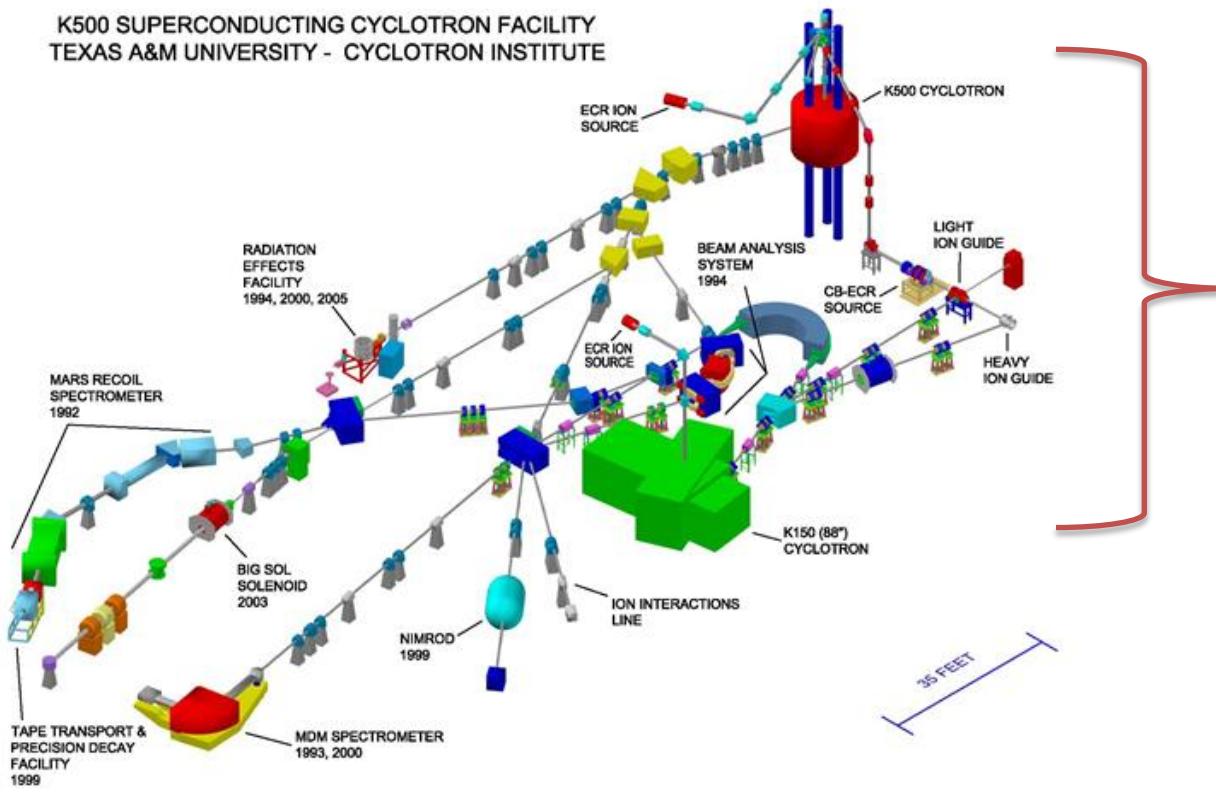


Clusterization and the Symmetry Energy in Low Density Nuclear Matter

J. Natowitz et al.



T-REX
[TAMU
Reaccelerated
EXotics]

**CYCLOTRON
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The Symmetry Energy Problem Density Dependence ?

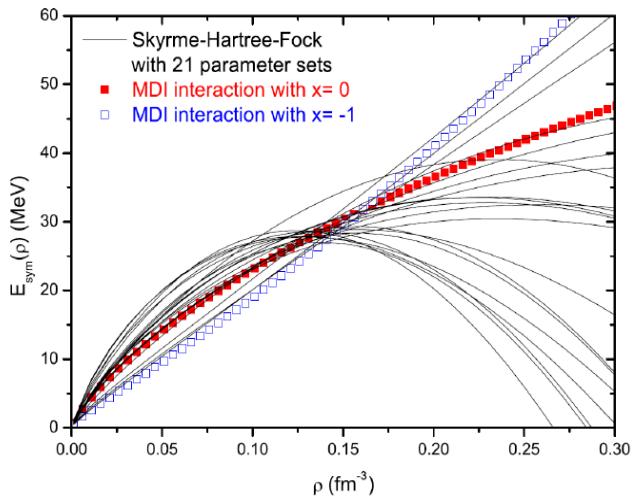


Fig. 3. Density dependence of the symmetry energy predicated by various interactions.

**Constraining the density dependence
of the symmetry energy away from
normal density is a complex problem-**

Paweł Danielewicz¹ and Jenny Lee²

Nuclear symmetry energy: An experimental overview

D V SHETTY^{1,*} and S J YENNELLO^{2,3}

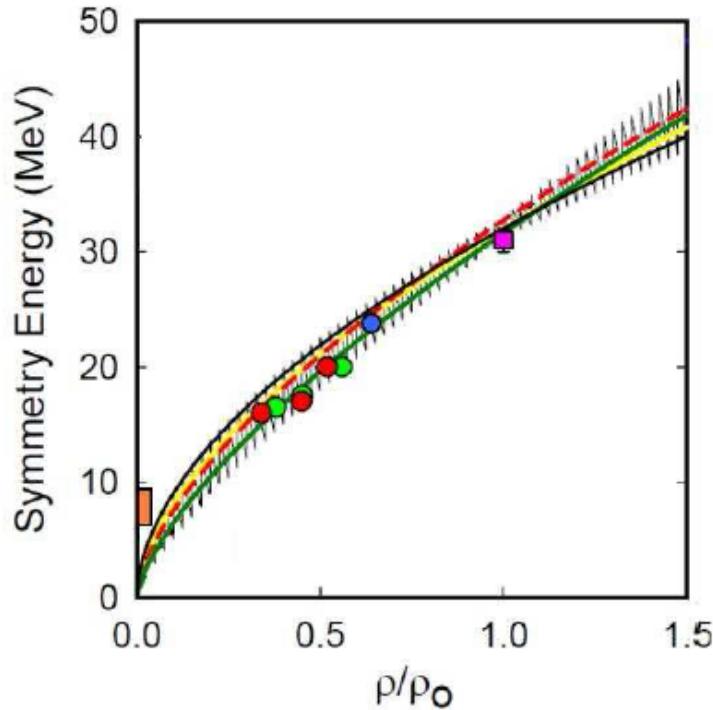


Figure 2. Density dependence of the symmetry energy extracted from different studies described in the text.

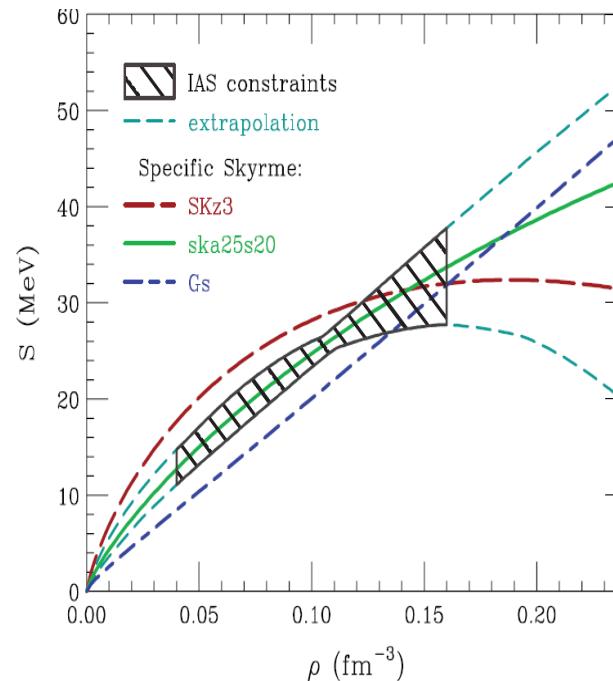
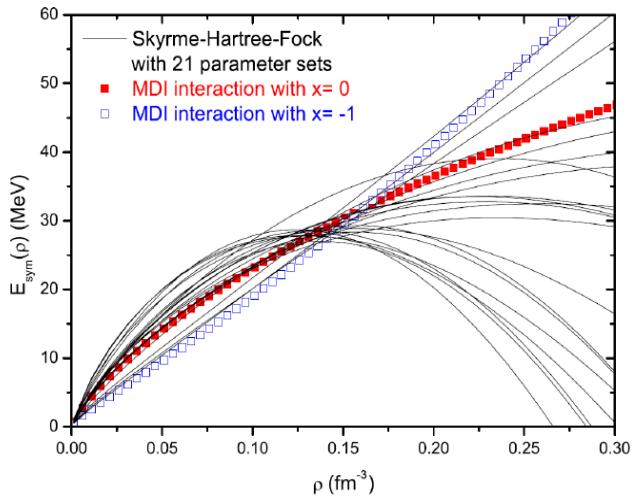


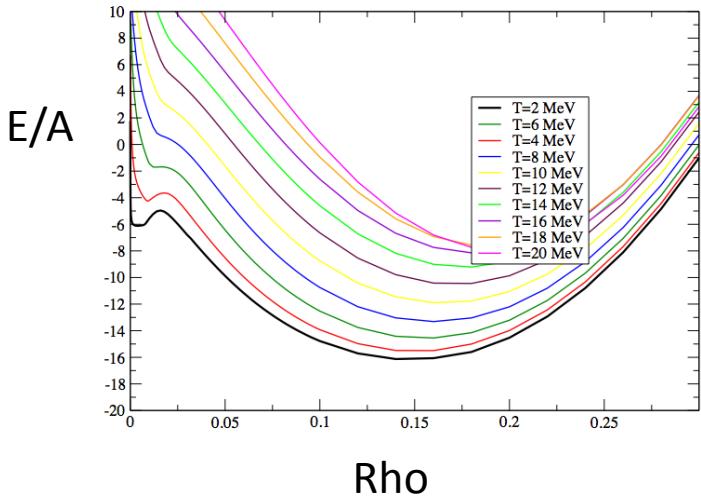
FIG. 15: Symmetry energy in uniform matter as a function of density. The hatched region represents IAS constraints. The short-dashed lines represent extrapolations of that region to supranormal, $\rho > \rho_0$, and low, $\rho < \rho_0/4$, densities. The solid, long-dashed and short-long-dashed lines represent the symmetry energies for the three Skyrme parametrizations represented in Fig. 14 (with symmetry coefficients).

[arXiv:1307.4130](https://arxiv.org/abs/1307.4130)

The Symmetry Energy Problem Density Dependence ?

