Critical Behavior of Nuclear Systems from Quantum Fluctuations

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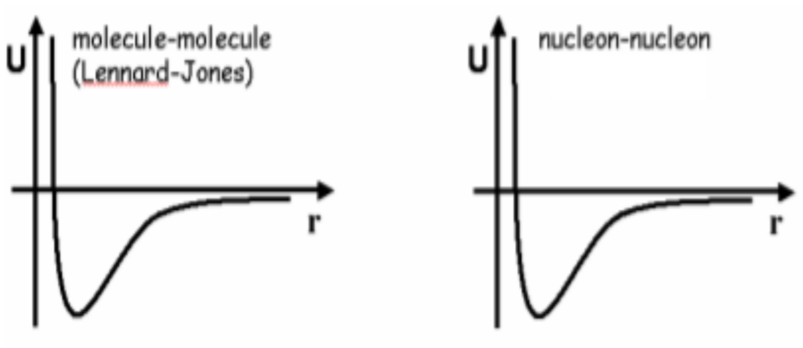
Cyclotron Institute
Texas A&M University

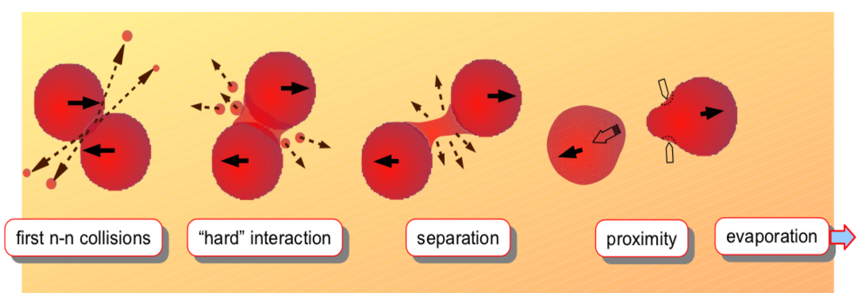
Outline

- ✧ **Introduction**
  - Nuclear liquid-gas phase transition
- ✧ **The Experiment**
  - Event selection and Source reconstruction
- ✧ **Extraction of Thermodynamic Quantities**
- ✧ **Results and Discussion**
- ✧ **Summary**

Nuclear liquid-gas phase transition

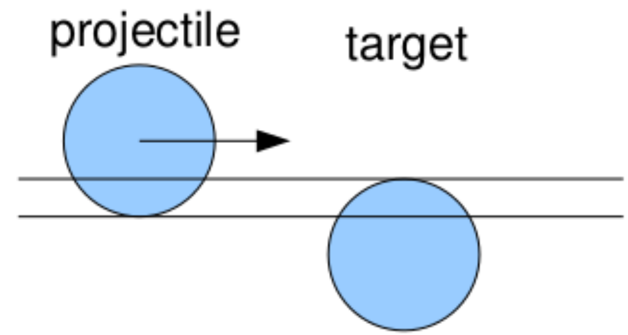
Nucleon-nucleon interaction similar to van der Waals interaction

