Tensor Force, Rearrangement & Symmetry Energy

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I will show that ...

- ✓ Tensor force plays a crucial role for both $E_{sym}(\rho)$ & $L(\rho)$

- ✓ Negligible contribution of rearrangement term to $E_{sym}(\rho)$ & $L(\rho)$

based on:

Phys. Rev. C 84, 062801 (R) (2011)

In preparation...

Symmetry Energy & Tensor Force

"Analysis of the contribution of the different terms of the NN force to E_{sym} and L"

using:

- ✓ BHF with Av18 + UIX
- ✓ Hellmann-Feynman Theorem

In collaboration with:

C. Providência

A. Polls

BHF approximation of ANM

Energy per particle

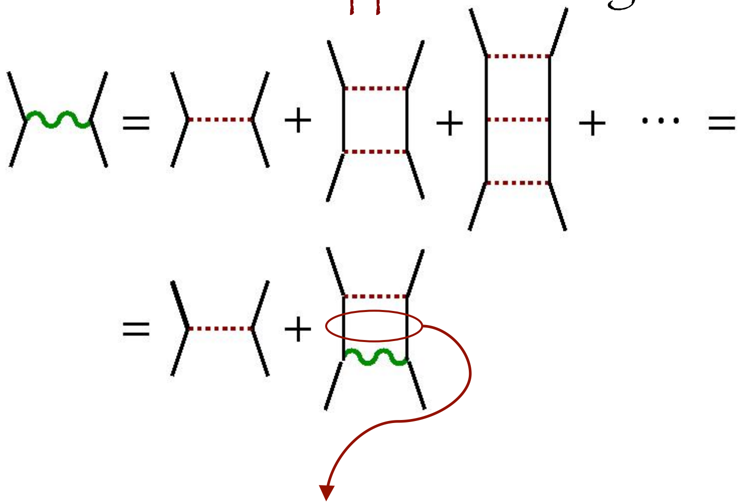
- $$\frac{E}{A}(\rho,\beta) = \frac{1}{A}\sum_{\tau}\sum_{k\le k_{F_\tau}}\left(\frac{\hbar^2 k^2}{2m_\tau} + \frac{1}{2}\mathrm{Re}\left[U_\tau(\vec{k})\right]\right)$$

Infinite sumation of two-hole line diagrams

Bethe-Goldstone Equation

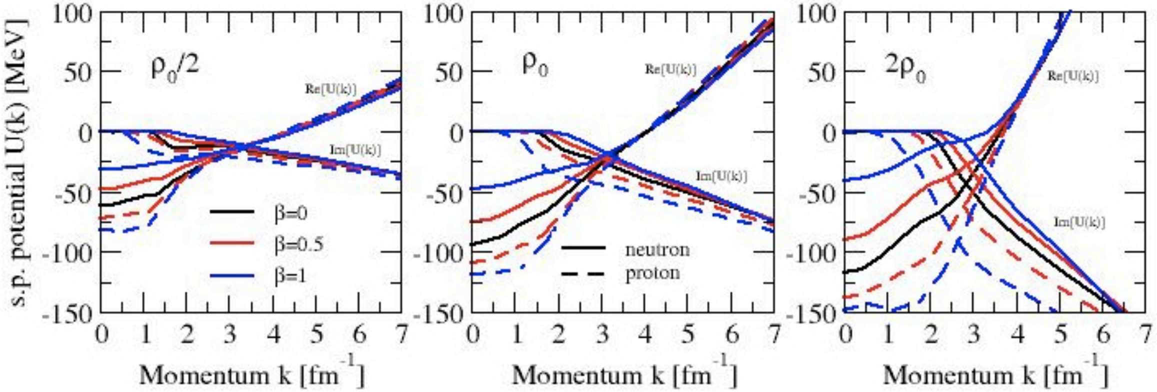
- $$G(\omega) = V + V\frac{Q}{\omega - E - E' + i\eta}G(\omega)$$
- $$E_\tau(k) = \frac{\hbar^2 k^2}{2m_\tau} + \mathrm{Re}\left[U_\tau(k)\right]$$
- $$U_\tau(k) = \sum_{\tau'}\sum_{k'\le k_{F_{\tau'}}}\left\langle \vec{k}\vec{k}'\right| G(\omega = E_\tau(k) + E_{\tau'}(k'))\left|\vec{k}\vec{k}'\right\rangle_{\mathcal{A}}$$

