A Simple Pairing Approach on the Linear Symmetry Energy Near N=Z

Ian Bentley and Stefan Frauendorf

3rd International Symposium on Nuclear Symmetry Energy
NSCL/FRIB, East Lansing, Michigan

Motivation

- We are interested in determining the origin of the linear symmetry energy term.
- The strong interaction creates a contribution to the binding energy which is of the form:

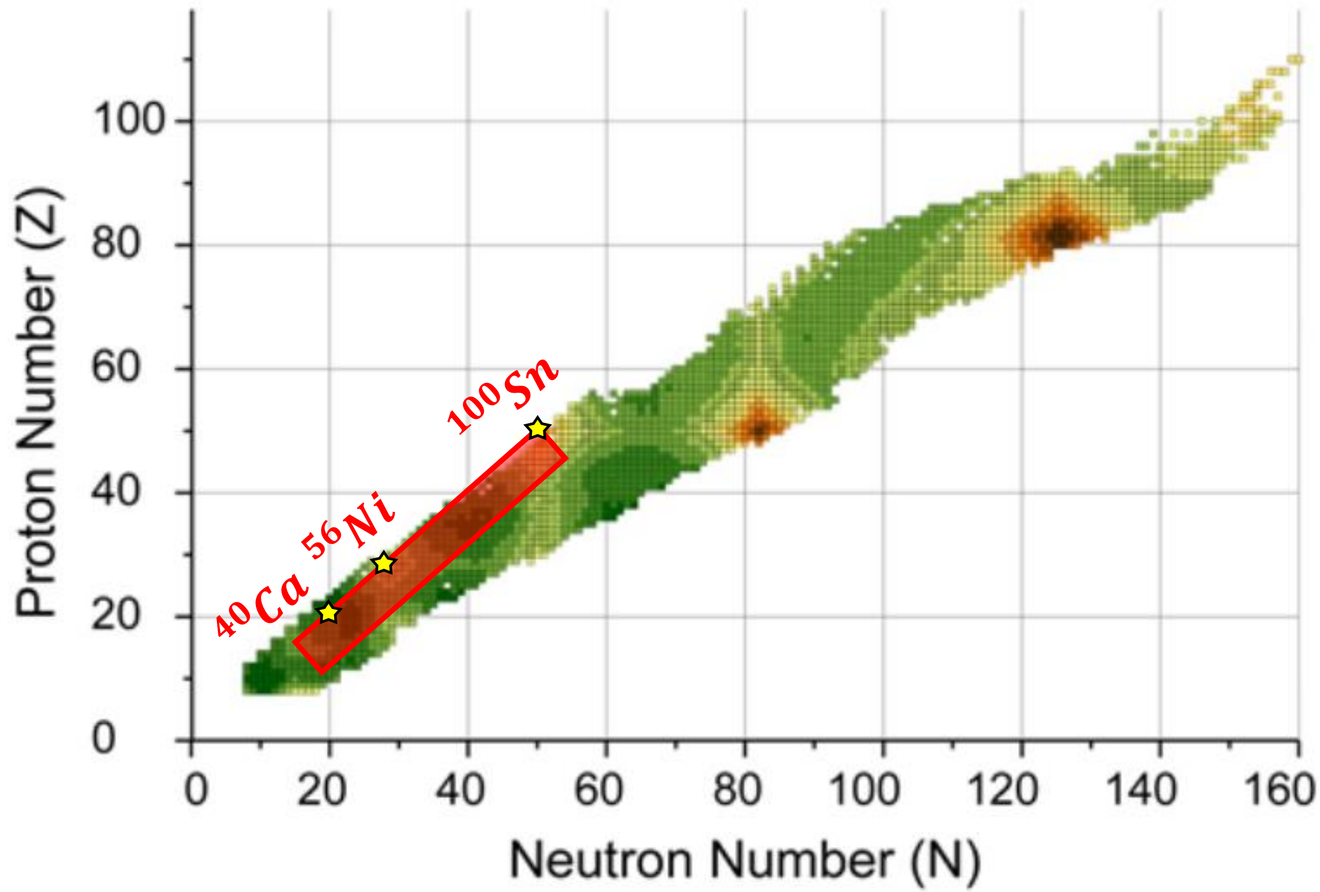
E_s=c$|T_Z|(|T_Z|+X) + E_{shell}$, where $T_Z = \frac{N-Z}{2}$.

- Approximate techniques (RPA, BCS, etc.) have critical behavior ($\Delta(G) = 0$) in which the pairing correlations are not well defined.
- We have chosen to solve this problem exactly by diagonalization of the pairing Hamiltonian.
- Only a few levels (6 or 7) nearest the Fermi surface are included.

$E_s = c|T_Z|(|T_Z| + X) + E_{shell}$

Region of Interest

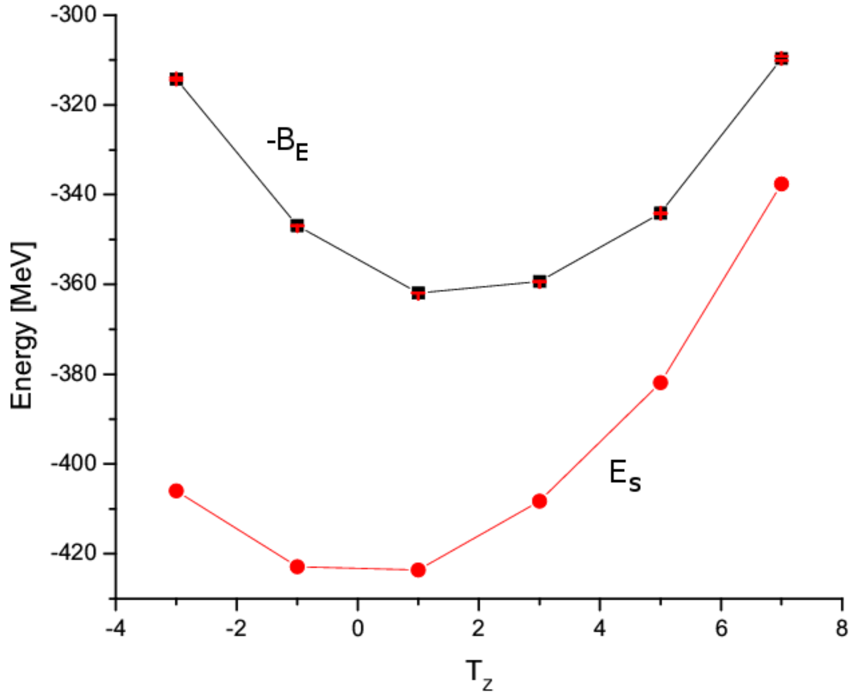
- Experimental evidence of a linear term symmetry energy term exists for $28 \leq A \leq 100$.
- More N=Z measurements are needed.
- There are 3 doubly magic nuclei which exhibit unique behavior.

