

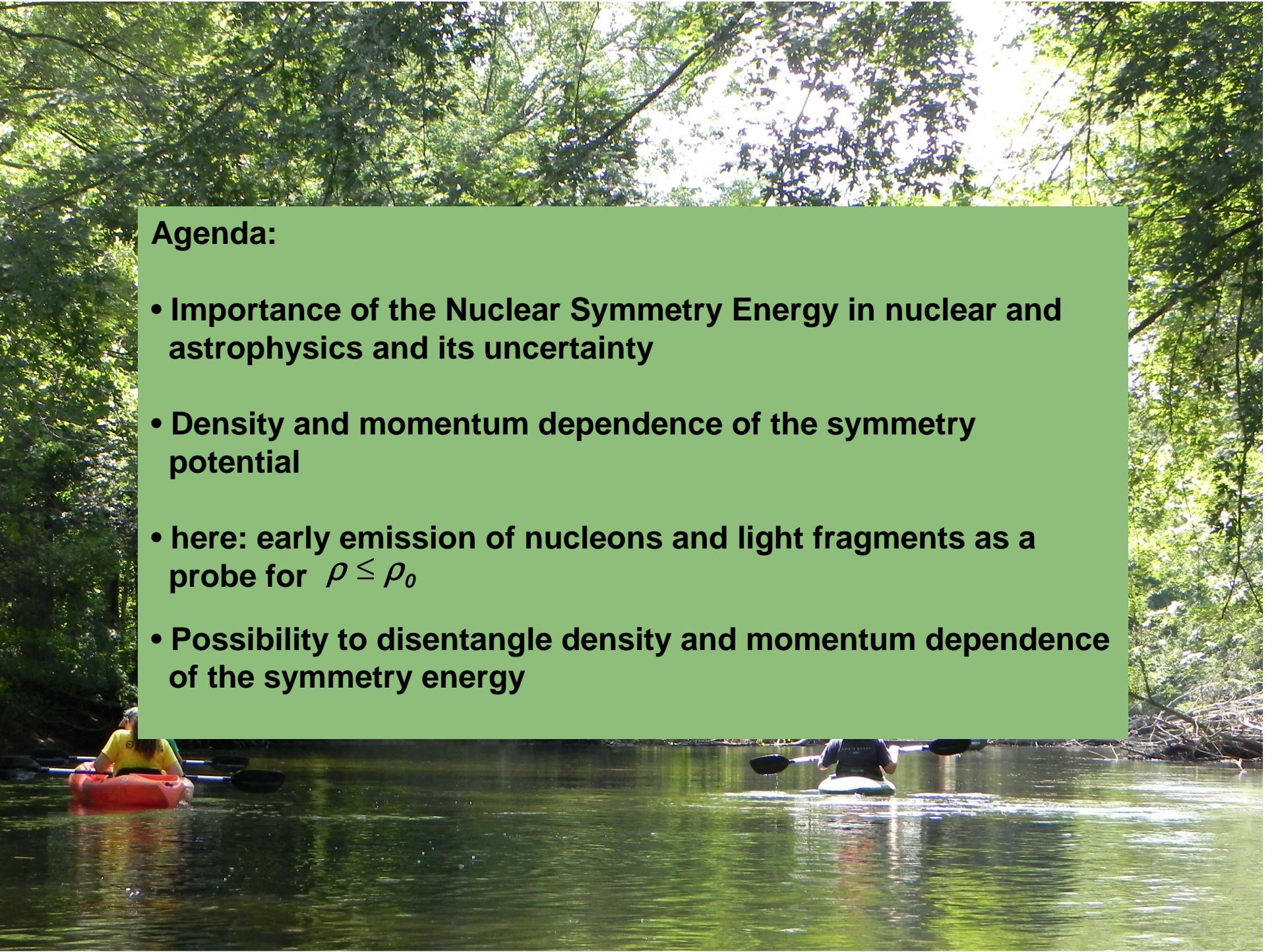
Light Fragment Production in Heavy Ion Collisions and the Symmetry Energy

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with

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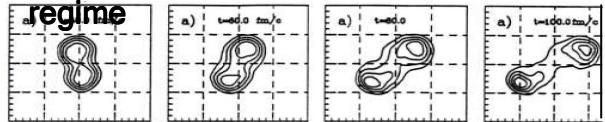
Agenda:

- Importance of the Nuclear Symmetry Energy in nuclear and astrophysics and its uncertainty
- Density and momentum dependence of the symmetry potential
- here: early emission of nucleons and light fragments as a probe for $\rho \leq \rho_0$
- Possibility to disentangle density and momentum dependence of the symmetry energy

Investigations of the Nuclear Symmetry Energy in different density regions

$$E(\rho, \delta)/A = E(\rho) + E_{\text{sym}}(\rho)\delta^2 + O(\delta^4) + \dots$$

heavy ion collisions in the Fermi energy regime



$$\delta = \frac{\rho_n - \rho_p}{\rho}$$

Isospin Transport properties,
(Multi-)Fragmentation
Pre-equilibrium emission

$E_{\text{sym}}(\rho_B)$ (MeV)

100

80

60

40

20

0

0

1

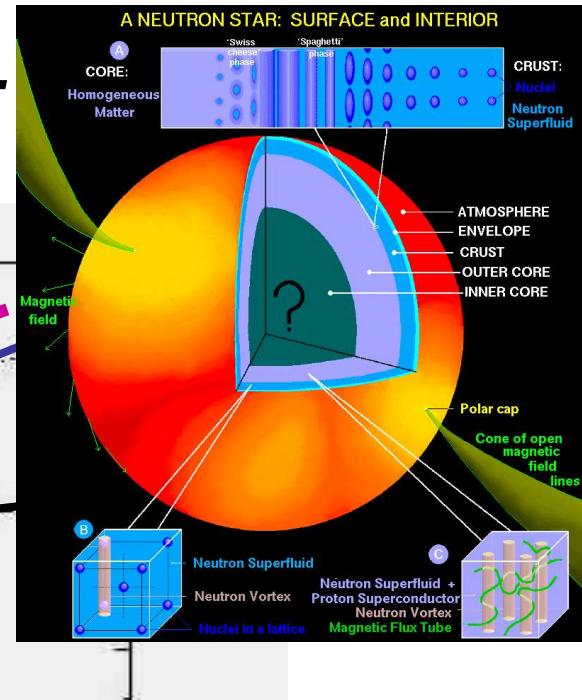
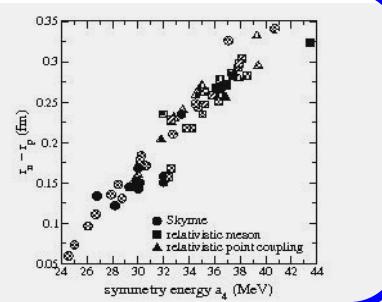
2

ρ_B/ρ_0

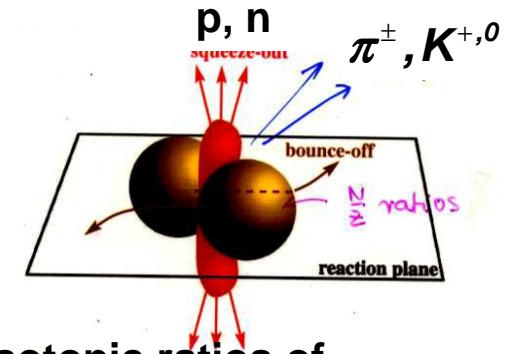
Asy-soft

Asy-stiff

Nuclear structure
(neutron skin thickness, Pygmy DR,
IAS)
Slope of Symm Energy

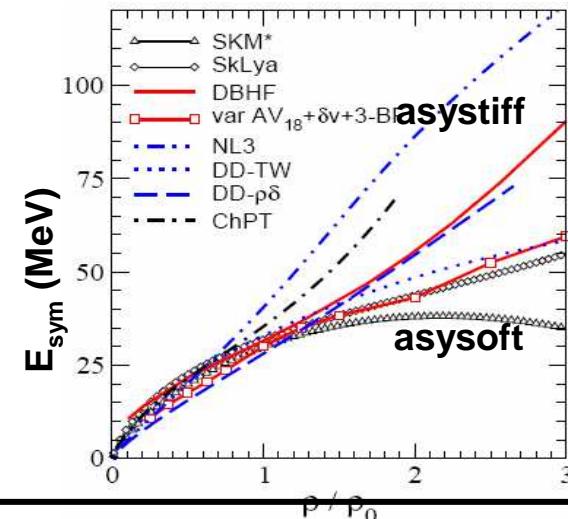
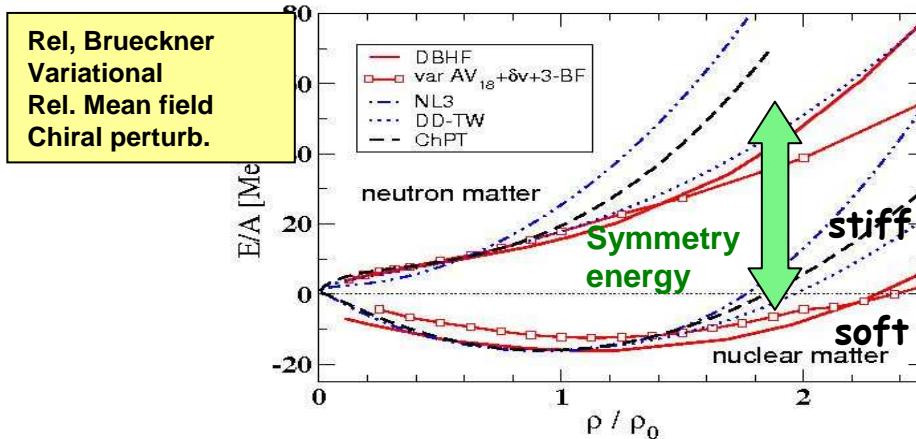


rel. heavy ion collisions



Isotopic ratios of
flow, particle production

The Nuclear Symmetry Energy in different „realistic“ models



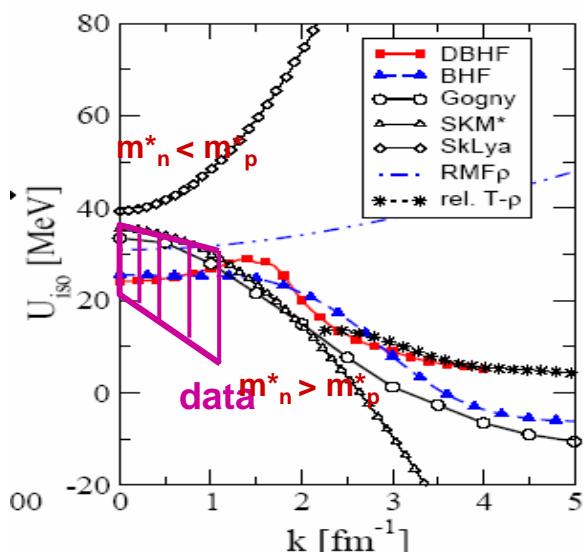
C. Fuchs, H.H. Wolter, EPJA 30(2006)5

Momentum dependence of symmetry potential (isoscalar and isovector)

$$\varepsilon(\rho[f(\rho, k)]; \delta) \rightarrow U(\rho, k; \delta) = \frac{\partial \varepsilon(\rho, \delta)}{\partial f(\rho, k)} = \underbrace{U_0(\rho, k) + U_{\text{sym}}(\rho, k)(\tau\delta) + \dots}_{U_\tau(\rho, k)}$$

