Symmetry Energy and Neutron Star Structure

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  - Nuclear Experimental Constraints on the Symmetry Energy
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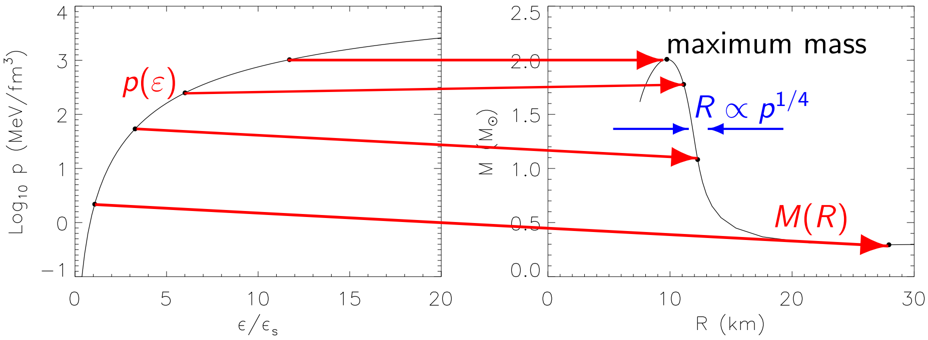
Neutron Star Structure

Tolman-Oppenheimer-Volkov equations

$$\frac{dp}{dr} = -\frac{G}{c^4}\frac{(mc^2 + 4\pi pr^3)(\varepsilon + p)}{r(r - 2Gm/c^2)}$$

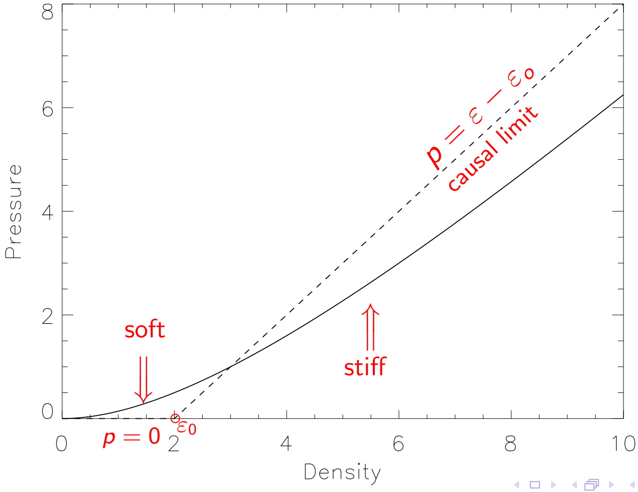
$$\frac{dm}{dr} = 4\pi\frac{\varepsilon}{c^2}r^2$$



Equation of State ← Observations

Extremal Properties of Neutron Stars

- The most compact and massive configurations occur when the low-density equation of state is "soft" and the high-density equation of state is "stiff" (Koranda, Stergioulas & Friedman 1997).



ε_0 is the only EOS parameter

The TOV solutions scale with ε_0

$w = \varepsilon/\varepsilon_0$
$y = p/\varepsilon_0$
$x = r\sqrt{G\varepsilon_0}/c^2$
$z = m\sqrt{G^3\varepsilon_0}/c^2$

Extremal Properties of Neutron Stars

The maximum mass configuration is achieved when $x_R = 0.2404$, $w_c = 3.034$, $y_c = 2.034$, $z_R = 0.08513$.

A useful reference density is the nuclear saturation density (interior density of normal nuclei):
$\rho_s = 2.7 \times 10^{14}$ g cm^{-3}, $n_s = 0.16$ baryons fm^{-3}, $\varepsilon_s = 150$ MeV fm^{-3}

- $M_{\rm max} = 4.1\ (\varepsilon_s/\varepsilon_0)^{1/2} M_\odot$ (Rhoades & Ruffini 1974)
- $M_{B,\rm max} = 5.41\ (m_B c^2/\mu_o)(\varepsilon_s/\varepsilon_0)^{1/2} M_\odot$
- $R_{\rm min} = 2.82\ GM/c^2 = 4.3\ (M/M_\odot)$ km
- $\mu_{b,\rm max} = 2.09$ GeV
- $\varepsilon_{c,\rm max} = 3.034\ \varepsilon_0 \simeq 51\ (M_\odot/M_{\rm largest})^2\ \varepsilon_s$
- $p_{c,\rm max} = 2.034\ \varepsilon_0 \simeq 34\ (M_\odot/M_{\rm largest})^2\ \varepsilon_s$
- $n_{B,\rm max} \simeq 38\ (M_\odot/M_{\rm largest})^2\ n_s$
- $\mathrm{BE}_{\rm max} = 0.34\ M$
- $P_{\rm min} = 0.74\ (M_\odot/M_{\rm sph})^{1/2}(R_{\rm sph}/10\ {\rm km})^{3/2}\ {\rm ms} = 0.20\ (M_{\rm sph,max}/M_\odot)\ {\rm ms}$