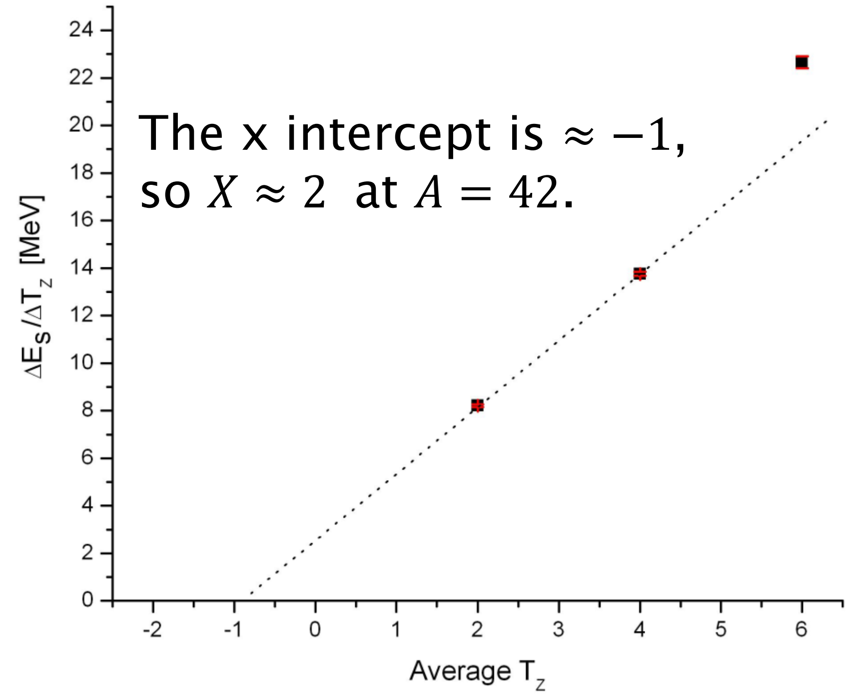


SAINT MARY'S COLLEGE NOTRE DAME, IN

$$E_s = c|T_Z|(|T_Z| + X) + E_{shell}$$

The Determination of X





The x intercept is ≈ -1, so $X \approx 2$ at $A = 42$.

$$\begin{aligned} E_S(Z+1,A) - E_S(Z-1,A) &= B_E(Z+1,A) - B_E(Z-1,A) \\ &- 2\Big(1.412(\pm 0.014) Z A^{-\frac{1}{3}} \\ &- 0.610(\pm 0.048) Z A^{-1} \\ &- 0.719(\pm 0.007) Z^{\frac{1}{3}} A^{-\frac{1}{3}}\Big)\ [MeV] \end{aligned}$$

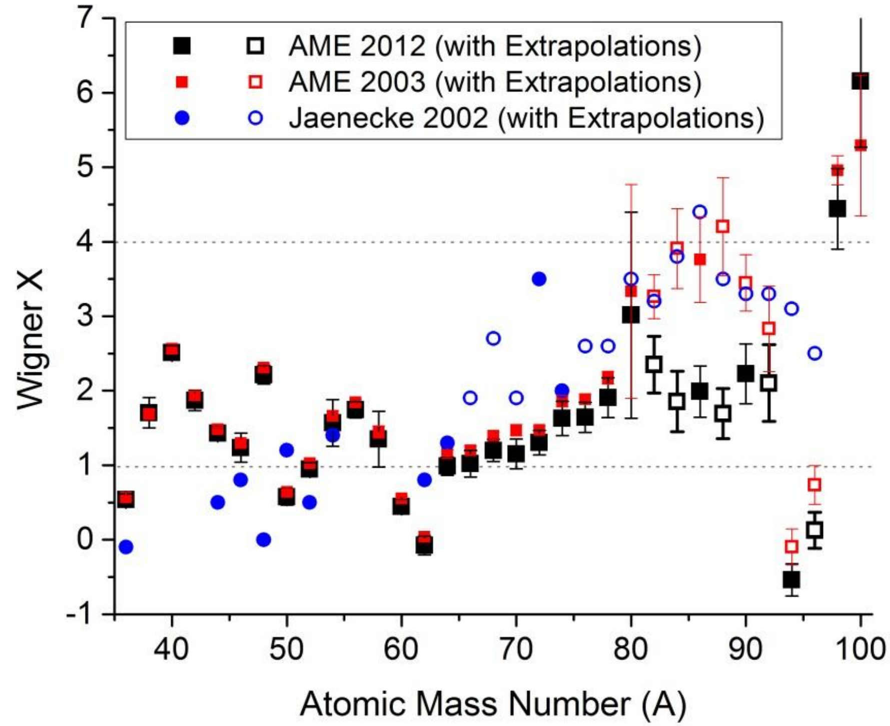
- Estimate the Coulomb energy using mirror nuclei.
- Remove Coulomb contribution.
- Use differences in the symmetry energy to find X.

$$\begin{aligned} \frac{\partial E_S}{\partial T_Z}\Big|_{T_Z=\bar{T}_Z} &= \frac{\Delta E_S}{\Delta T_Z}\Big|_{T_Z=\bar{T}_Z} \\ &= 2c(|\bar{T}_Z| + X/2) + E'_{shell} \end{aligned}$$

SAINT MARY'S COLLEGE NOTRE DAME, IN

The Evolving X

- This investigation was inspired by the work of Jänecke et al. who observed a shift in the linear term from $X \approx 1$ to $X \approx 4$.
- Using the 2003 Atomic Mass Evaluation (AME), we have observed roughly the same transition.
  - Different Coulomb fits are used and some new measurements are included.
  - At $A = [94{,}96]$, $X \approx 1$ is new.
- The AME 2012, contains many important changes:
  - new masses at ^{86}Mo and ^{90}Ru,
  - cause new extrapolations in the $80 \leq A \leq 90$ region for ^{82}Zr, ^{84}Mo, ^{88}Ru, and ^{92}Pd.



$$X_E(A) = \left(\frac{6E_S(T_z=0) - 8E_S(T_z=2) + 2E_S(T_z=4)}{-E_S(T_z=0) + 2E_S(T_z=2) - E_S(T_z=4)} \right)$$

$$X_O(A) = \left(\frac{8E_S(T_z=1) - 12E_S(T_z=3) + 4E_S(T_z=5)}{-E_S(T_z=1) + 2E_S(T_z=3) - E_S(T_z=5)} \right)$$

The Hamiltonian

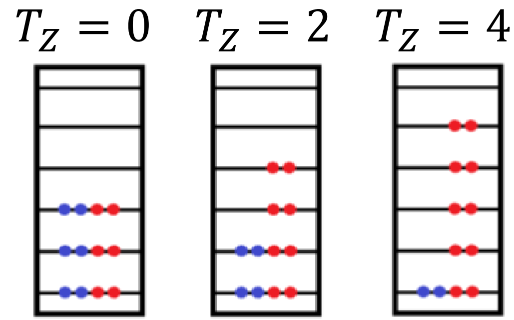
- The Hamiltonian used in this evaluation is of the form:

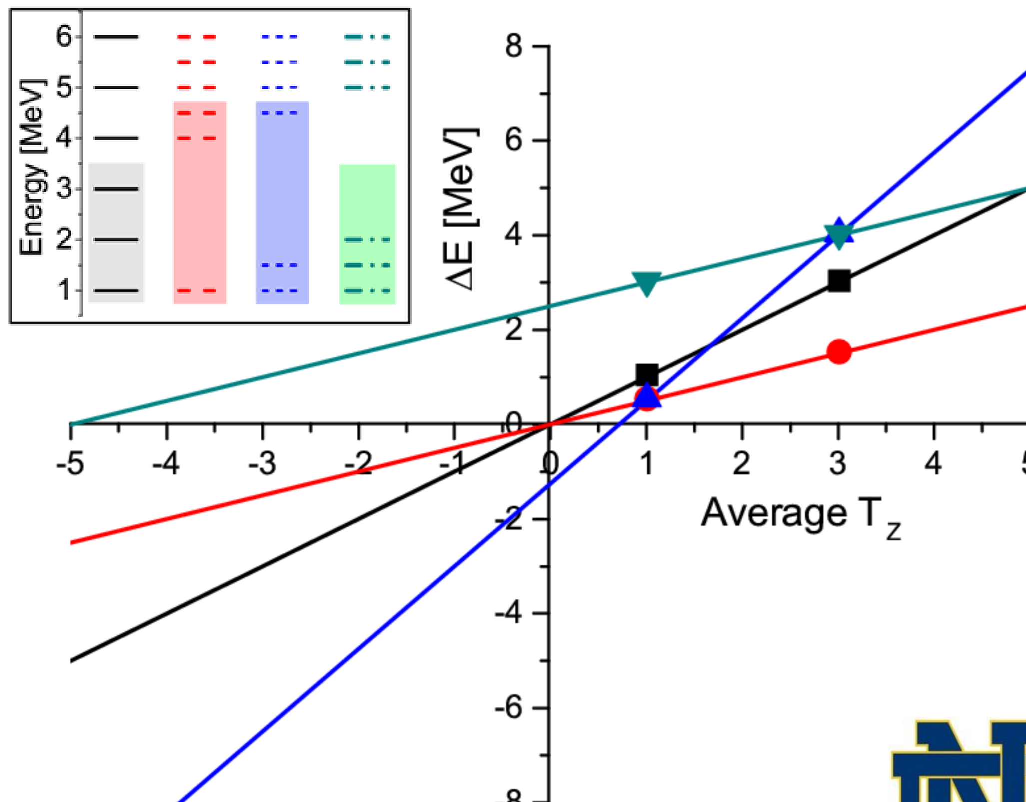
$$\hat{H} = \sum \epsilon_k \hat{N}_k - G_V \sum \left(\hat{P}^\dagger{}_{k(pp)} \hat{P}_{k'(pp)} + \hat{P}^\dagger{}_{k(pn)} \hat{P}_{k'(pn)} + \hat{P}^\dagger{}_{k(nn)} \hat{P}_{k'(nn)} \right)$$
$$-G_s \sum \left(\hat{S}^\dagger{}_{k(pn)} \hat{S}_{k'(pn)} \right) + \mathrm{C} \vec{T} \cdot \vec{T}$$

- The first term is the sum over occupied levels.
- The second term is the monopole isovector interaction.
- The monopole isoscalar interaction term contains only anti-aligned spin pairs.
- The spin aligned pairs carry angular momentum and should be largely decoupled from the ground state.
- The final term accounts for the isospin dependence of the single particle levels.
- Neergård has determined that this gives a contribution of the form $CT(T+1)$, resulting from a collective rotation in isospace.

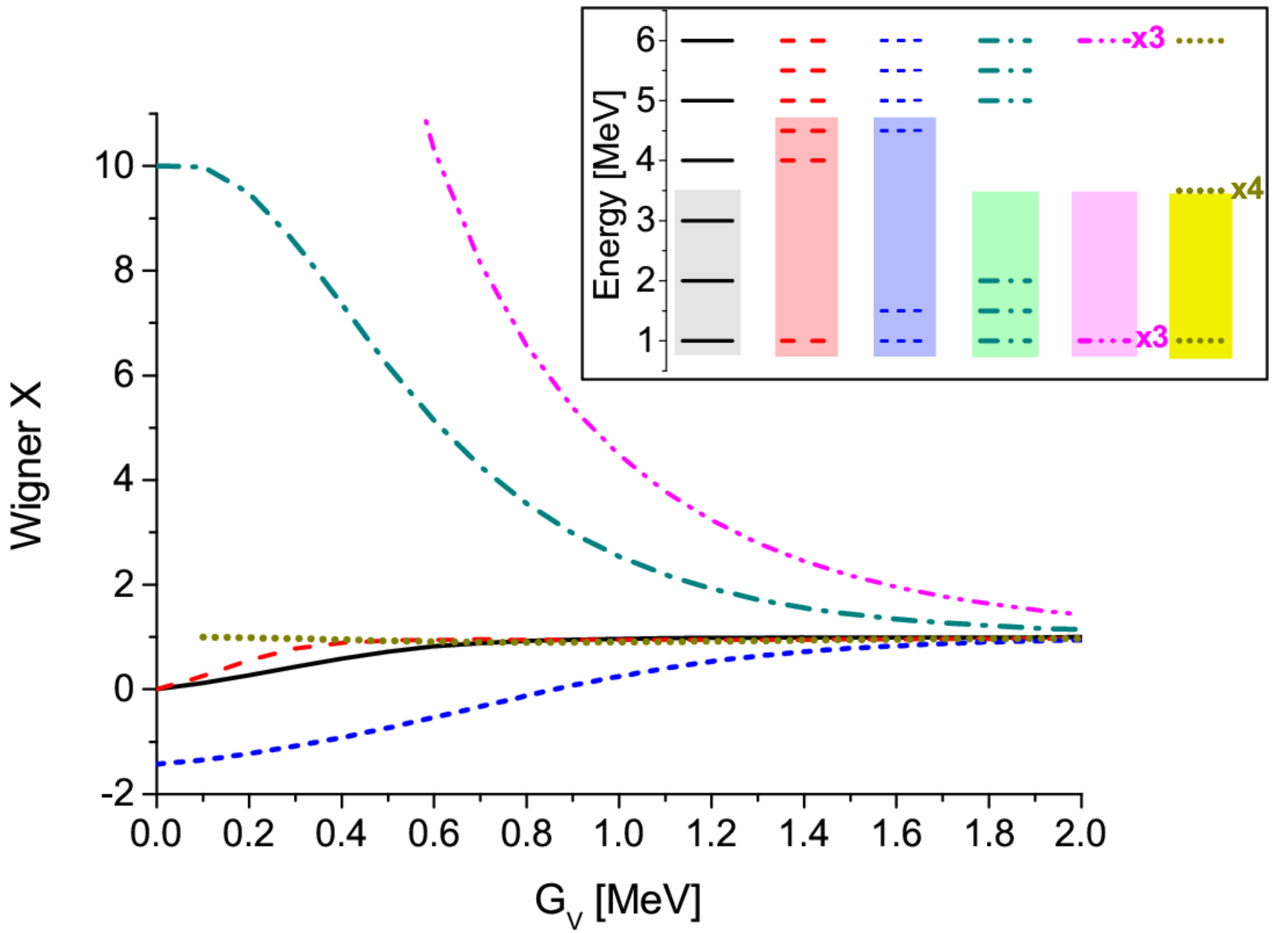
The No Pairing Limit ($G_V = G_S = 0$) with $C = 0$

- In the no pairing limit, the level distribution alone determines X.
- This is because as T_Z increases a pair of protons is removed and a pair of neutrons is added.
- The values of X ranges from large positive values to small negative values.
- If the levels are completely degenerate, then E_S is a constant, and X is ill-defined because the slope is zero.





