f(\vec{r},\vec{p})=\sum_i \frac{1}{(\pi\hbar)^3}\exp\left[-\frac{(\vec{r}-\vec{r}_i)^2}{2\sigma_r{}^2}-\frac{2\sigma_r{}^2}{\hbar^2}(\vec{p}-\vec{p}_i)^2\right]$$

Hamiltonian equations

$$\dot{\vec{p}}_i=-\frac{\partial H}{\partial r_i},\qquad \dot{\vec{r}}_i=\frac{\partial H}{\partial p_i}$$

$$H=T+U_{Coul}+U_2+U_3+U_{sym}+U_{sur}+U_{MDI}$$

two-body collision

$$N+N\to N+N,\quad N+N\to N+\Delta,$$
$$N+\Delta\to N+\Delta,\quad N+\Delta\to N+N,$$
$$\Delta+\Delta\to\Delta+\Delta,\ \cdots$$

Fermionic nature

Pauli blocking

phase-space density constraint

Difference in EOS

$$\dot{\vec{p}}_i = -\frac{\partial H}{\partial r_i}, \quad \dot{\vec{r}}_i = \frac{\partial H}{\partial p_i}$$

$$H = T + U_{Coul} + U_2 + U_3 + U_{MDI} + U_{sym} + U_{sur}$$

depend → EOS of symmetric nuclear matter

Form 1

$$U = \alpha(\rho/\rho_0) + \beta(\rho/\rho_0)^{\gamma} + \delta \ln^2(\varepsilon(\rho/\rho_0)^{2/3} + 1)\rho/\rho_0$$

Soft EOS plus MDI: K=200 MeV
Hard EOS plus MDI: K=380 MeV

J. Aichelin, Phys. Rep. 202 (1991) 233

K (MeV)	α (MeV)	β (MeV)	γ	δ (MeV)	ε	EOS
200	−390	320	1.14	1.57	21.54	SM
380	−130	59	2.09	1.57	21.54	HM

Form 2

derived directly from the Skyrme energy-density functional

$$U = \alpha(\rho/\rho_0) + \beta(\rho/\rho_0)^{\gamma} + g_{\tau}(\rho/\rho_0)^{5/3}$$

$$\frac{\alpha}{2} = \frac{3}{8} t_0 \rho_0, \quad \frac{\beta}{1+\gamma} = \frac{t_3}{16} \rho_0^{\gamma},$$

K=200 ~ 380 MeV

$$g_{\tau} = \frac{3}{80}(3t_1 + 5t_2 + 4x_2 t_2)(\frac{3}{2}\pi^2)^{2/3} \rho_0^{5/3}$$

N. Wang et al. PRC 65.064608; Z.Q. Feng et al., NPA 750 (2005)

Difference in symmetry energy

$$\dot{\vec{p}}_i = -\frac{\partial H}{\partial \vec{r}_i}, \quad \dot{\vec{r}}_i = \frac{\partial H}{\partial \vec{p}_i}$$

$$H = T + U_{Coul} + U_2 + U_3 + U_{MDI} + U_{sym} + U_{sur}$$

$$U_{sym} = \int V_{sym} d^3 r$$

V_{sym} is the potential energy density

$$\delta = \frac{\rho_n - \rho_p}{\rho}$$

Form 1

$$V_{sym} = \frac{C_{sym}}{2\rho_0}\rho^2\delta^2 \quad \text{or} \quad V_{sym} = \frac{C_{sym}}{2\rho_0}(\rho^2 - \kappa_s(\nabla\rho)^2)\delta^2$$

N. Wang et al. PRC 85, 034601(2012)

Form 2

$$V_{sym} = \frac{C_{sym}}{2}\left(\frac{\rho}{\rho_0}\right)^{\gamma_i}\rho\delta^2$$

Y. X. Zhang PRC 85, 051602(2012);
J. Pu PRC 87, 047603(2013)

Form 3

$$V_{sym} = \left[a_{sym}\left(\frac{\rho}{\rho_0}\right) + b_{sym}\left(\frac{\rho}{\rho_0}\right)^2\right]\rho\delta^2$$

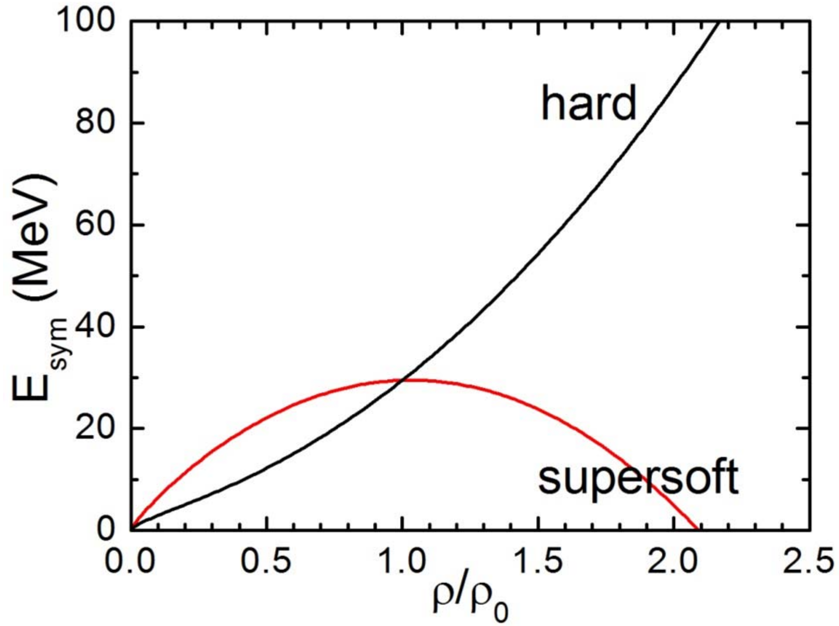
Z. Q. Feng, PRC 87, 064605(2013)

Form 4

$$V_{sym} = (A\rho^2 + B\rho^{1+\gamma} + C\rho^{8/3})\delta^2$$

Y. X. Zhang et al. PhysRevC 74, 014602(2006)

Change the density dependence of the symmetry energy



Difference in surface energy

$$\dot{\vec{p}}_i = -\frac{\partial H}{\partial r_i}, \quad \dot{\vec{r}}_i = \frac{\partial H}{\partial p_i}$$

$$H = T + U_{Coul} + U_2 + U_3 + U_{MDI} + U_{sym} + U_{sur}$$

reflect the surface effects in the finite system

Form 1

Yukawa interaction

Aichelin, Phys. Rep. 202 (1991) 233

$$U_{Yuk} = \frac{V_y}{2m}\sum_{i\neq j}\frac{1}{r_{ij}}\exp(Lm^2)[\exp(-mr_{ij})erfc(\sqrt{L}m - r_{ij}/\sqrt{4L})$$
$$-\exp(mr_{ij})erfc(\sqrt{L}m + r_{ij}/\sqrt{4L})]$$

Form 2

Derived from the Skyrme energy-density functional

$$U_{sur} = \int \frac{g_{sur}}{2}(\nabla\rho)^2 d^3r$$

N. Wang et al. PRC 85, 034601(2012)

Difference in momentum dependent interactions

$$\dot{\vec{p}}_i = -\frac{\partial H}{\partial r_i}, \quad \dot{\vec{r}}_i = \frac{\partial H}{\partial p_i}$$

$$H = T + U_{Coul} + U_2 + U_3 + \boxed{U_{MDI}} + U_{sym} + U_{sur}$$

Form 1

Isospin independent

$$U_{md} = \frac{1}{2\rho_0}\sum_{N1}\frac{1}{16\pi^6}\int d^3p_1 d^3p_2 f_{N1}(\vec{p}_1) f_{N2}(\vec{p}_2)$$

$$\times 1.57[\ln(1+5\times10^{-4}(\Delta p)^2)]^2$$

Y. X. Zhang, PRC 85, 051602(2012)

Form 2

Isospin dependent

$$U_{md} = \frac{1}{2\rho_0}\sum_{i,j,i\neq j}\sum_{\tau,\tau'} C_{\tau,\tau'}\delta_{\tau,\tau_i}\delta_{\tau',\tau'_j}\int d^3p d^3p' d^3r$$

$$\times f_i(\vec{r},\vec{p},t)\left\{\ln\left[\varepsilon(\vec{p}-\vec{p}')^2+1\right]\right\}^2 f_j(\vec{r},\vec{p}',t)$$