Causality and the Maximum Mass

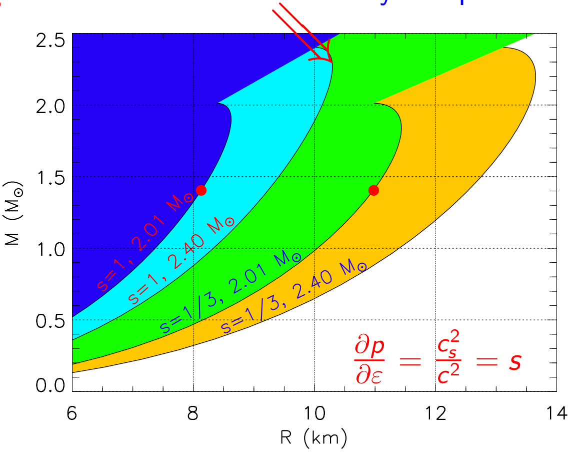
A precise radius and mass measurement sets an upper limit to the maximum mass.

A small radius measurement implies a small maximum mass.

$1.4 M_\odot$ stars must have $R > 8.15 M_\odot$.

A measured $R < 11$ km for a $1.4 M_\odot$ star rules out a strange quark matter star, and, effectively, also a hybrid quark/hadron star.

$M - R$ curves for maximally compact EOS



Mass-Radius Diagram and Theoretical Constraints

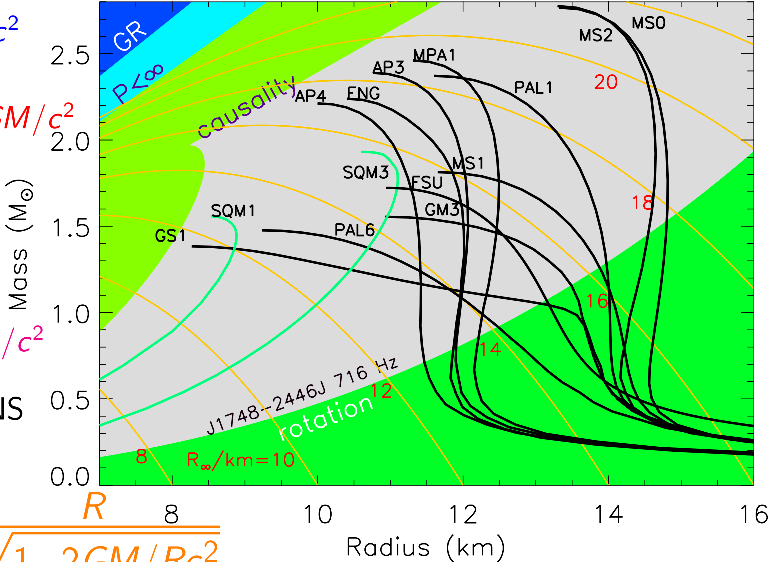
GR:
$R > 2GM/c^2$

$P < \infty$:
$R > (9/4)GM/c^2$

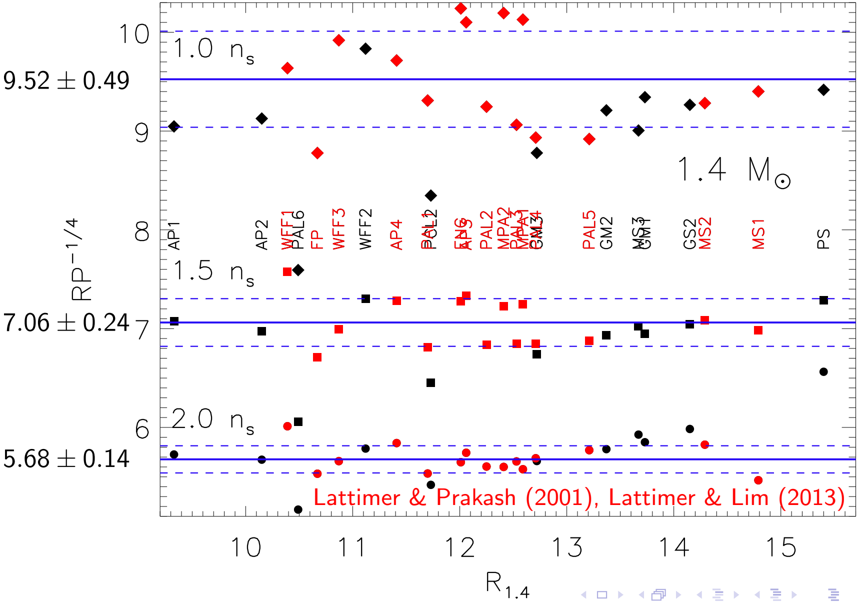
causality:
$R \gtrsim 2.9GM/c^2$

— normal NS
— SQS

— $R_\infty = \dfrac{R}{\sqrt{1-2GM/Rc^2}}$



The Radius – Pressure Correlation



Neutron Star Structure

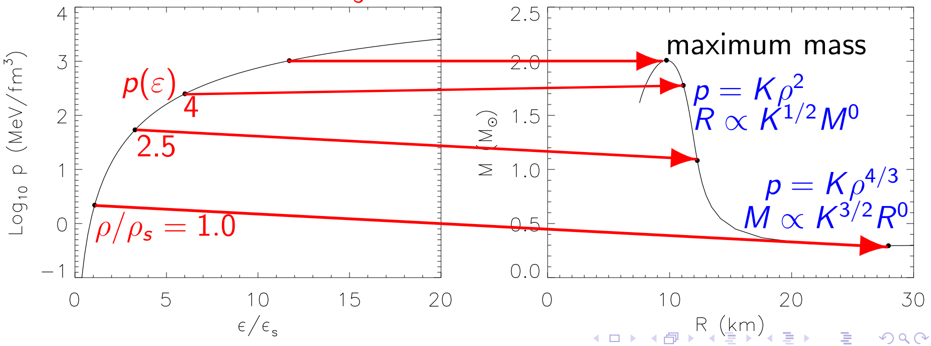
Newtonian Gravity:

$$\frac{dp}{dr} = -\frac{Gm\rho}{r^2}; \qquad \frac{dm}{dr} = 4\pi\rho r^2; \qquad \rho c^2 = \varepsilon$$

Newtonian Polytrope:

$$p = K\rho^\gamma; \qquad M \propto K^{1/(2-\gamma)} R^{(4-3\gamma)/(2-\gamma)}$$

$$\rho < \rho_s:\ \gamma \simeq \frac{4}{3}; \qquad \rho > \rho_s:\ \gamma \simeq 2$$



Nuclear Symmetry Energy

Defined as the difference between energies of pure neutron matter ($x = 0$) and symmetric ($x = 1/2$) nuclear matter.

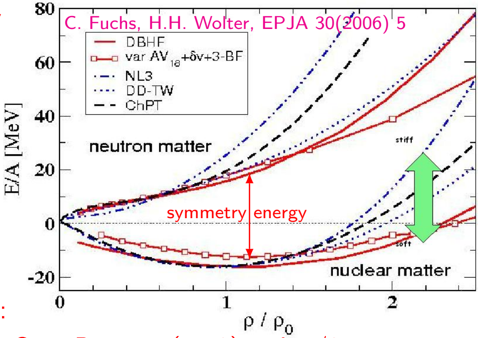