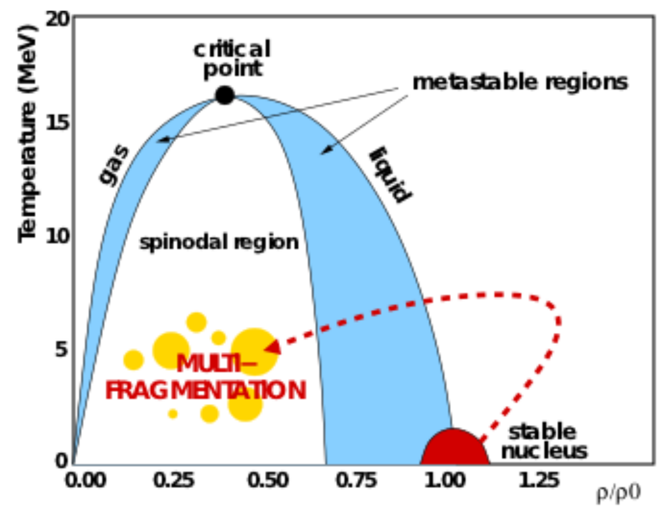




Slides from A. Olmi, INFN-Firenze

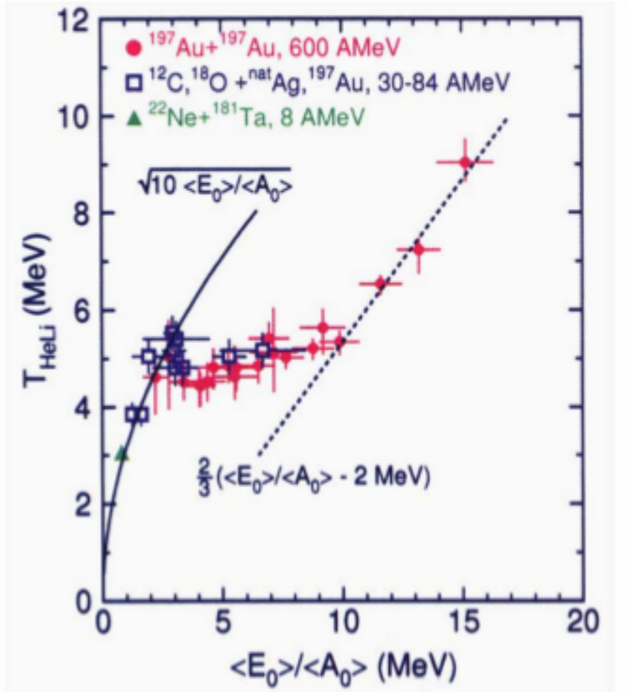
Non-central collisions





Previous Works: Indicators of phase transition

Caloric curve



IG. 2. Caloric curve of nuclei determined by the dependence f the isotope temperature T_{HeLi} on the excitation energy per ucleon. The lines are explained in the text.

J. Pochodzalla *et al.*, PRL **75**, 1040 (1995)

Negative Heat capacity

Au+Au
@ 35 A.MeV

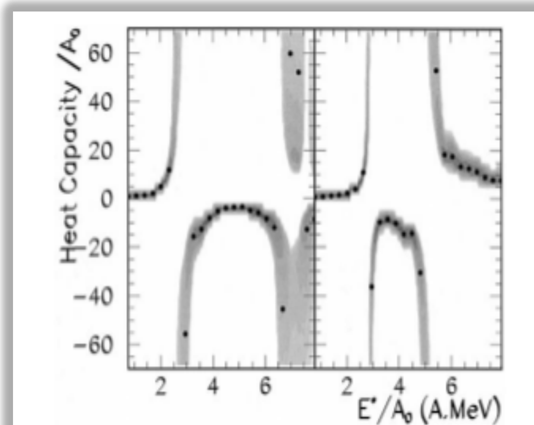


Fig. 4. Heat capacity (solid symbols) per nucleon from Eq. (2). The panel on the left (right) refers to the freeze-out hypothesis I (II). The grey contour indicates the confidence region for C_t (see text).

M. D'Agostino *et al.*, PLB **473**, 219 (2000)

Power law dependence

P+Xe
@ 80-350 GeV

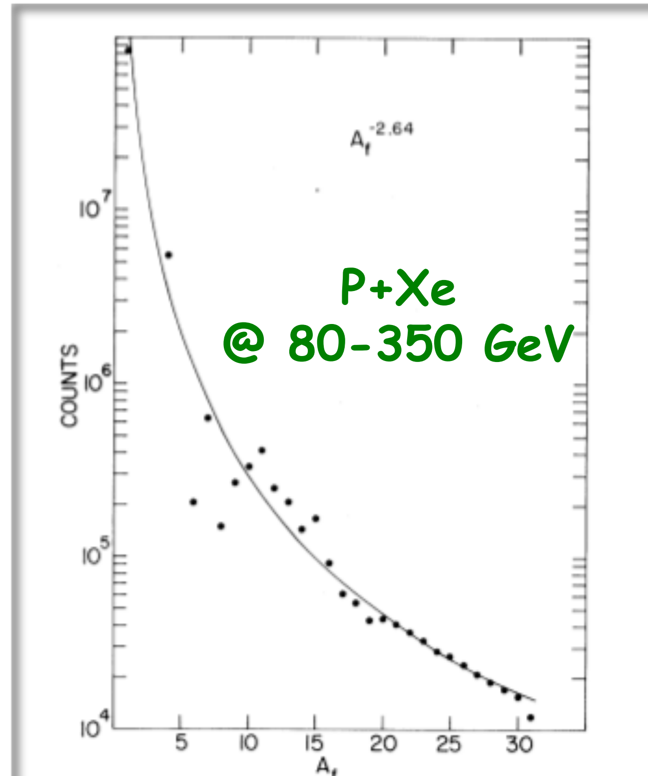
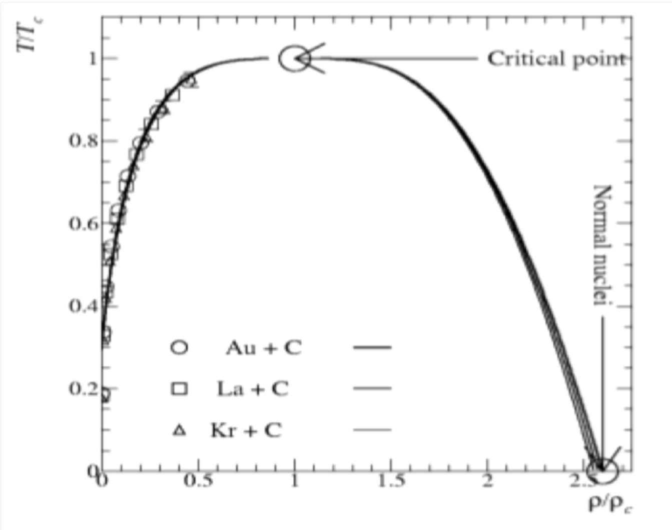
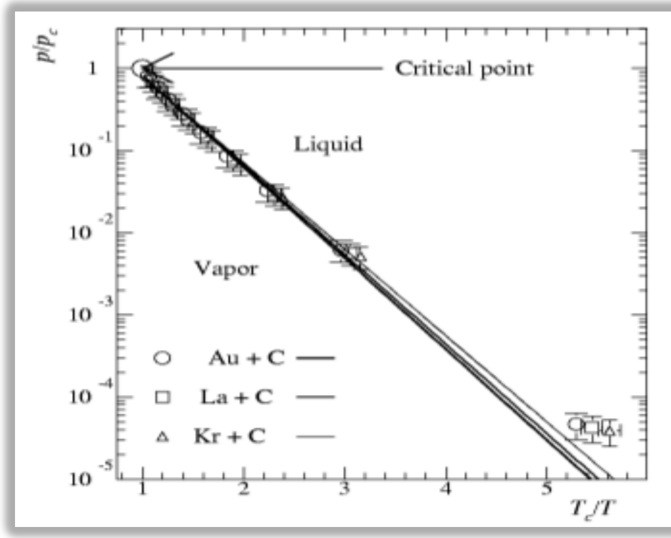