Effective masses and
effective proton-neutron
mass splitting

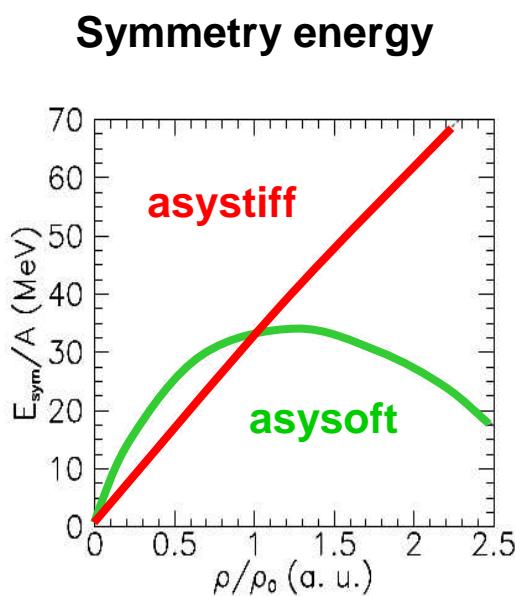
$$\frac{m^*_\tau}{m} = \left(1 + \frac{m}{\hbar^2 k} \frac{\partial U_\tau}{\partial k} \right)^{-1}$$



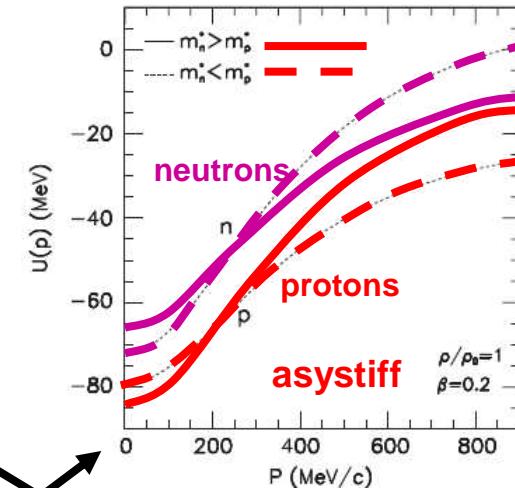
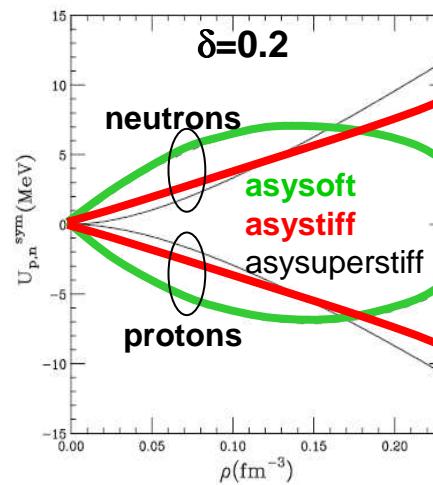
Even no agreement on ordering of n/p effective masses
between Skyrme, RMF, BHF, DBHF

Why is symmetry energy so uncertain??
 → In-medium ρ mass, and short range isovector tensor correlations (e.g. B.A. Li, PRC81 (2010));
 → use heavy ion collisions to investigate in the laboratory

Example of the isovector dependence for a particular parametrization
 (used in the following results, BGBD, Bombaci-Gale-Bertsch-DasGupta)

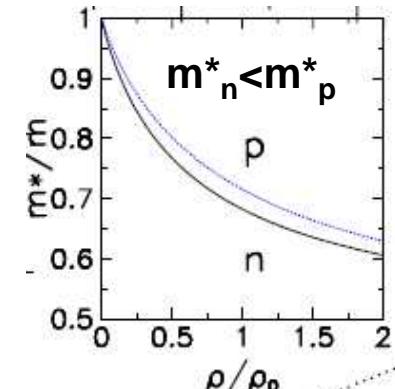
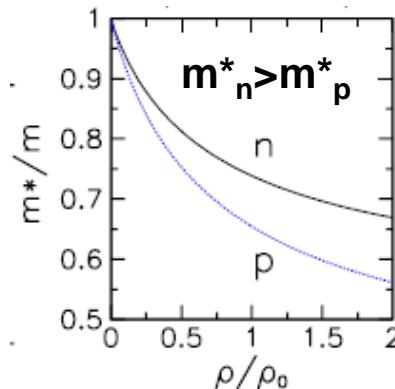


Neutron/proton potentials as a fct of density for $p=p_F$ (left) momentum for $\rho=\rho_0$ (right)



effects of the density and momentum dependence can be of the same order

**Effective masses
(for asystiff and $p=p_F$)**



Symmetry energy and symmetry potentials (effective masses)

B.A. Li, X. Han, [arXiv:1304.3368](https://arxiv.org/abs/1304.3368)

establish a relation $\{E_{\text{sym}}, L\} \leftrightarrow \{U_{\text{sym}}, m^*_{n,p}\}$ using the Hugenholtz-Van Hove theorem

$$E_{\text{sym}}(\rho) = \frac{1}{3} \frac{\hbar^2 k_F^2}{2m_0^*} + \frac{1}{2} U_{\text{sym}}(\rho, k_F),$$

$$L(\rho) = \frac{2}{3} \frac{\hbar^2 k_F^2}{2m_0^*} + \frac{3}{2} U_{\text{sym}}(\rho, k_F) + \frac{\partial U_{\text{sym}}}{\partial k} \Big|_{k_F} k_F,$$

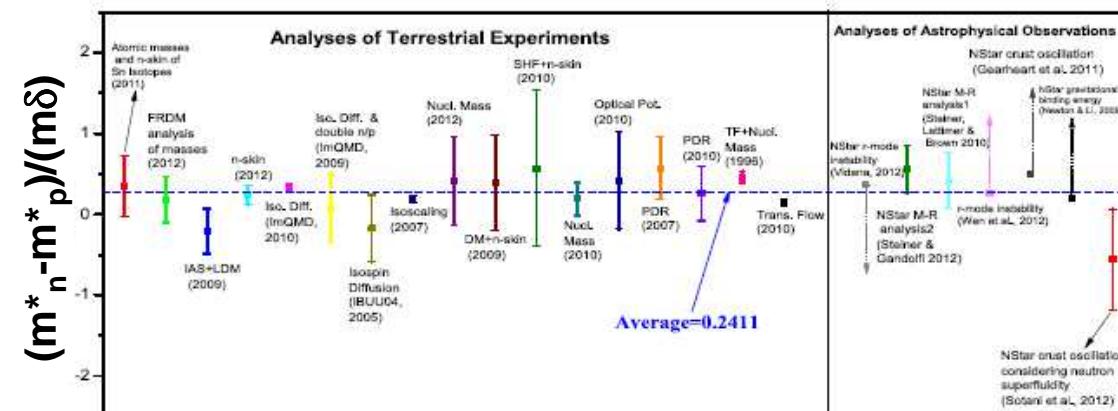
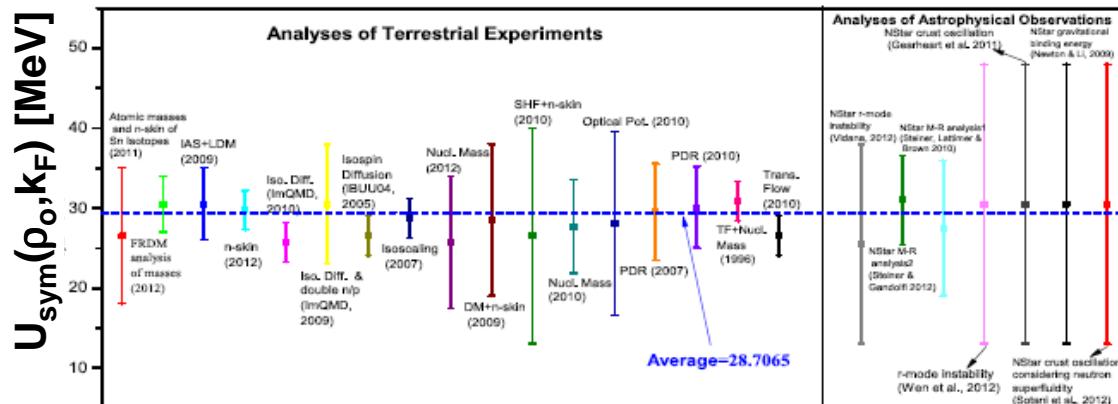
$$U_{\text{sym}}(\rho_0, k_F) = 2 \left[E_{\text{sym}}(\rho_0) - \frac{1}{3} \frac{m}{m_0^*} E_F(\rho_0) \right],$$

$$\frac{dU_{\text{sym}}}{dk} \Big|_{k_F} (\rho_0) = \left[L(\rho_0) - 3E_{\text{sym}}(\rho_0) + \frac{1}{3} \frac{m}{m_0^*} E_F(\rho_0) \right] / k_F,$$

Use existing determinations in the literature for $\{E_{\text{sym}}, L\}$ to obtain values for $\{U_{\text{sym}}, m^*_{n,p}\}$

Without going into details, shows appearance of some consensus on the symmetry potential and the effective mass, but also the large scatter (esp. for the effective mass splitting)