$$S(\rho) = E(\rho, x = 0) - E(\rho, x = 1/2)$$

Expanding around the saturation density (ρ_s) and symmetric matter ($x = 1/2$)

$$E(\rho, x) = E(\rho, 1/2) + (1-2x)^2 S_2(\rho) + \ldots$$

$$S_2(\rho) = \mathbf{S_v} + \frac{\mathbf{L}}{3}\frac{\rho - \rho_s}{\rho_s} + \ldots$$

$$\mathbf{S_v} \simeq 31 \text{ MeV}, \quad \mathbf{L} \simeq 50 \text{ MeV}$$



Connections to pure neutron matter:

$$E(\rho_s, 0) \approx S_v + E(\rho_s, 1/2) \equiv S_v - B, \qquad p(\rho_s, 0) = L\rho_s/3$$

Neutron star matter (in beta equilibrium):

$$\frac{\partial(E + E_e)}{\partial x} = 0, \quad p(\rho_s, x_\beta) \simeq \frac{L\rho_s}{3}\left[1 - \left(\frac{LS_v}{\hbar c}\right)^3 \frac{4 - 3S_v/L}{3\pi^2 \rho_s}\right]$$

Determining Symmetry Parameters from Nuclear Masses

From liquid drop model:

$$E_{\rm sym}(N,Z) = (S_v A - S_s A^{2/3}) I^2$$

$$\chi^2 = \sum_i (E_{ex,i} - E_{sym,i})^2 / \mathcal{N}\sigma_D^2$$

$$\chi_{vv} = \frac{2}{\mathcal{N}\sigma_D^2} \sum_i I_i^4 A_i^2$$

$$\chi_{ss} = \frac{2}{\mathcal{N}\sigma_D^2} \sum_i I_i^4 A_i^{4/3}$$

$$\chi_{vs} = \frac{2}{\mathcal{N}\sigma_D^2} \sum_i I_i^4 A_i^{5/3}$$

$$\sigma_{S_v} = \sqrt{\frac{2\chi_{ss}}{\chi_{vv}\chi_{ss} - \chi_{sv}^2}} \simeq 2.3\,\sigma_D$$

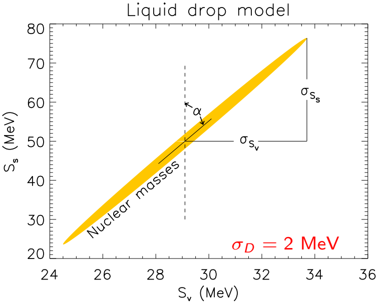
$$\sigma_{S_s} = \sqrt{\frac{2\chi_{vv}}{\chi_{vv}\chi_{ss} - \chi_{sv}^2}} \simeq 13.2\,\sigma_D$$