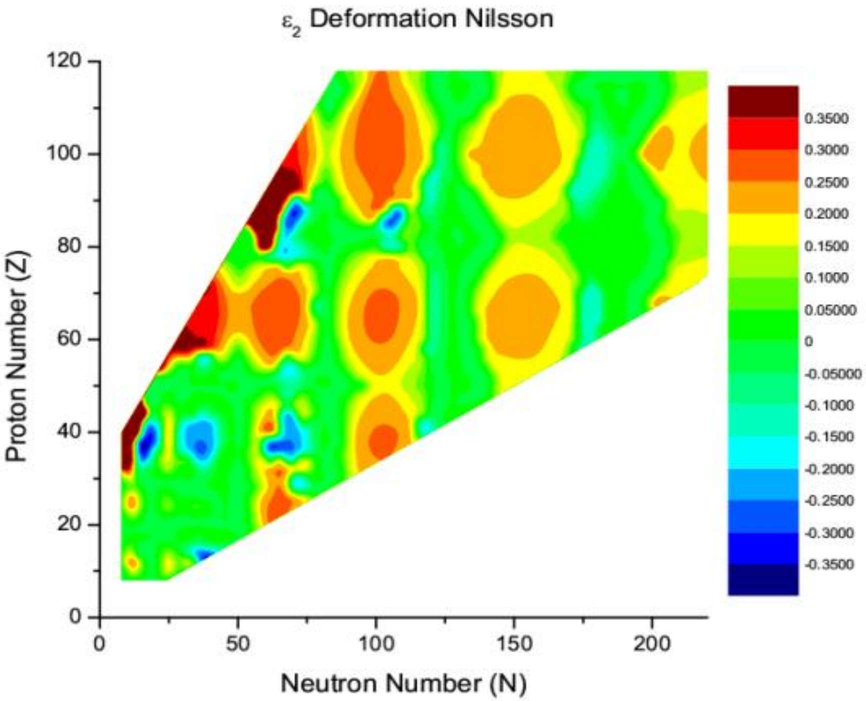
Schematic Calculations



- This explains the observed up-down feature seen near doubly magic nuclei (^{40}Ca, ^{56}Ni and ^{100}Sn).
- More realistic levels might not be static.
- We will need to know the equilibrium deformations, and then generate the corresponding levels.

Deformations and Levels

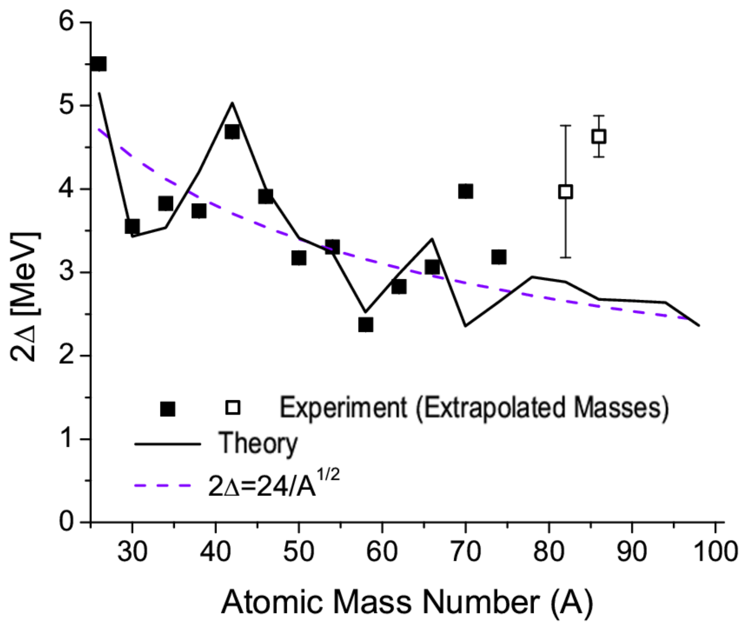
- A mixed micro-macro method can be used to determine the equilibrium deformations.
  - This involves a deformed liquid droplet.
  - Strutinski renormalization is used to combine the microscopic contributions with the macroscopic ones.
  - BCS pairing is used and best fits Δ_n and Δ_p are used given by Möller and Nix.
- The deformation parameters $(\varepsilon_2, \varepsilon_4, \gamma)$ determine the potential energy surface.
- The equilibrium deformation corresponds to the minimum.
- The resulting deformations are comparable to the state of the art (e.g. FRDM, HFB-21).
- The levels used in the pairing calculations correspond to the average of the proton and neutron Nilsson levels nearest the $N = Z$ Fermi surface.



Determining $G_V(A)$ and $C(A)$ with $G_S=0$

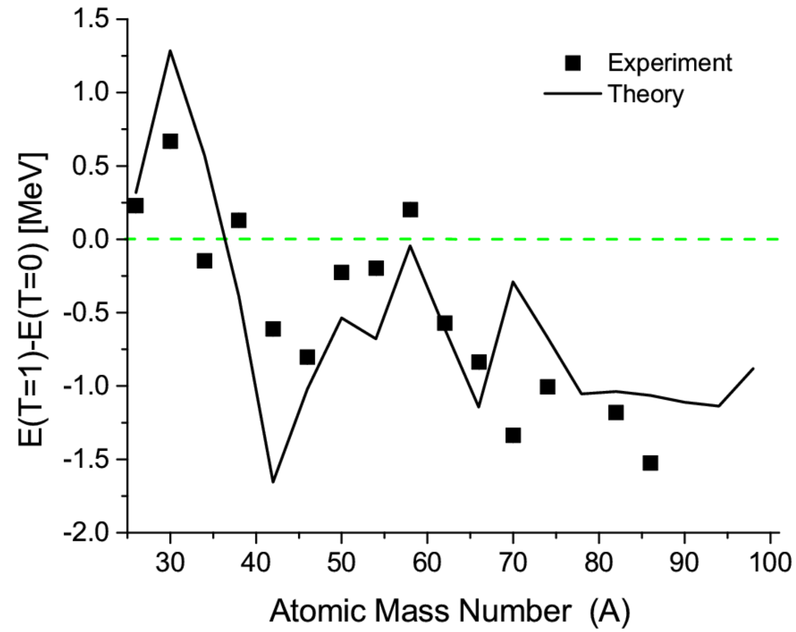
Even-even, odd-odd T=0 pairing gap at N=Z:

$$2\Delta(N,Z) = \frac{1}{2}[B_E(N-1,Z-1) - 2B_E(N,Z) + B_E(N+1,Z+1)]$$



$$G_V(A) = 13.9\ A^{-3/4}\ [MeV]$$

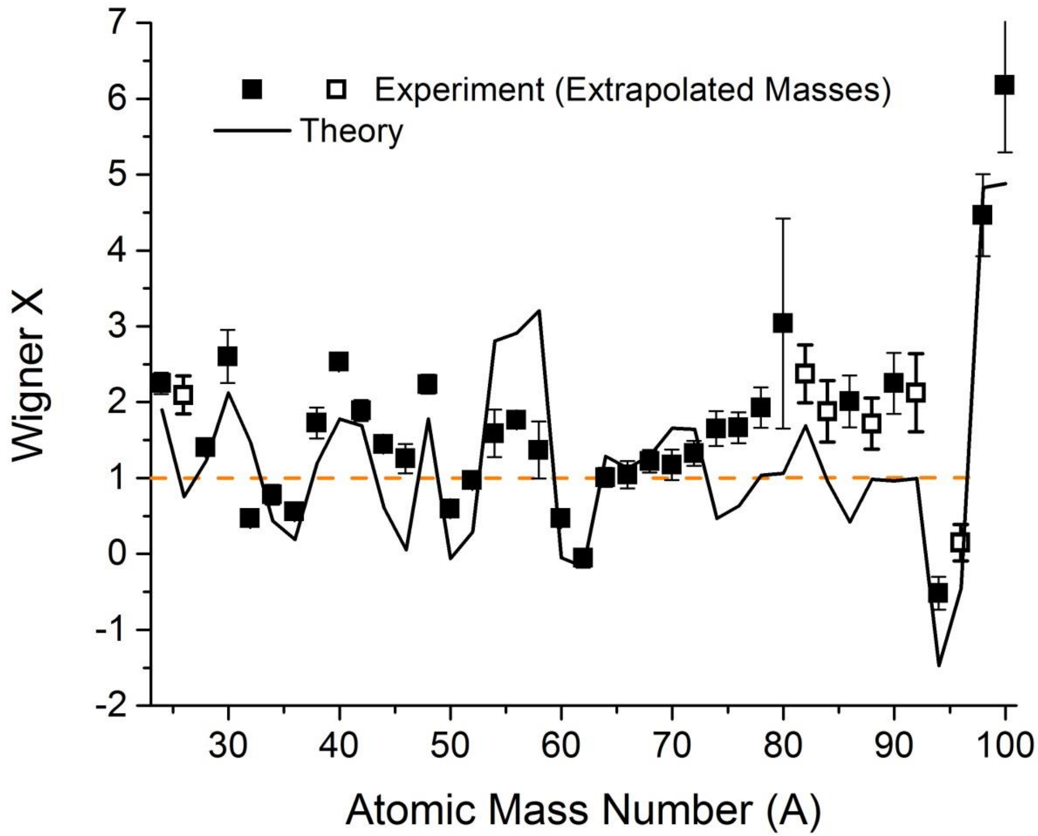
Low lying isospin of odd-odd N=Z nuclei.



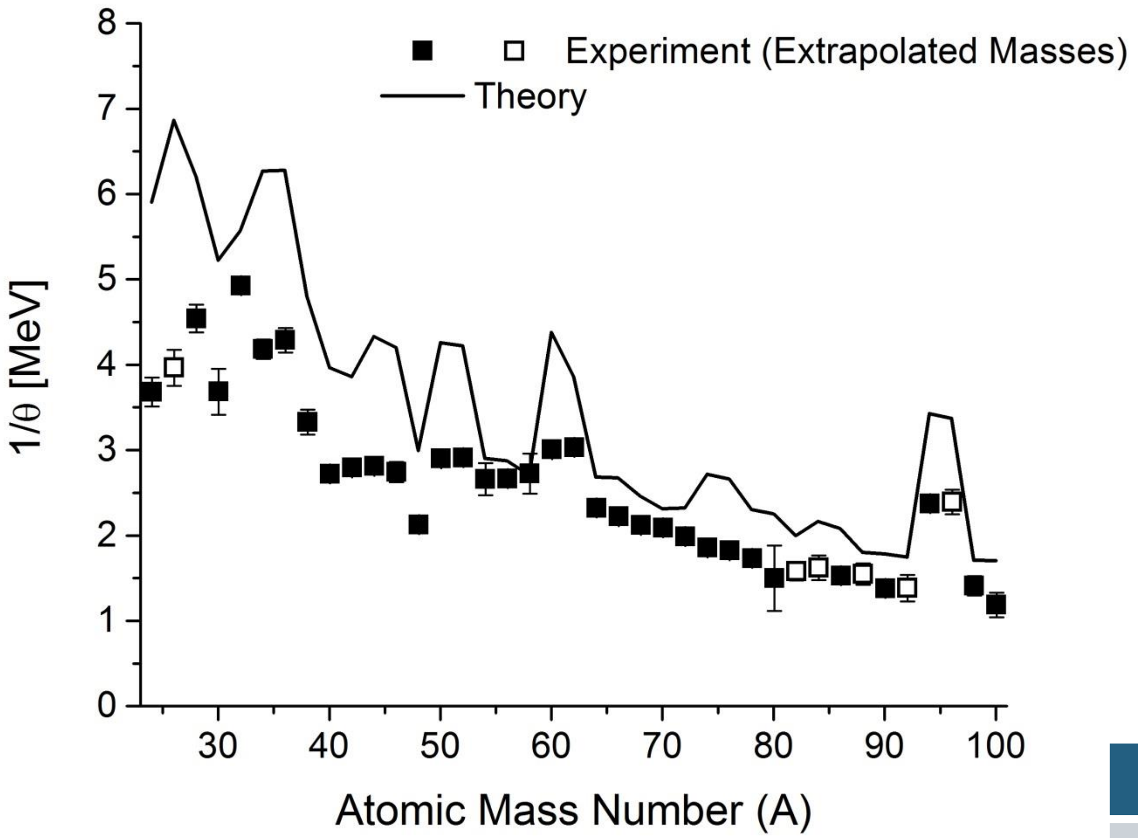
$$C(A) = 58.9\ A^{-1}\ [MeV]$$

Resulting X with $G_S=0$

- The values of the Wigner X are in good agreement with what is observed experimentally.
- All of the up-down features coming from changes in level density are reproduced, but some not always to the exact value.
- Discrepancies are likely the result from the using model deformations.