→ need to constrain this better

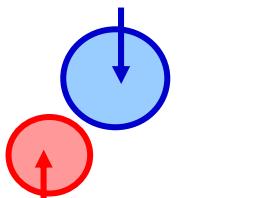


Isospin-related Observables in Low Energy Heavy Ion Collisions

Coulomb barrier to Fermi energies

peripheral

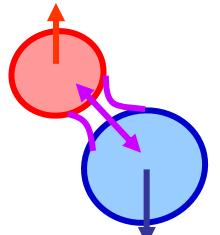
Isospin migration



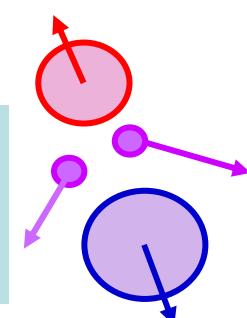
deep-inelastic

pre-equil.
dipole

N/Z of PLF
residue
= isospin
diffusion

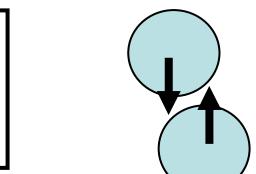


N/Z of neck
fragment and
velocity
correlations



central

Isospin
fractionation,
multifragm



pre-equil.
light
particles

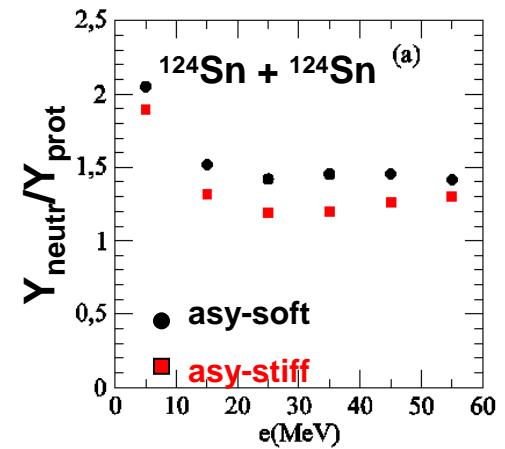
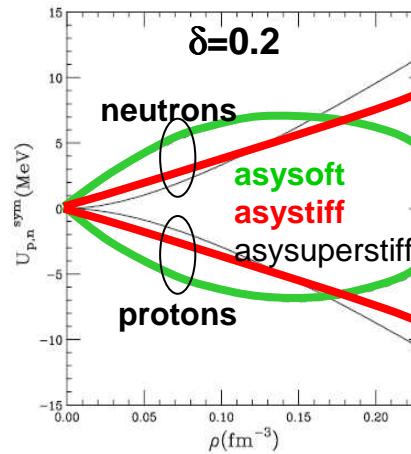
N/Z ratio of
IMF's

Discuss here

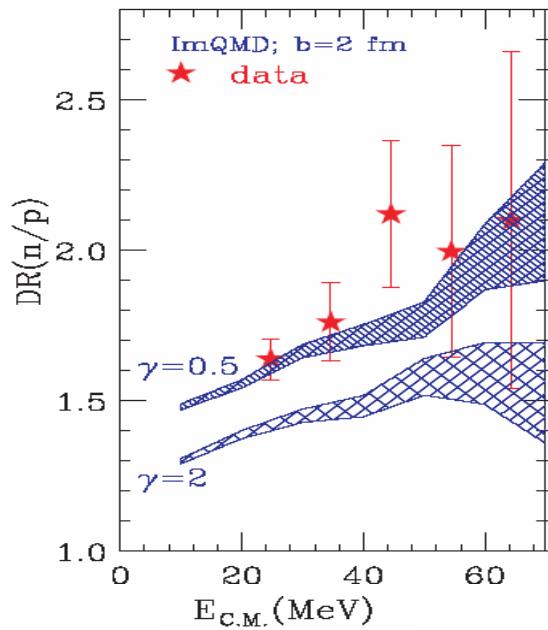
Pre-equilibrium particle emission: n/p ratio

Early emitted neutrons and protons reflect difference in potentials in expanded source, esp. ratio $Y(n)/Y(p)$.
more emission for asy-soft, since symm potential higher

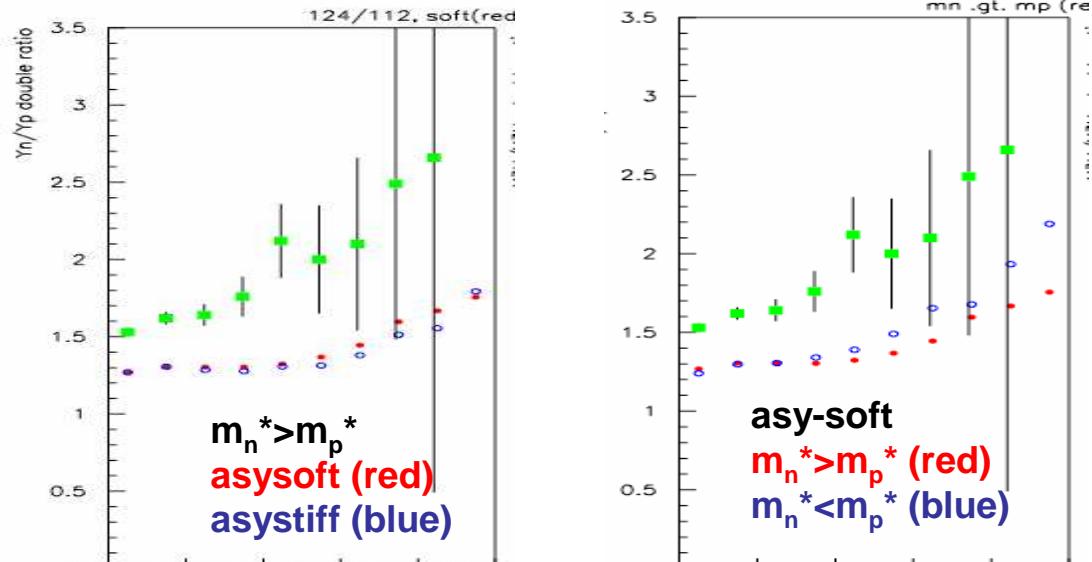
Neutrons difficult to measure, $\frac{^{124}\text{Sn} + ^{124}\text{Sn}}{^{112}\text{Sn} + ^{112}\text{Sn}}$
thus „Double Ratios“



Data: Famiano et al. PRL 06
Calc.: Danielewicz, et al. 07



M. Zielinska-Pfabe, et al., AsyEoS-Meeting, Noto 2010



Similar qualitatively but smaller effect

Effect of mass splitting of same magnitude

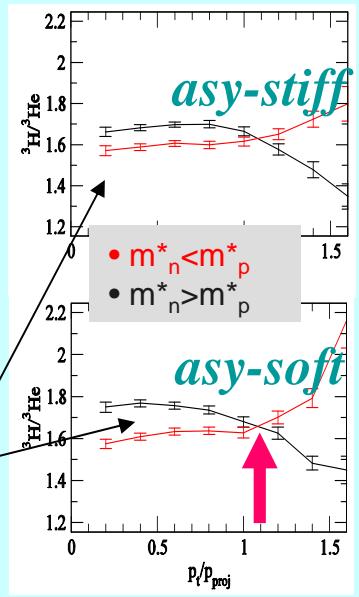
→ A sensitive observable, but perhaps double ratio not optimal. Light charged particles?

Previous studies of t/3He ratios:

**Yield ratios,
mainly studied
at high energy**

$^{197}\text{Au} + ^{197}\text{Au}$
600 AMeV $b=5 \text{ fm}$,
 $y(0) \leq 0.3$

**effect of
effective mass
more
prominent than
of asystiffness**



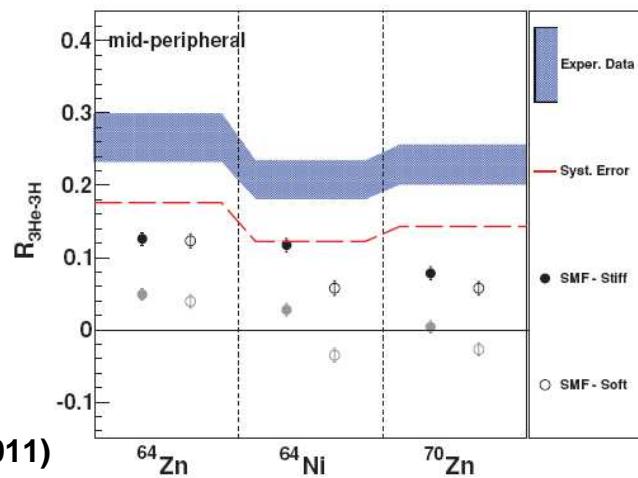
(V.Giordano, et al., PRC 81(2010))

**Flow ratios
At low energy**

$$R_{^3\text{He}-^3\text{H}} = \frac{F(^3\text{He}) - F(^3\text{H})}{F(^3\text{He}) + F(^3\text{H})},$$

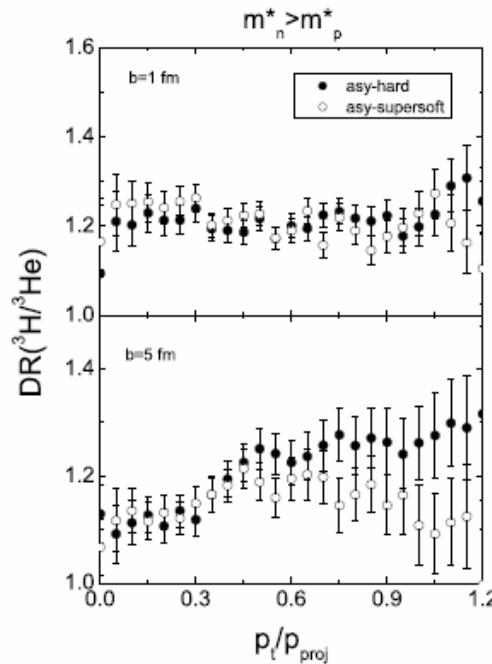
**Effect of symm energy
seen, but far away from
data**