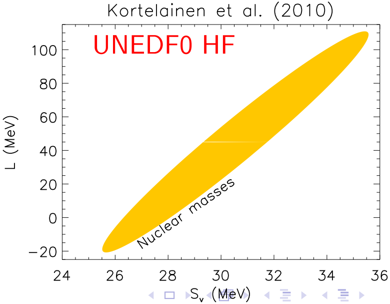
$$\alpha = \frac{1}{2}\tan^{-1}\frac{2\chi_{vs}}{\chi_{vv} - \chi_{ss}} \simeq 9^\circ.8$$

$$r_{vs} = -\frac{\chi_{vs}}{\sqrt{\chi_{vv}\chi_{ss}}} \simeq 0.997$$

Liquid droplet model:

$$E_{\rm sym}(N,Z) = \frac{S_v A I^2}{1 + (S_s/S_v) A^{-1/3}}$$

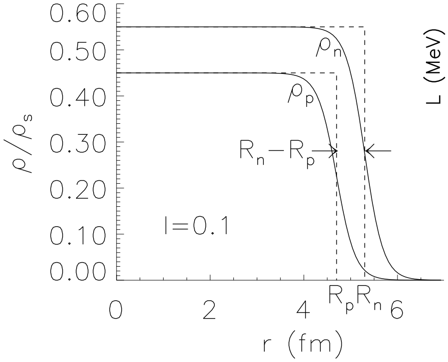


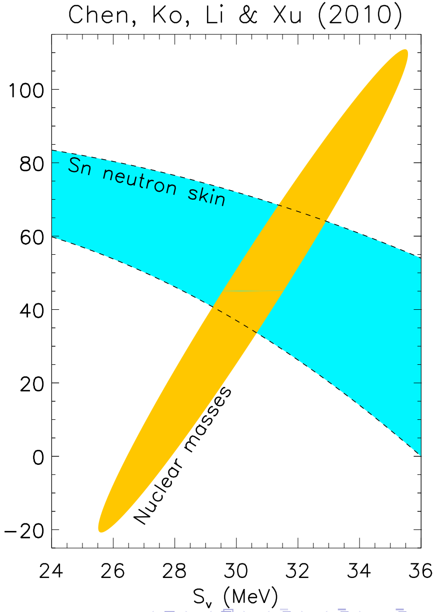


Neutron Skin Thickness

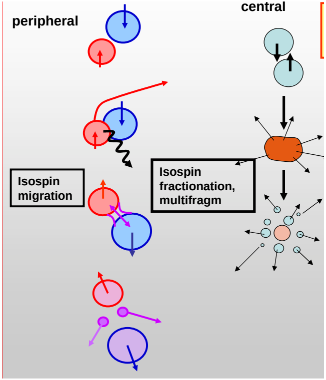
$$\frac{R_n - R_p}{r_o} \simeq \sqrt{\frac{4}{15}} \frac{S_s I}{S_v + S_s A^{-1/3}}$$