Resulting Slope with $G_S=0$



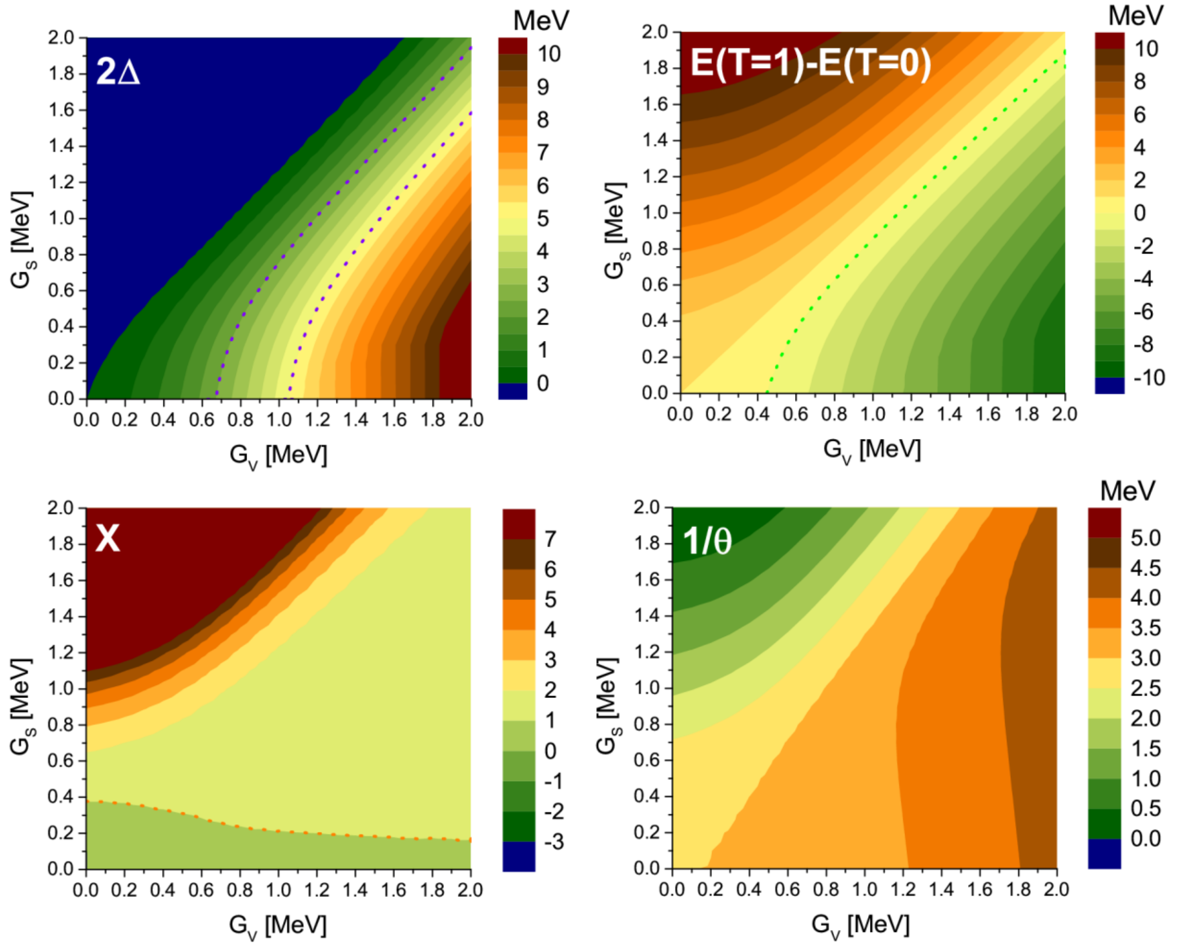
- The isomoment of inertia, does not come out well.
- This is a result of performing the calculations with so few levels.
- The pairing correlations at high T_Z have been artificially diminished.

T_Z	$\#_{Even}$	$\#_{Odd}$
0,1	3647	3647
2,3	1001	1890
4,5	70	210

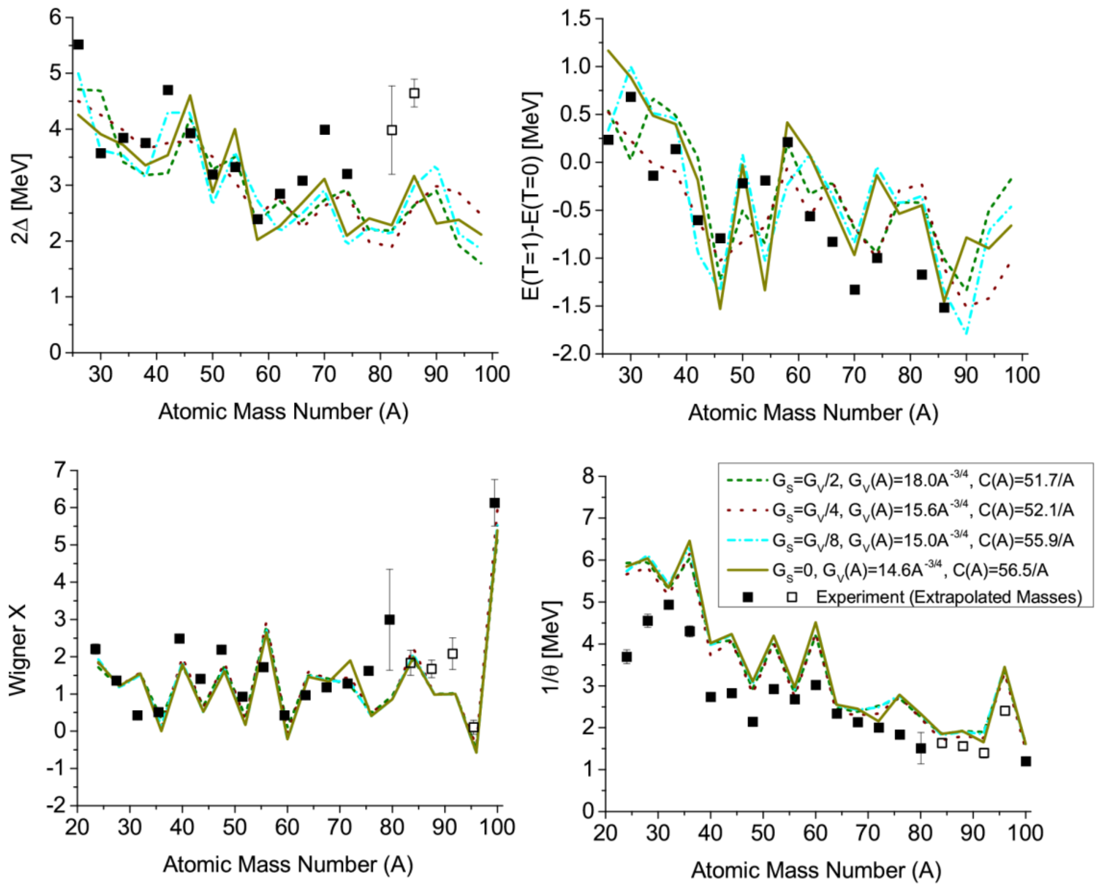
SAINT MARY'S COLLEGE NOTRE DAME, IN

Including Isoscalar Interactions

- The isoscalar interaction can also be varied.
- These are the results for six evenly spaced levels and with $C = 1\ MeV$.
- Fitting the pairing gap and the low lying isospin severely limits the phase space.
- The ground state isospin further limits the range.
- The results in the allowed region are very similar.