Partial sumation of pp ladder diagrams



- ✓ Pauli blocking
- ✓ Nucleon dressing

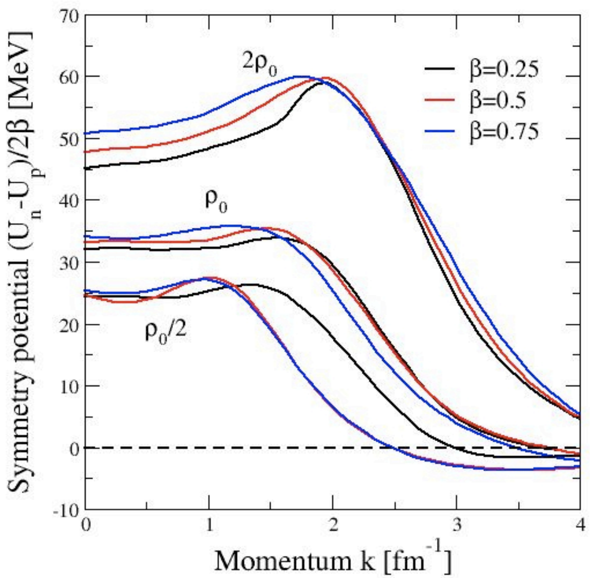
BHF nucleon mean field in ANM



Isospin splitting of mean field in ANM

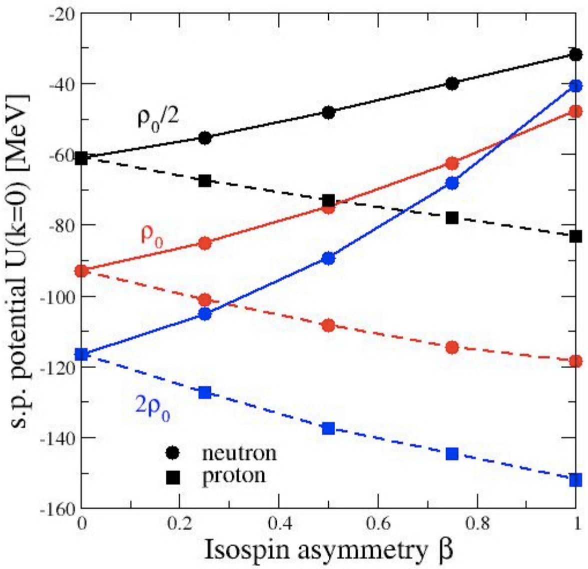
$$U_n \sim U_0 + U_{sym}\beta$$

$$U_p \sim U_0 - U_{sym}\beta$$



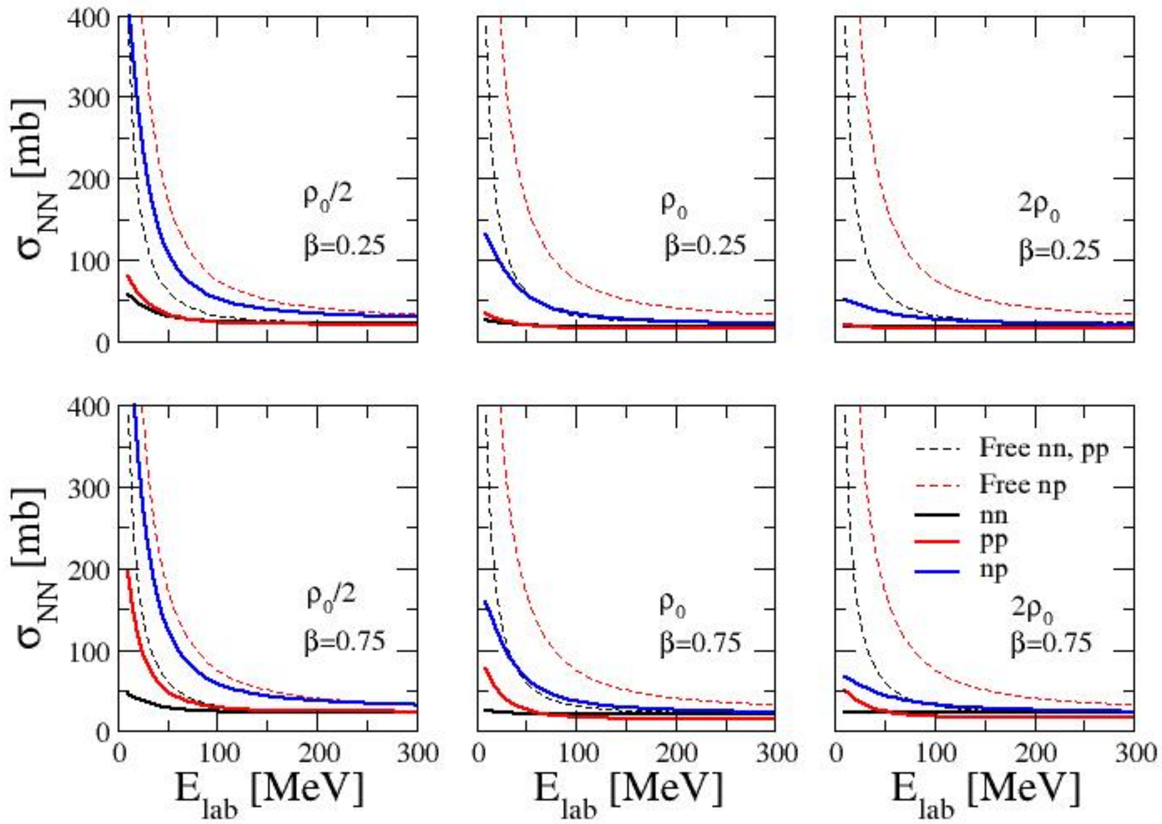
Symmetry potential

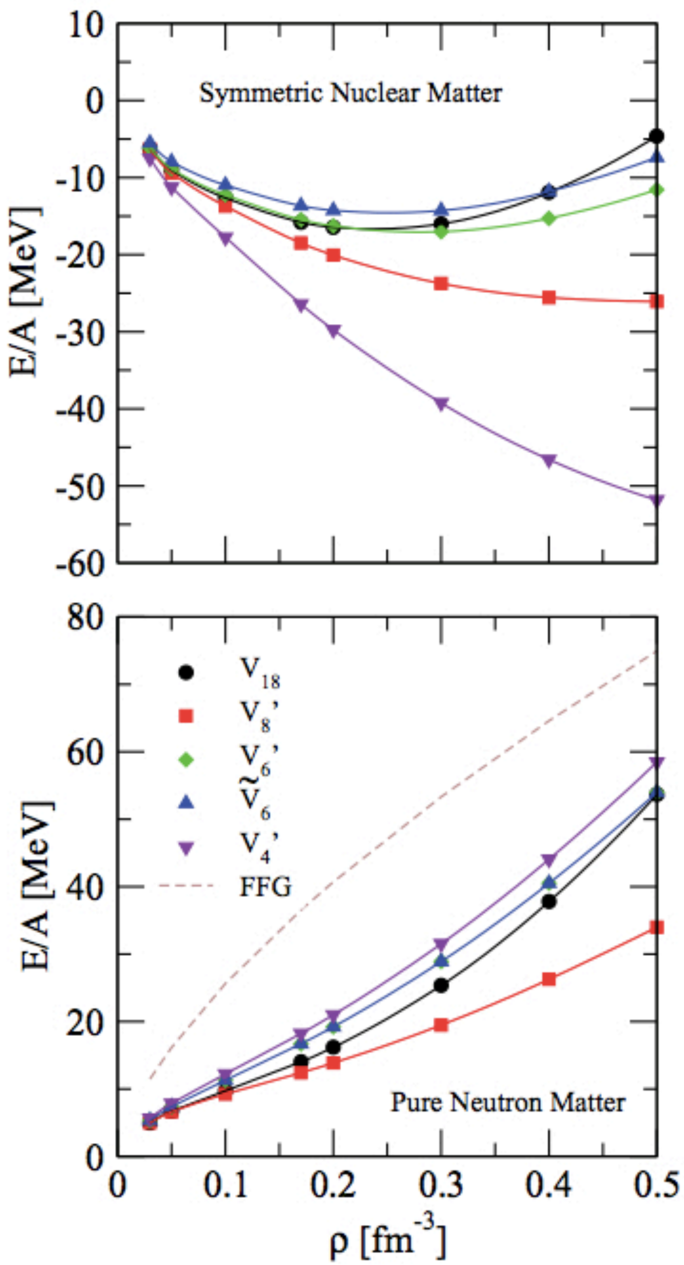
$$U_{sym} = \frac{U_n - U_p}{2\beta}$$



G-matrix gives access to in-medium NN cross sections

$$\sigma_{\tau\tau'} = \frac{m^*_\tau m^*_{\tau'}}{16\pi^2\hbar^4} \sum_{LL'SJ} \frac{2J+1}{4\pi} \left| G^{LL'SJ}_{\tau\tau' \to \tau\tau'} \right|^2, \quad \tau\tau' = nn, pp, np$$