Z. Q. Feng, PLB 707,83(2012)

Difference in two-body collisions

Nucleon-nucleon cross section in free space

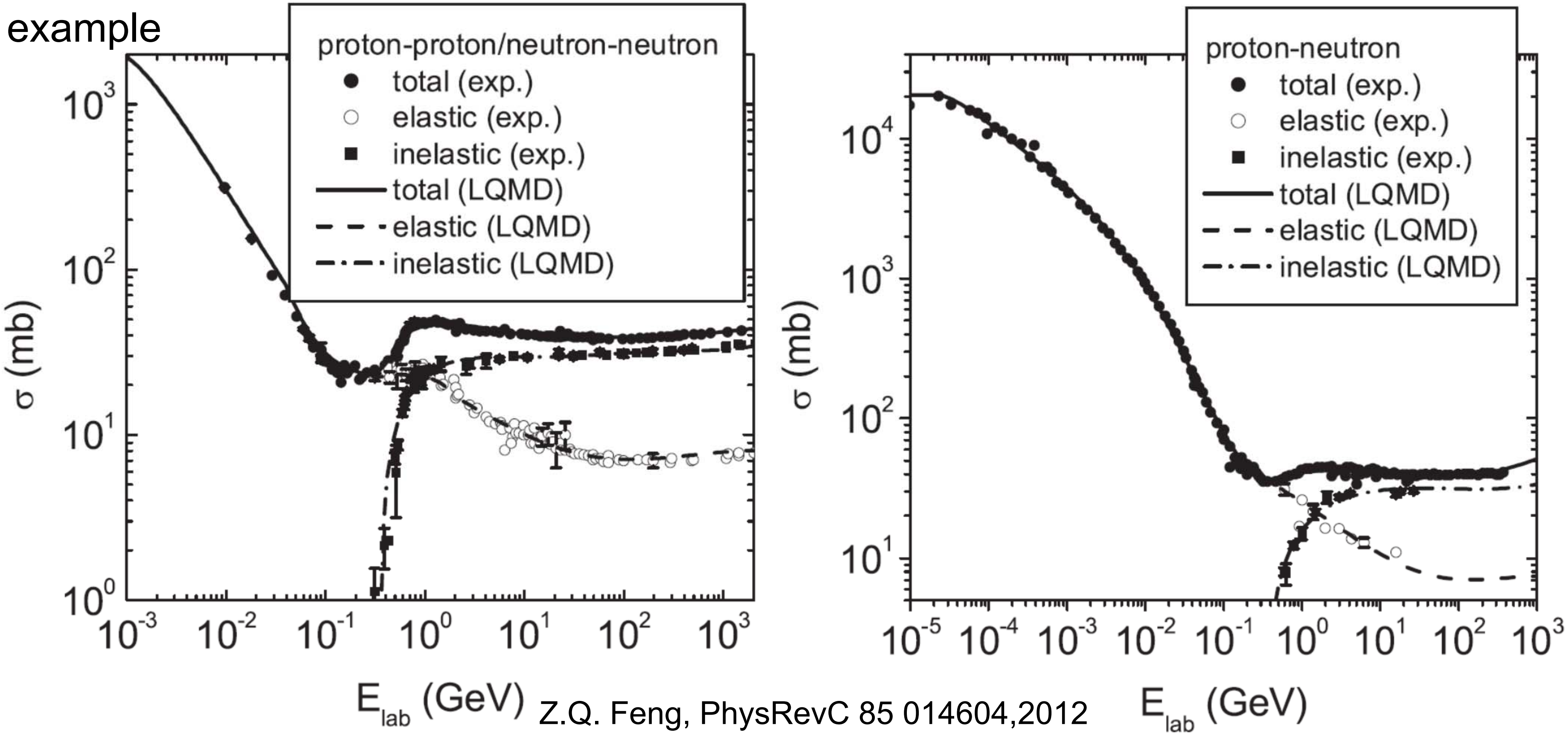
isospin dependent

difference in parametrizatoin
but fit to the experimental data

isospin independent

$$\sigma_{nn} = \sigma_{pp} = \sigma_{np}$$

example



Z.Q. Feng, PhysRevC 85 014604,2012

Difference in in-medium NN cross sections

$$\sigma^*_{nn} = f_{medium}\sigma^{free}_{nn}$$

Form 1

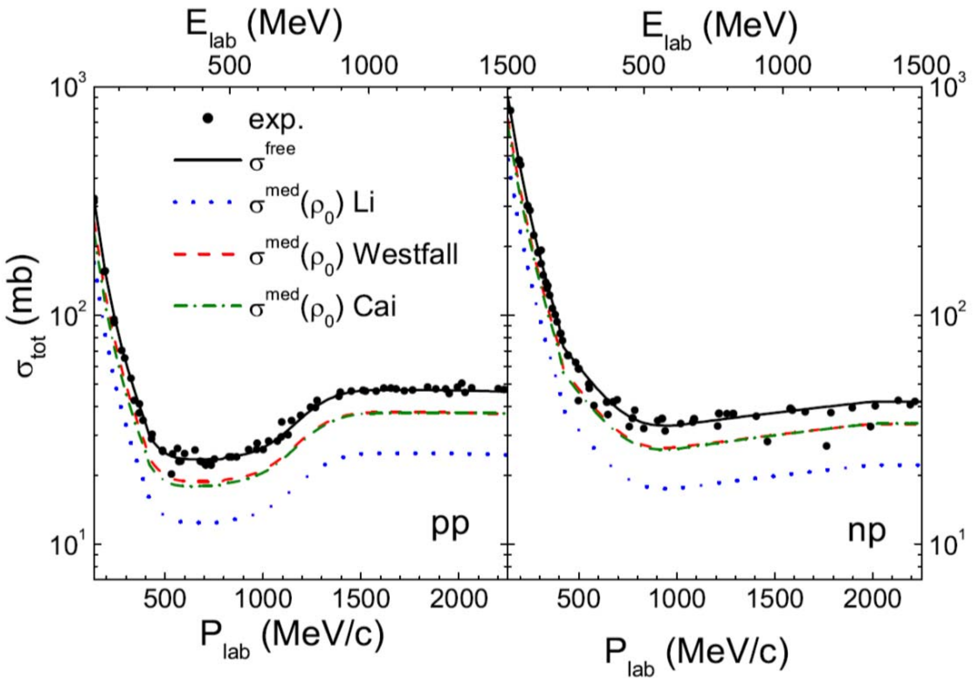
$$\sigma^*_{nn} = (1-\eta\frac{\rho}{\rho_0})\sigma^{free}_{nn} \qquad \eta = 0.2$$

G. D. Westfall et al. PRL 71, 1986(1993)

Form 2

$$\sigma^*_{nn} = (\frac{m^*(\rho)}{m})^2\sigma^{free}_{nn}$$

Q.F. Li et al.,PRC 62 014606(2000)
D. Persram et al.,PRC 65 064611(2002)



Form 3

Yuan. PhysRevC 81 034913,2012

$$\sigma^*_{nn} = F(u,\alpha,p)\sigma^{free}_{nn}$$

$$F(u,\alpha,p) = F_u^p F_\alpha^p$$

$$\begin{cases} F_u^p = 1 + \frac{2}{3}\left[\exp(-u/0.54568)-1\right]/\left[1+(\mathrm{p_{NN}}/p_0)\kappa\right], & \mathrm{p_{NN}} \le 1\mathrm{GeV/c} \\ F_\alpha^p = 1 + \left[0.85\tau_{ij}\eta\alpha/(1+3.25u)\right]/\left[1+(\mathrm{p_{NN}}/p_0)\kappa\right], & \mathrm{p_{NN}} \le 1\mathrm{GeV/c} \\ F_u^p = F_\alpha^p = 1, & \mathrm{p_{NN}} > 1\mathrm{GeV/c} \end{cases}$$

Difference in Fermionic nature

phase-space density constraint $\overline{f}_i \leq 1$ (for all nucleon)

$$\overline{f}_i = \sum_j \delta_{\tau_i \tau_j} \delta_{s_i s_j} \int_{h^3} f_j(\vec{r}, \vec{p}) d^3 r d^3 p$$

Pauli blocking

- block with a probability $P_{block} = 1 - [1 - \min(P_1, 1)][1 - \min(P_2, 1)]$