FIG. 4. Mass yield of fragments from xenon vs mass number, A_f, corrected for effects discussed in text.

J.E. Finn *et al.*, PRL **49**, 1321 (1982)

J. B. Elliot *et al.* , PRC **67**, 024609 (2003)





Estimates are made of T-ρ and P-T coexistence curves of finite neutral nuclear matter.

Current work->adding a constraint on the source asymmetry.

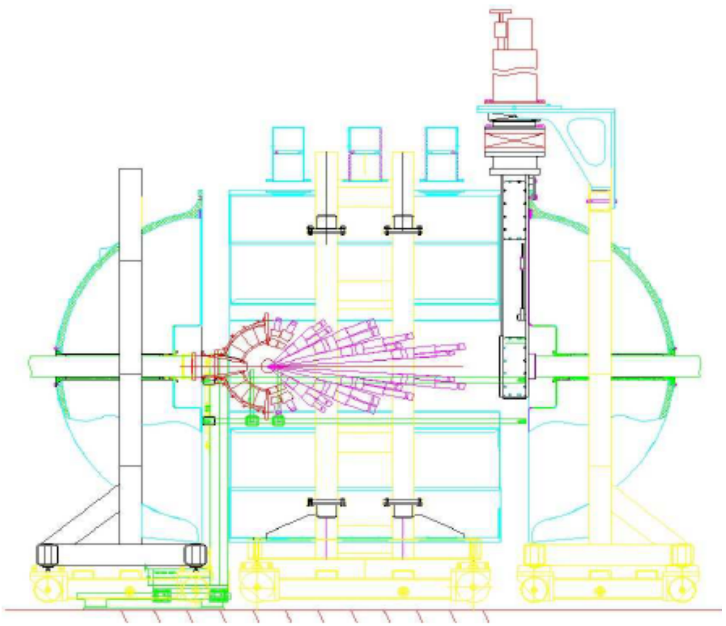
Extraction of thermodynamic quantities through quantum fluctuations