Brueckner-Hatree-Fock:

✓ gives total energy

But

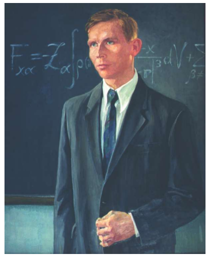
✓ does not gives separately neither <T> nor <V> because it does not provide the correlated many-body wave function

However

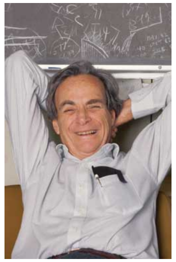
Hellmann-Feynman theorem can be used to calculate <V>

Hellmann-Feynman theorem

Proven independently by many-authors:
Güttinger (1932), Pauli (1933), Hellmann (1937), Feynman (1939)



H. Hellmann



R. P. Ferynman

$$\frac{dE_\lambda}{d\lambda} = \frac{\langle \psi_\lambda | \frac{d\hat{H}_\lambda}{d\lambda} | \psi_\lambda \rangle}{\langle \psi_\lambda | \psi_\lambda \rangle}$$

- Writting the nuclear matter Hamiltonian as: $\hat{H} = \hat{T} + \hat{V}$
- Defining a λ-dependent Hamiltonian: $\hat{H}_\lambda = \hat{T} + \lambda \hat{V}$

$$\rightarrow \langle \hat{V} \rangle = \frac{\langle \psi | \hat{V} | \psi \rangle}{\langle \psi | \psi \rangle} = \left(\frac{dE_\lambda}{d\lambda} \right) \Bigg|_{\lambda=1}$$

Kinetic and Potential energy contributions

	E_{NM}	E_{SM}	E_{sym}	L
<T>	53.321	54.294	-0.973	14.896
<V>	-34.251	-69.524	35.273	51.604
Total	19.070	-15.230	34.300	66.500

- Potential contribution to

✓ E_{sym}: almost equal to total E_{sym}

✓ L: ~78% of the total L

- Kinetic contribution to

✓ E_{sym}: very small and negative in contrast to FFG result (~14.4 MeV) ➔ strong isospin dependence of short range NN correlations

✓ L: smaller than FFG one (~29.2 MeV)

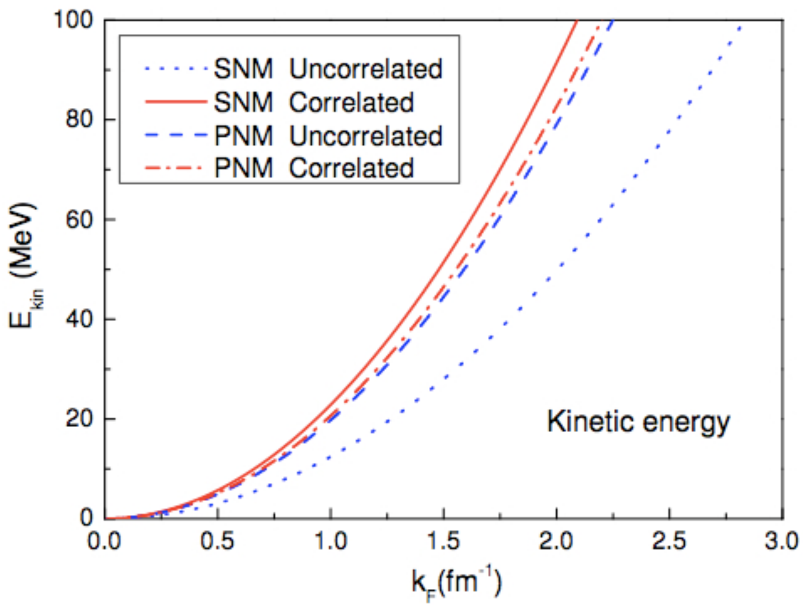


Figure from Xu & Li, arXiv:1104.2075v1 (2011)

Spin-Isospin channel & partial wave decomposition

(S,T)	E_{NM}	E_{SM}	E_{sym}	L
(0,0)	0	5.600	-5.600	-21.457
(0,1)	-29.889	-23.064	-6.825	-17.950
(1,0)	0	-49.836	49.836	90.561
(1,1)	-4.362	-2.224	-2.138	0.450
<V>	-34.251	-69.524	35.273	51.604

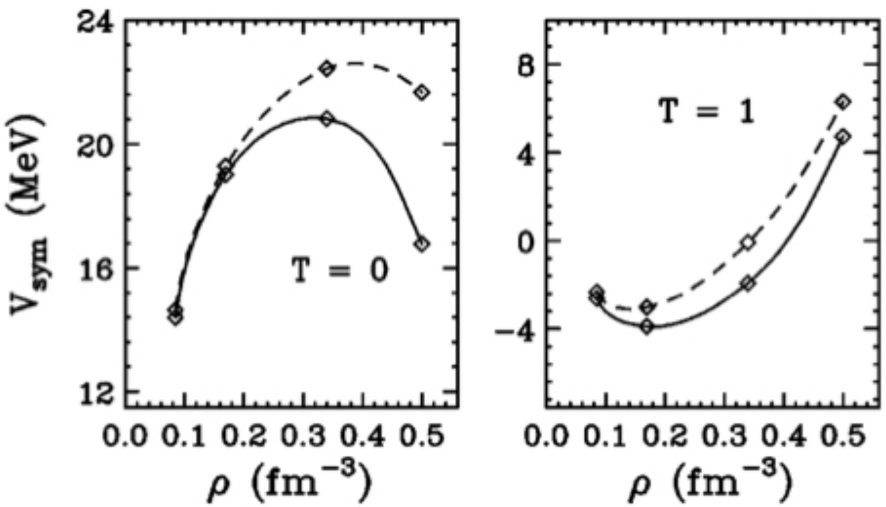
- ✓ Largest contribution from S=1, T=0 channel
- ✓ Similar T=1 channel contributions to E_{NM} and E_{SM} which almost cancel out in E_{sym}