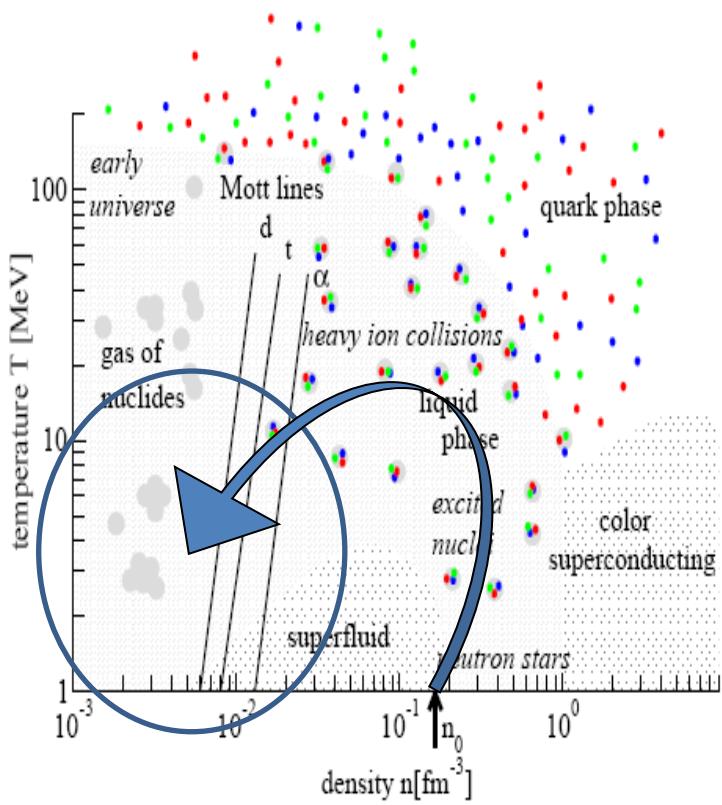


Constraining the density dependence of the symmetry energy away from normal density is a complex problem-



While low density situation would appear to be easier to constrain- cluster formation changes the medium This leads to additional complexity (opportunity)

CLUSTER FORMATION Modifies Nuclear EOS



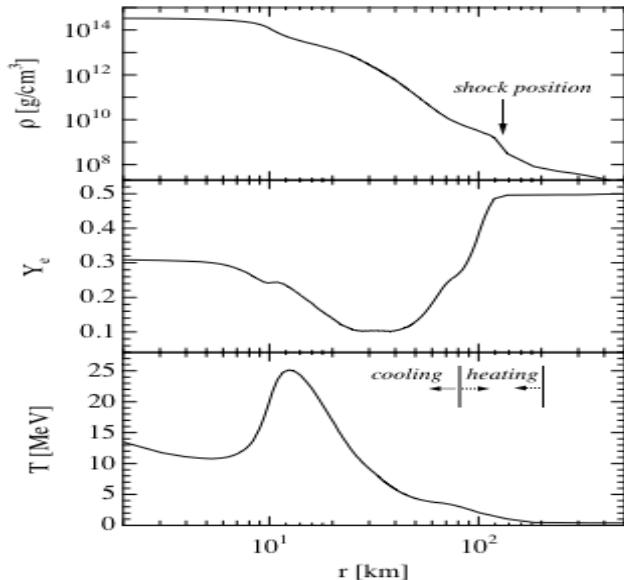
M. Beyer et al.,
Phys.Lett. B488, 247-253 (2000)

S. Typel, et al., ArXiv 0908.2344v1
August 2009

Astrophysical Implications, e.g., Core-collapse Supernovae

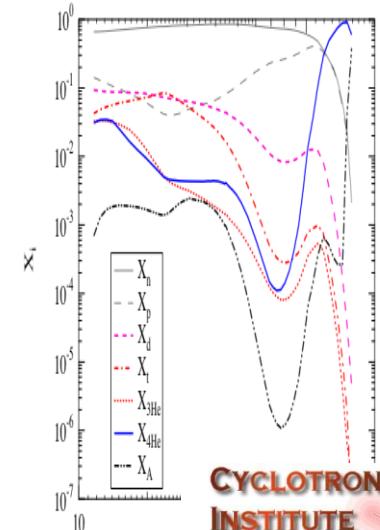
K.Sumiyoshi et al.,
Astrophys.J. **629**,
922 (2005)

Density, electron fraction, and temperature profile of a 15 solar mass supernova at 150 ms after core bounce --as function of the radius.

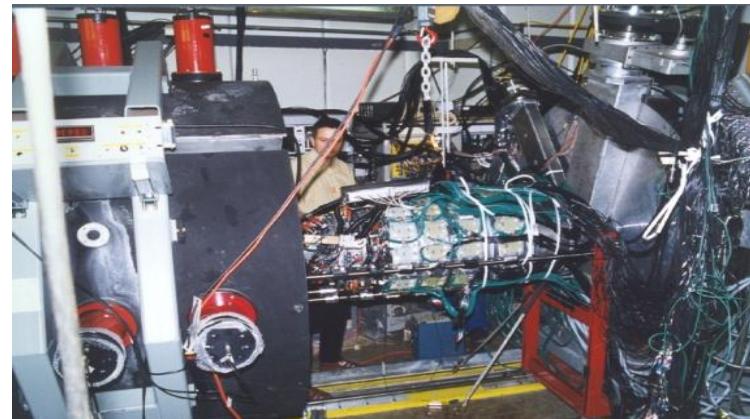
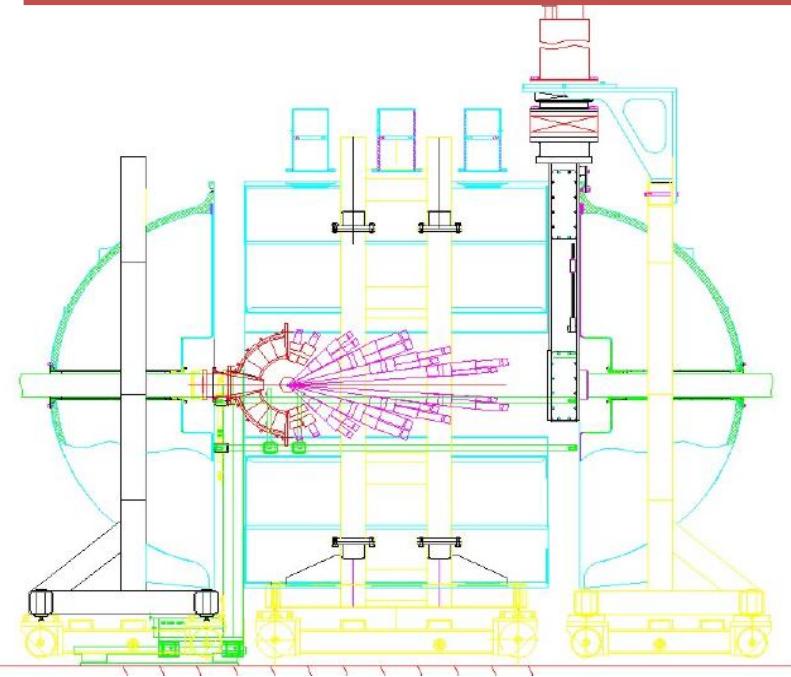


K.Sumiyoshi, G.
Roepke
PRC 77, 055804
(2008)

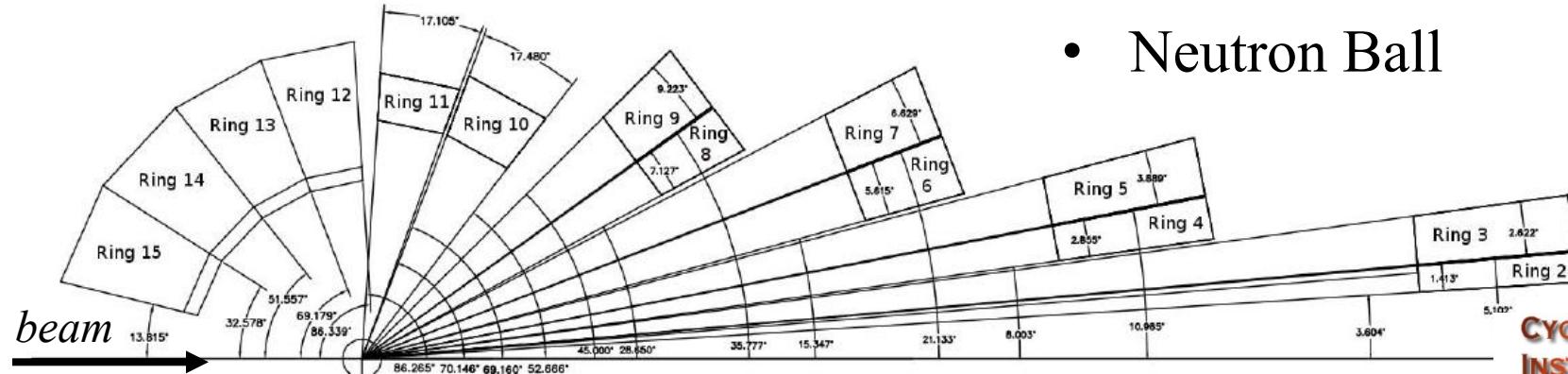
**cluster
formation
Influences
neutrino
flux**



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- 14 Concentric Rings
- Silicon-CsI
- 3.6-167 degrees
- Neutron Ball



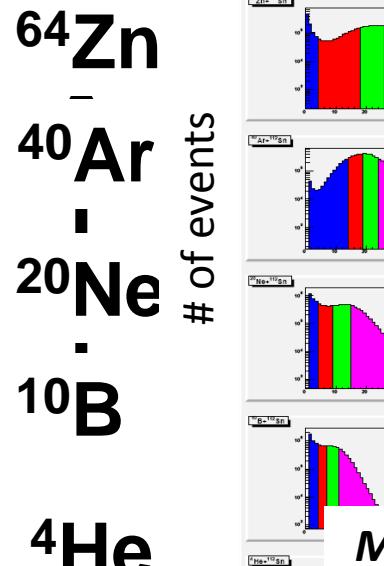
Light Charged Particle Emission - High Total Multiplicity Collisions

Thesis – L. Qin

TAMU-

Projectile Energy - 47A MeV

112Sn 124Sn



Charged-particle Multiplicity

Velocity Plots- Light Charged Particles

Global Fits
3 Sources

Velocity Plot Protons
 $^{40}\text{Ar} + ^{124}\text{Sn}$

From Fitting

Experiment

Sampling the GAS-
early emission faster
particles

IV

N

V parallel

V perpendicular

Evaporation-like

TLF

Sampling the Liquid
– late emission

Sum of Source Fits

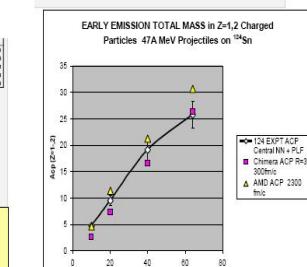
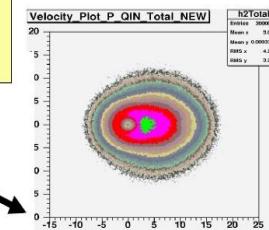


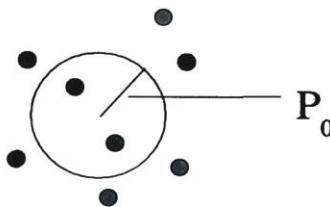
Fig. 34. Sum of neutron and charged particle multiplicity distribution of each reaction system. Bin4 corresponds to the most violent collision events, Bin3 corresponds to the semiviolent events, Bin2 corresponds to the semiperipheral events, and Bin1 corresponds to the peripheral events.