Z.Kohley, et al., PRC83, 044601 (2011)



symmetric collisions, 35 AMeV

197Au+197Au, 400 AMeV



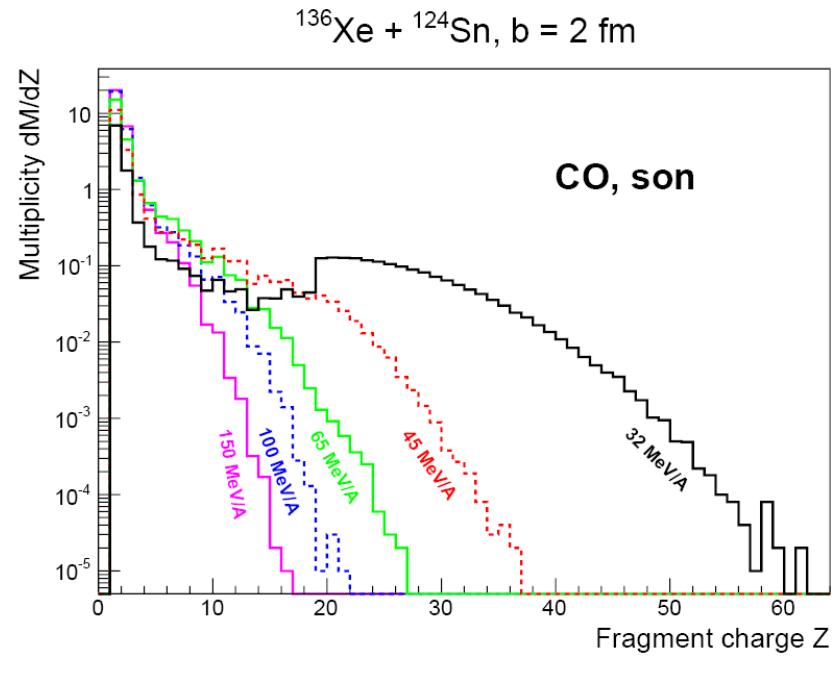
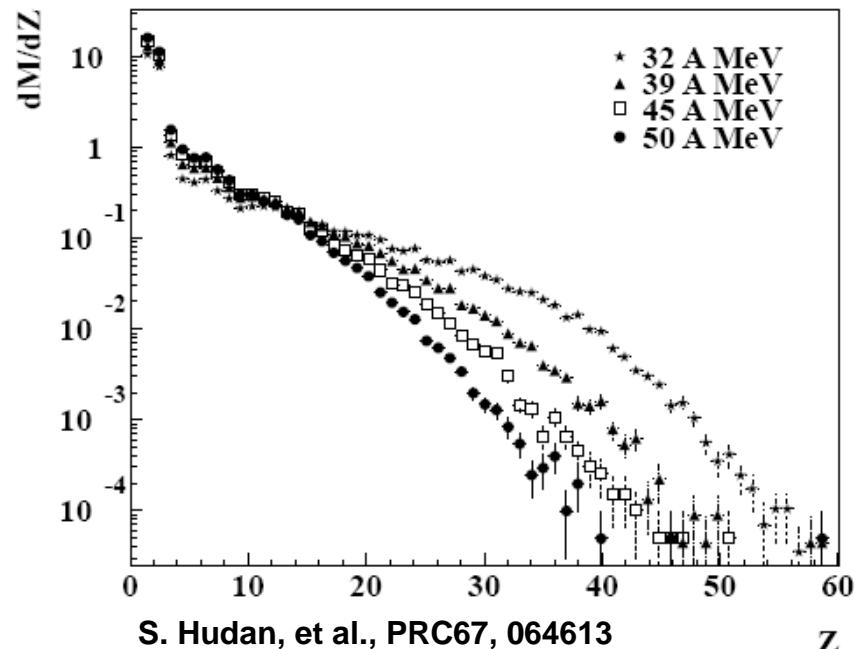
Z.Q. Feng, NPA878, 3 (2012)

**Also studies at higher energy,
e.g. G.C.Yong, et al., PRC80,
044608 (2009)**

**Situation not so clear,
More data desirable
Systematic study useful**

Study of Light Fragment Emission: $^{136,124}\text{Xe} + ^{124,112}\text{Sn}$, $E = 32,..,150 \text{ AMeV}$,
 Prelim. data from R. Bougault, et al., GANIL

**Cluster recognition by two methods: Coalescence (CO) in phase space
 Density cut (DC, „gas“ (p.n) and „liquid“)
 Global charge distributions agree reasonably**



**Calculations with variation
 of symmetry energy
 and effective mass splitting**

asy-soft	so	n	$m_n^* > m_p^*$
asy-Stiff	st	p	$m_p^* > m_n^*$

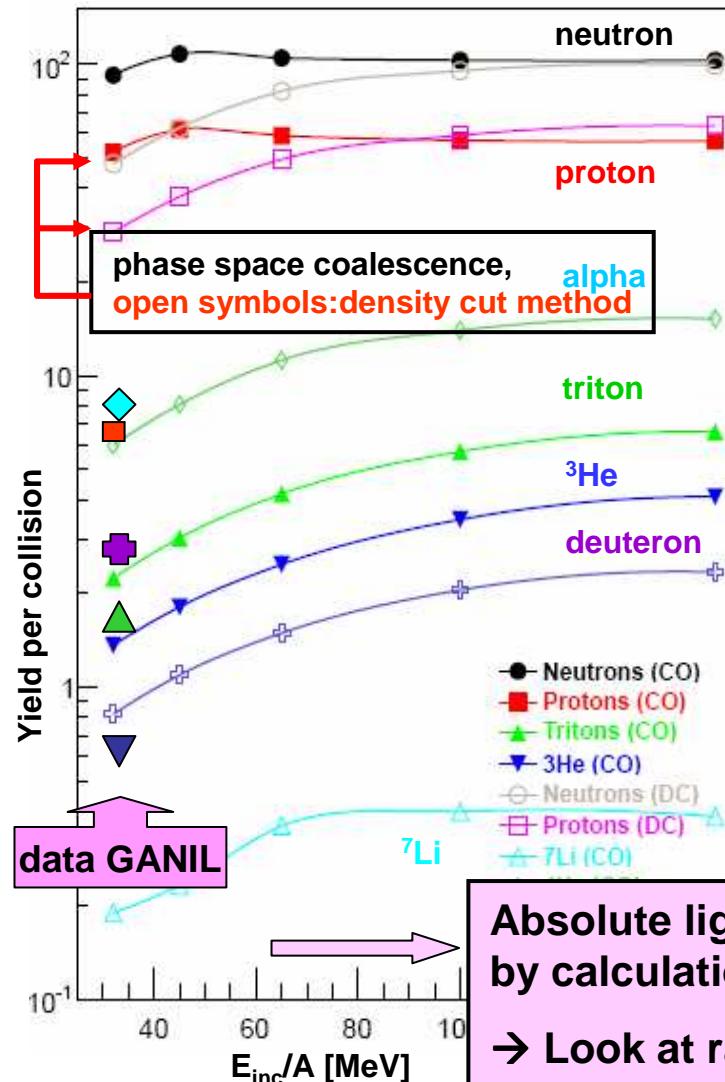
son
stn
sop
stp

Study of Light Fragment Emission: $^{136,124}\text{Xe} + ^{124,112}\text{Sn}$, $E = 32,..,150 \text{ AMeV}$,

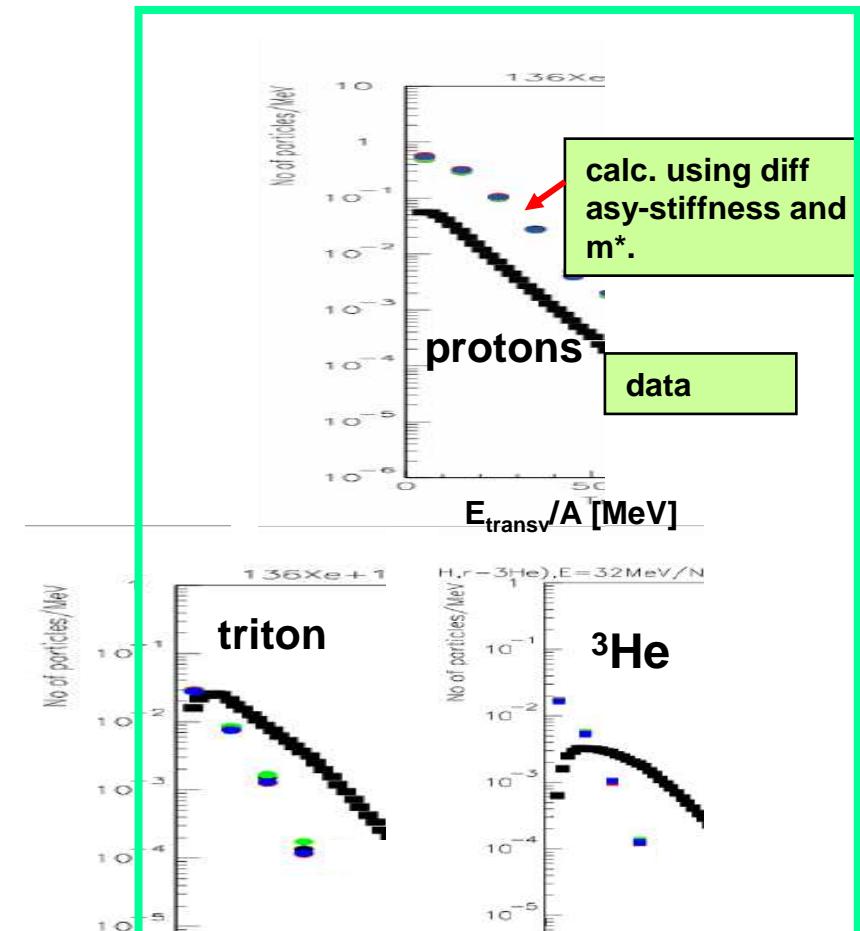
Yields and spectra in comp. to experiment

$^{136}\text{Xe} + ^{124}\text{Sn}$ (n-rich)

Total yields



Transverse energy spectra



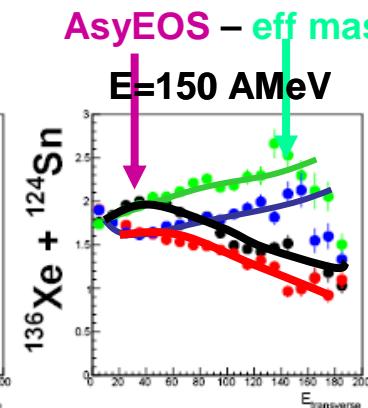
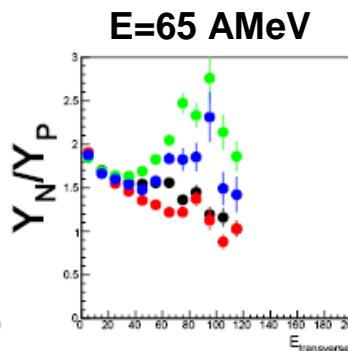
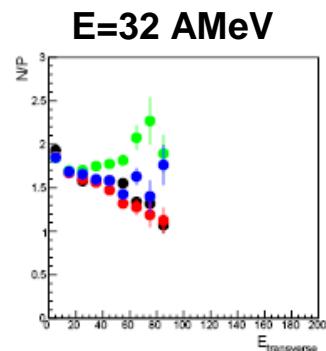
Absolute light fragment yields and spectra not well reproduced by calculations – important to improve!