  Aichelin, Phys. Rep. 202 (1991) 233

  $$\mathrm{P_i} = \sum_{k, k \neq i}^{A} \frac{1}{(\pi \hbar)^3} \exp\left[-\frac{(\vec{r}_i - \vec{r}_k)^2}{2\sigma_r^2}\right] \exp\left[-\frac{(\vec{p}_i - \vec{p}_k)^2}{2\sigma_p^2}\right]$$

- Final states satisfy $\overline{f}_i \leq 1$ (for two nucleon)

  Su, Zhang, PhysRevC 87 017602,2013

- Final states satisfy

  Li PhysRevC 83, 044617, 2011

  $$\eta < \psi$$

  $$\psi = \sum_i \exp\left(-\frac{(\vec{r} - \vec{r}_i)^2}{4L^2}\right) \exp\left(-\frac{L^2 (\vec{p} - \vec{p}_i)^2}{\hbar^2}\right)$$

  $$\eta = a_{fit} + b_{fit} \sum_i \exp\left(-\frac{(\vec{r} - \vec{r}_i)^2}{2L^2}\right)$$

- Final states satisfy $\frac{4\pi}{3} r_{ij}^3 \frac{4\pi}{3} p_{ij}^3 \geq \frac{h^3}{4}$

  LI, PhysRevC 64 064612, 2001

2.2 Boltzmann-like models

Boltzmann-like: f(r,p,t) one body phase space density

$$\left(\frac{\partial}{\partial t}+\frac{\vec{p}}{m}\cdot\nabla_r-\nabla_r U\left(\hat{f}\right)\cdot\nabla_p\right)\hat{f}\left(\vec{r},\vec{p},t\right)=K\left(\hat{f}\right)+\delta K\left(\vec{r},\vec{p},t\right)$$

$$f(\vec{r},\vec{p})=\frac{1}{\tilde{N}}\sum\delta(\vec{r}-\vec{r}_i)\delta(\vec{p}-\vec{p}_i)$$

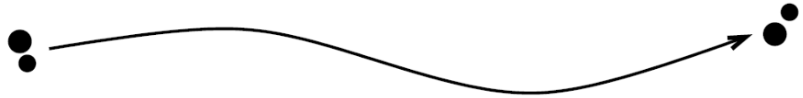
Two-body collision: occurs between test part.

Version: IBUU04, BLE,……

The differences among the Vlasov, BUU and BL models

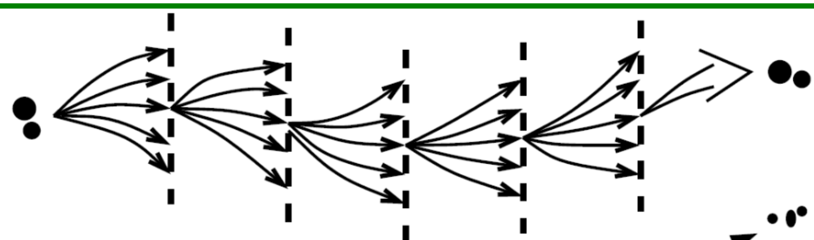
Vlasov

$$\left(\frac{\partial}{\partial t}+\frac{\vec{p}}{m}\cdot\nabla_r-\nabla_r U\left(\hat{f}\right)\cdot\nabla_p\right)\hat{f}\left(\vec{r},\vec{p},t\right)=0$$



BUU

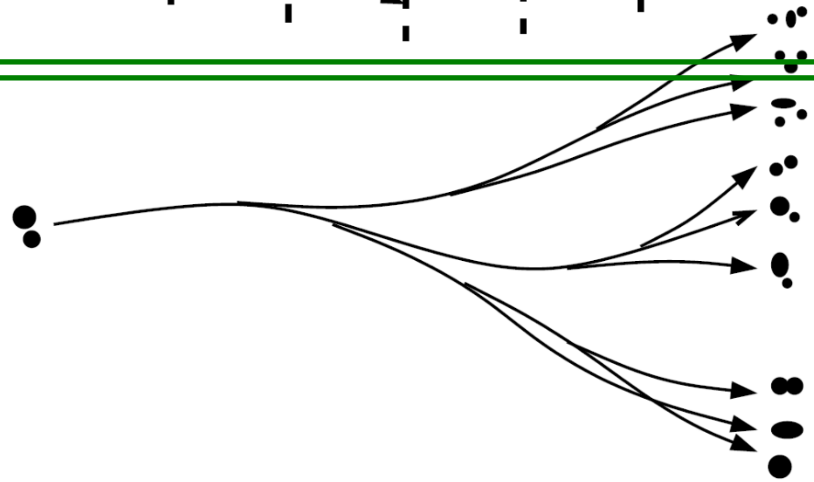
$$\left(\frac{\partial}{\partial t}+\frac{\vec{p}}{m}\cdot\nabla_r-\nabla_r U\left(\hat{f}\right)\cdot\nabla_p\right)\hat{f}\left(\vec{r},\vec{p},t\right)=K\left(\hat{f}\right)$$



BL

$$\left(\frac{\partial}{\partial t}+\frac{\vec{p}}{m}\cdot\nabla_r-\nabla_r U\left(\hat{f}\right)\cdot\nabla_p\right)\hat{f}\left(\vec{r},\vec{p},t\right)=K\left(\hat{f}\right)$$

$$+\delta K\left(\vec{r},\vec{p},t\right)$$



F. S. Zhang and L.X. Ge, Nuclear Multifragmentation, Science Press, Beijing,1998
A. Ono, J. Randrup, Eur. Phys. J. A 30 (2006) 109

IBUU Model for HIC's

J. Aichelin, G.F. Bertsch, S. Das Gupta, W. Bauer, Bao-An Li, ……

Phase-space distributions f(r, p, t) satify the **Boltzmann equation**

$$\left(\frac{\partial}{\partial t}+\frac{\vec{p}}{m}\cdot\nabla_r-\nabla_r U(f)\cdot\nabla_p\right)f(\vec{r},\vec{p},t)=K(f)$$

Solve the Boltzmann equation using **test particle method**
Isospin-dependent **initialization**
Isospin- (momentum-) dependent **mean field potential**

$$V=V_0+\frac{1}{2}(1-\tau_z)V_c+V_{sym}$$

Isospin-dependent **N-N cross sections**

a. Experimental free space N-N cross section σ_{exp}
b. In-medium N-N cross section from the Dirac-Brueckner approach based on Bonn A potential σin-medium
c. Mean-field consistent cross section due to m*

Isospin-dependent **Pauli Blocking**

Has been extended to IBUU04 by Bao-An Li

IBU04--- potential

Isospin- and momentum-dependent potential (MDI)

$$U(\rho,\delta,\vec{p},\tau,x) = A_u(x)\frac{\rho_{\tau'}}{\rho_0} + A_l(x)\frac{\rho_\tau}{\rho_0} + B\left(\frac{\rho}{\rho_0}\right)^\sigma (1-x\delta^2) - 8\tau x \frac{B}{\sigma+1}\frac{\rho^{\sigma-1}}{\rho_0^{\sigma}}\delta\rho_{\tau'}$$

$$+\frac{2C_{\tau',\tau}}{\rho_0}\int d^3p' \frac{f_\tau(\vec{r},\vec{p})}{1+(\vec{p}-\vec{p}')^2/\Lambda^2} + \frac{2C_{\tau',\tau}}{\rho_0}\int d^3p' \frac{f_{\tau'}(\vec{r},\vec{p}')}{1+(\vec{p}-\vec{p}')^2/\Lambda^2}.$$

$$A_u(x) = -95.98 - x\frac{2B}{\sigma+1},\quad A_l(x) = -120.57 + x\frac{2B}{\sigma+1}$$

Das/Gupta/Gale/Li, PRC67,034611 (2003)