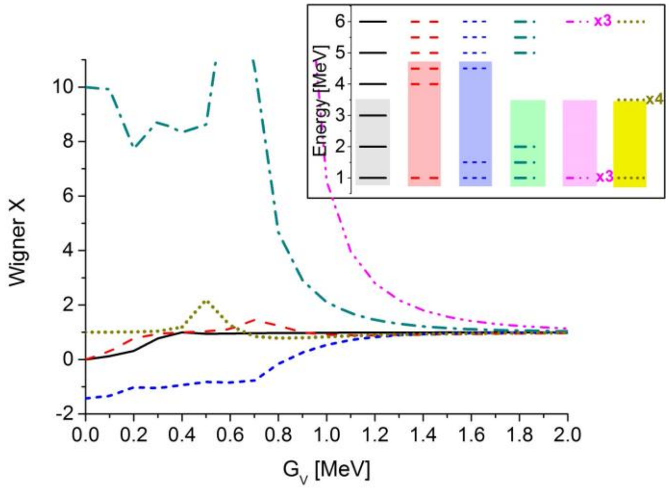


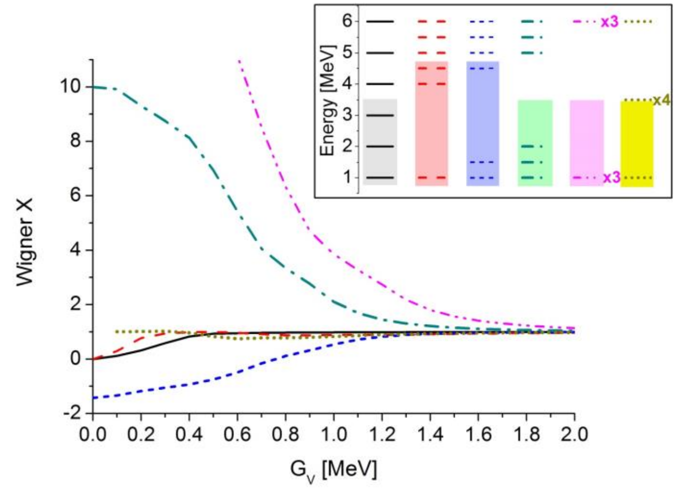
Fixed Ratio Calculations



- With the isoscalar interactions included, only six level calculations can be used.
- Fixed ratios of $\frac{G_S}{G_V}$ will allow for the same two variables, $G_V(A)$ and $C(A)$, to be fit.
  - Again, 2Δ is used to fit $G_V(A)$,
  - And the low lying isospin with the fit $G_V(A)$ is used to find $C(A)$.
- Note that only even T_Z chains are included for X and $\frac{1}{\theta}$.
- The resulting X is the same!

X from the RPA





- The Random Phase Approximation contains critical phemenona.
- The resulting X suffers as a result.
- Neergård, has created a technique that interpolates over the critical region.
  - This calculation is in good agreement with the exact calculation.
  - And it is capable of involving many levels.
- Calculations involving this technique and 50 level calculations are underway.

Measurement Wish List

- In order to verify the $A = 80$ results are fluctuations around $X = 1$, not $X = 4$, measurements of $N \approx Z$ nuclei would be helpful.
- Specifically, high precision measurements of ^{80}Zr, ^{82}Zr, ^{84}Mo, ^{88}Ru, and ^{92}Pd are needed.
- The trend above $A = 100$, is not known so low $T_Z \approx 0$ nuclei would be interesting.
- As would high $T_Z \approx 5$, below $A = 24$.
- It would also be interesting to do a study of the mirror nuclei up to $T_Z \approx -4$, currently ^{19}Mg has the lowest value in the AME 2012 measurements at $T_Z \approx -2.5$.

Conclusions

- The observed fluctuations in the linear symmetry energy term are related to the level distribution near the Fermi surface.
- The Wigner X can be calculated using Nilsson levels and a simple pairing Hamiltonian.
- Boundary issues are particularly important for the isomoment of inertia.
- The four observables of interest are relatively insensitive to the pairing correlations caused by the simple spin zero isoscalar term.
- $G_S \leq G_V$ is constrained by fitting the pairing gap.

Thank you!

Binding Energy Formula

- Even-even and odd-odd mass parabola separation can be measured for neighboring nuclei.
- Mirror nuclei have constant A, $|T_z|$, and are both even-even or odd-odd.
- An isobaric chain of even-even nuclei provides insight to the structure of the symmetry term, with the Coulomb energy removed.
- Shell effects are not removed in any of these…

$$B_E = a_v A - a_s A^{2/3} - a_{sym}(A)\,|T_z|(|T_z|+X) \pm a_p A^{-1/2} + E_{shell}$$

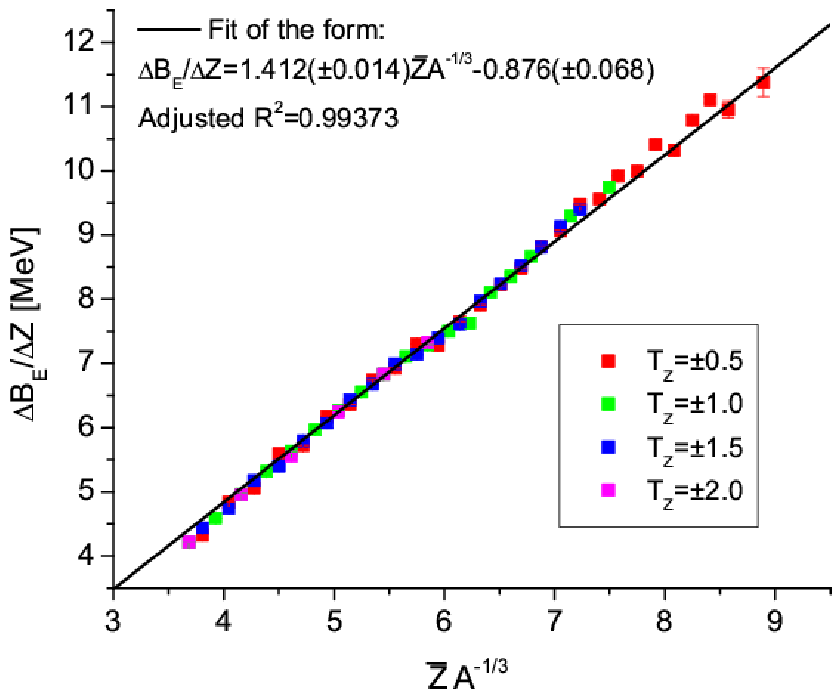
$$-\frac{3}{5}\frac{q^2}{4\pi\epsilon_0 R_e} Z^2 A^{-1/3}\left(1 - \frac{5}{6}\left(\frac{d\pi}{R_e}\right)^2 A^{-2/3} - 5\left(\frac{3}{16\pi}\right)^{2/3} Z^{-2/3}\right)$$