**Analysis of IV(NN) Source
- As a Nascent fireball**

Density Extraction Method Coalescence Model

Radius (momentum space) T.C. Awes et al, Phys. Rev. C24 1981 89

$$\frac{d^2 N(Z, N, E_A)}{dE_A d\Omega} = \frac{A^{-1}}{N! Z!} \left(\frac{\frac{4}{3}\pi P_0^3}{[2m^3(E - E_C)]^{1/2}} \right)^{A-1} \left(\frac{d^2 N(0, 1, E)}{dEd\Omega} \right)^N \left(\frac{d^2 N(1, 0, E)}{dEd\Omega} \right)^Z$$



$$\frac{d^2 N(Z, N, E_A)}{dE_A d\Omega} = R_{np}^N \frac{A^{-1}}{N! Z!} \left(\frac{\frac{1}{3}\pi P_0^3}{[2m^3(E - E_C)]^{1/2}} \right)^{A-1} \left(\frac{d^2 N(1, 0, E)}{dEd\Omega} \right)^A$$

Volume (coordinate space):

→Mekjian:
(thermal model)

$$V = \left(\frac{Z! N! A^3}{2^A} \right) (2s+1) e^{E_0/T} \left[\frac{3h^3}{4\pi P_0^3} \right]^{1/(A-1)}$$

PHYSICAL REVIEW C

VOLUME 17, NUMBER 3

MARCH 1978

Explosive nucleosynthesis, equilibrium thermodynamics, and relativistic heavy-ion collisions

A. Z. Mekjian

Nuclear Science Division, Lawrence Berkeley Laboratory, University of California, Berkeley, California 94720
and Department of Physics, Rutgers University, New Brunswick, New Jersey 08903

Coalescence Model Analysis

NN Source Evolution

PHYSICAL REVIEW C 72, 024603 (2005)

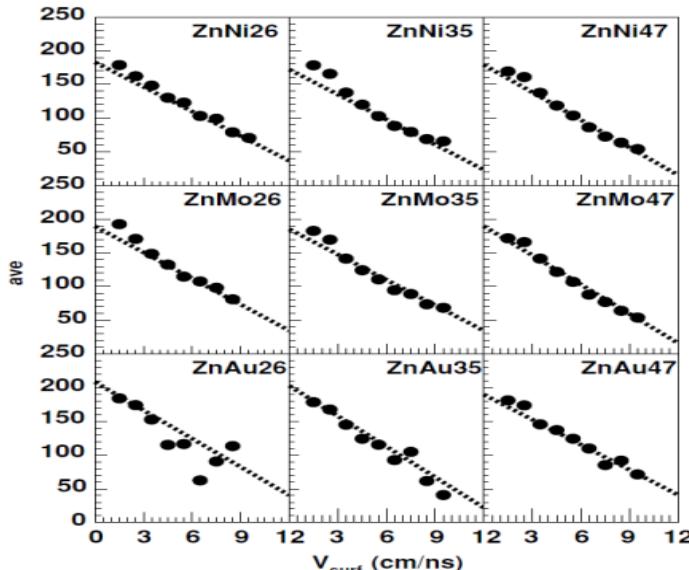


FIG. 2. Correlation of average emission time with surface velocity for early emitted nucleons as calculated by the AMD-V code. Solid symbols depict the results for the nine different reactions as labeled. Dashed lines indicate the linear fits assumed for time calibrations.

$$T_{\text{HHe}} = \frac{14.3}{\ln(\sqrt{9/8}(1.59 R_{v_{\text{surf}}}))}. \quad (1)$$

If Y represents a cluster yield, $R_{v_{\text{surf}}} = Y(^2\text{H})Y(^4\text{He})/Y(^3\text{H})Y(^3\text{He})$ for clusters with the same surface velocity. The constants 14.3 and 1.59 reflect binding energy, spin, masses and mass differences of the ejectiles. Eq. (1) differs from the usual formulation only by the factor of $\sqrt{9/8}$ appearing in the logarithm term in the denominator.

$v_{\text{surf}} \sim \text{time}$

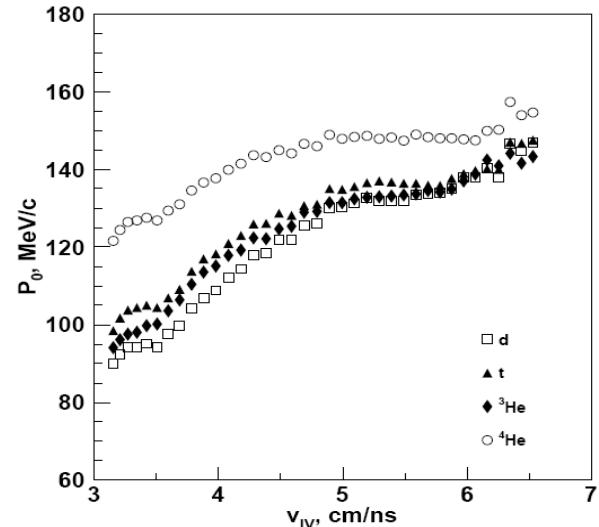


FIG. 1: Coalescence parameters, P_0 as a function of surface velocity in the intermediate velocity source frame. Reaction: 47 A MeV $^{40}\text{Ar} + ^{112}\text{Sn}$

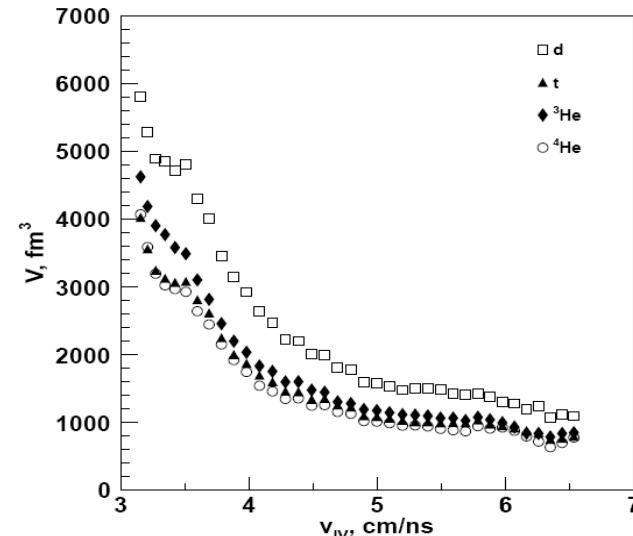


FIG. 2: Coalescence model volumes as a function of velocity in the intermediate velocity source frame. Reaction: 47 A MeV $^{40}\text{Ar} + ^{112}\text{Sn}$

Temperatures and Densities

^{40}Ar , $^{64}\text{Zn} + ^{112}\text{Sn}$, ^{124}Sn

- System starts hot
- As it cools,
it expands

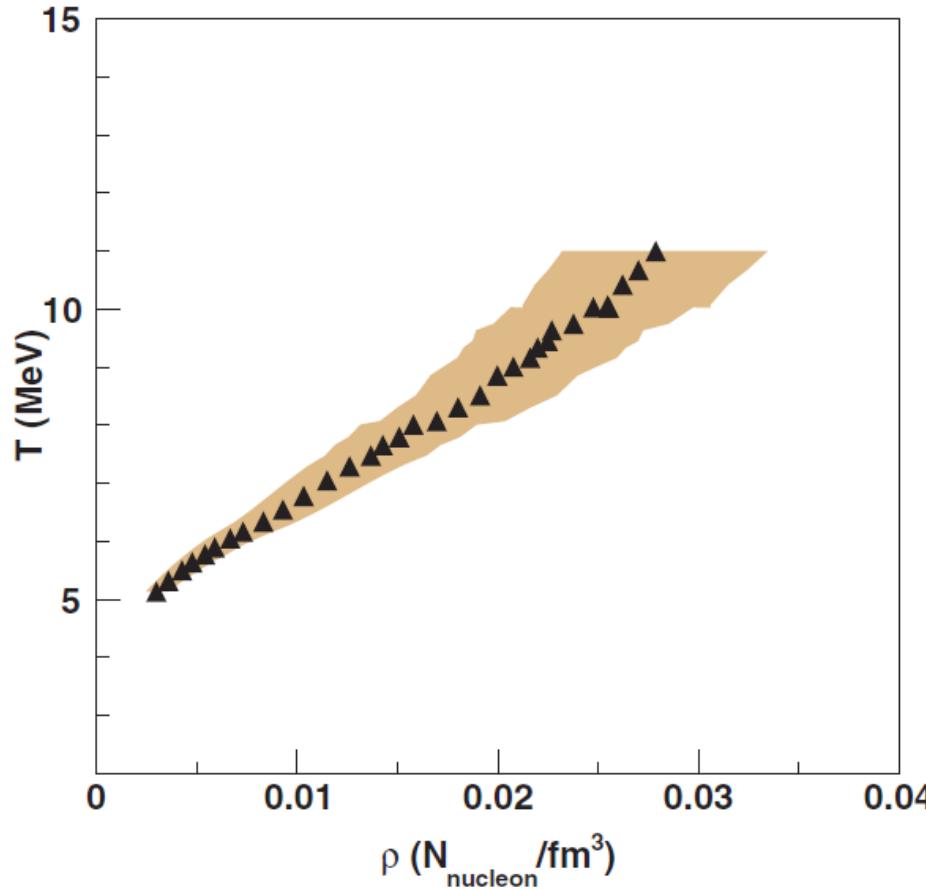


FIG. 3. (Color online) Temperatures and densities sampled by cluster emission from the expanding IV source.