→ Look at ratios: p/n, t/ ^3He single ratios and double ratios

Study of Light Fragment Emission: $^{136,124}\text{Xe} + ^{124,112}\text{Sn}$, $E = 32,..,150 \text{ AMeV}$,

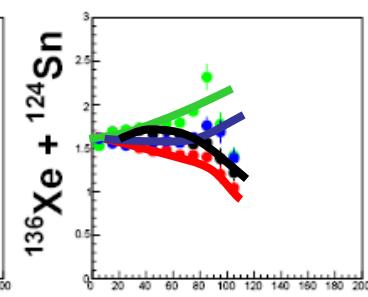
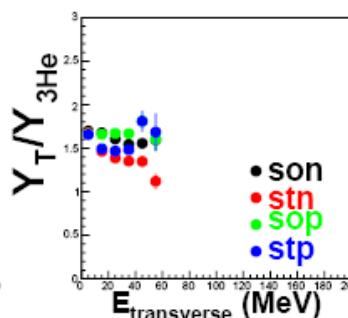
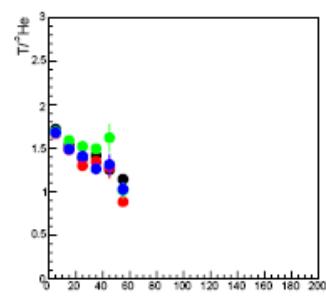
Single ratios

Single ratio n/p
neutron rich
 $^{136}\text{Xe} + ^{124}\text{Sn}$



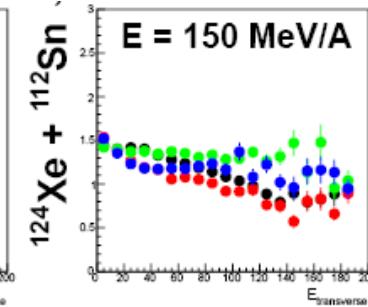
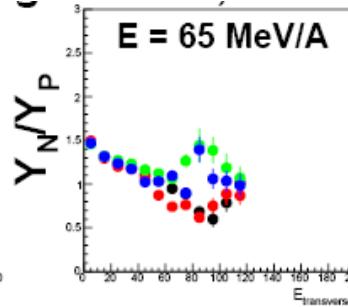
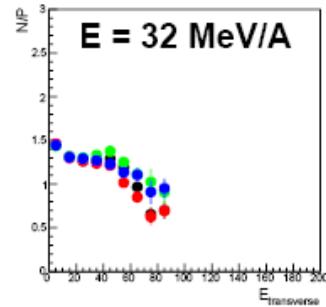
Possibility to separate density and momentum dependence of symmetry energy

Single ratio t/3He
neutron rich
 $^{136}\text{Xe} + ^{124}\text{Sn}$



Effects smaller for light clusters t/3He
improve statistics!

Single ratio n/p
neutron poor
 $^{124}\text{Xe} + ^{112}\text{Sn}$



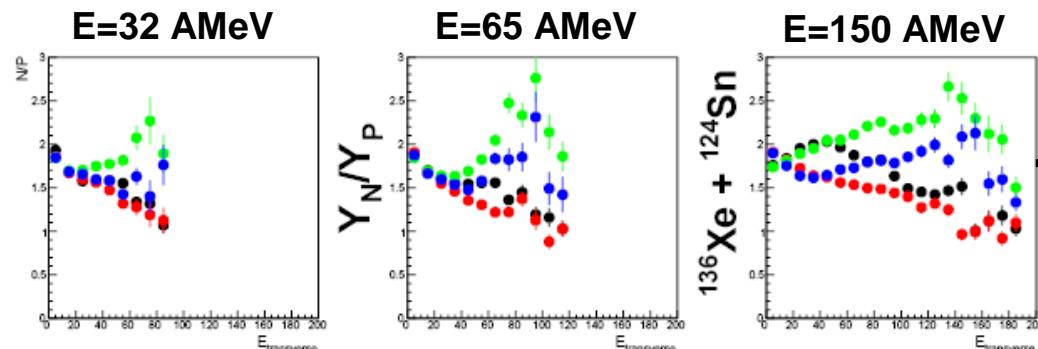
Smaller neutron excess: effects smaller

son: asysoft, $m_n^* > m_p^*$
stn: asystiff, $m_n^* > m_p^*$
sop: asysoft, $m_n^* < m_p^*$
stp: asystiff, $m_n^* < m_p^*$

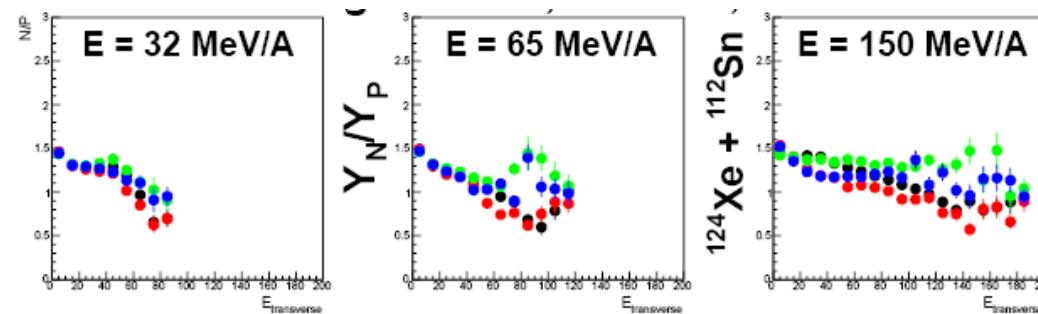
Study of Light Fragment Emission: $^{136,124}\text{Xe} + ^{124,112}\text{Sn}$, $E = 32,..,150 \text{ AMeV}$,

Double ratios

Single ratio n/p
neutron rich
 $^{136}\text{Xe} + ^{124}\text{Sn}$



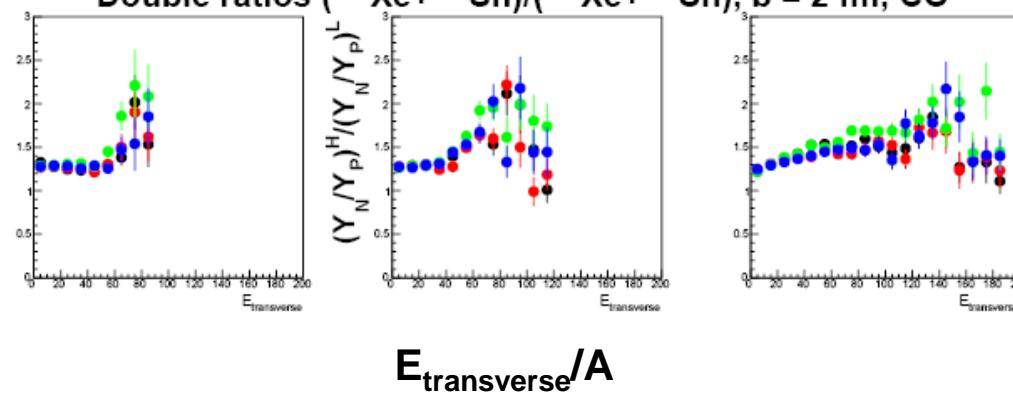
Single ratio n/p
neutron poor
 $^{124}\text{Xe} + ^{112}\text{Sn}$



Double ratio n/p
neutron rich
neutron poor

son: asysoft, $m_n^* > m_p^*$
stn: asystiff, $m_n^* > m_p^*$
sop: asysoft, $m_n^* < m_p^*$
stp: asystiff, $m_n^* < m_p^*$

Double ratios $(^{136}\text{Xe} + ^{124}\text{Sn}) / (^{124}\text{Xe} + ^{112}\text{Sn})$, $b = 2 \text{ fm}$, CO