MDI Interaction: Gogny

$$\rho_0 = 0.16\ \mathrm{fm}^{-3}$$

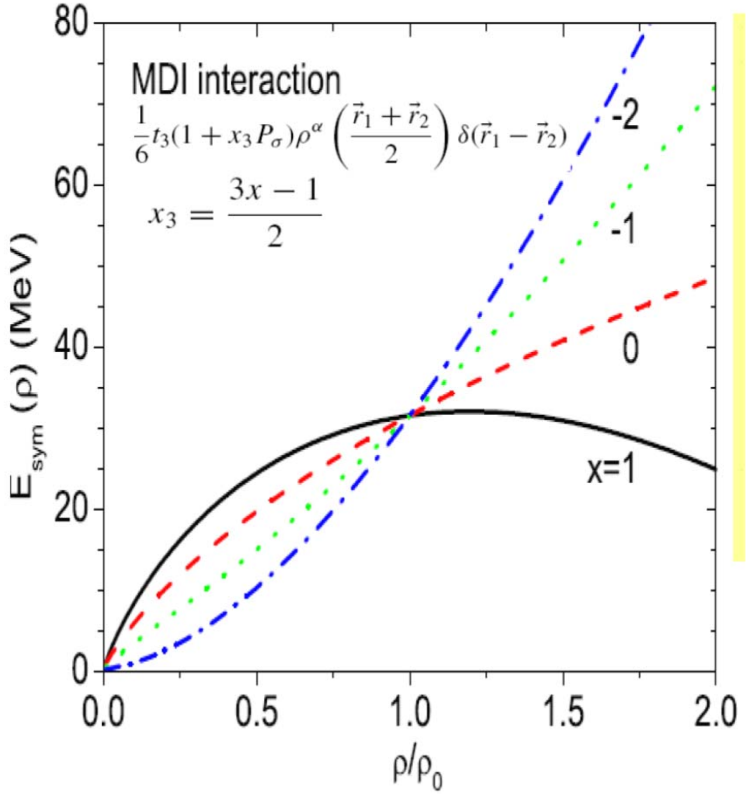
$$E(\rho_0)/A = -16\ \mathrm{MeV}$$

$$E_{sym}(\rho_0) = 31.6\ \mathrm{MeV}$$

$$\mathrm{K}_0 = 211\ \mathrm{MeV}$$

$$\mathrm{m}^*/\mathrm{m} = 0.68$$

Chen/Ko/Li, PRL94,032701 (2005);
Li/Chen,PRC72, 064611 (2005)



IBUU04

In-medium Nucleon-nucleon cross sections

Neglecting medium effects on the transition matrix

Medium effects: effective mass on the incoming current in initial state and level density of the final state

$$\sigma_{medium} / \sigma_{free} \approx \left(\frac{\mu^*_{NN}}{\mu_{NN}} \right)^2$$

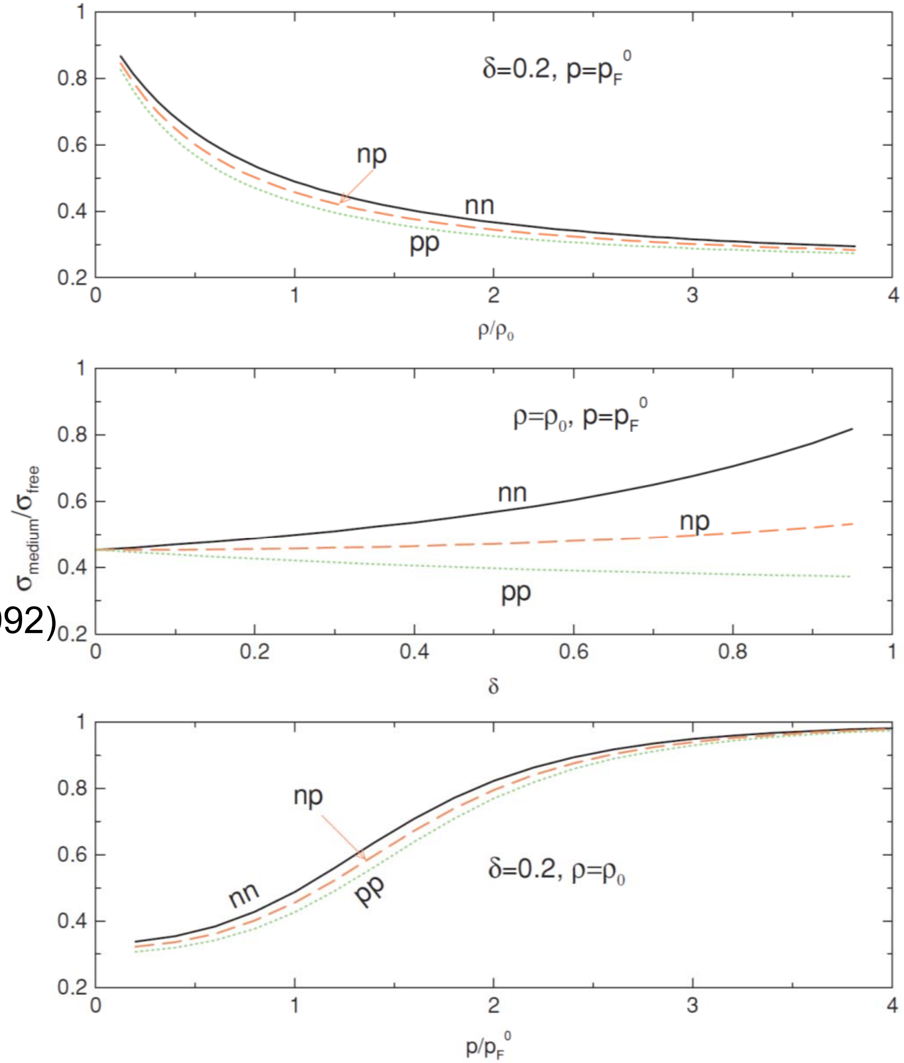
μ^*_{NN} is the reduced mass of the colliding pair NN in medium

J.W. Negele and K. Yazaki, PRL 47, 71 (1981)
V.R. Pandharipande and S.C. Pieper, PRC 45, 791 (1992)
M. Kohno et al., PRC 57, 3495 (1998)
D. Persram and C. Gale, PRC65, 064611 (2002).

1. In-medium cross sections are reduced
2. nn and pp cross sections are splitted due to the neutron-proton effective mass splitting in neutron-rich matter

Li/Chen, PRC72 (2005)064611

$\sigma_{medium} / \sigma_{free}$



BUU (f)→ BL($\hat{f}$)

Phase-space distributions f(r, p, t) satify the **Boltzmann equation**

$$\left(\frac{\partial}{\partial t}+\frac{\vec{p}}{m}\cdot\nabla_r-\nabla_r U\left(\hat{f}\right)\cdot\nabla_p\right)\hat{f}\left(\vec{r},\vec{p},t\right)=K\left(\hat{f}\right) + \delta K(\mathbf{r},\mathbf{p},t)$$

Isospin- (momentum-) dependent **mean field potential**

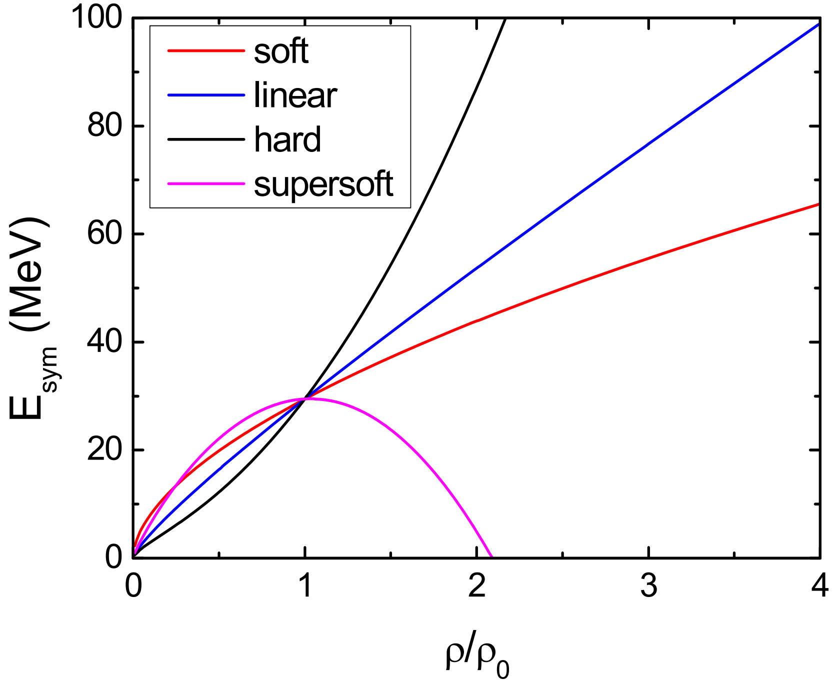
$$U_\tau(\rho,\delta,\vec{p}) = \alpha\frac{\rho}{\rho_0}+\beta\left(\frac{\rho}{\rho_0}\right)^\gamma + E_{sym}^{loc}(\rho)\delta^2+\frac{\partial E_{sym}^{loc}(\rho)}{\partial\rho}\rho\delta^2$$

$$+E_{sym}^{loc}(\rho)\rho\frac{\partial\delta^2}{\partial\rho_\tau}+U_{MDI}$$

$$E_{sym}^{pot}(\rho)=38.9\left(\frac{\rho}{\rho_0}\right)-18.4\left(\frac{\rho}{\rho_0}\right)^{2.14}-3.8\left(\frac{\rho}{\rho_0}\right)^{5/3}$$

$$E_{sym}^{pot}(\rho)=14.7\left(\frac{\rho}{\rho_0}\right)^{\gamma_s}$$

$$E_{sym}(\rho_0)=29.4MeV$$



$$U_{MDI}=1.57\ln^2[0.0005\Delta\vec{p}^2+1]\frac{\rho}{\rho_0}$$

B.A. Bian, et al., Nucl. Phys. A807,71(2008); W.J. Xie, et al., Phys. Lett. B 718,1510(2013)

BL
The inelastic channels and cross sections

$$NN \leftrightarrow N\Delta, NN \leftrightarrow NN^{*}, NN \leftrightarrow \Delta\Delta,$$

$$\Delta \leftrightarrow N\pi, N^{*} \leftrightarrow N\pi$$

The parameterized cross sections for each channel to produce resonances are obtained in terms of the one-boson exchange model.