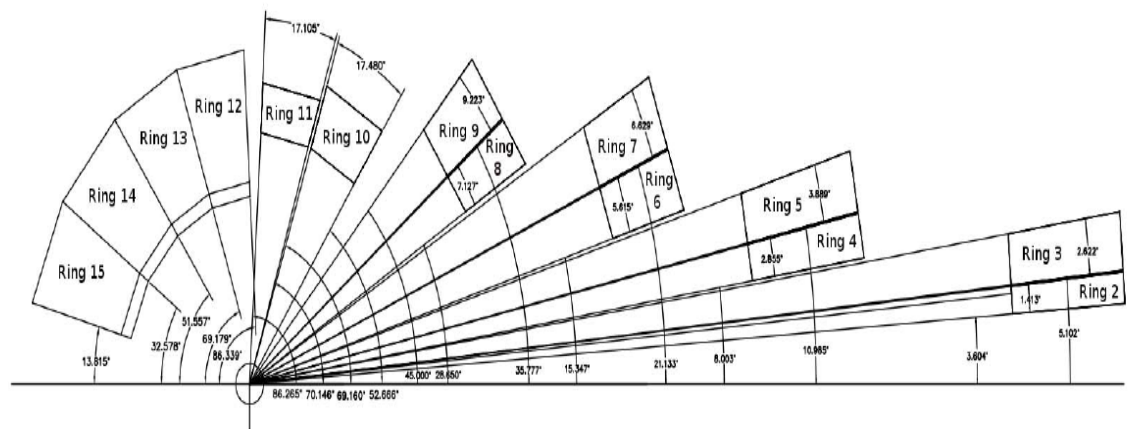
The Experiment

$^{64}Ni + ^{64}Ni$
$^{64}Zn + ^{64}Zn$
$^{70}Zn + ^{70}Zn$
@ 35 A.MeV

NIMROD-ISiS Array

- ✓ Detection arrays arranged 14 Rings
- ✓ 3.6-167 degrees
- ✓ Full Silicon Coverage (4π)
- ✓ Isotopic resolution to Z=17
- ✓ Elemental resolution to $Z_{projectile}$
- ✓ Neutron Ball (4π)





Event Selection & Source Reconstruction

Selection of QP source by means of Several cuts

- Exclude fragments that do not originate from a QP

$$m_- \le v_z / v_{z,PLF} \le m_+$$

m±=1±0.65 for Z=1, 1±0.60 for Z=2 and 1±0.45 for Z≥3

- Select events with a well-measured QP

$$54 \le \sum_i^{CP} A_i + M_n \le 64$$

- Select events with near-zero average momentum quadrupole

$$-0.3 \le \log Q \le 0.3$$

$$Q = \frac{\sum p_{z,i}^2}{\frac{1}{2}\sum p_{T,i}^2}$$

- Data sorted in 4 asymmetry $m_s=(N_s-Z_s)/A_s$ bins of the QP

- Excitation energy of the QP

$$E_{QP}^* = \sum_i^{M_{CP}} K_{T,i} + M_n \langle K_n \rangle - Q$$

- Multiplicity of free neutrons

$$M_{QP} = \frac{M_{exp} - M_{bkg}}{\left(\varepsilon_{QP} + \frac{N_T}{N_P}\varepsilon_{QT}\right)\varepsilon_{lab} / \varepsilon_{sim}}$$

S. Wuenschel *et al.*, PRC **79**, 061602 (2009)
J.C. Steckmeyer *et al.*, NPA **686**, 537 (2001)
D. Lacroix *et al.*, PRC **69**, 054604 (2004)
P. Marini *et al.*, NIMA **707**, 80 (2013)