$$r_o = \left(\frac{3}{4\pi\rho_s}\right)^{1/3} \simeq 1.14 \text{ fm}$$

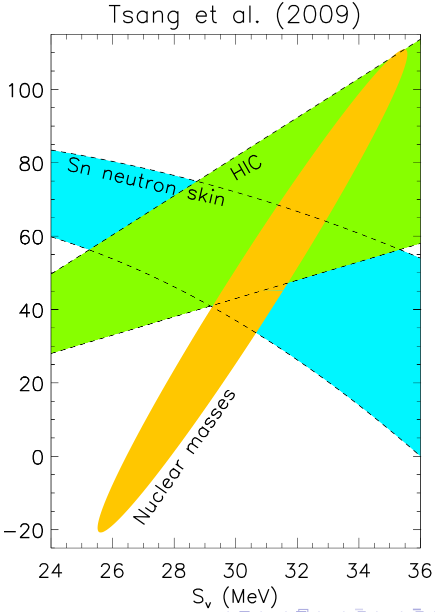




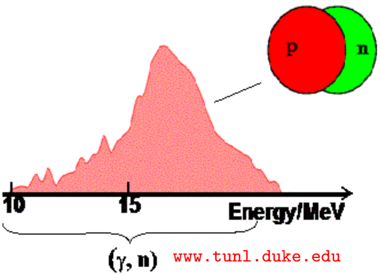
Heavy Ion Collisions



Wolter, NuSYM11



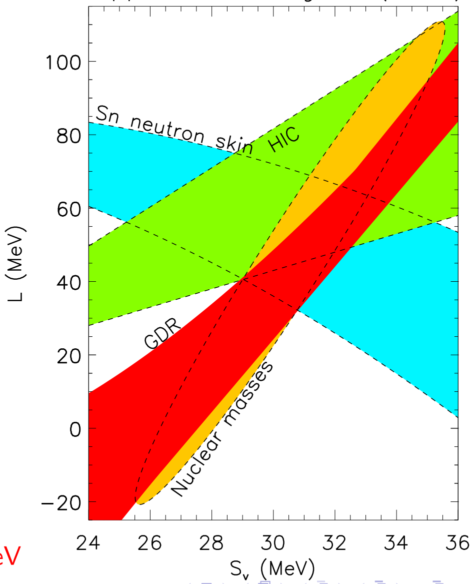
Giant Dipole Resonances



$$E_{-1} \propto \sqrt{\frac{S_v}{1+\frac{5S_s}{3S_v}A^{-1/3}}}$$

23.3 MeV$< S_2(0.1\ \mathrm{fm}^{-3}) <$24.9 MeV

Trippa, Colo & Vigezzi (2008)



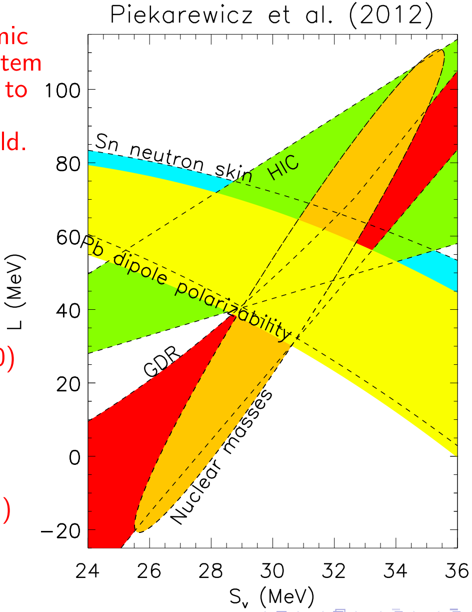
Dipole Polarizability

The linear response, or dynamic polarizability, of a nuclear system excited from its ground state to an excited state, due to an external oscillating dipolar field.

α_D and $R_n - R_p$ in ^{208}Pb are 98% correlated

Reinhard & Nazawericz (2010)

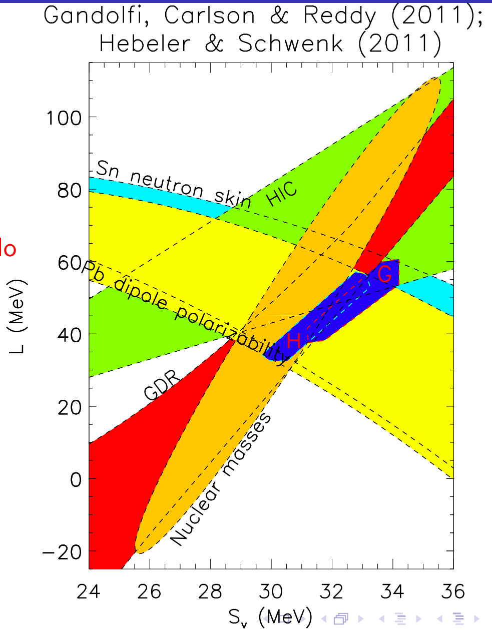
Data from Tamii et al. (2011)



Theoretical Neutron Matter Calculations

H&S: Chiral Lagrangian

GC&R: Quantum Monte Carlo



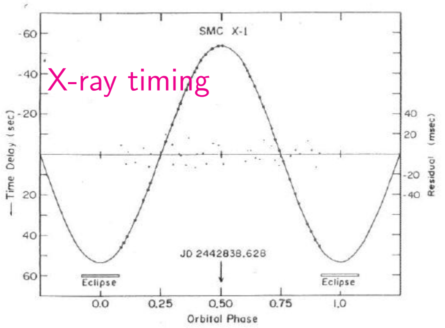
Binary Mass Measurements

Mass function

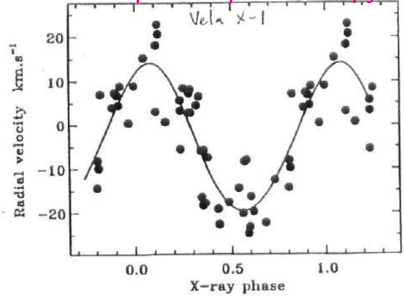
$$f(M_1) = \frac{P(v_2 \sin i)^3}{2\pi G} = \frac{(M_1 \sin i)^3}{(M_1+M_2)^2} < M_1$$

$$f(M_2) = \frac{P(v_1 \sin i)^3}{2\pi G} = \frac{(M_2 \sin i)^3}{(M_1+M_2)^2} < M_2$$

In an X-ray binary, $v_{optical}$ has the largest uncertainties. In some cases $\sin i \sim 1$ if eclipses are observed. If no eclipses observed, limits to i can be made based on the estimated radius of the optical star.