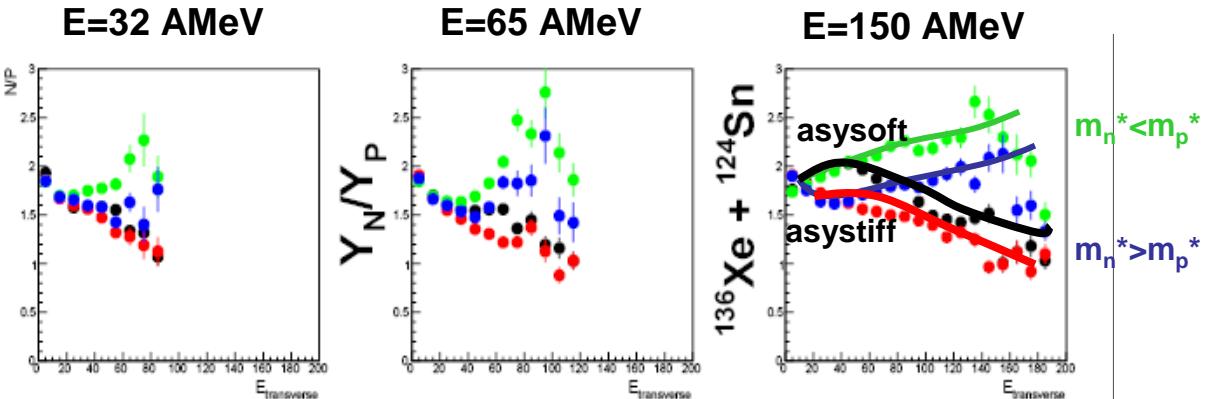


Ratio

Double ratio also shows effect but less sensitive to symmetry energy

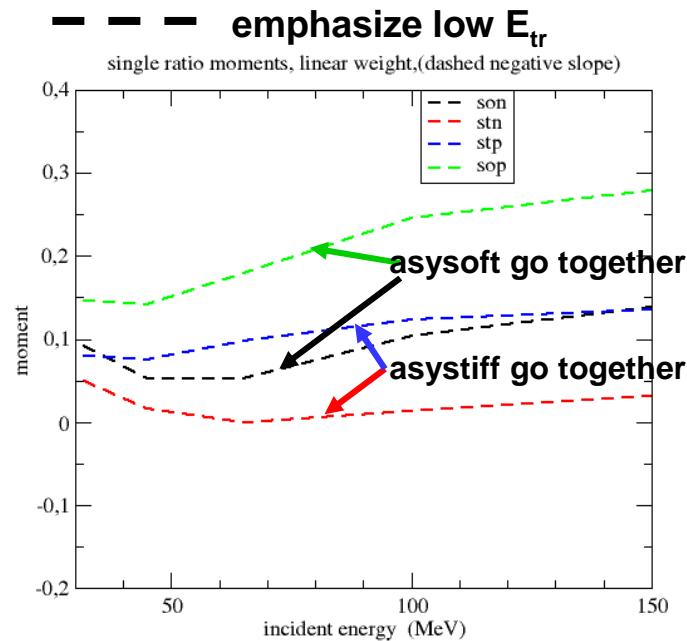
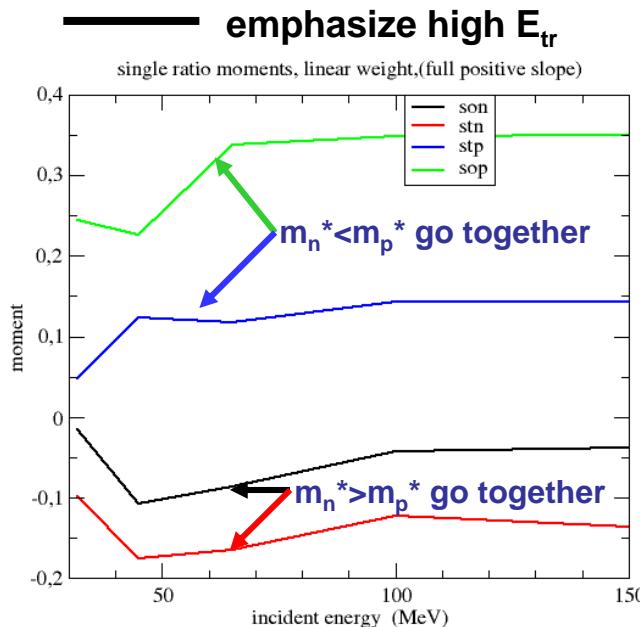
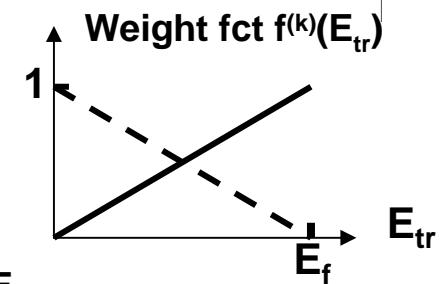
Moments of single ratios: characterize behavior on asy-EoS and m^*

son: asysoft, $m_n^* > m_p^*$
stn: asystiff, $m_n^* > m_p^*$
sop: asysoft, $m_n^* < m_p^*$
stp: asystiff, $m_n^* < m_p^*$



Calculate moments of the single ratios, e.g. for n/p ratio

$$R_{\alpha}^{(k)}(E_{inc}) = \frac{1}{E_f} \int_0^{E_f} dE_{tr} f^{(k)}\left(\frac{E_{tr}}{E_f}\right) \left(R_{\alpha}(E_{tr}, E_{inc}) - \frac{N}{Z} \right)$$

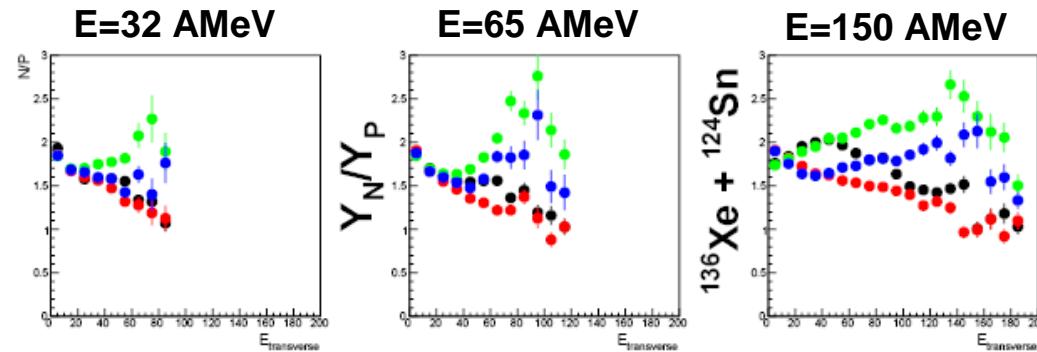


Characteristic patterns depending on asy-EoS and m^* : perhaps an observable?

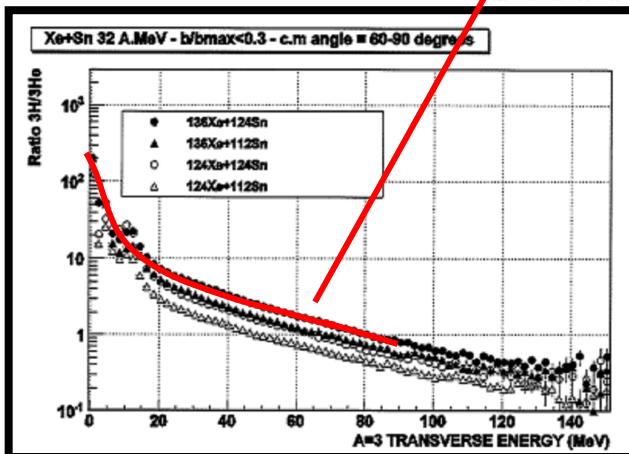
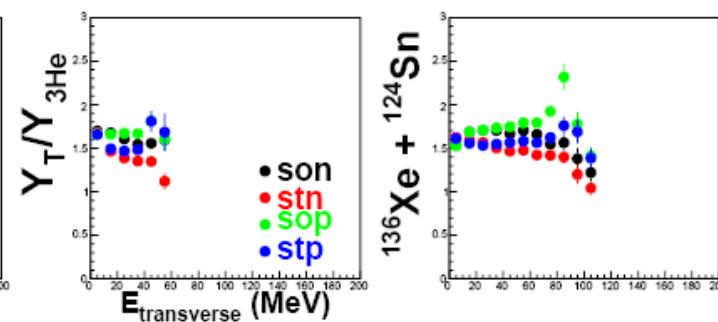
Comparison to data $^{136,124}\text{Xe} + ^{124,112}\text{Sn}$, $E = 32, \dots, 150$ AMeV
 data R. Bougault, A. Chbihi (Ganil, prelim, IWM11)

son: asysoft, $m_n^* > m_p^*$
 stn: asystiff, $m_n^* > m_p^*$
 sop: asysoft, $m_n^* < m_p^*$
 stp: asystiff, $m_n^* < m_p^*$

Single ratio n/p
 neutron rich
 $^{136}\text{Xe} + ^{124}\text{Sn}$



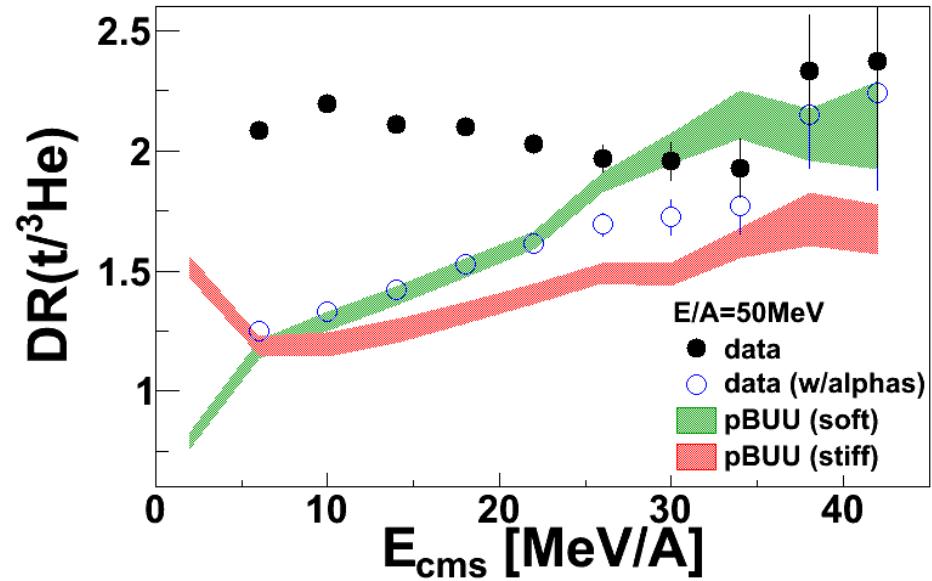
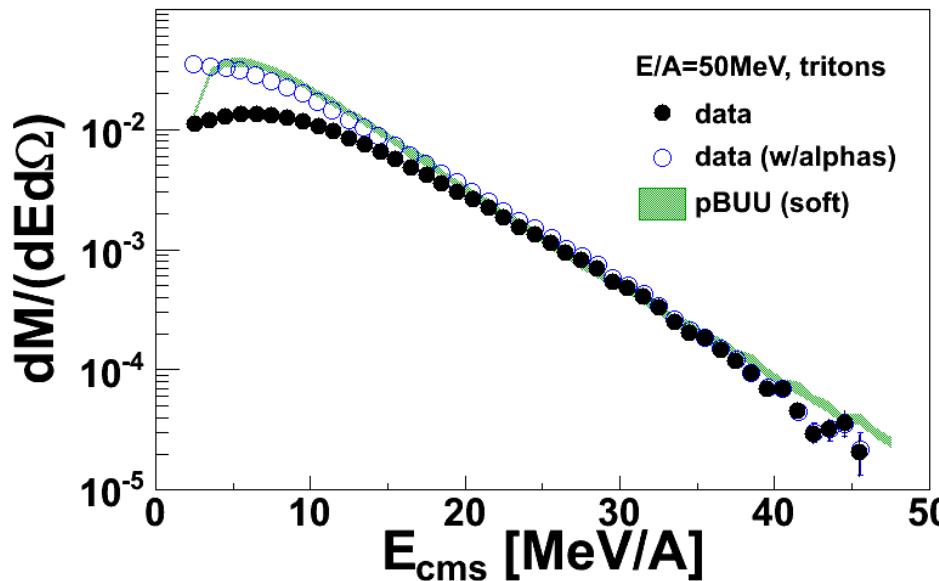
Single ratio t/3He
 neutron rich
 $^{136}\text{Xe} + ^{124}\text{Sn}$



Prelim. data
 136+124, 32 AMeV,
 INDRA
 favors asy-stiff with
 $m_n^* > m_p^*$

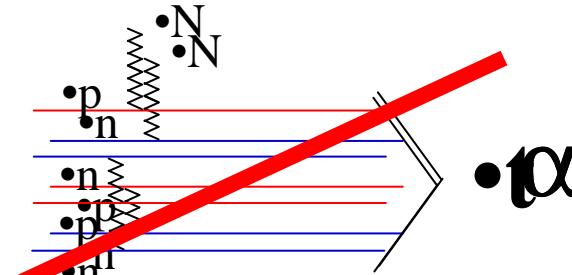
The single t/3He ratio seems to be a promising observable

Similar work at MSU: W.Lynch, INPC, Florence, 2013: $^{124,112}\text{Sn} + ^{124,112}\text{Sn}$, 50 AMeV
Calculations using P. Danielewicz code with clusters



- Illustrate with Danielewicz BUU with cluster production.
- Approx. QM description of cluster production up to A=3 (but not beyond)
- $m^* = 0.7m_0$, $m_p^* = m_n^*$
- Calculations underpredict the double-ratio

Figure courtesy of Z. Chajecki.