The resonance lifetime are calculated according to the following figure.

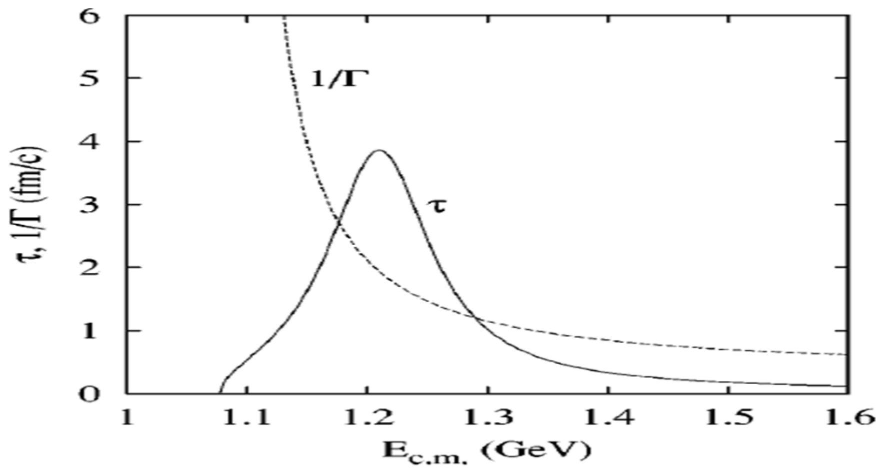


FIG. 1. The inverse width $1/\Gamma$ (dashed line), where Γ is given by Eq. (11), and the lifetime [solid line, Eq. (2)] of the Δ resonance as functions of the total c.m. energy of the pion and nucleon.

Z.Q. Feng, Phys. Rev. C 82 (2010) 057901; S. Huber, et al. Nucl. Phys. A573, (1994) 587;
A.B. Larionov, et al., Phys. Rev. C66,(2002)054604

BL---deal with the fluctuation term

$$\left(\frac{\partial}{\partial t}+\frac{\vec{p}}{m}\cdot\nabla_r-\nabla_r U\left(\hat{f}\right)\cdot\nabla_p\right)\hat{f}\left(\vec{r},\vec{p},t\right)=K\left(\hat{f}\right)$$

$$+\delta K\left(\vec{r},\vec{p},t\right)$$

The fluctuating collision term can be interpreted as a stochastic force acting on density and is characterized by a correlation function,

$$\left\langle\delta K\left(\vec{r}_1,\vec{p}_1,t_1\right)\delta K\left(\vec{r}_2,\vec{p}_2,t_2\right)\right\rangle=C\left(\vec{p}_1,\vec{p}_2\right)\delta\left(\vec{r}_1-\vec{r}_2\right)\delta\left(t_1-t_2\right).$$

A projection method is used in the BL model. The fluctuations are projected on a set of low-order multipole moments of the momentum distribution,

$$C_{LML'M'}\left(\vec{r},t\right)=\int d\vec{p}\,d\vec{p}\,'Q_{LM}\left(\vec{p}\right)Q_{L'M'}\left(\vec{p}\,'\right)C\left(\vec{p},\vec{p}\,'\right)$$

$$=\int d\vec{p}_1 d\vec{p}_2 d\vec{p}_3 d\vec{p}_4 \Delta Q_{LM}\Delta Q_{L'M'}\times W\left(12,34\right)f_1 f_2\left(1-f_3\right)\left(1-f_4\right).$$

$$\hat{Q}_{20}\left(\vec{r},t+\Delta t\right)=Q_{20}\left(\vec{r},t+\Delta t\right)+\sqrt{\Delta t C_{20}\left(\vec{r},t\right)}W_2,$$

$$\hat{Q}_{30}\left(\vec{r},t+\Delta t\right)=Q_{30}\left(\vec{r},t+\Delta t\right)+\sqrt{\Delta t C_{30}\left(\vec{r},t\right)}W_3.$$

F.S. Zhang et al. Phys. Rev. C 51(1995) 3201, Y. Abe, S. Ayik, et, al. Phys. Rep. 275 (1996)49

Developing processes from BL,IBL to ImIBL

BL $U(\rho) = \alpha \frac{\rho}{\rho_0} + \beta(\frac{\rho}{\rho_0})^{\gamma}$

No isospin effects,no inelastic channels

F.S. Zhang, Phys. Rev. C51,3201(1995)

IBL $U(\rho, \delta) = \alpha \frac{\rho}{\rho_0} + \beta(\frac{\rho}{\rho_0})^{\gamma} + U_{sym}$

No inelastic channels

B.A. Bian, et al., Nucl. Phys. A807,71(2008)

ImIBL $U_{\tau}(\rho, \delta, \vec{p}) = \alpha \frac{\rho}{\rho_0} + \beta(\frac{\rho}{\rho_0})^{\gamma} + U_{sym} + U_{MDI}$

$$NN \leftrightarrow N\Delta, NN \leftrightarrow NN^*, NN \leftrightarrow \Delta\Delta,$$

$$\Delta \leftrightarrow N\pi, N^* \leftrightarrow N\pi$$

Inelastic channels and MDI

W.J. Xie, et al., Phys. Lett. B 718,1510(2013)

Outline

1.Introduction

2.Different transport models used in China

3.Observables for symmetry energy

How to use those models to probe the nuclear symmetry energy with heavy-ion collisions ?

3.1 At sub-saturation densities (know something) X √

- Global nucleon optical potentials from n/p-nucleus and (p,n) reactions
- **Thickness of n-skin in ^{208}Pb measured using various approaches and sizes of n-skins of unstable nuclei from total reaction cross sections**
- **n/p ratio of FAST, pre-equilibrium nucleons**
- **Isospin fractionation and isoscaling in nuclear multifragmentation**
- **Isospin diffusion/transport**
- **Neutron-proton differential flow**
- **Neutron-proton correlation functions at low relative momenta**
- **$t/^3He$ ratio and their differential flow**