	E_{NM}	E_{SM}	E_{sym}	L
$^3S_1$	0	-45.810	45.810	71.855
$^3D_1$	0	-0.981	0.981	-3.739
$\Sigma_{\text{rest up to J=8}}$	-34.251	-22.733	-11.518	-16.512

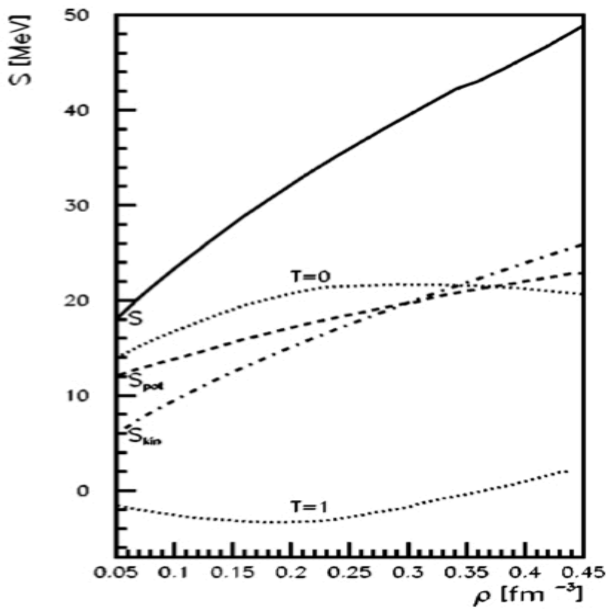
- ✓ Main contribution from $^3S_1$-$^3D_1$ p.w. (not present in NM)

In agreement with previous BHF calculations

- Zuo et al., (1999): Av14 & Paris



- Dieperink et al., (2003): Reid93



Note that in these works:

$$V_{sym}(\rho) = \frac{1}{A}\sum_{k\le k_{F_n}}\left(\frac{1}{2}\mathrm{Re}\left[U_n(\vec{k},\beta=1)\right]\right) - \frac{1}{A}\sum_{\tau=n,p}\sum_{k\le k_{F_\tau}}\left(\frac{1}{2}\mathrm{Re}\left[U_\tau(\vec{k},\beta=0)\right]\right)$$

NM correlation energy $\left(\neq \langle V_{NM}\rangle\right)$ SM correlation energy $\left(\neq \langle V_{SM}\rangle\right)$

Few words on the NN and NNN forces used ...

- Argonne V18 (Av18) NN potential

$$V_{ij} = \sum_{p=1,18} V_p(r_{ij}) O_{ij}^p$$

$$O_{ij}^{p=1,14} = \left[1, (\vec{\sigma}_i \cdot \vec{\sigma}_j), S_{ij}, \vec{L}\cdot\vec{S}, L^2, L^2(\vec{\sigma}_i \cdot \vec{\sigma}_j), (\vec{L}\cdot\vec{S})^2\right] \otimes \left[1, (\vec{\tau}_i \cdot \vec{\tau}_j)\right]$$

$$O_{ij}^{p=15,18} = \left[T_{ij}, (\vec{\sigma}_i \cdot \vec{\sigma}_j) T_{ij}, S_{ij} T_{ij}, (\tau_{zi} + \tau_{zj})\right]$$

- Urbana IX (UIX) NNN potential

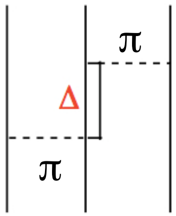
$$V_{ijk}^{UIX} = V_{ijk}^{2\pi} + V_{ijk}^{R}$$

$V_{ijk}^{2\pi}$: Attractive Fujita-Miyazawa force

$$V_{ijk}^{2\pi} = A \sum_{cyclic} \left(\{X_{ij}, X_{jk}\}\{\vec{\tau}_i \cdot \vec{\tau}_j, \vec{\tau}_j \cdot \vec{\tau}_k\} + \frac{1}{4}[X_{ij}, X_{jk}][\vec{\tau}_i \cdot \vec{\tau}_j, \vec{\tau}_j \cdot \vec{\tau}_k] \right)$$

$$X_{ij} = Y(m_\pi r_{ij})\vec{\sigma}_i \cdot \vec{\sigma}_j + T(m_\pi r_{ij}) S_{ij}$$

$$Y(x) = \frac{e^{-x}}{x}\left(1 - e^{x^2}\right) \quad T(x) = \left(1 + \frac{3}{x} + \frac{3}{x^2}\right)\frac{e^{-x}}{x}\left(1 - e^{x^2}\right)^2$$



V_{ijk}^{R} : Repulsine & Phenomenological

$$V_{ijk}^{R} = B \sum_{cyclic} T^2(r_{ij}) T^2(r_{jk})$$

Reduced to an effective density-dependent 2BF

$$U_{NN}^{eff}(\vec{r}_{ij}) = \int V^{UIX}(\vec{r}_i, \vec{r}_j, \vec{r}_k)\, n(\vec{r}_i, \vec{r}_j, \vec{r}_k)\, d^3\vec{r}_k \longrightarrow O_{ij}^{p=1,3} = 1, (\vec{\sigma}_i \cdot \vec{\sigma}_j)(\vec{\tau}_i \cdot \vec{\tau}_j), S_{ij}(\vec{\tau}_i \cdot \vec{\tau}_j)$$