- Alpha production not included in the model
 \Rightarrow alphas end up being t or ${}^3\text{He}$
- Check: combine experimental alpha spectra with tritons and helium-3 and compare to the model predictions.
- Need to extend cluster production past A=4.

Conclusions:

- The nuclear symmetry potential is density and momentum dependent; both behaviors are not well known from nuclear matter theories
- both are important for isospin sensitive observables in HIC; one should study many observables simultaneously to constrain the symmetry energy
- the ratios of p/n and light fragment spectra are promising to disentangle the density and momentum behaviour.
new observable: moments of ratios?
- more experimental data desirable (FRIB!)
- but also improved treatment of cluster production in transport calc.

Thank you

backup

Effective masses:

Non-relativistic mass

$$m_{NR}^* = \left[m + \frac{1}{k} \frac{dU_{sp}}{dk} \right]^{-1} = k \left[\frac{dE}{dk} \right]^{-1}$$

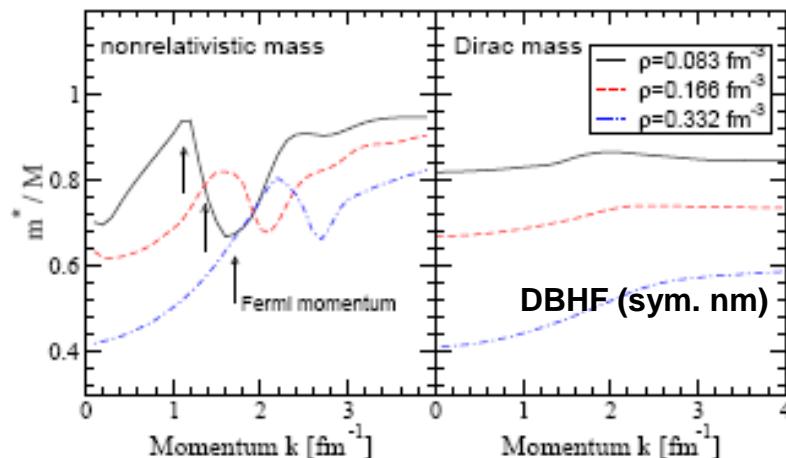
Parametrizes non-locality in space (k-mass) and time (E-mass)

Dirac mass $m_D^* = m + \Sigma_s$

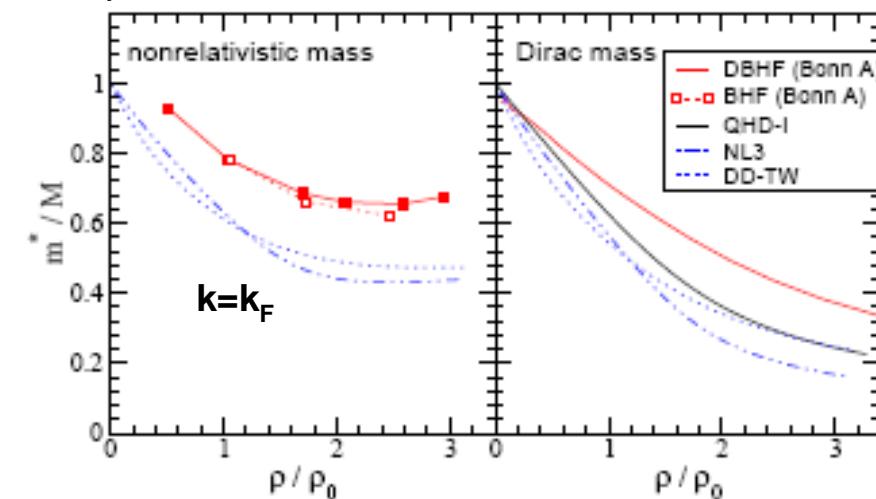
Includes part of the interaction;

$$\text{relativistically } U_{sp} \approx \frac{m_D^*}{E^*} \Sigma_s + \Sigma_0$$

k-dependence



p-dependence



Effective mass splitting

$$m_{NR,n}^* \Leftrightarrow m_{NR,p}^* \quad m_{D,n}^* \Leftrightarrow m_{D,p}^*$$

BHF:	>
RMF ($\rho + \delta$)	<
DBHF (fit S)	>
DBHF (project)	>

C. Fuchs, H.H. Wolter, EPJA 30(2006)5

- | | |
|---|------------------------|
| < | Baran, PhysRep 410 |
| > | Sammarucca, PRC67 |
| < | v. Dalen, Fuchs, PRL95 |

No agreement on ordering of n/p effective masses!

Example of the isovector dependence for a particular parametrization (used in the following results, BGBD, Bombaci-Gale-Bertsch-DasGupta)

Energy density: kinetic + potential terms

$$\varepsilon_{kin}(\rho) = \frac{3}{10} \rho E_F [(1 + \beta)^{5/3} + (1 - \beta)^{5/3}] = \frac{3}{5} E_F + \frac{1}{3} \rho E_F \beta^2 \quad \beta = \frac{N-Z}{A}$$

Bombaci-Gale-Bertsch-Das Gupta (BGBD) interaction

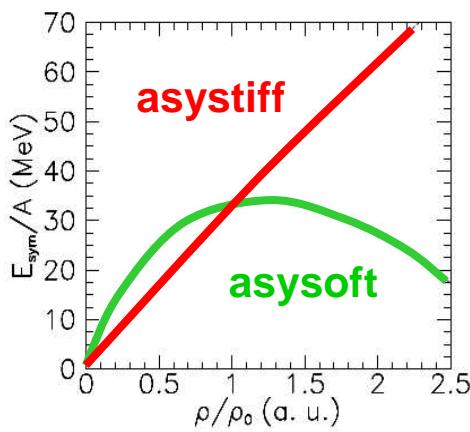
isoscalar	isovector
$\varepsilon_A = \frac{A}{2} \frac{\rho^2}{\rho_0} - \frac{A}{3} \left(\frac{1}{2} + x_0 \right) \frac{\rho^2}{\rho_0} \beta^2$	$\varepsilon_B = \frac{B}{\sigma+1} \frac{\rho^{\sigma+1}}{\rho_0^\sigma} - \frac{2}{3} \frac{B}{\sigma+1} \left(\frac{1}{2} + x_3 \right) \frac{\rho^{\sigma+1}}{\rho_0^\sigma} \beta^2$

Momentum Dependence (MD): isoscalar + isovector

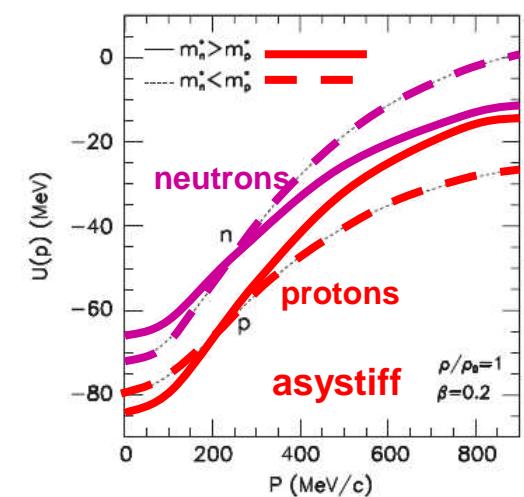
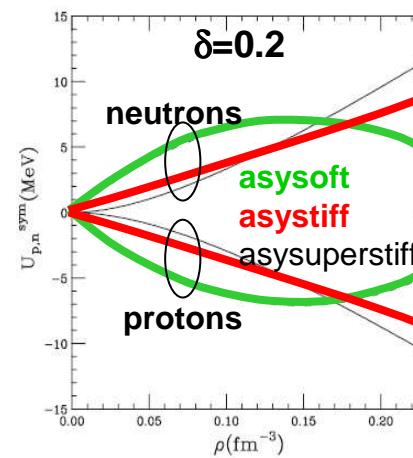
$$\varepsilon_{C,z} = \frac{8}{5\rho_0} (C + 2z) I_{np} + \frac{2}{5\rho_0} (3C - 4z) (I_{nn} + I_{pp})$$

$$I_{tt'} = \left[\frac{2}{(2\pi)^3} \right]^2 \int d^3k d^3k' f_t(\vec{r}, \vec{k}) f_{t'}(\vec{r}, \vec{k}') g(\vec{k}, \vec{k}')$$