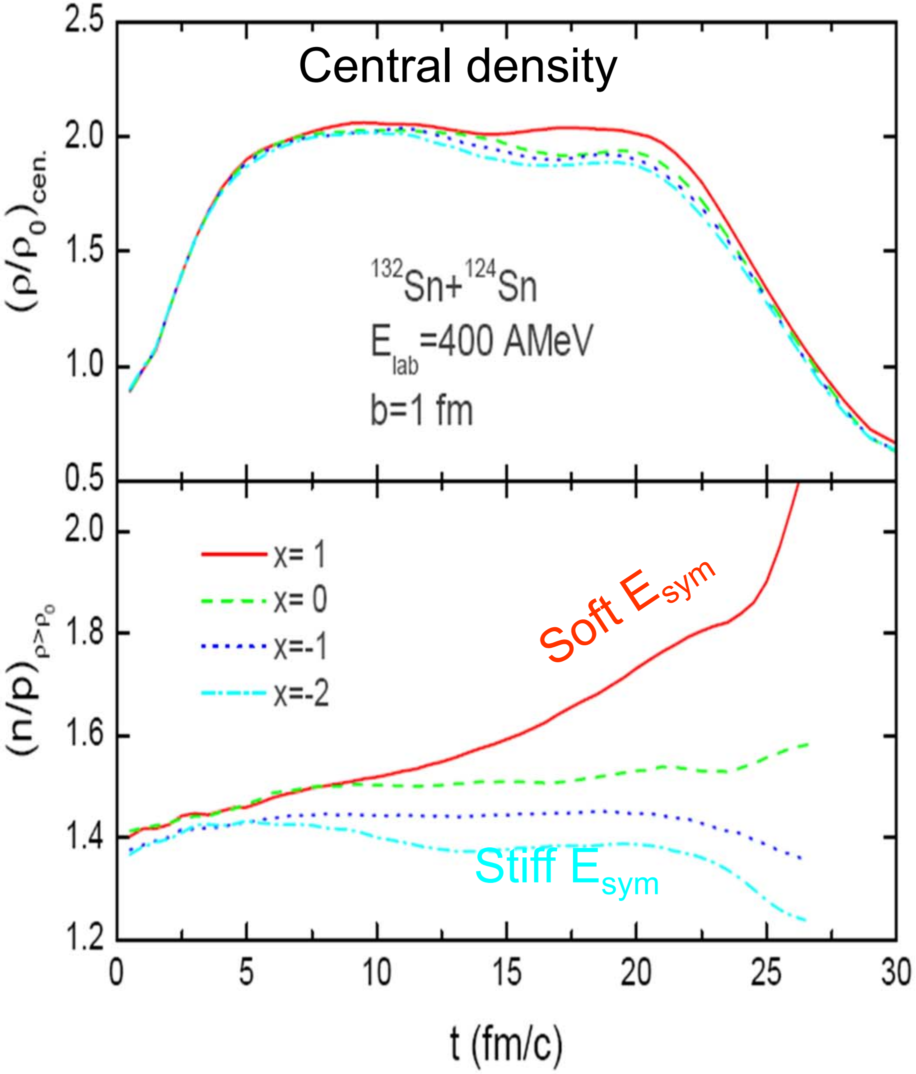
3.2 Towards supra-saturation densities (know nothing) X X

- **π^-/π^+ ratio, K^+/K^0 ?**
- **Neutron-proton differential transverse flow**
- **n/p ratio of squeezed-out nucleons perpendicular to the reaction plane**
- **Nucleon elliptical flow at high transverse momentum**
- **t-^3He differential and difference transverse flow**

(1) Correlations of multiple observables are important
(2) Detecting neutrons simultaneously with charged particles is critical

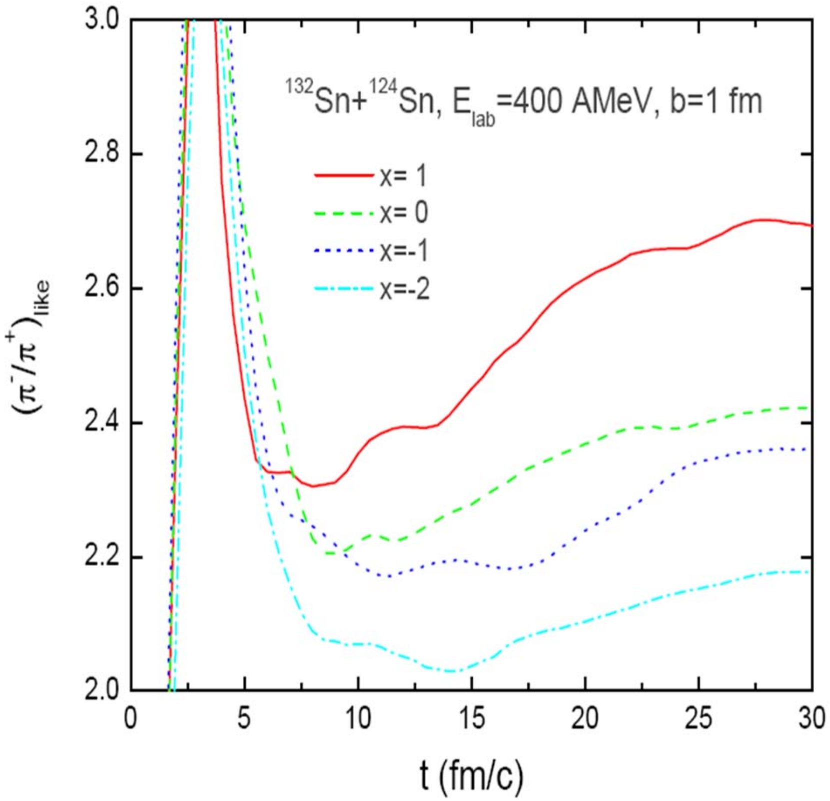
B.A. Li, L.W. Chen and C.M. Ko, *Physics Reports 464, 113 (2008)*

$$E(\rho,\delta) = E(\rho,0) + E_{sym}(\rho)\delta^2$$



n/p ratio at supra-normal densities

With the soft symmetry energy the *n/p* ratio of the high-density region can be significantly higher than that of the reaction system.

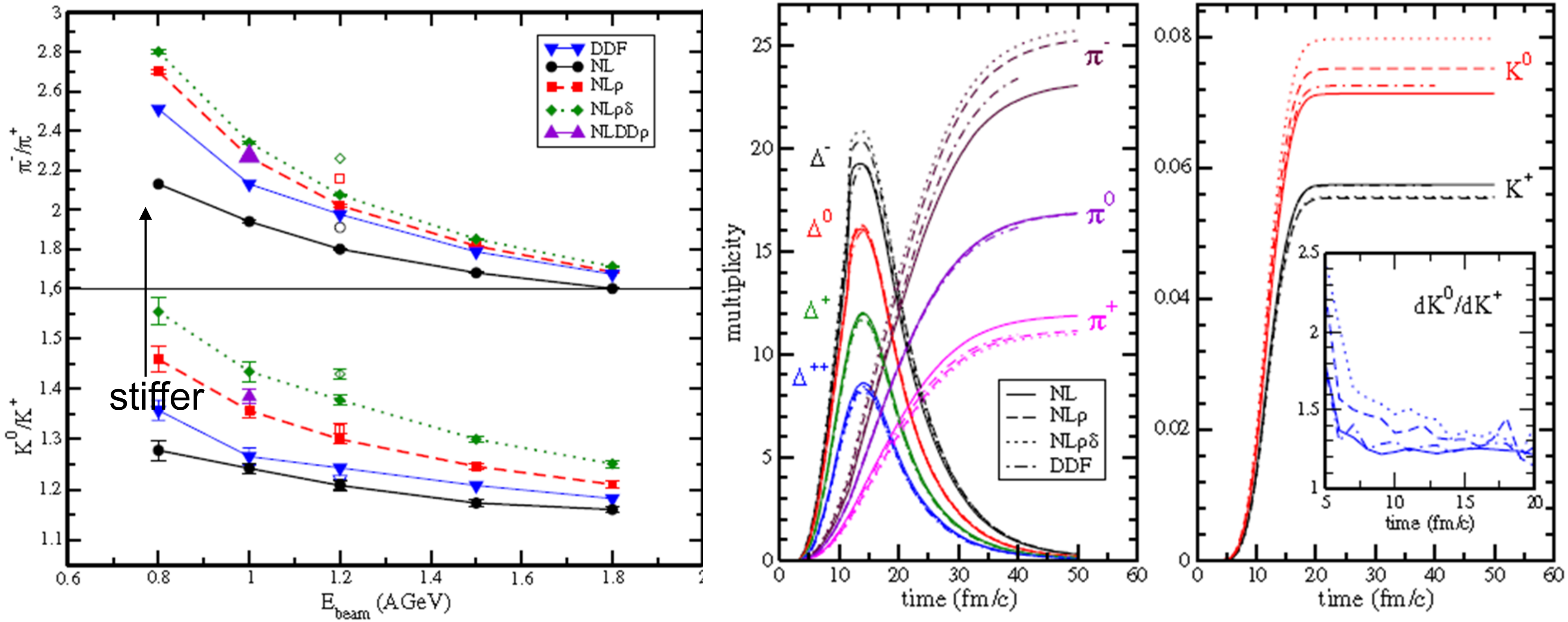


B.A. Li et al, Phys. Rev. C 71 (2005) 014608

PRL 97, 202301 (2006) PHYSICAL REVIEW LETTERS week ending 17 NOVEMBER 2006

Isospin Effects on Subthreshold Kaon Production at Intermediate Energies

G. Ferini,[1] T. Gaitanos,[2] M. Colonna,[1] M. Di Toro,[1,*] and H. H. Wolter[2]
[1]*Università di Catania and INFN, Laboratori Nazionali del Sud, 95123 Catania, Italy*
[2]*Department für Physik, Universität München, D-85748 Garching, Germany*