Temperature and Density from Quantum Fluctuations

Momentum quadrupole fluactuation

The quadrupole momentum distribution

$$Q_{xy} = p_x^2 - p_y^2$$

Contains information on the temperature through its fluctuations

$$\sigma_{xy}^2 = \int d_p^3 (p_x^2 - p_y^2) f(p)$$

Assuming a Fermi-Dirac distribution

$$\sigma_{xy}^2 = (2mT)^2 F_{QC}$$

F_{QC} is the quantum correction factor

$$F_{QC} = 0.2\left(\frac{T}{\varepsilon_f}\right)^{-1.71} + 1$$

Multiplicity fluctuation

✧ Similar derivation is given for the multiplicity fluctuation σ^2_N

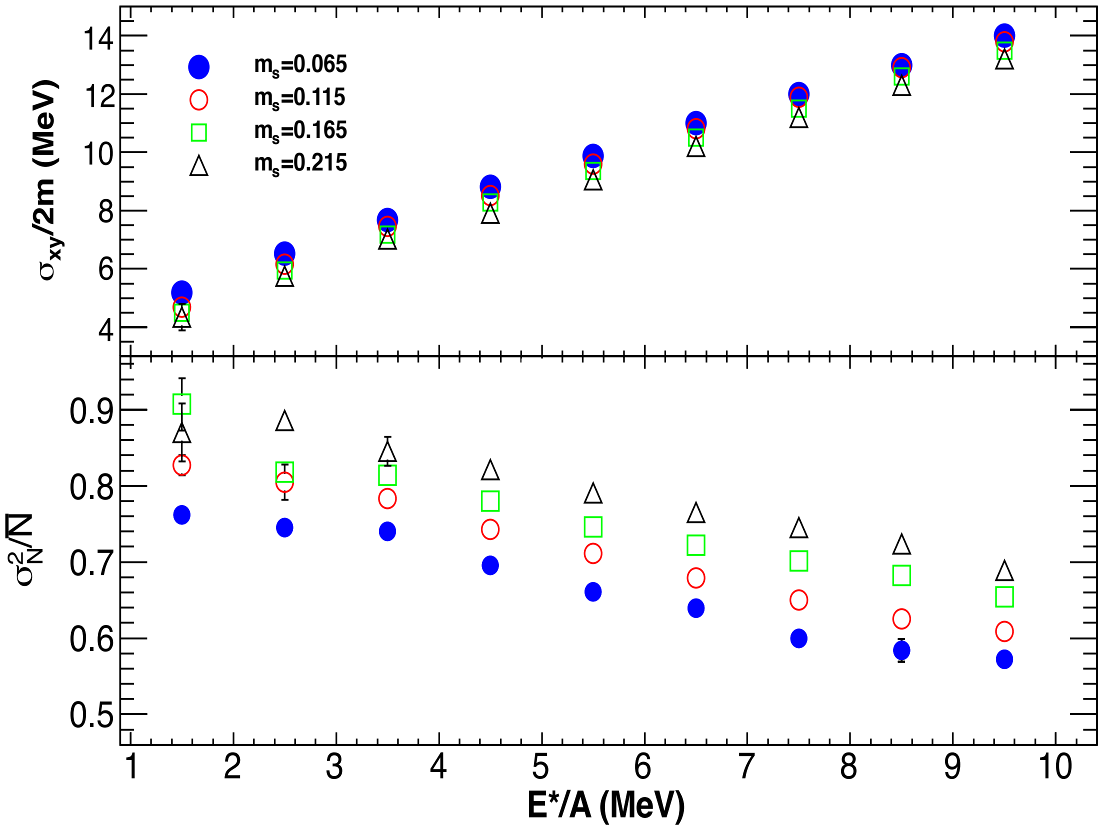
$$\frac{T}{\varepsilon_f} = -0.442 + \frac{0.442}{\left(1 - \frac{\sigma_N^2}{\bar{N}}\right)^{0.656}} + 0.345\frac{\sigma_N^2}{\bar{N}} - 0.12\left(\frac{\sigma_N^2}{\bar{N}}\right)^2$$

$$\varepsilon_f \approx 36\left(\rho / \rho_0\right)^{2/3}$$

H. Zheng & A. Bonasera, PLB **696**, 178 (2011)
H. Zheng & A. Bonasera, PRC **86**, 027602 (2012)

Classical temperatures and normalized multiplicity fluctuations

$m_s=(N_s-Z_s)/A_s$



$54 \leq A_{QP} \leq 64$
4 narrow asymmetry bins
Proton as probe particle

Large asymmetry->
Low temperatures

Asymmetry dependence of Nuclear Caloric Curves
(McIntosh's talk on Friday)

Classical case: $\sigma^2_N/\langle N\rangle \approx 1$
Suppression of multiplicity Fluctuations
-> Similar to quenching in Trapped Fermi gases

T. Müller *et al.*, PRL **105**, 040401

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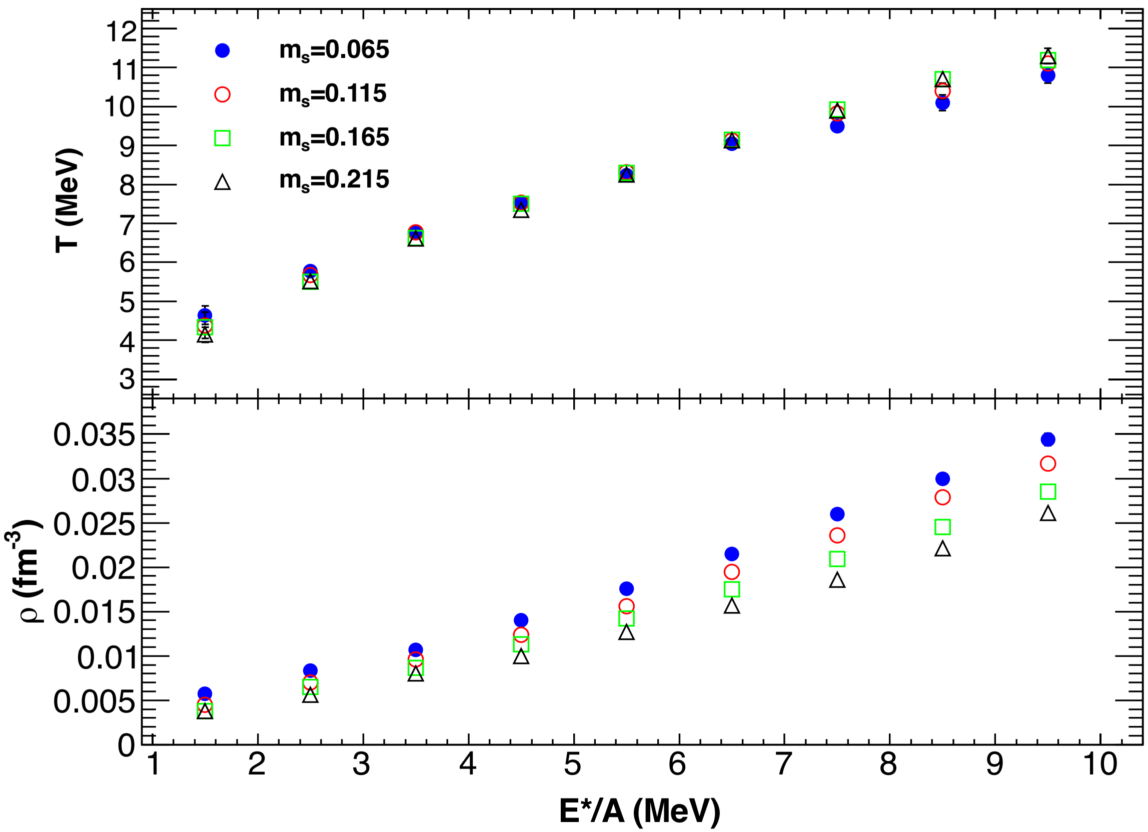
Extracted Temperatures and Densities