Contributions from different terms of the NN force

	E_{NM}	E_{SM}	E_{sym}	L
$\langle V_1 \rangle$	−31.212	−32.710	1.498	−5.580
$\langle V_{\vec{\tau}_i\cdot\vec{\tau}_j} \rangle$	−4.957	3.997	−8.954	−20.383
$\langle V_{\vec{\sigma}_i\cdot\vec{\sigma}_j} \rangle$	−0.319	−0.382	0.063	2.392
$\langle V_{(\vec{\sigma}_i\cdot\vec{\sigma}_j)(\vec{\tau}_i\cdot\vec{\tau}_j)} \rangle$	−5.724	−11.388	5.664	2.521
$\langle V_{S_{ij}} \rangle$	−0.792	1.912	−2.704	−4.998
$\langle V_{S_{ij}(\vec{\tau}_i\cdot\vec{\tau}_j)} \rangle$	−4.989	−37.592	32.603	47.095
$\langle V_{\vec{L}\cdot\vec{S}} \rangle$	−7.538	−1.754	−5.784	−12.251
$\langle V_{\vec{L}\cdot\vec{S}(\vec{\tau}_i\cdot\vec{\tau}_j)} \rangle$	−2.671	−6.539	3.868	3.969
$\langle V_{L^2} \rangle$	11.850	13.610	−1.760	1.521
$\langle V_{L^2(\vec{\tau}_i\cdot\vec{\tau}_j)} \rangle$	−2.788	0.270	−3.058	−14.262
$\langle V_{L^2(\vec{\sigma}_i\cdot\vec{\sigma}_j)} \rangle$	1.265	1.383	−0.118	1.405
$\langle V_{L^2(\vec{\sigma}_i\cdot\vec{\sigma}_j)(\vec{\tau}_i\cdot\vec{\tau}_j)} \rangle$	0.051	0.008	0.043	−0.341
$\langle V_{(\vec{L}\cdot\vec{S})^2} \rangle$	4.194	5.682	−1.488	−0.327
$\langle V_{(\vec{L}\cdot\vec{S})^2(\vec{\tau}_i\cdot\vec{\tau}_j)} \rangle$	5.169	−6.190	11.359	31.368
$\langle V_{T_{ij}} \rangle$	0.003	0.039	−0.036	−0.022
$\langle V_{(\vec{\sigma}_i\cdot\vec{\sigma}_j)T_{ij}} \rangle$	−0.017	−0.106	0.089	0.042
$\langle V_{S_{ij}T_{ij}} \rangle$	0.004	0.079	−0.075	−0.124
$\langle V_{(\tau_{z_i}+\tau_{z_j})} \rangle$	−0.084	−0.001	−0.083	−0.331
$\langle U_1 \rangle$	2.985	3.251	−0.266	−0.630
$\langle U_{(\vec{\sigma}_i\cdot\vec{\sigma}_j)(\vec{\tau}_i\cdot\vec{\tau}_j)} \rangle$	2.254	3.999	−1.745	−7.228
$\langle U_{S_{ij}(\vec{\tau}_i\cdot\vec{\tau}_j)} \rangle$	−0.935	−7.092	6.157	27.768

✓ Largest contribution from tensor components

- E_{sym}: 36.056 (Total: 34.4)
- L: 69.968 (Total: 66.5)

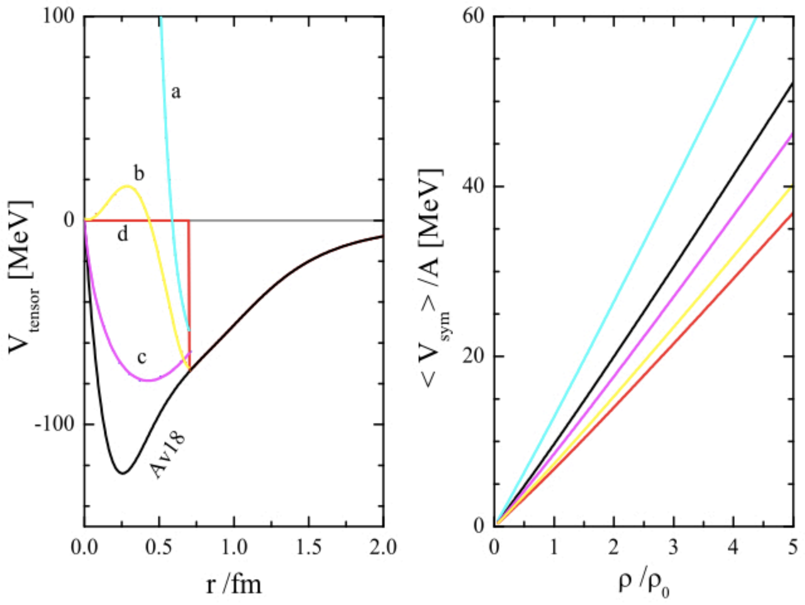
✓ Contributions from other terms negligible (e.g. charge symmetry breaking terms) or almost cancel out

In agreement with other calculations ...

- A. Li & B.A. Li (2011):

$$\left\langle V_{sym} \right\rangle \approx \frac{12}{E} \left\langle V_{tensor}^{2}(r) \right\rangle$$

Brown & Machleidt, PRC 50, 1731 (1993)



- Sammarruca (2011): DBHF

Large contribution of π to E_{sym}

Potential	$U^{\pi}_{\mathrm{NM}} - U^{\pi}_{\mathrm{SNM}}$	$U^{\rho}_{\mathrm{NM}} - U^{\rho}_{\mathrm{SNM}}$	$U^{\delta}_{\mathrm{NM}} - U^{\delta}_{\mathrm{SNM}}$
Bonn B	20.78	−5.90	−6.78
Bonn A	15.98	−4.68	−2.80
Bonn C	24.42	−5.48	−10.24

Summarizing …

- Main contribution to E_{sym} and L from < V >. Kinetic energy contribution < T > to E_{sym} very small and negative.

- Partial wave & spin-isospin channel decompositon of E_{sym} & L ➔ major contribution from (S=1, T=0) channel.

- Dominant effect of the tensor force: critical role in the determination of $E_{sym}(\rho)$.