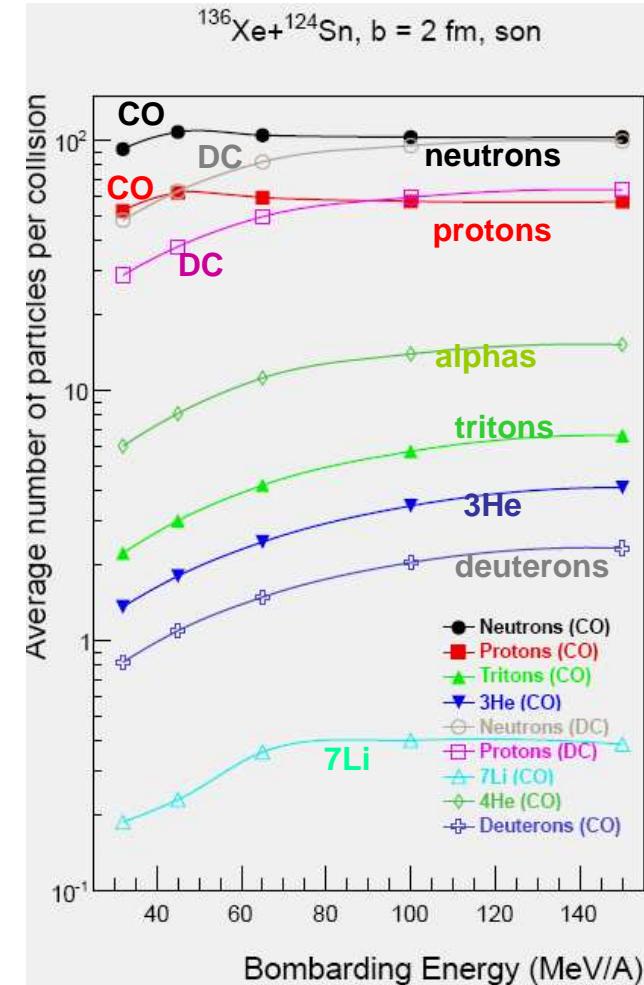
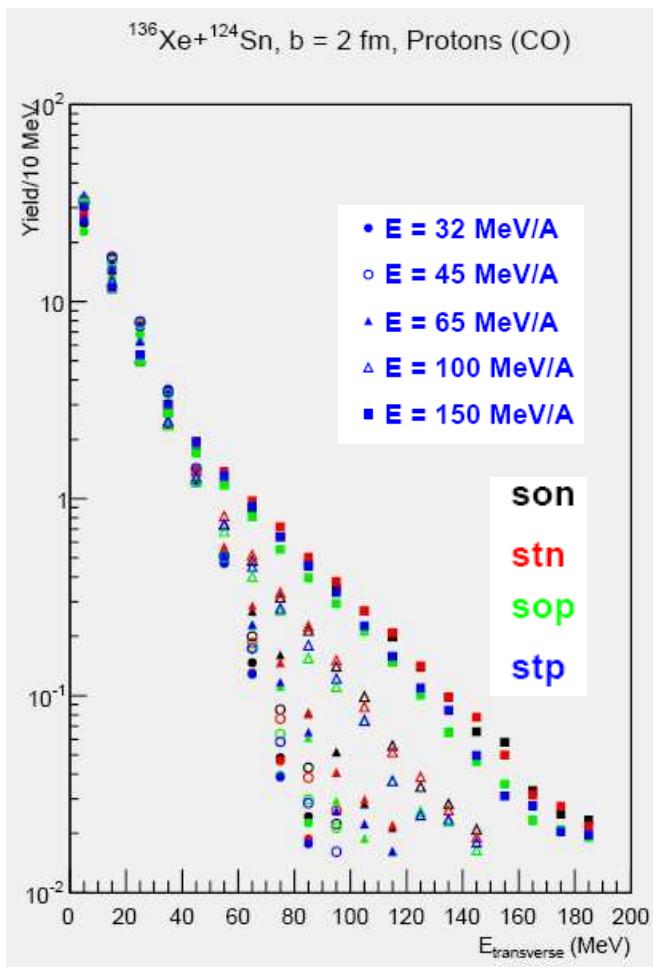
Symmetry energy



Neutron/proton potentials
as fct as fct of density for $p=p_F$ (left)
and of momentum for $\rho=\rho_0$ (right)



Study of Light Fragment Emission: $^{136,124}\text{Xe} + ^{124,112}\text{Sn}$, $E = 32,..,150$ AMeV , Spectra and multiplicities



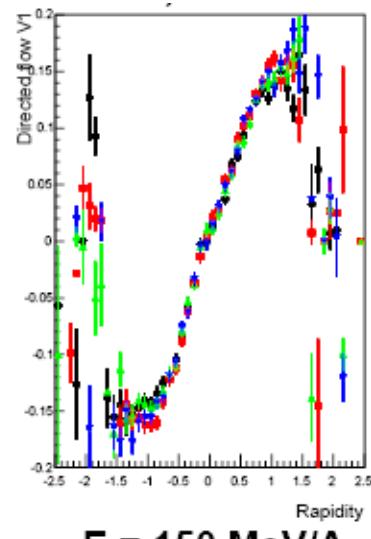
Steeper with increasing incident energy
effect of AsyEoS and m_{eff} seen

BE effects seen in particle yields
Coalescence (CO) has too many free
nucleons

Study of Light Fragment Emission: $^{136,124}\text{Xe} + ^{124,112}\text{Sn}$, $E = 32,..,150 \text{ AMeV}$,
Proton flows

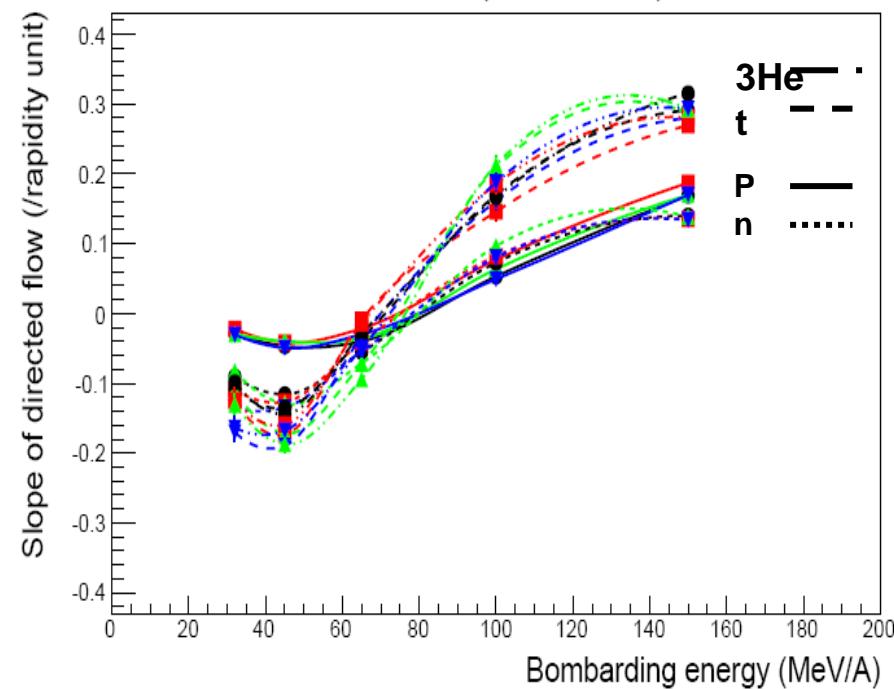
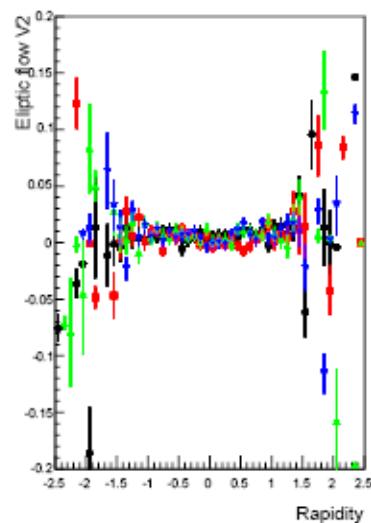
Flows: protons, $^{136}\text{Xe} + ^{124}\text{Sn}$, $b = 2 \text{ fm}$, CO

Directed flow



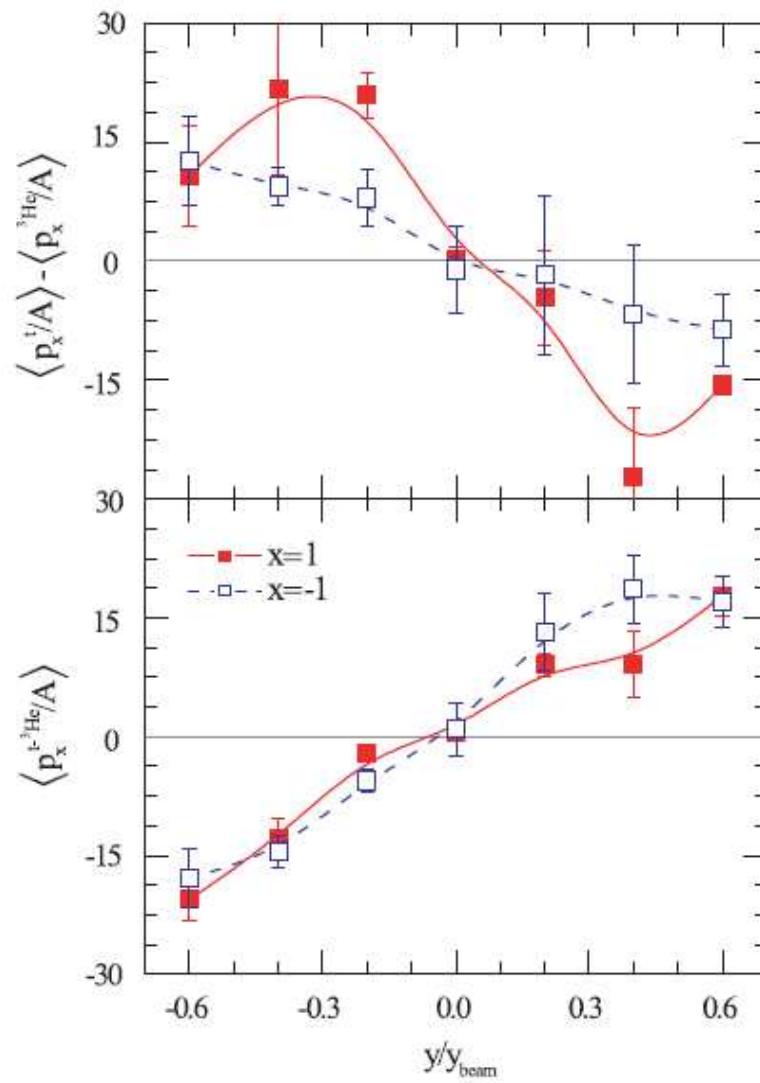
$E = 150 \text{ MeV}/\text{A}$

Elliptic flow



- Heavier particles have larger flow (known)
- AsyEoS differences small
- Look at differences or differential flow

Differential flow



G.C.Yong, et al., PRC80, 044608 (2009)

$^{132}\text{Sn} + ^{124}\text{Sn}$, 400 AMeV