Astrophysical Equations of State - Alpha Mass Fractions

29

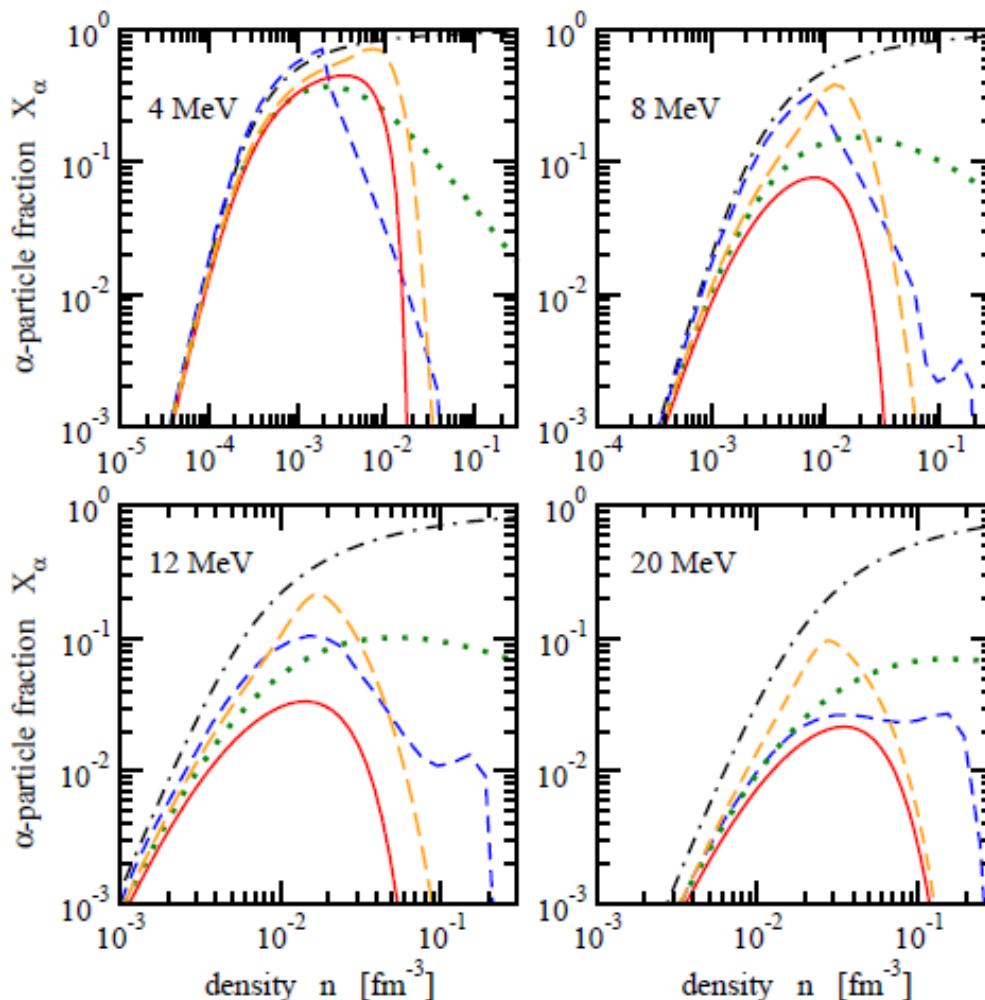


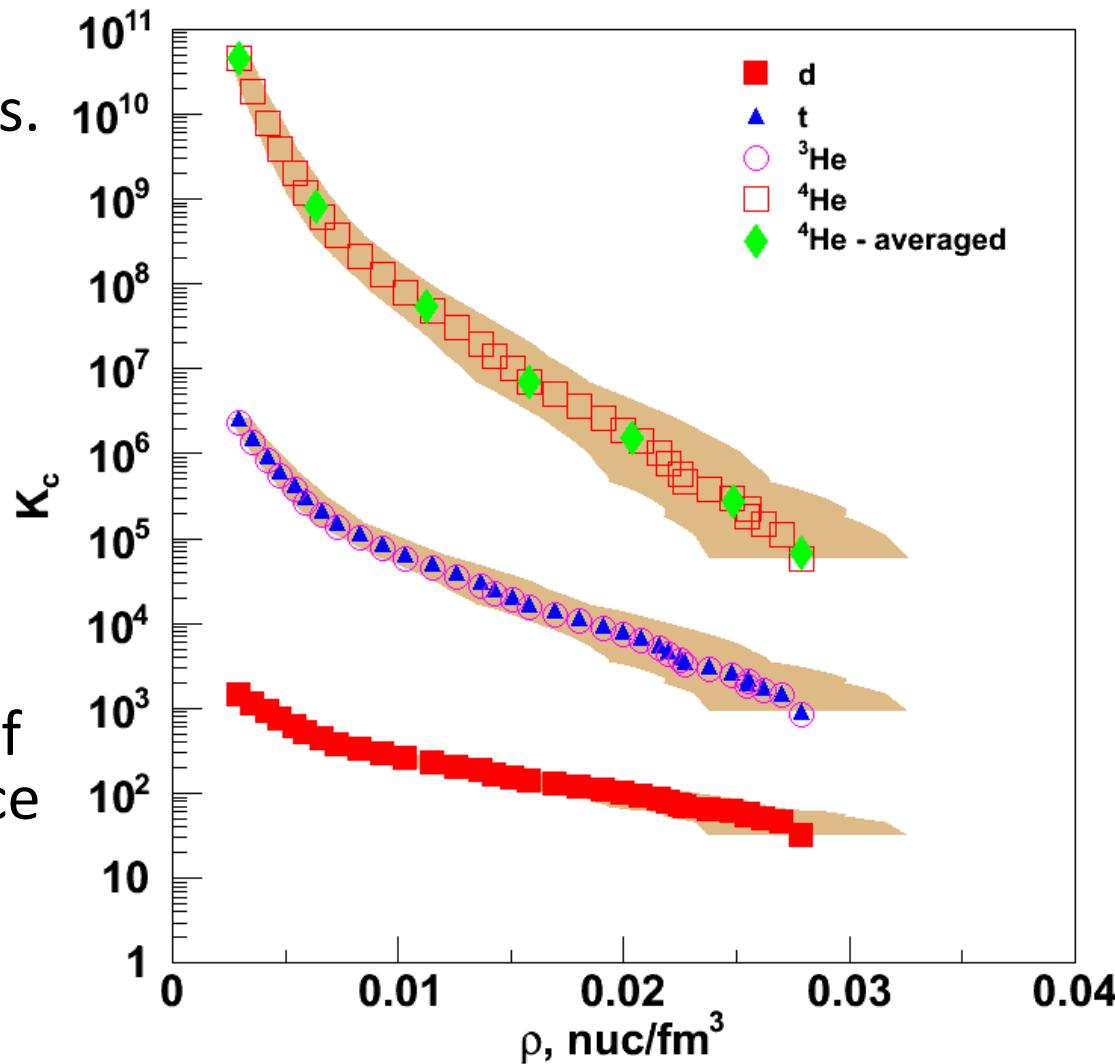
FIG. 15: (Color online) Comparison of α -particle fractions in symmetric nuclear matter as a function of the density at four temperatures for the virial expansion (black dashed-dotted lines), NSE (green dotted lines), the EoS of Shen et al. [29] (blue dashed lines), the generalized RMF model (red solid lines) and the QS approach (orange dashed lines). Note the different scales on the x-axes.

S. Typel, G. Roepke, T. Klahn
D. Blaschke and H.H. Wolter
ArXiv 0908.2344v1
August 2009

Equilibrium constants

- Many tests of EOS are done using mass fractions.
- Various calculations include various different competing species.
- In calculations, if any relevant species is not included, mass fractions are not accurate.
- **Equilibrium constants** should be independent of proton fraction and choice of competing species.

() —————



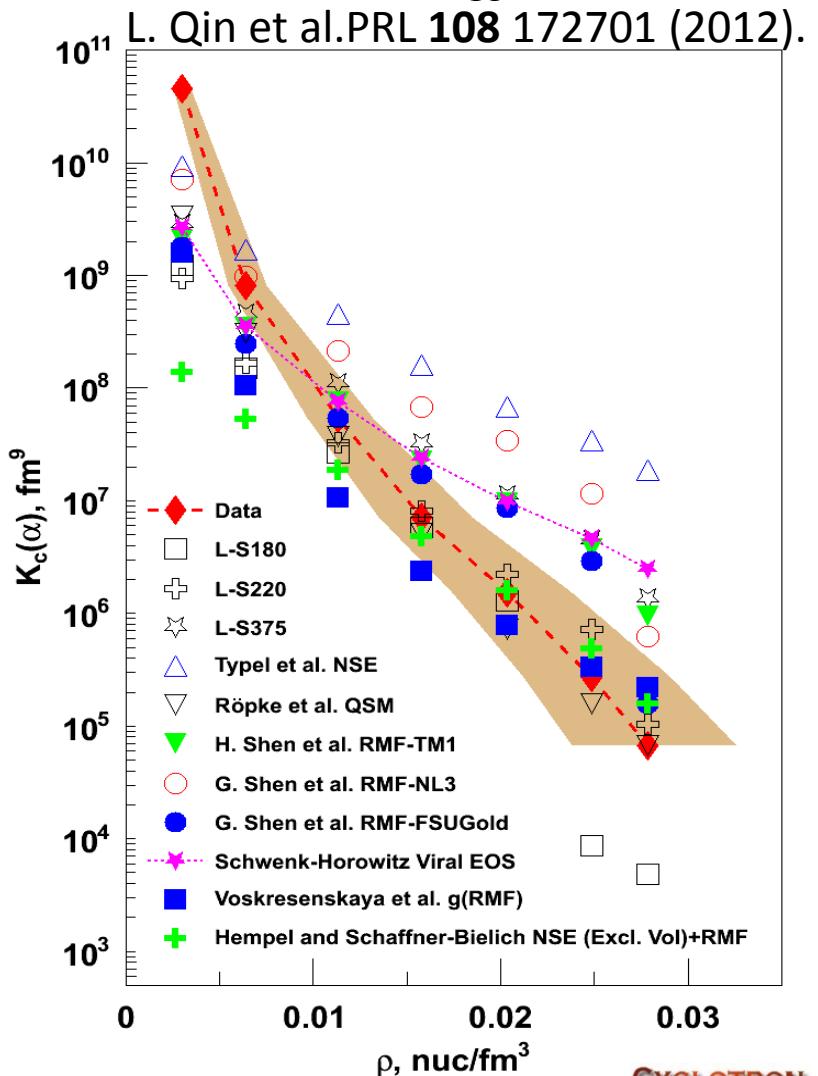
Test of Astrophysical Equations of State

Equilibrium Constant, K_α

- Many tests of EOS are done using mass fractions. Various calculations include various different competing species., if all relevant species are not included, mass fractions are not accurate.
- Equilibrium constants, e.g.,
$$(\quad) \quad \underline{\quad}$$

should be independent of proton fraction and choice of competing species.