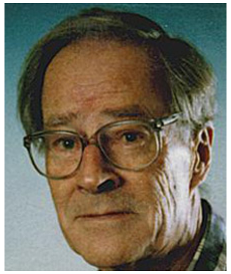
Symmetry Energy & Rearrangement

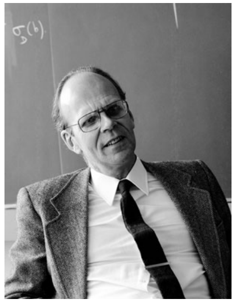
"Analysis of the contribution of the rearrangement term to E_{sym} and L"

using again:

✓ BHF with Av18 + UIX

Hugenholtz-Van Hove theorem


N. M. Hugenholtz


L. Van Hove

Powerful tool to check thermodynamical consistency

- Original HvH theorem

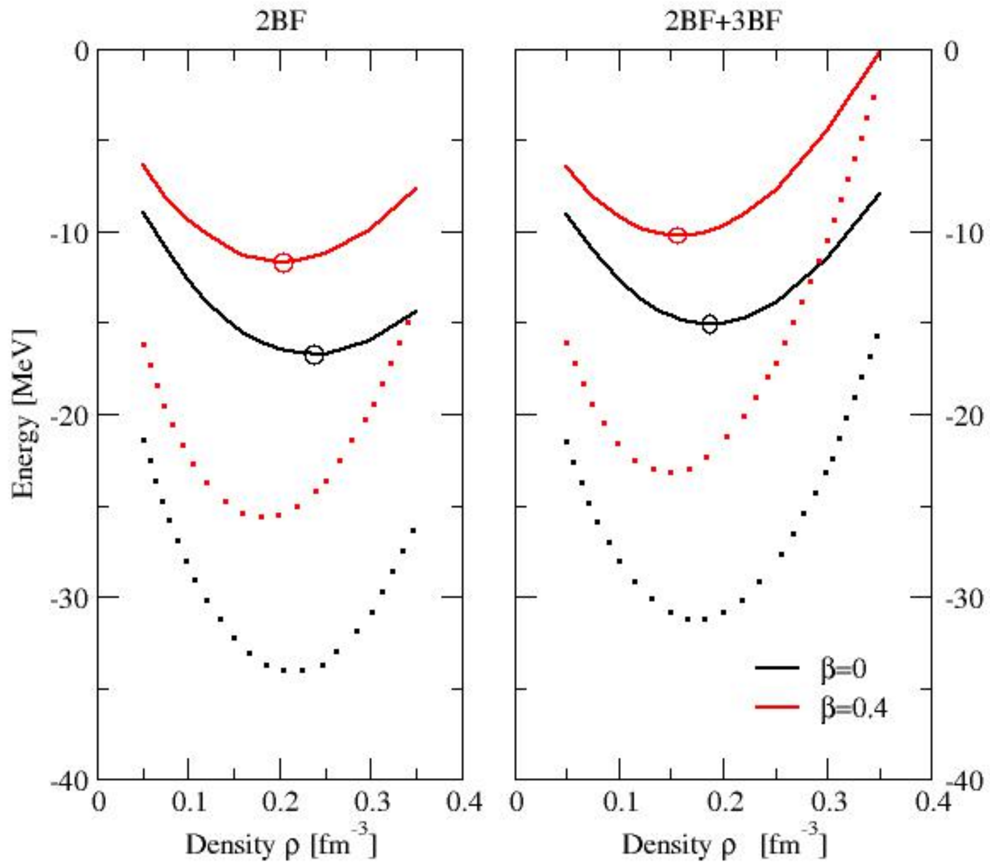
at saturation

$$\frac{E}{A}(\rho)+\rho\frac{\partial(E/A)}{\partial\rho}\bigg|_{\Omega}=\frac{\partial E}{\partial A}\bigg|_{\Omega}=\varepsilon_F(\rho) \rightarrow \frac{E}{A}(\rho_0)=\varepsilon_F(\rho_0)$$

- Generalized HvH theorem Satpathy & Nayak, PRL 51, 1243 (1983)

$$\frac{E}{A}(\rho)+\rho\frac{\partial(E/A)}{\partial\rho}\bigg|_{\Omega}=x_n\varepsilon_n+x_p\varepsilon_p=\frac{1}{2}\left[(1+\beta)\varepsilon_n+(1-\beta)\varepsilon_p\right]$$

BHF & Hugenholtz-Van Hove theorem



It is well known that BHF:

- ✓ Violates HvH theorem by about 20 MeV
- ✓ Thermodynamical consistency restored by including <u>rearrangement</u> contribution to the single-particle energy

Rearrangement contribution of the s.p. energy

- BHF total energy

$$E = \sum_{\tau i} n_{\tau i} \frac{\hbar^2 k_i^2}{2m} + \frac{1}{2\Omega} \sum_{\tau' i \tau'' j} n_{\tau' i} n_{\tau'' j} \langle \tau' i \tau'' j | G | \tau' i \tau'' j \rangle_A$$

- Single-particle energy

BHF potential ($U_{BHF,\tau}(k)$)

$$\varepsilon_{\tau k} = \frac{\delta E}{\delta n_{\tau k}} = \frac{\hbar^2 k^2}{2m} + \frac{1}{\Omega} \sum_{\tau' i} n_{\tau' i} \langle \tau' i \tau k | G | \tau' i \tau k \rangle_A$$

$$+ \frac{1}{2\Omega} \sum_{\tau' i \tau'' j} n_{\tau' i} n_{\tau'' j} \frac{\delta \langle \tau' i \tau'' j | G | \tau' i \tau'' j \rangle_A}{\delta n_{\tau k}}$$

Rearrangement ($U_{rearr,\tau}(k)$)
(infinite # of terms)

Using:

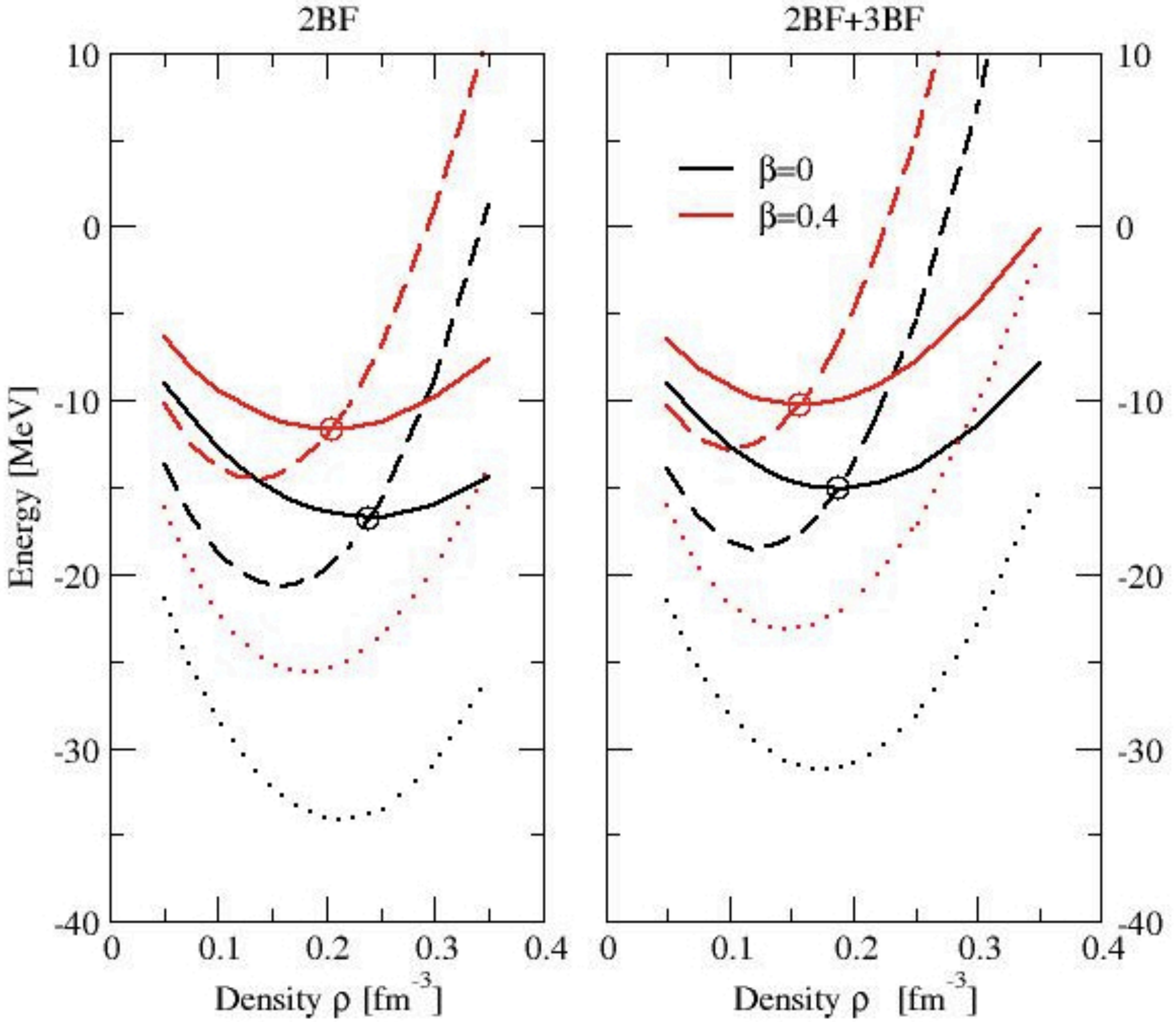
$$\frac{\delta}{\delta n_{\tau k_{F_\tau}}} \sim \frac{\partial}{\partial k_{F_\tau}} \frac{\partial k_{F_\tau}}{\partial \rho_\tau} \frac{\delta \rho_\tau}{\delta n_{\tau k_{F_\tau}}} = \frac{k_{F_\tau}}{3\rho_\tau} \frac{1}{\Omega} \frac{\partial}{\partial k_{F_\tau}}$$

we evaluate the s.p. energy including U_{rearr} at the Fermi surface

$$\varepsilon_{\tau k_{F_\tau}} \sim \frac{\hbar^2 k_{F_\tau}^2}{2m} + \underbrace{\frac{1}{\Omega} \sum_{\tau' i} n_{\tau' i} \left\langle \tau' i \tau k_{F_\tau} \middle| G \middle| \tau' i \tau k_{F_\tau} \right\rangle_A}_{U_{BHF,\tau}(k_{F\tau})}$$

$$+ \underbrace{\frac{k_{F_\tau}}{3\rho_\tau} \frac{1}{2\Omega^2} \sum_{\tau' i \tau'' j} n_{\tau' i} n_{\tau'' j} \frac{\partial \left\langle \tau' i \tau'' j \middle| G \middle| \tau' i \tau'' j \right\rangle_A}{\partial k_{F_\tau}}}_{U_{rearr,\tau}(k_{F\tau})}$$

Identifying $\varepsilon_{\tau k \tau}$ with the Fermi energy the HvH theorem is fulfilled



Rearrangement & Symmetry Energy

Using:

- $\Delta\mu = \mu_n - \mu_p = \frac{2}{\rho}\frac{\partial(E/\Omega)}{\partial\beta}$
- $\frac{E}{\Omega}(\rho) = e_o(\rho) + e_{sym}(\rho)\beta^2$

$\rightarrow$

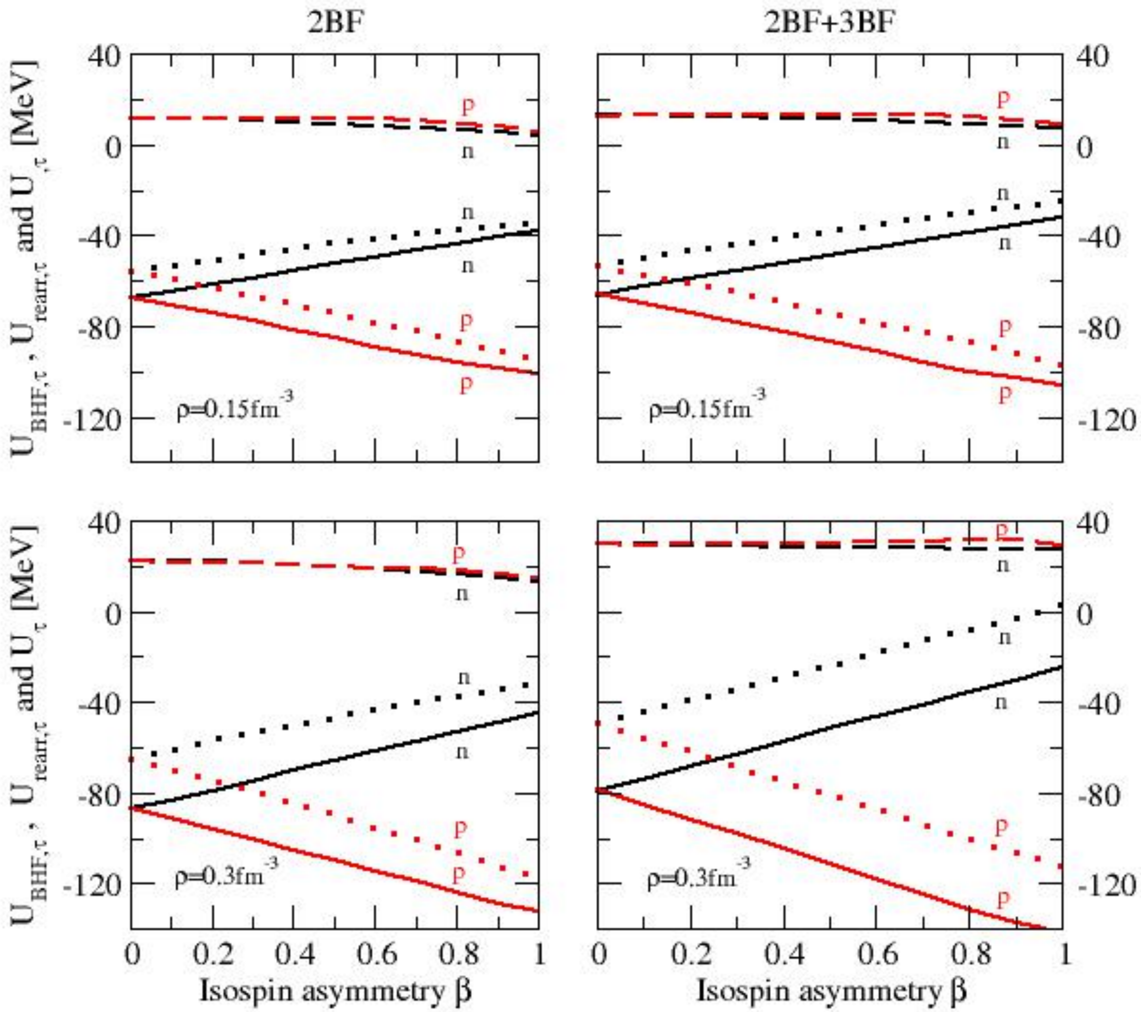
$$\Delta\mu = \frac{4}{\rho}\beta e_{sym}(\rho) = 4\beta E_{sym}(\rho)$$