Coulomb Fit

$$E_{CMS} = \frac{3}{5}\frac{q^2}{4\pi\epsilon_0 R_0} Z^2 A^{-1/3}\left(1 - \frac{5}{6}\left(\frac{d\pi}{R_0}\right)^2 A^{-2/3} - 5\left(\frac{3}{16\pi}\right)^{2/3} Z^{-2/3}\right)$$

$$\Delta E_{CMS} = \frac{\Delta E}{\Delta Z} \approx \left.\frac{\partial E}{\partial Z}\right|_{Z=\bar{Z}} = \frac{3}{5}\frac{q^2}{4\pi\epsilon_0 R_0} 2\bar{Z} A^{-1/3}\left(1 - \frac{5}{6}\left(\frac{d\pi}{R_0}\right)^2 A^{-2/3} - \frac{10}{3}\left(\frac{3}{16\pi}\right)^{2/3} \bar{Z}^{-2/3}\right)$$

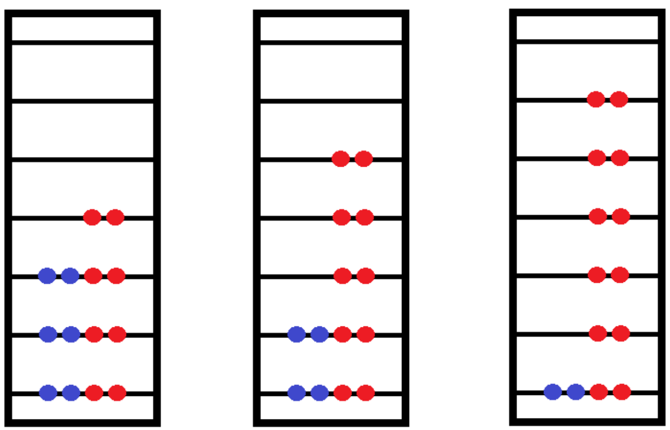
$\underline{\bar{Z} A^{-1/3}}$ $\quad$ $\cancel{\left(\frac{\bar{Z}}{A}\right)}$ $\quad$ $\cancel{\left(\frac{\bar{Z}}{A}\right)^{1/3}}$



Levels Used

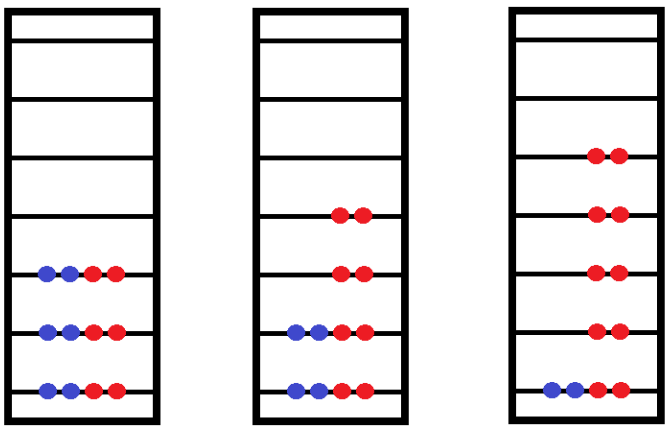
- The key to isospin conservation is to always use the same levels.
- Effort was made to not move the window.

Slope and X (Odd T_z chain)



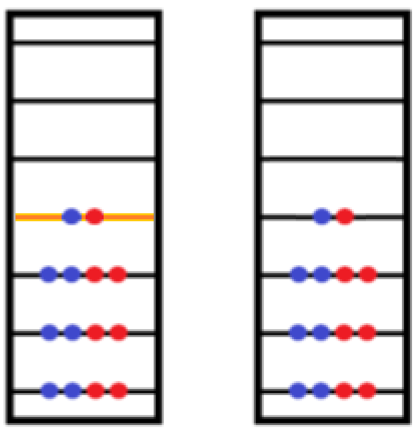
- $T_z = [1,3,5]$ for a seven level calculation.
- These are organized so that at there will be at least one pair and one hole one each level.
- Each of these also include addition of $CT(T+1)$ term.

Slope and X (Even T_z chain)



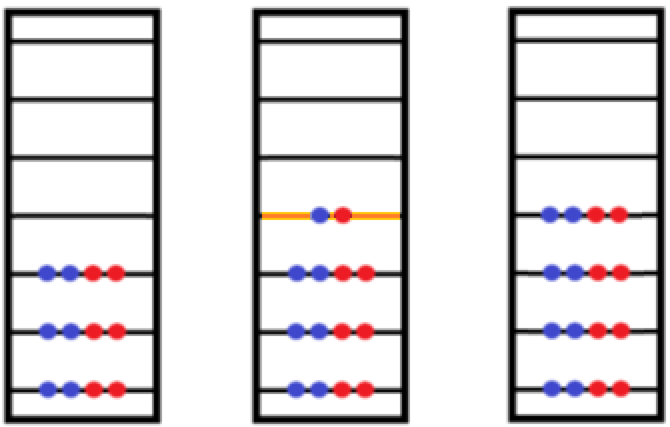
- $T_z = [0,2,4]$ for a seven level calculation.
- It was arbitrarily chosen to use 3 occupied and 4 unoccupied levels at $N = Z$.
- The other choice gives comparable results depending on the spacing of those top and bottommost levels.
- Each of these also include addition of $CT(T+1)$ term.

Isospin Inversion



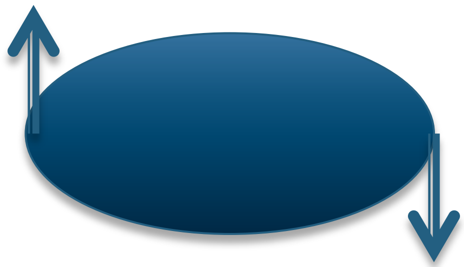
- The isospin inversion involves a blocked level.
- Blocking will remove a level and a 6 level calculation is performed.
- The blocked pair is added back in.
- The unblocked calculation involves the addition of the $CT(T+1)$ term.

Delta N=Z



- The N=Z configurations for a seven level calculation.
- These are centered about the OO nucleus.
- Again, blocking is used.
- Only the T=0 states are compared, so this does not depend on C.

Deformed nucleus



Isovector pair field

rotation in ordinary space

rotational energy:

$$E(I) = < H > + \frac{I(I+1)}{2q}$$

rotation in abstract isospace

isorotational energy :

$$E(T) = < H > + \frac{T(T+1)}{2\theta}$$

Limit of strong symmetry breaking: Wigner X=1
("large deformation" in isovector space)
The experimental X often close to 1, but not as close as for ordinary rotation. Weak deformation.