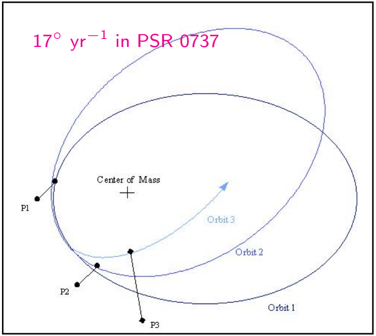
Optical spectroscopy



Pulsar Mass Measurements

Mass functions for pulsars are precisely measured.

In some cases, the rate of periastron advance and the Einstein gravitational redshift/time dilation term are known:



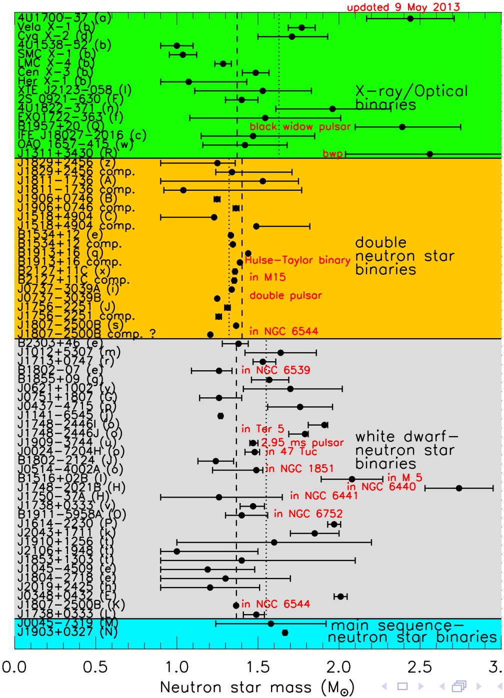
$$\dot\omega = \frac{3}{1-e^2}\left(\frac{2\pi}{P}\right)^{5/3}\left(\frac{GM}{c^2}\right)^{2/3}$$

$$\gamma = \left(\frac{P}{2\pi}\right)^{1/3} e M_2 (2M_2 + M_1)\left(\frac{G}{M^2c^2}\right)^{2/3}$$

Gravitational radiation leads to orbit decay:

$$\dot P = -\frac{192\pi}{5c^5}\left(\frac{2\pi G}{P}\right)^{5/3}(1-e^2)^{-7/2}\left(1+\frac{73}{24}e^2+\frac{37}{96}e^4\right)\frac{M_1M_2}{M^{1/2}}$$

In some cases, can also constrain Shapiro time delay, $r(M_2, e, \sin i)$ is magnitude and $s = \sin i$ is shape parameter.



vanKerkwijk 2010

Romani et al. 2012

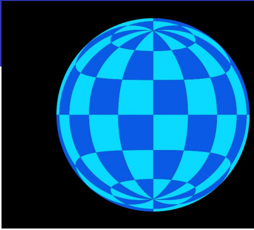
Although simple average mass of w.d. companions is 0.23 $M_\odot$ larger, weighted average is 0.04 $M_\odot$ smaller

Demorest et al. 2010

Antoniadis et al. 2013

Champion et al. 2008

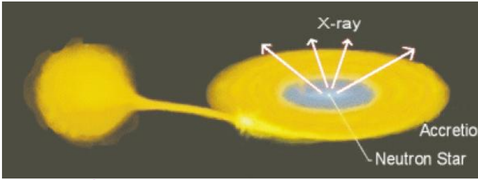
Simultaneous Mass/Radius Measurements



- The measurement of flux ($F_\infty = \frac{R_\infty}{D}\sigma T_{\rm eff}^4$) and temperature ($T_c \propto \lambda_{\rm max}^{-1}$) yields an apparent angular size (pseudo-BB):

$$\frac{R_\infty}{D} = \frac{R}{D}\frac{1}{\sqrt{1-2GM/Rc^2}}$$

- Observational uncertainties include distance D, interstellar absorption N_H, atmospheric composition

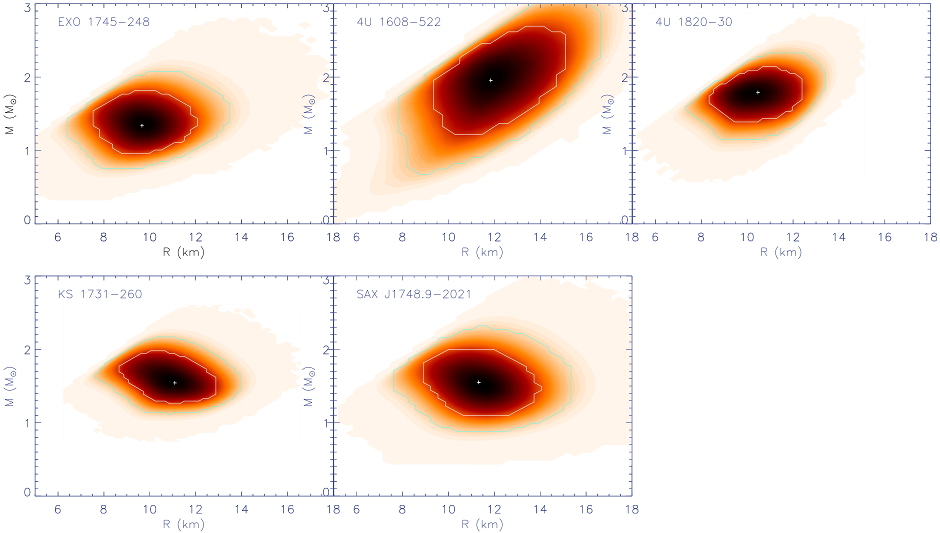


Best chances for accurate radius measurement:

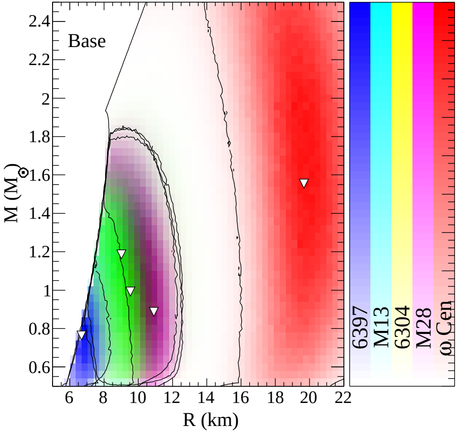
- Nearby isolated neutron stars with parallax (uncertain atmosphere)
- Quiescent low-mass X-ray binaries (QLMXBs) in globular clusters (reliable distances, low B H-atmospheres)
- Bursting sources (XRBs) with peak fluxes close to Eddington limit (where gravity balances radiation pressure)

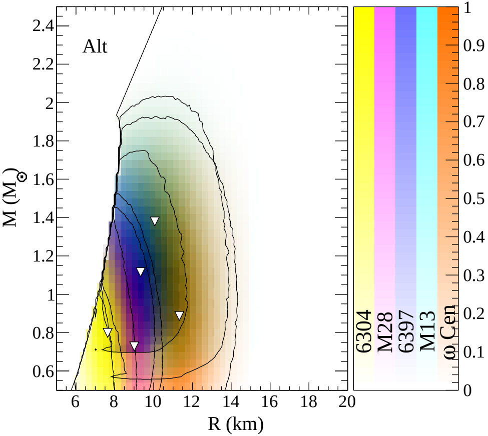
$$F_{\rm Edd} = \frac{cGM}{\kappa D^2}\sqrt{1-2GM/Rc^2}$$

$M-R$ PRE Burst Estimates



$M - R$ QLMXB Estimates





Guillot et al. (2013)
M, R, N_H from X-ray fit

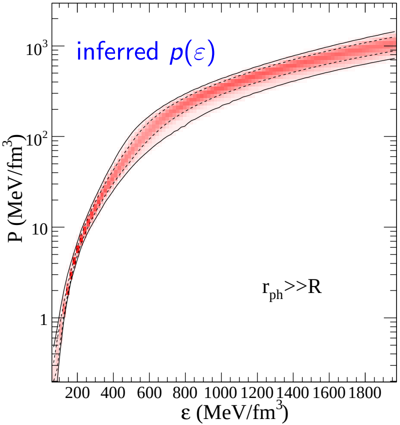
relative Bayes factor: 1

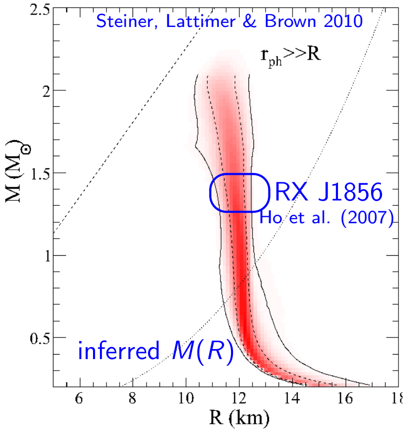
Lattimer & Steiner (2013)

M, R from X-ray fit
N_H from Dickey & Lockman (1990)
relative Bayes factor: $8 \cdot 10^5$

Bayesian TOV Inversion

- $\varepsilon < 0.5\varepsilon_0$: Known crustal EOS
- $0.5\varepsilon_0 < \varepsilon < \varepsilon_1$: EOS parametrized by K, K', S_v, γ
- Polytropic EOS: $\varepsilon_1 < \varepsilon < \varepsilon_2$: n_1; $\varepsilon > \varepsilon_2$: n_2
- EOS parameters $K, K', S_v, \gamma, \varepsilon_1, n_1, \varepsilon_2, n_2$ uniformly distributed
- $M_{\rm max} \geq 1.97$ $M_\odot$, causality enforced
- All stars equally weighted





Astronomical Observations

$11.0 < R_{1.4}/\mathrm{km} < 12.3$ (68%)
Steiner et al. (2010)