Taking: $\mu_\tau \equiv \varepsilon_{\tau k_{F_\tau}} = \frac{\hbar^2 k_{F_\tau}^2}{2m} + U_{BHF}(k_{F_\tau}) + U_{rearr}(k_{F_\tau})$

$$E_{sym}(\rho) = \frac{1}{4\beta}\left[\left(\frac{\hbar^2 k_{Fn}^2}{2m} - \frac{\hbar^2 k_{Fp}^2}{2m}\right) + \left(U_{BHF}^{n}(k_{F_n}) - U_{BHF}^{p}(k_{F_p})\right) + \left(U_{rearr}^{n}(k_{Fn}) - U_{rearr}^{p}(k_{F_p})\right)\right]$$

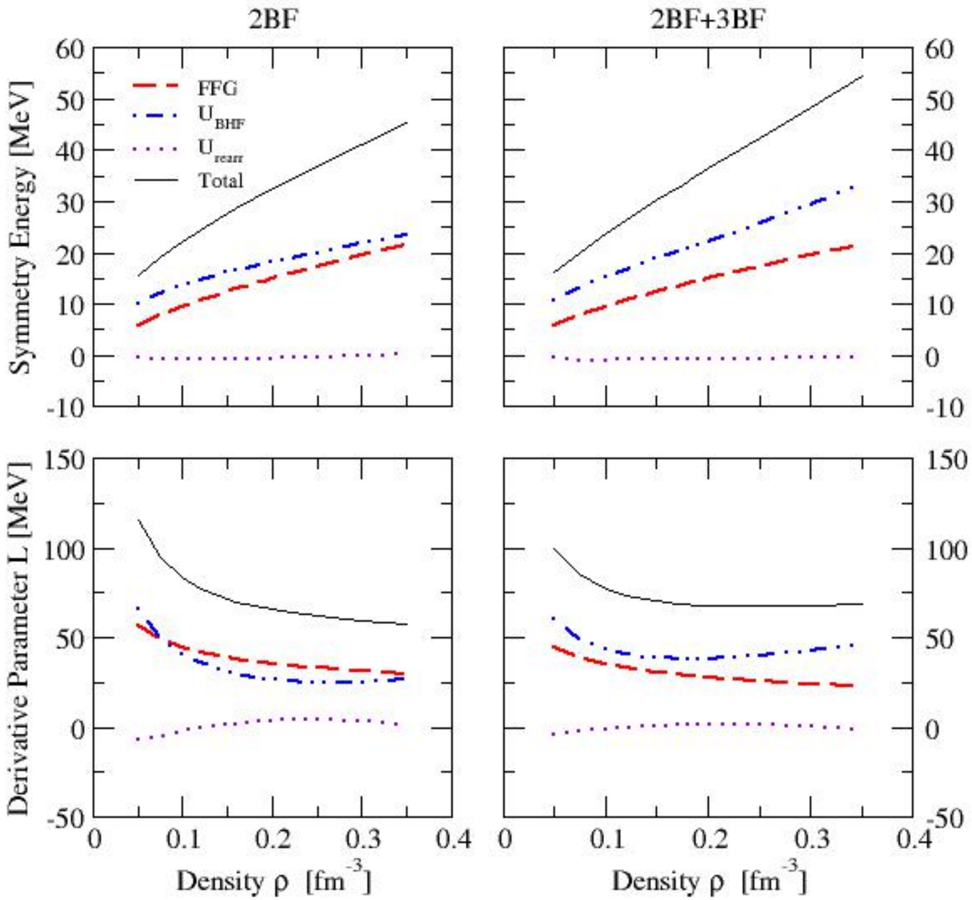
Free Fermi Gas — BHF — Rearrangement

Rearrangement contribution to the single-particle potential



- ✓ Almost constant with isospin asymmetry
- ✓ Very similar for neutrons & protons

Rearrangement contribution to E_{sym} & L



	ρ_0	ρ_0	FFG	U_{BHF}	U_{rearr}	Total
2BF	E_{sym}	0.240	16.7	19.5	-0.4	35.8
	L		33.5	25.3	4.3	63.1
2+3BF	E_{sym}	0.187	14.4	20.6	-0.7	34.3
	L		29.2	35.8	1.5	66.5

✓ Negligible contribution to both E_{sym} & L

Summarizing …

- Rearrangement contribution to neutron & proton s.p. potentials:
  - ✓ Almost constant with β
  - ✓ Similar for neutrons and protons

- Negligible contribution of rearrangement term to both $E_{sym}(\rho)$ & $L(\rho)$

COMPETE
PROGRAMA OPERACIONAL FACTORES DE COMPETITIVIDADE