$54 \leq A_{QP} \leq 64$
4 narrow asymmetry bins
Proton as probe particle

$m_s=(N_s-Z_s)/A_s$



Asymmetry dependence
->less pronounced on T

Larger asymmetry
->lower density

NuSYM13

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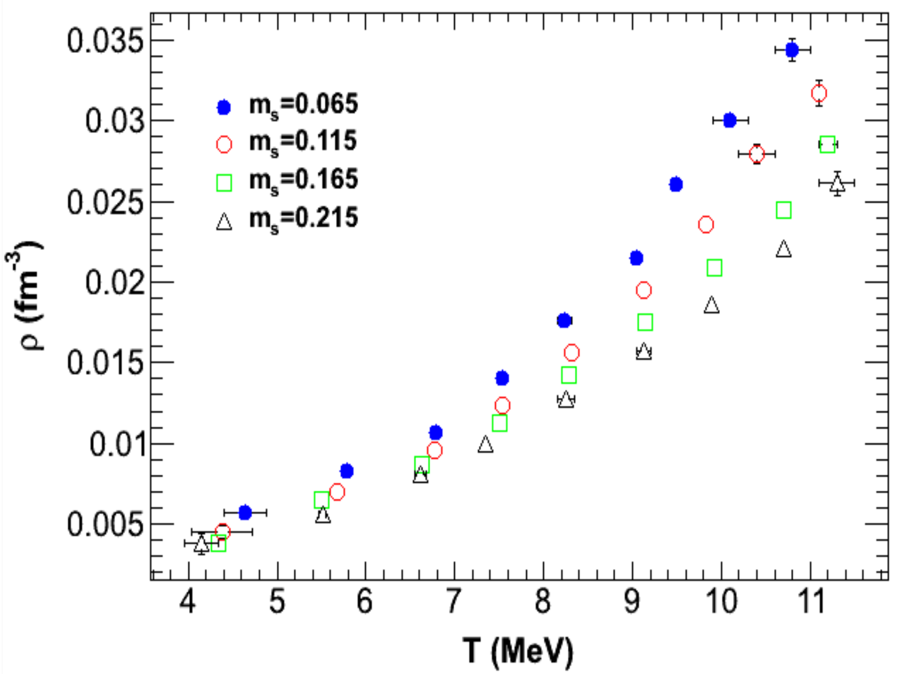
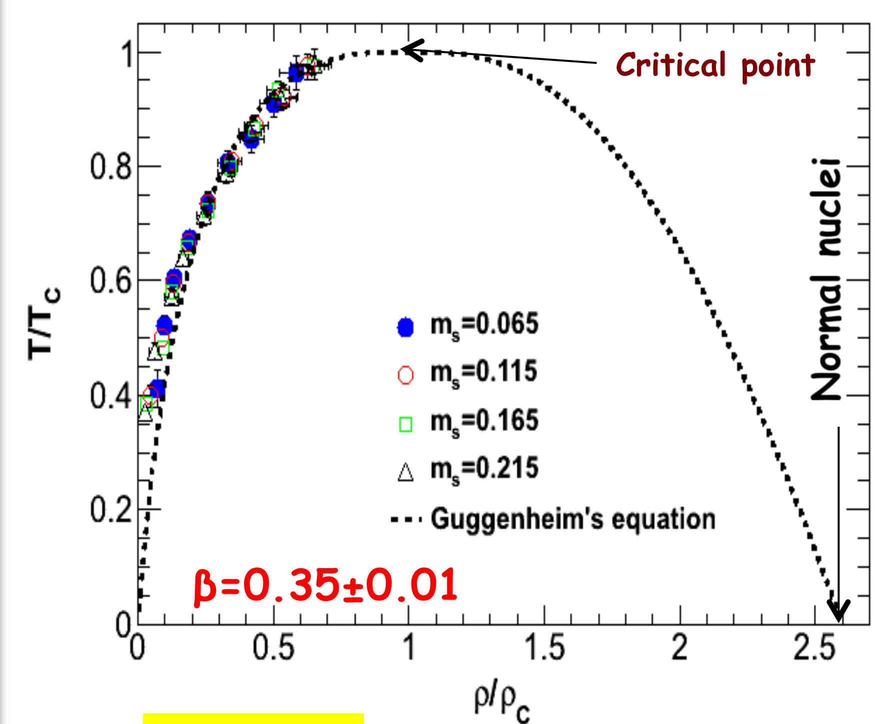
Liquid-gas coexistence curve