- Models converge at lowest densities, but many are significantly above data at higher density
- Lattimer & Swesty with $K=180, 220$ show reasonable agreement with data (excluded volume)
- QSM (Roepke et al.) with in-medium binding energy shifts works well



Light clusters in nuclear matter: Excluded volume versus quantum many-body approaches

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Jürgen Schaffner-Bielich†

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Stefan Typel‡

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Gerd Röpke§

Institut für Physik, Universität Rostock, Universitätsplatz 3, 18051 Rostock, Germany

Clusterized nuclear matter in the (proto-)neutron star crust and the symmetry energy

[arXiv:1307.4202](https://arxiv.org/abs/1307.4202)

[Ad. R. Raduta](#), [F. Aymard](#), [F. Gulminelli](#)

A contribution to the upcoming EPJA Special Volume on Nuclear Symmetry Energy

IN MEDIUM BINDING ENERGIES and MOTT TRANSITION

G. Roepke and Collaborators

M. Beyer et al.,

Phys.Lett. B488, 247-253 (2000)

FIGURES

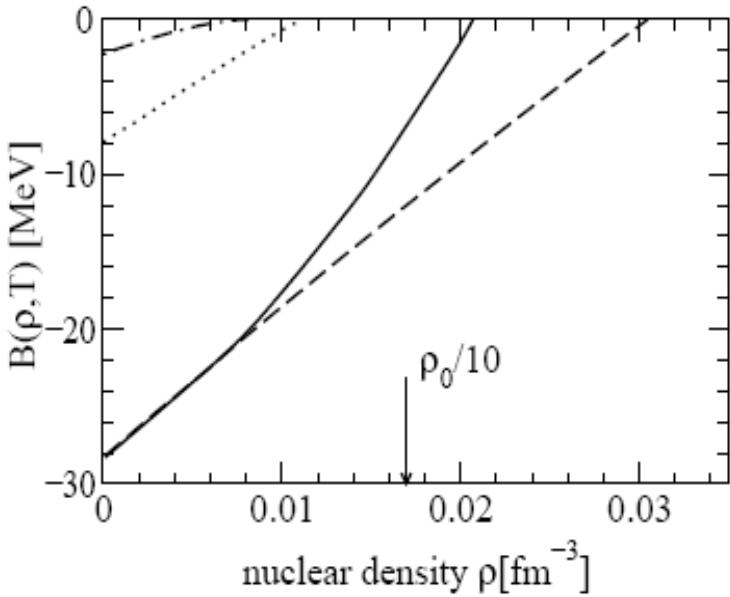
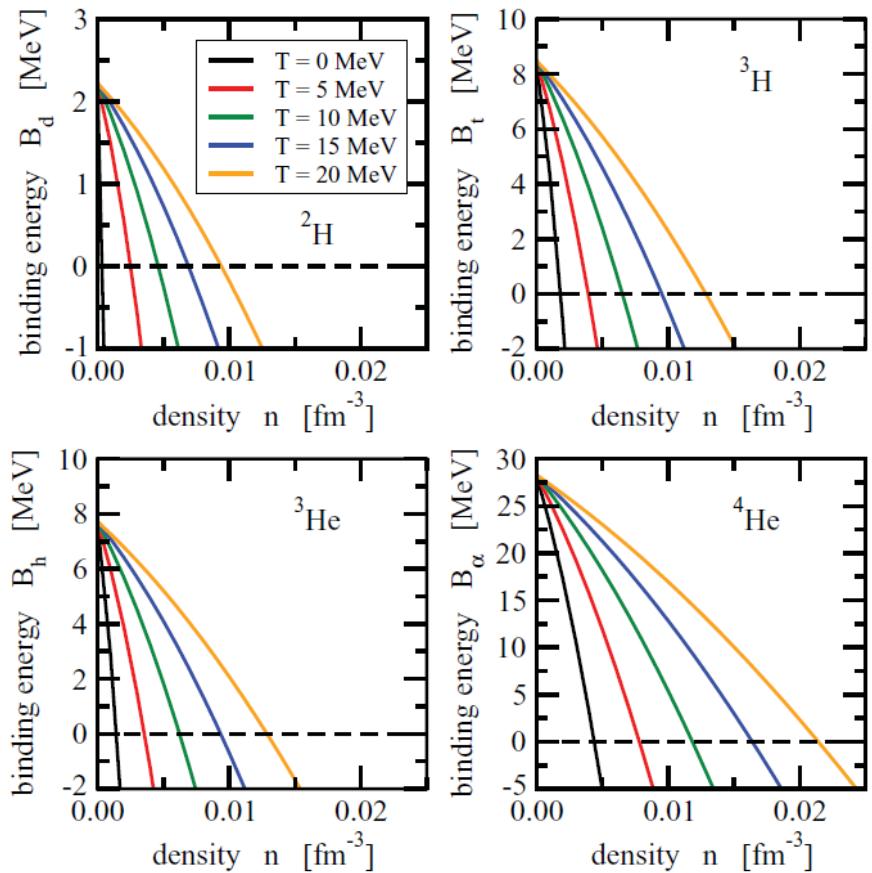


FIG. 1. Binding energy of an α -like cluster with zero c.m. momentum embedded in symmetric nuclear matter at a temperature of $T = 10$ MeV as a function of nucleon density. Solid line: Yamaguchi potential, renormalized to experimental binding energy at zero density. Dashed line: perturbation approach. For comparison, the medium dependent binding energies of the deuteron (dashed-dotted) and triton (dotted) are also shown.



Roepke et al- May 2010

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DERIVING MOTT POINTS

$$K_c(A, Z) = \rho_{(A, Z)} / [(\rho_p)^Z (\rho_n)^N]$$

– Albergo et al.,

IL NUOVO CIMENTO

VOL. 89 A, N. 1

1 Settembre 1985

$$\rho(A, Z) = \frac{N(A, Z)}{V} = \frac{A^{\frac{3}{2}} \lambda_T^{3(A-1)} \omega(A, Z)}{(2s_p+1)^Z (2s_n+1)^{A-Z}} \times \rho_p^Z \rho_n^{A-Z} \exp \frac{B(A, Z)}{T} \quad (4)$$

In this expression, $\lambda_T = 2\pi/MT)^{1/2}$ is the thermal wavelength of a nucleon, s_p and s_n are the proton and neutron spins, T is the temperature and $B(A, Z)$ is the cluster binding energy. The term $\omega(A, Z)$ is the internal partition function of the cluster, taken here to 1 for the $Z = 1$ and $Z = 2$ clusters considered.

Mixing Entropy Term --Added Contribution to Free Energy (Hirsch et al., PRC 29, Feb. 1984)

$$\Delta F = T(Z \ln(Z/A) + N \ln(N/A)) \quad (5)$$

where once again Z , N and A are those of the cluster being formed [20]. As mixing is a spontaneous process the free energy of mixing is negative and therefore favors the cluster formation.

Use NSE (Albergo) Relation, Add Mixing Entropy term (Minich) , Solve for B(T)

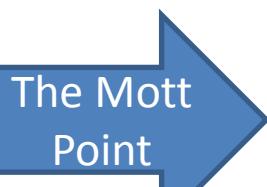
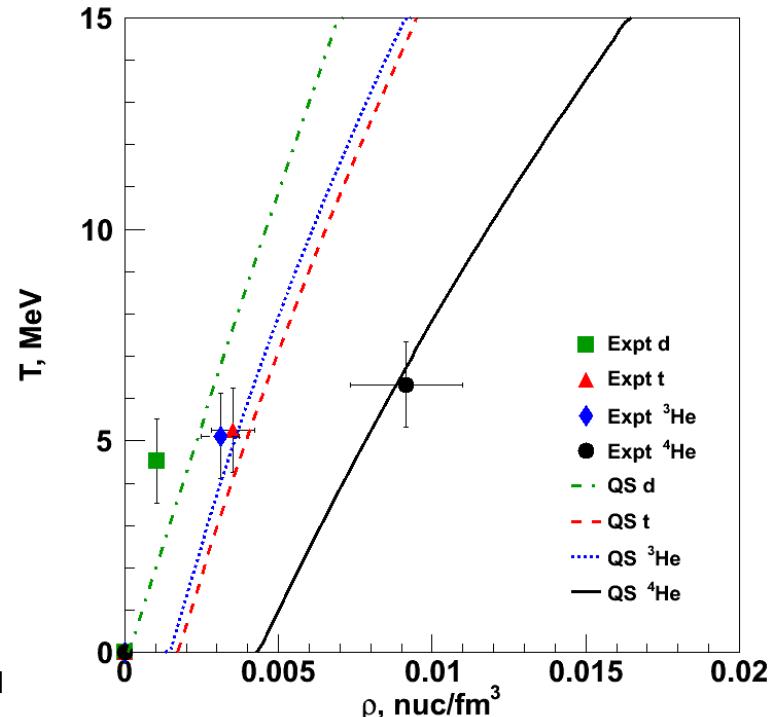
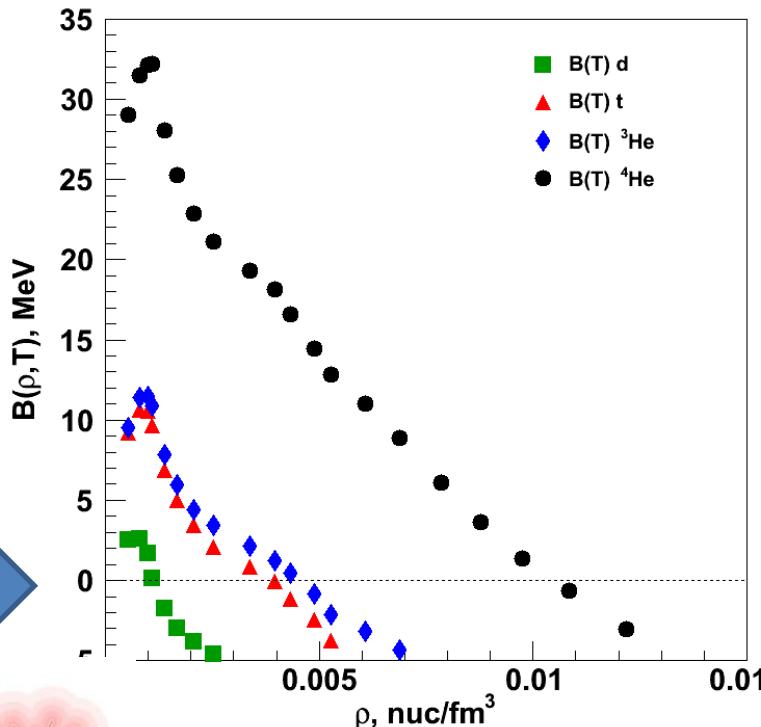
$$\ln[K_c/C(T)] = B/T - Z \ln(Z/A) - N \ln(N/A) \quad (6)$$

where $C(T)$ includes all terms on the right hand side of Eq. (4) except the exponential term. Using the experimentally determined equilibrium constants and temperatures we then solve this expression to obtain the apparent binding energies, $B(\rho, T)$, of the clusters for the different temperatures and densities sampled in the experiments.