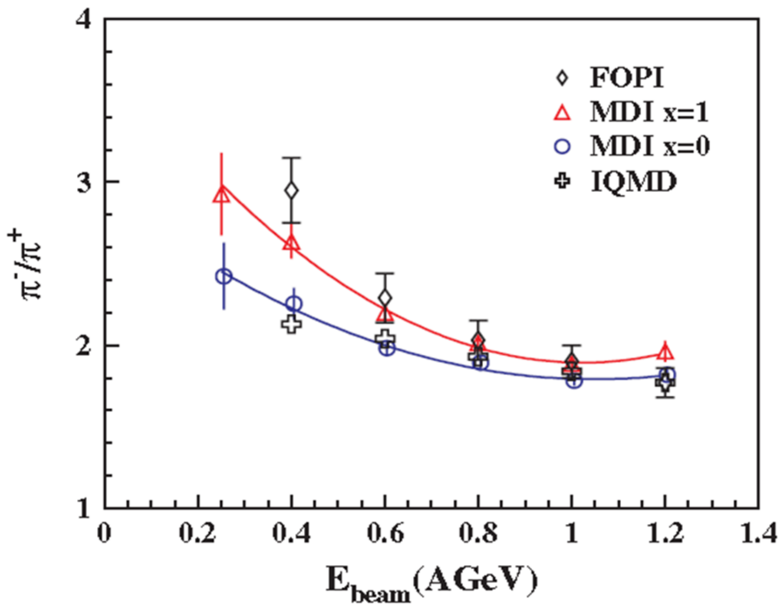


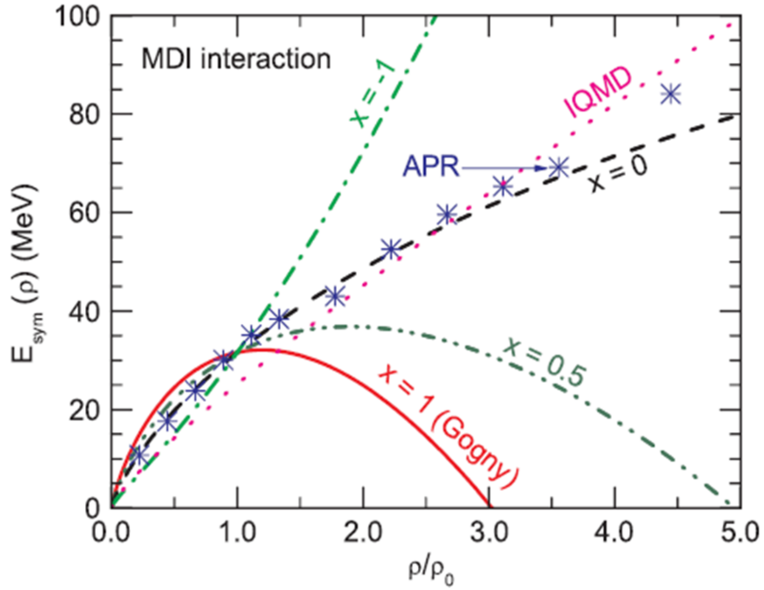
Based on RBUU model, by calculating the excitation function of pion and kaon ratio, due to the inclusion of delta meson field, a stiffer symmetry energy results in a larger pion and kaon ratios.

PRL 102, 062502 (2009) PHYSICAL REVIEW LETTERS week ending 13 FEBRUARY 2009

Circumstantial Evidence for a Soft Nuclear Symmetry Energy at Suprasaturation Densities

Zhigang Xiao,[1] Bao-An Li,[2,*] Lie-Wen Chen,[3] Gao-Chan Yong,[4] and Ming Zhang[1]
[1]*Department of Physics, Tsinghua University, Beijing 100084, P.R. China*
[2]*Department of Physics, Texas A&M University-Commerce, Commerce, Texas 75429-3011, USA*
[3]*Institute of Theoretical Physics, Shanghai Jiao Tong University, Shanghai 200240, P.R. China*





Based on the IBUU04 model, by calculating the excitation function of pion ratio, a very soft symmetry energy was obtained. A soft symmetry energy results in a larger pion ratio.

FOPI data, W. Reisdorf, et al., Nucl. Phys. A 781, 459 (2007).

Physics Letters B 683 (2010) 140–144

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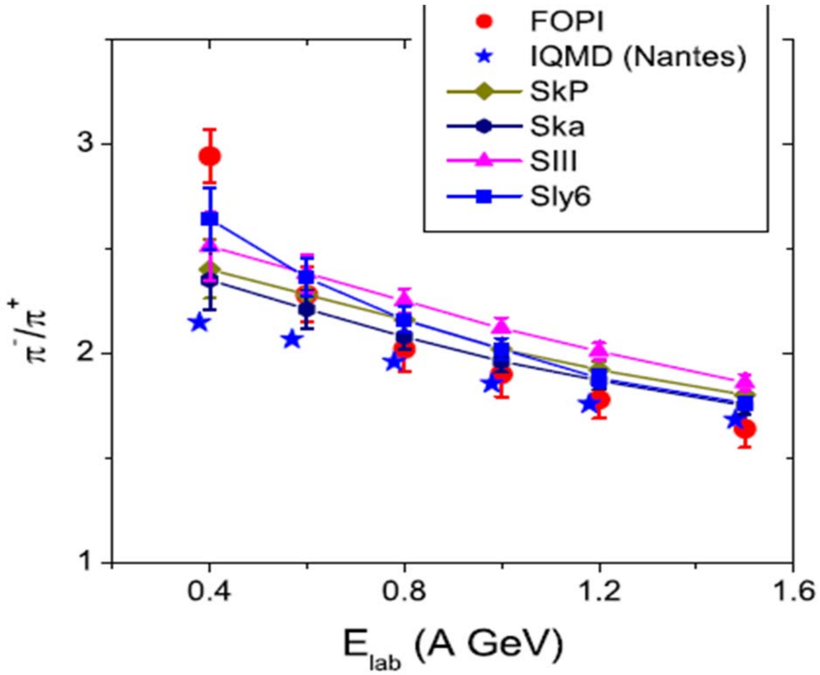
www.elsevier.com/locate/physletb

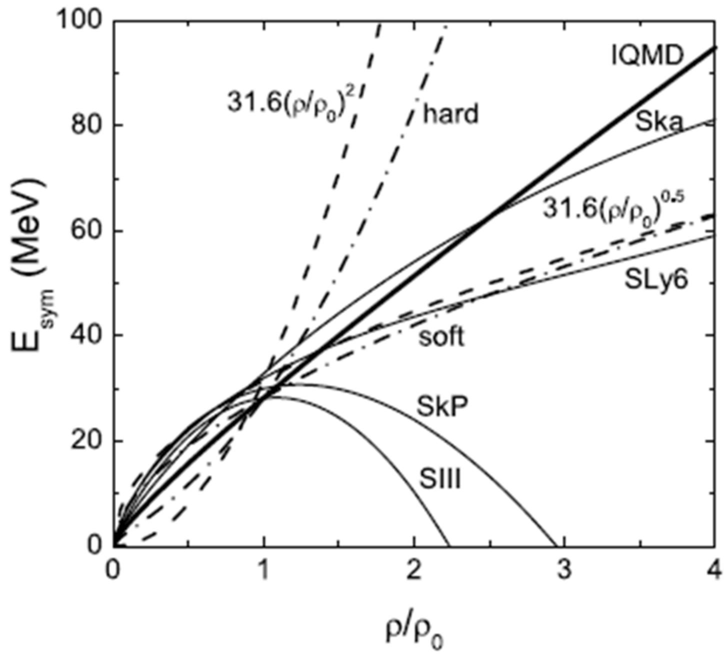
Probing high-density behavior of symmetry energy from pion emission in heavy-ion collisions

Zhao-Qing Feng *, Gen-Ming Jin

Institute of Modern Physics, Chinese Academy of Sciences, Lanzhou 730000, People's Republic of China

FOPI data, W. Reisdorf, et al., Nucl. Phys. A 781, 459 (2007).





Based on the LQMD model, a hard symmetry energy results in a larger pion ratio. A hard symmetry energy was predicted by comparing with the FOPI data.