T and ρ plotted in scaled units
⇒ Data collapses on a single line
⇒ Scaling law obeyed



Guggenheim scaling

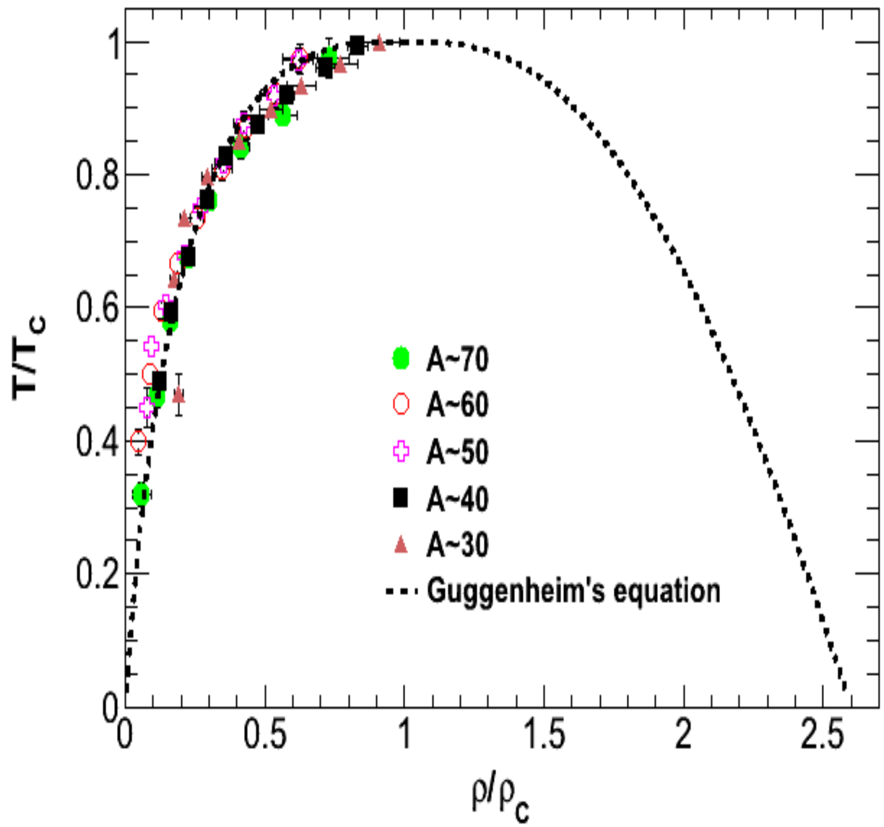
$$\frac{\rho_{l,g}}{\rho_c} = 1 + b_1\left(1 - \frac{T}{T_c}\right) \pm b_2\left(1 - \frac{T}{T_c}\right)^{\beta}$$