Density Dependent Binding Energies of Light Clusters – The Mott Point

- Successful QSM Model of Roepke et al. Incorporates In-Medium modification of Cluster Binding Energies . Clusters become unbound wrt medium at the Mott Point
- Mott points determined in reactions of 47AMeV ^{40}Ar and ^{64}Zn projectiles with $^{112}, ^{124}\text{Sn}$ are in close agreement with the theoretical estimates

K. Hagel et al., PRL **108** 062702 (2012).

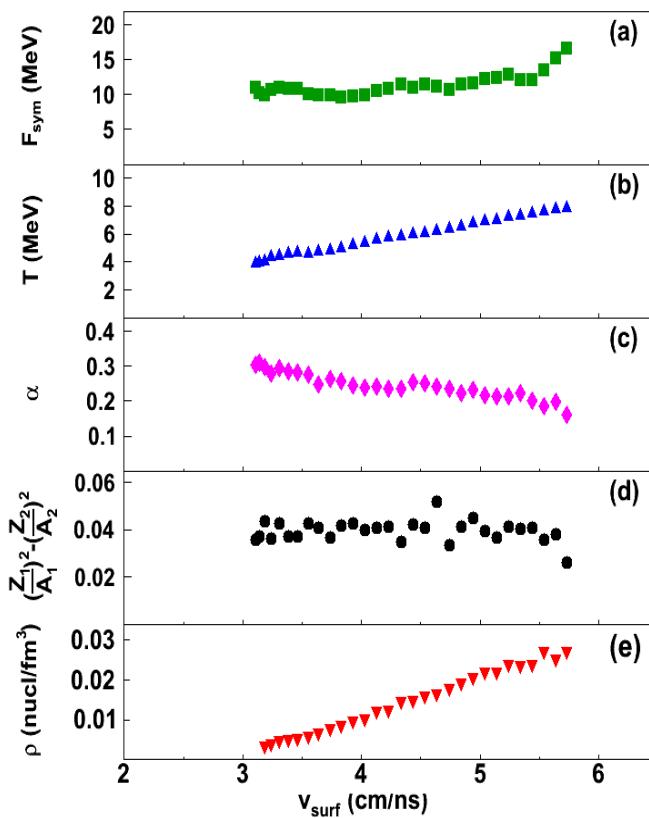


Isoscaling Analysis and Symmetry Energy

An isoscaling analysis of Light Cluster Yield Ratios For Different N/Z Systems Allows Determination of The Symmetry Free Energy.

$$\frac{Y_2}{Y_1} = Ce^{\{[\mu_2(n)-\mu_1(n)]N+[\mu_2(p)-\mu_1(p)]Z\}/T} = Ce^{\alpha N + \beta Z},$$

————— [(—) (—)]



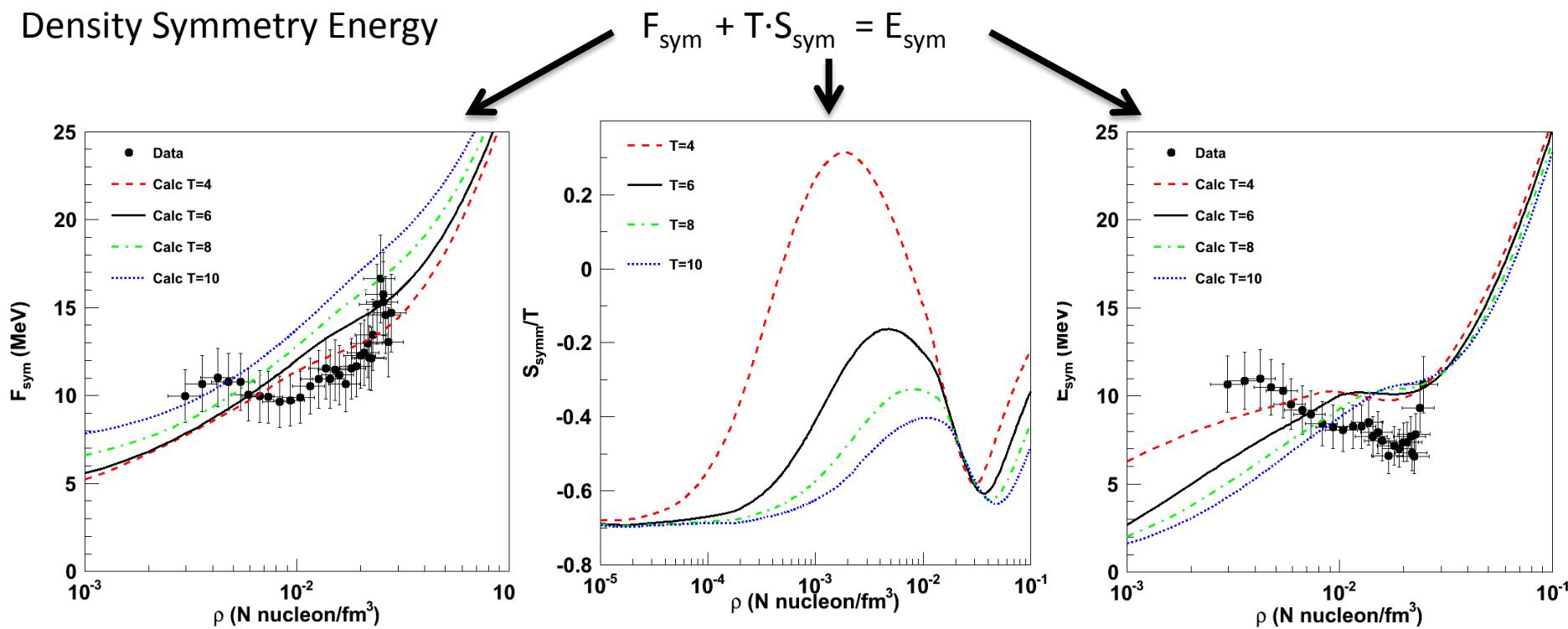
R. Wada et al. Phys. Rev. C 85, 064618 (2012)

- Both T and ρ are changing
- T, ρ increase with v_{surf}
- α extracted from light particle yields decreases with v_{surf}
- F_{sym} Relatively Flat

Symmetry Energy at Low Density

R. Wada et al. Phys. Rev. C 85, 064618 (2012)

At Low Density The Symmetry Energy is Determined by Cluster Formation. Analysis of Cluster Yield Ratios For Different N/Z Systems Allows Determination of The Symmetry Free Energy. Employment of Entropies Calculated with the QSM Model of Roepke, Typel et al (shown to be appropriate by other measured quantities) Allows Extraction of The LOW Density Symmetry Energy



See also S. Typel et al., Phys. Rev. C 81, 015803 (2010).
J.B. Natowitz et al., Phys. Rev. Lett. 104:202501 (2010).

Symmetry energy of dense inhomogeneous matter

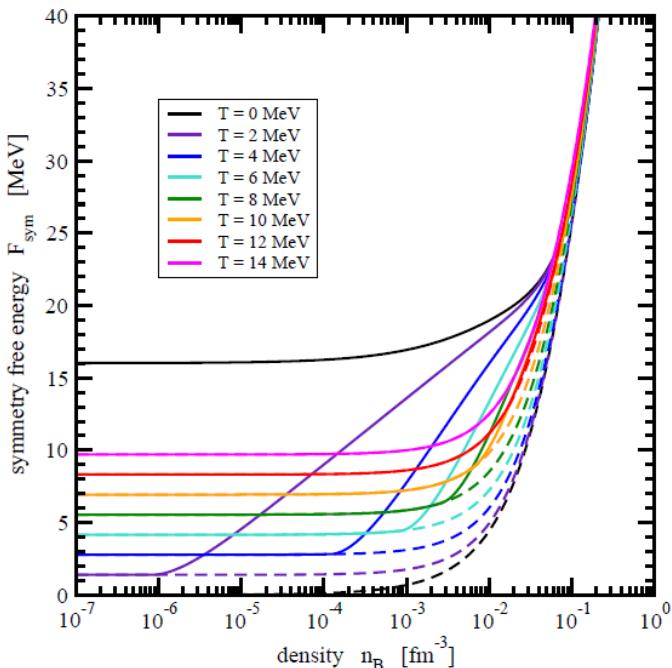


Fig. 10. Symmetry free energy F_{sym} in nuclear matter without (dashed lines) and with (full lines) liquid-gas phase transition as a function of the baryon density n_B for various temperatures using the finite difference definition of the symmetry free energy.

S. Typel¹, H.H. Wolter², G. Röpke³, and D. Blaschke^{4,5}

Prepared for EPJA July 2013

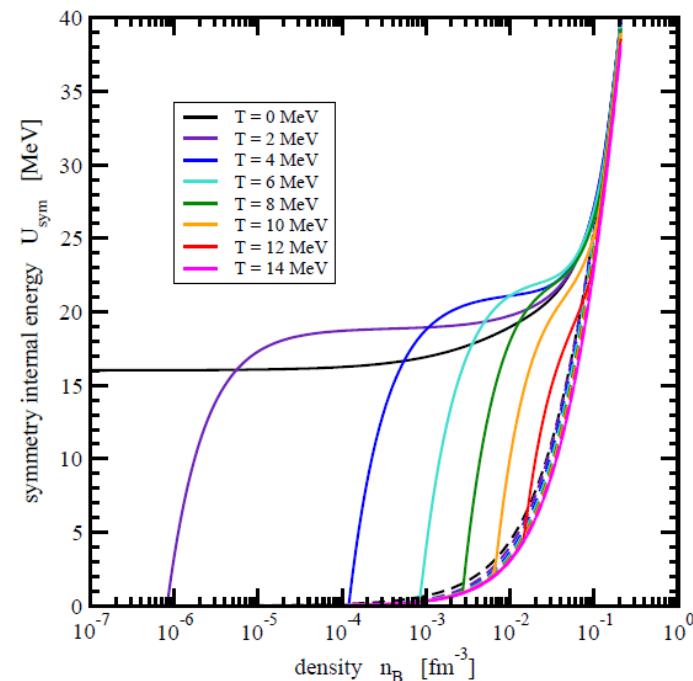


Fig. 11. Symmetry internal energy U_{sym} in nuclear matter without (dashed lines) and with (full lines) liquid-gas phase transition as a function of the baryon density n_B for various temperatures using the finite difference definition of the symmetry internal energy.