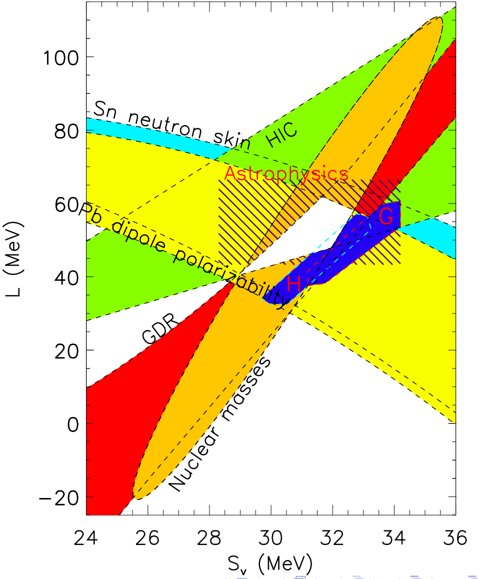
$10.6 < R_{1.4}/\mathrm{km} < 12.5$ (95%)
Steiner et al. (2013)

43 MeV$\lesssim L \lesssim$ 66 MeV

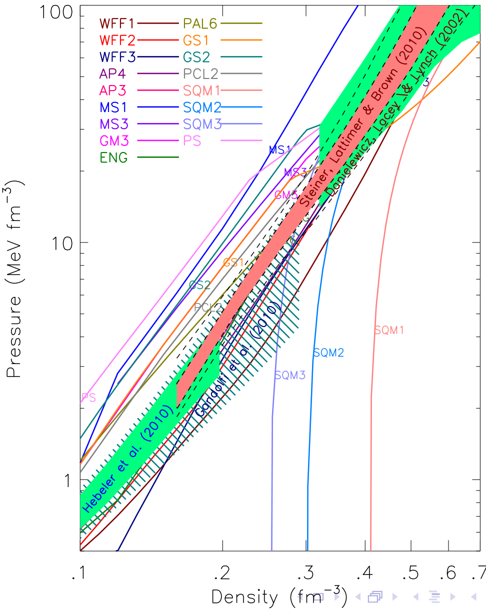
$10.8 < R_{1.4}/\mathrm{km} < 12.3$ (68%)
Gandolfi et al. (2012)

$9.7 < R_{1.4}/\mathrm{km} < 13.9$ (100%)
Hebeler et al. (2010)

$10.0 < R_{1.4}/\mathrm{km} < 13.7$ (100%)
Hebeler et al. (2013)

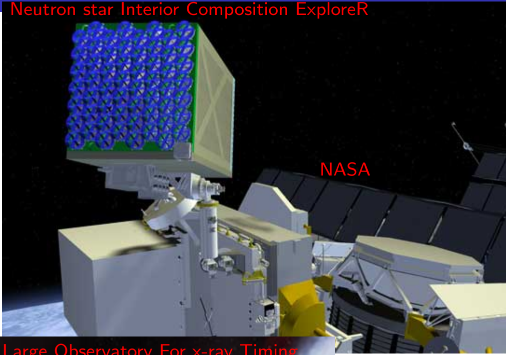


Consistency with Neutron Matter and Heavy-Ion Collisions



Additional Proposed Radius and Mass Constraints

- Pulse profiles
  Hot or cold regions on rotating neutron stars alter pulse shapes: NICER and LOFT will enable timing and spectroscopy of thermal and non-thermal emissions. Light curve modeling $\rightarrow M/R$; phase-resolved spectroscopy $\rightarrow R$.
- Moment of inertia
  Spin-orbit coupling of ultra-relativistic binary pulsars (e.g., PSR 0737+3039) vary i and contribute to $\dot{\omega}$: $I \propto MR^2$.
- Supernova neutrinos
  Millions of neutrinos detected from a Galactic supernova will measure BE$= m_B N - M$, $< E_\nu >$, τ_ν.
- QPOs from accreting sources
  ISCO and crustal oscillations



Constraints from Observations of Gravitational Radiation

Mergers:

Chirp mass $\mathcal{M} = (M_1 M_2)^{3/5} M^{-1/5}$ and tidal deformability $\lambda \propto R^5$ (Love number) are potentially measurable during inspiral.

$\bar{\lambda} \equiv \lambda M^{-5}$ is related to $\bar{I} \equiv I M^{-3}$ by an EOS-independent relation (Yagi & Yunes 2013). Both $\bar{\lambda}$ and $\bar{I}$ are also related to M/R in a relatively EOS-independent way (Lattimer & Lim 2013).

- Neutron star - neutron star: $M_{\rm crit}$ for prompt black hole formation, $f_{\rm peak}$ depends on R.
- Black hole - neutron star: $f_{\rm tidal\ disruption}$ depends on $R, a, M_{\rm BH}$. Disc mass depends on $a/M_{\rm BH}$ and on $M_{\rm NS} M_{\rm BH} R^{-2}$.

Rotating neutron stars: r-modes