Physics Letters B 718 (2013) 1510–1514

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Physics Letters B

www.elsevier.com/locate/physletb

Symmetry energy and pion production in the Boltzmann–Langevin approach

Wen-Jie Xie [a,b,d], Jun Su [a,b], Long Zhu [a,b], Feng-Shou Zhang [a,b,c,*]

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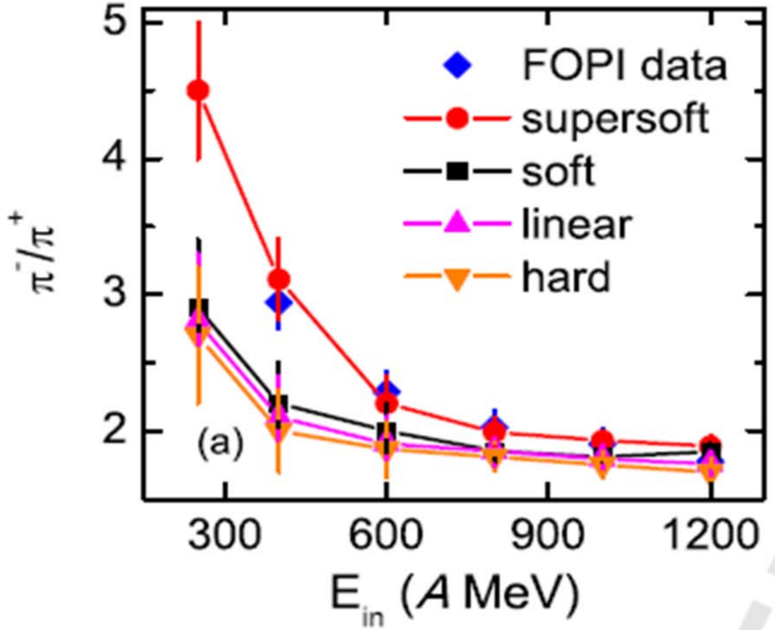
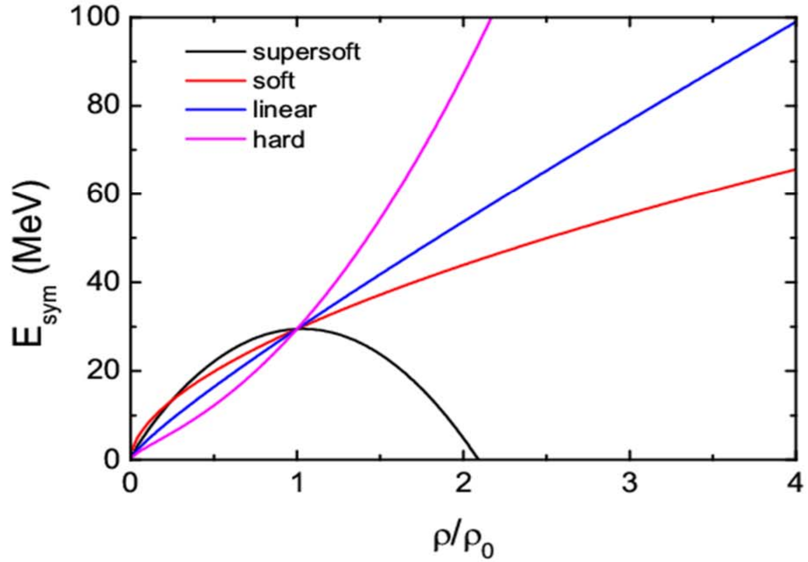
b Beijing Radiation Center, Beijing 100875, China
c Center of Theoretical Nuclear Physics, National Laboratory of Heavy Ion Accelerator of Lanzhou, Lanzhou 730000, China
d Department of Physics, Yuncheng University, Yuncheng 044000, China



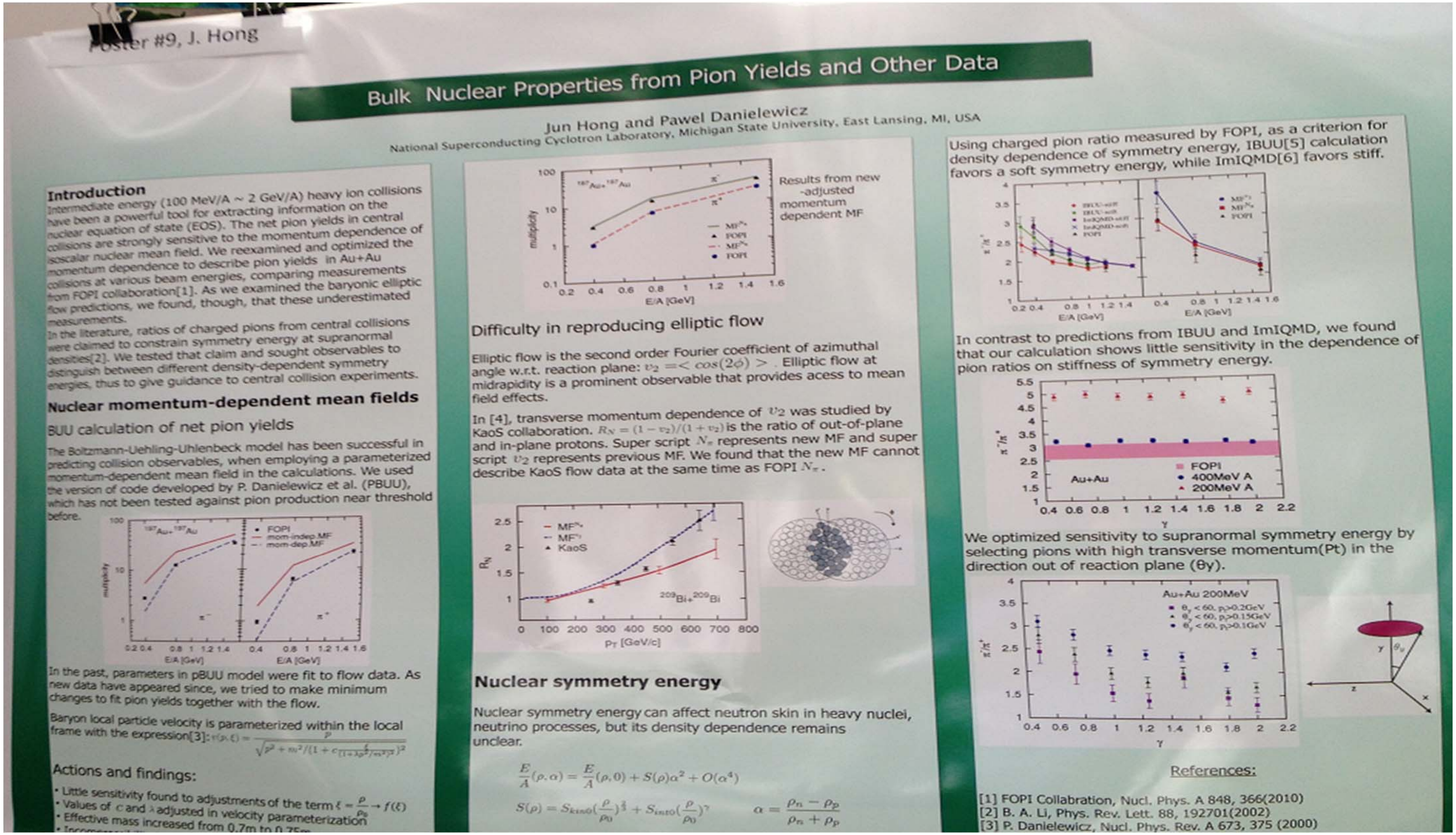


Based on the improved isospin-dependent BL model, a supersoft symmetry energy is obtained by calculating the excitation functions of pion ratios in comparison with the FOPI data, which is consistent with the IBUU04 calculations.

FOPI data, W. Reisdorf, et al., Nucl. Phys. A 781, 459 (2007).

Bulk nuclear properties from pion yields and other data
H. Jun and P. Danielewicz, by PBUU

PBUU: Danielewicz et al., NPA 533, 712 (1991)



How to manage this inconsistency:

transport models with the same + the simplest input quantities

1. Molecular dynamics like: N-body approaches
   CMD, QMD,
   CoMD,
   IQMD, LQMD, ImQMD,…

   AMD, FMD,…
2. Boltzmann-like: 1-body approaches
   IBUU (BNV, LV), IBL

Important: Input quantities, numerical treatments,…

Thank you for your attention