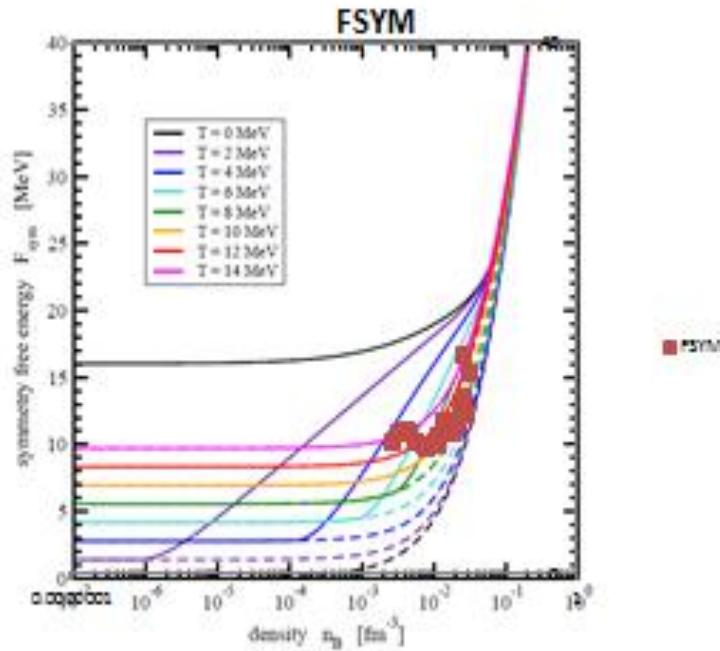


Fig. 10. Symmetry free energy F_{sym} in nuclear matter without (dashed lines) and with (full lines) liquid-gas phase transition as a function of the baryon density n_B for various temperatures using the finite difference definition of the symmetry free energy.

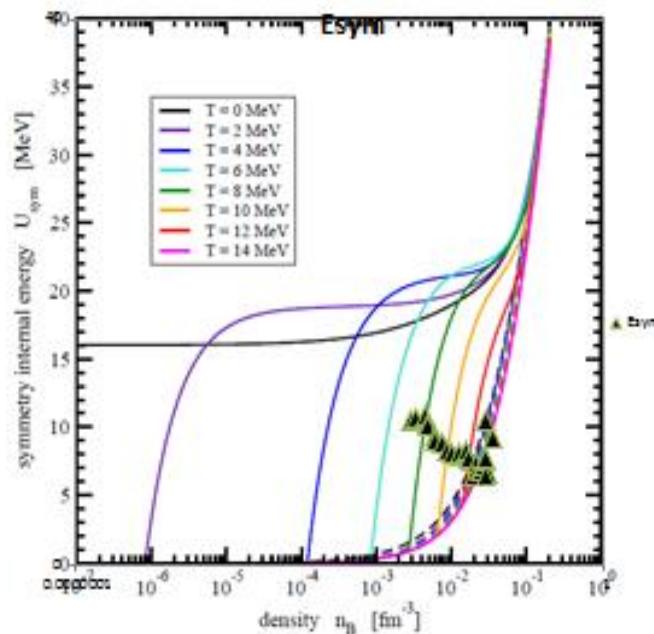


Fig. 11. Symmetry internal energy U_{sym} in nuclear matter without (dashed lines) and with (full lines) liquid-gas phase transition as a function of the baryon density n_B for various temperatures using the finite difference definition of the symmetry internal energy.

Symmetry Energy II: Isobaric Analog States

Paweł Danielewicz¹ and Jenny Lee²

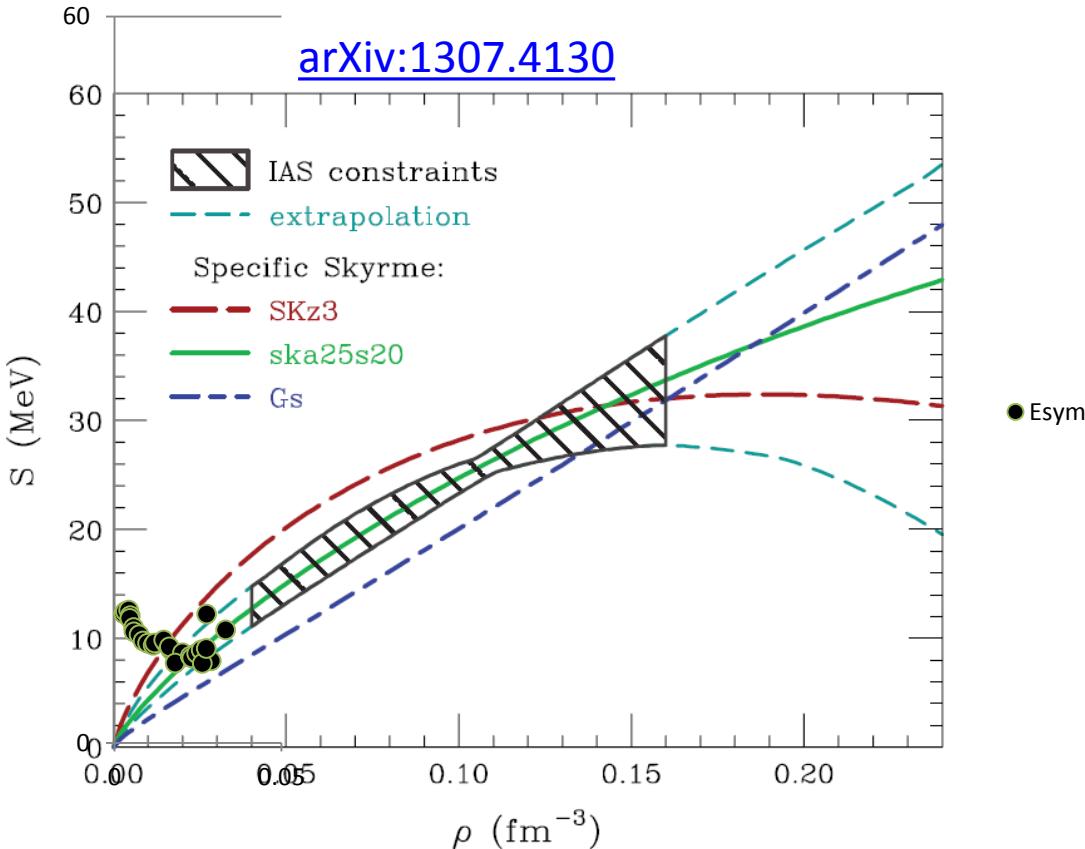


FIG. 15: Symmetry energy in uniform matter as a function of density. The hatched region represents IAS constraints. The short-dashed lines represent extrapolations of that region to supranormal, $\rho > \rho_0$, and low, $\rho < \rho_0/4$, densities. The solid, long-dashed and short-long-dashed lines represent the symmetry energies for the three Skyrme parametrizations represented in Fig. 14 (with symmetry coefficients).

Density Determinations in heavy Ion Collisions – G Roepke et al.
Submitted To Phys. Rev. C July 2013

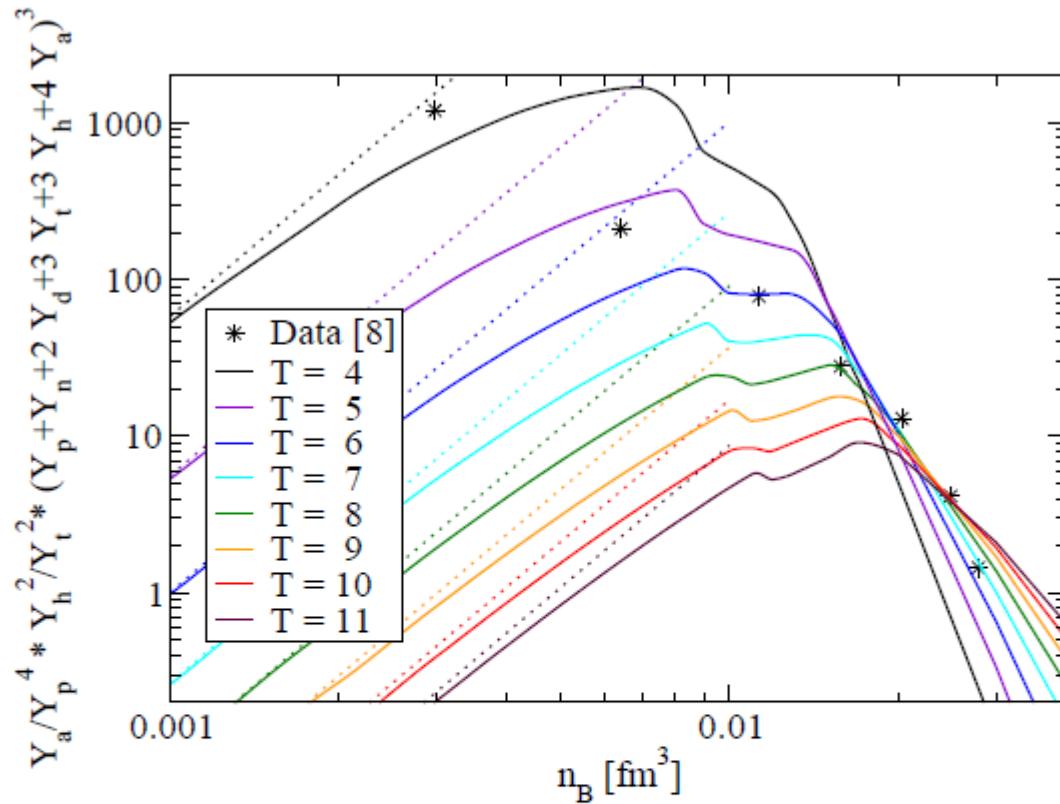


FIG. 2: (Color online) Chemical constant \tilde{K}_α , Eq. (11), as function of density and temperature (T in MeV). Data (stars) [8] for $T = 5, 6, 7, 8, 9, 10, 11 \text{ MeV}$ (increasing density) in comparison with the NSE values (thin dotted lines) and QS calculations (bold straight lines).

Density Determinations in heavy Ion Collisions – G Roepke et al.
Submitted To Phys. Rev. C July 2013

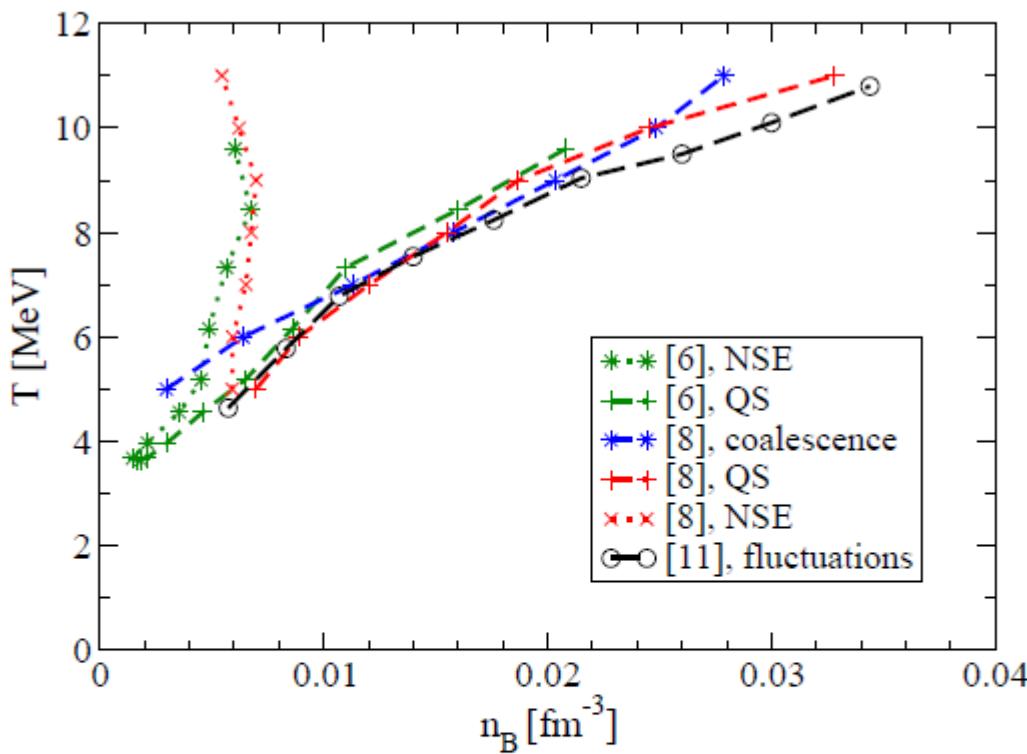


FIG. 3: (Color online) Baryon density derived from yields of light elements. Data according to [6, 8, 11] are compared with the results of the analysis of yields using NSE and QS calculations for \tilde{K}_α .

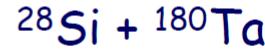
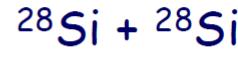
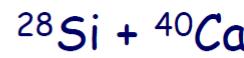
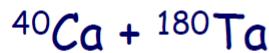
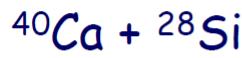
See Also
J. Mabiala et al.
arXiv1208.3480v1

And this workshop

IN PROGRESS - COLLISIONS AND DISASSEMBLY OF ALPHA-CONJUGATE NUCLEI

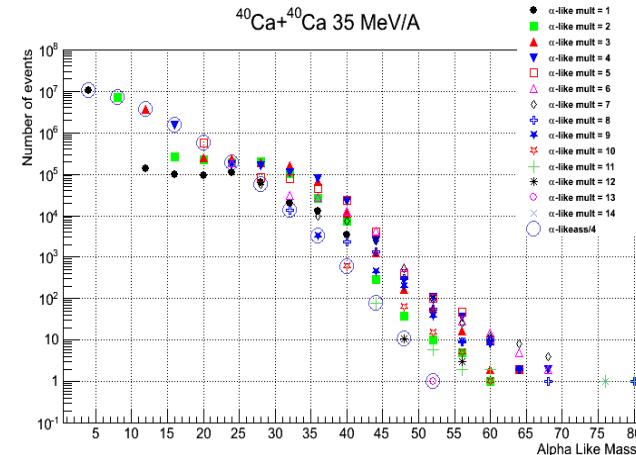
1. Premise: Interest in clusterization in low density nuclear matter pursued in collisions of alpha-conjugate nuclei. Explore role of clusterization in dynamics and disassembly. Study exotic dynamically formed alpha clustered states

2. METHOD: Employing TAMU NIMROD detector system to observe production and disassembly of alpha-clustered entities. Characterize role of clusterization in reaction mechanisms. Inquire into Bose condensate possibilities.

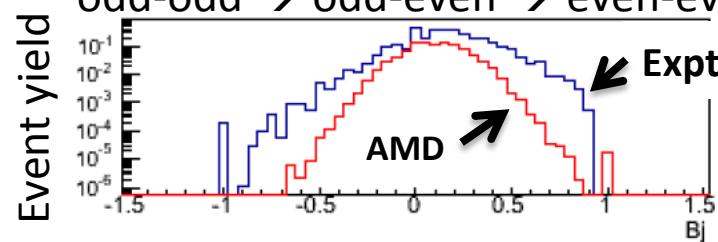


at 10, 25 and 35 MeV/A

3. Preliminary Results I: Global characterization of exit channels in multiplicities of α -like (alpha or alpha-conjugate ejectiles) indicates larger contribution of such ejectiles than obtained in theoretical (AMD) estimates.



odd-odd \rightarrow odd-even \rightarrow even-even

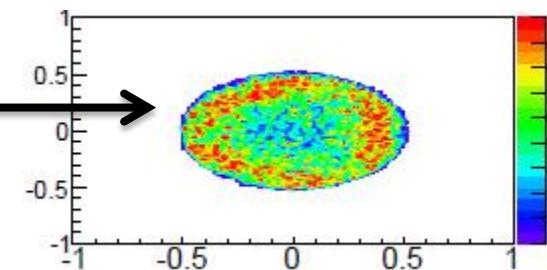
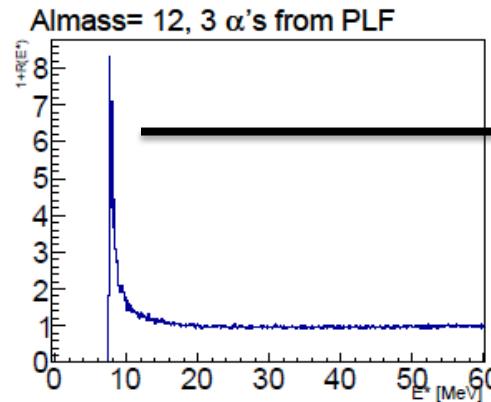
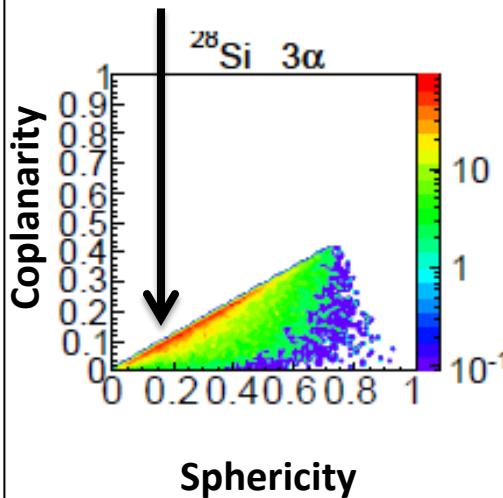


$$Bj = \frac{1}{M} \sum_{i=1}^M \frac{(-1)^{Z_i} + (-1)^{N_i}}{2}$$

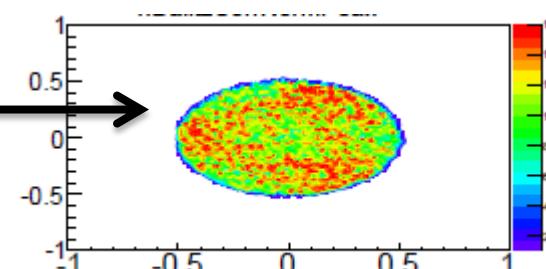
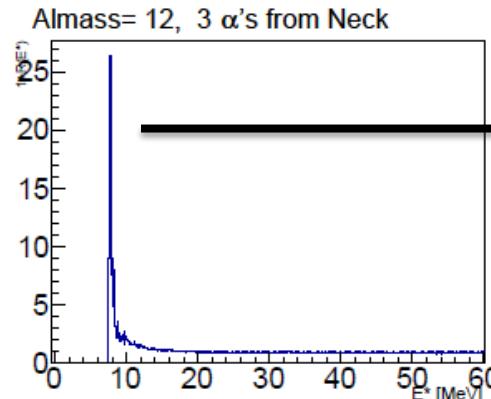
IN PROGRESS - COLLISIONS AND DISASSEMBLY OF ALPHA-CONJUGATE NUCLEI

3. Preliminary Results II: Investigation of mid-peripheral collisions reveals large cross sections for formation of alpha-conjugate neck structures associated with leading heavy alpha-conjugate fragment, ($^{28}\text{Si}, 3\alpha$), ($^{24}\text{Mg}, 4 \alpha$), ($^{20}\text{Ne}, 5 \alpha$) etc. **Disassembly indicates important role of initial neck geometry and/or proximity effects in neck disassembly.**

ROD-LIKE EVENTS



Energy Dalitz Plot
3 α From PLF



Energy Dalitz Plot
3 α From Neck

3 Energy Correlation Functions

ANALYSIS CONTINUES

Deformation and cluster structures in ^{12}C studied with configuration mixing using Skyrme interactions

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arXiv 1304.5297v1

22 April 2013

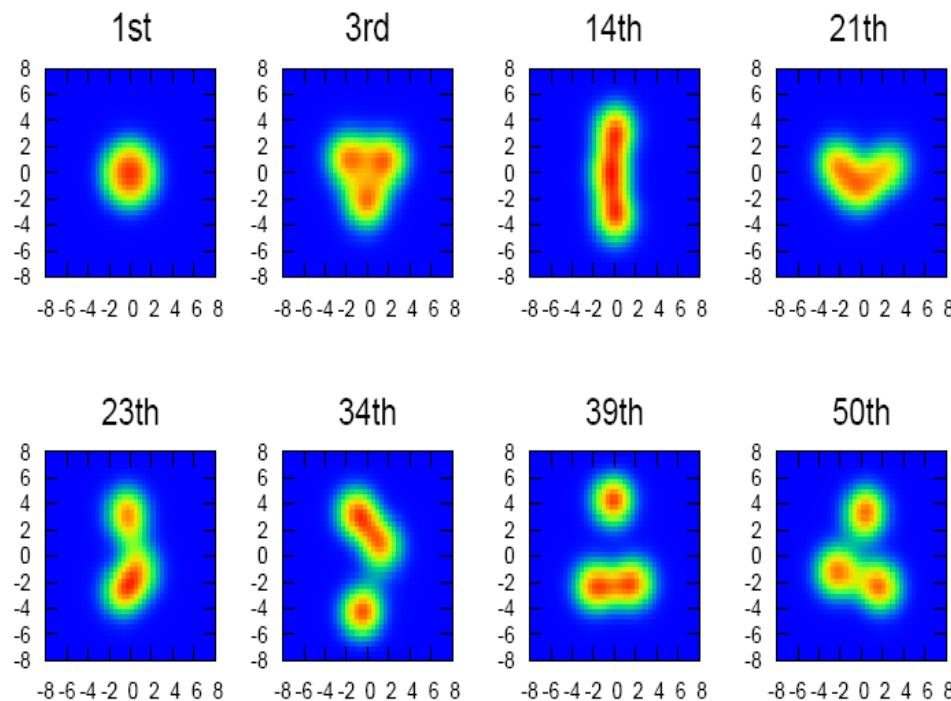


FIG. 1. Contour plots of nuclear densities of the stored SDs for ^{12}C . A sequential number of the SD is indicated in the top of the panel. Units of vertical and horizontal axes are fm.

- Summary
- For near symmetric nuclear matter at densities $\sim .03 \leq \rho/\rho_0 \leq 0.2$, We
- Measure Alpha Mass Fractions, Equilibrium constants-Test Astrophysical Equations of State
- Determine Density dependence of Binding energies - Extract Mott Points
- Determine Symmetry Free Energies
- Good overall agreement with model which includes clusterization and in-medium effects on clusters.
- Thank You

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