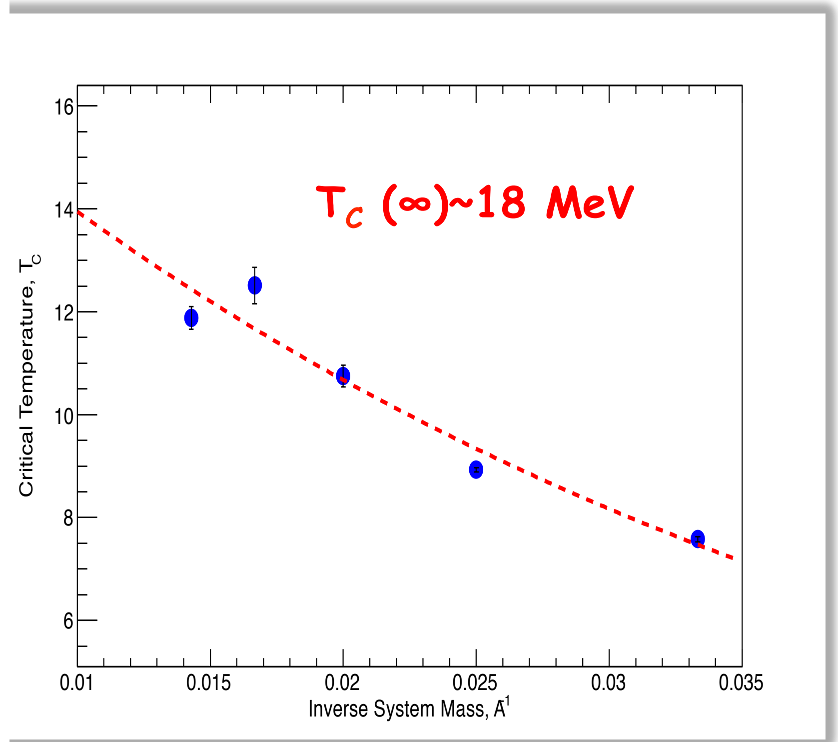
$$b_1 = 0.3$$
$$b_2 = b_1 + 1$$

$$\rho_c \sim \rho_0 / 3$$

E.A. Guggenheim, J. Chem. Phys. **13**, 253 (1945)
J.B. Elliot et al, PRC **67**, 024609 (2003)

Finite size effects: How to scale to infinite system





$$\frac{\rho_{l,g}}{\rho_c} = 1 + b_1\left(1 - \frac{T}{T_c}\right) \pm b_2\left(1 - \frac{T}{T_c}\right)^{\beta}$$

J.B. Elliot et al, PRC 87, 054622 (2013)

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Fisher's droplet model and EOS

P. Finicchiaro et *al.*, NPA 600 (1996) 236-250

- Pressure calculated making use of grand partition function from Fisher's droplet model

$$P = T\rho M_0 / M_1$$

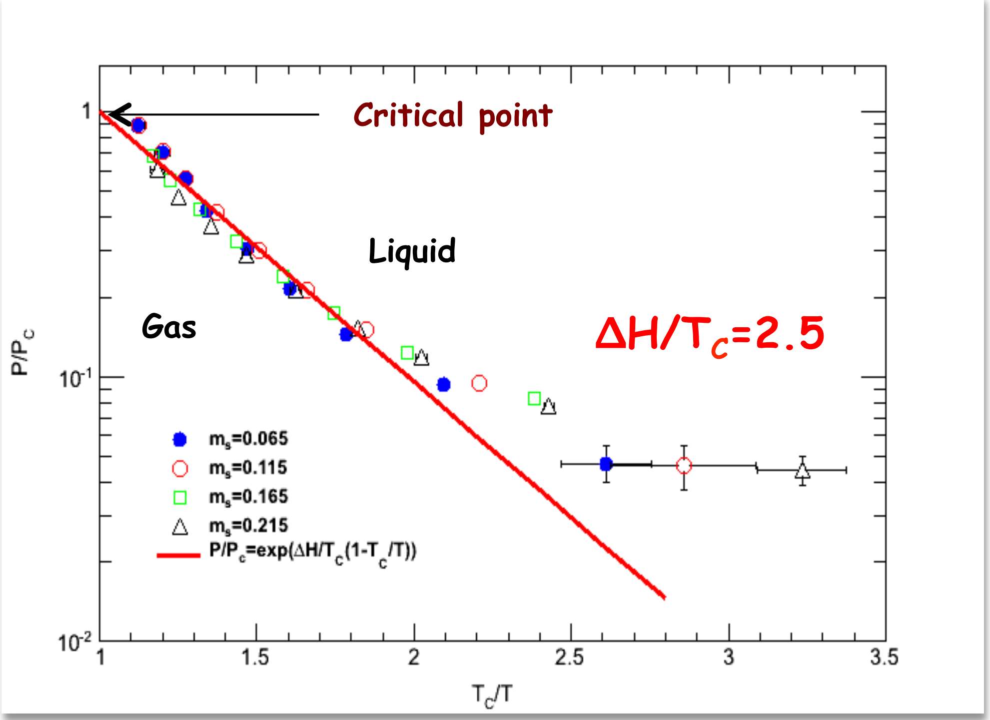
- The EOS expressed in terms of moments of fragment size distribution

$$M_k = \sum_{A \neq A_{max}} A^k Y(A)$$

- Clausius-Clapeyron equation

$$\frac{P}{P_c} = \exp\left[\Delta H / T_c (1 - T_c / T)\right]$$

- Describes several fluids for T≤Tc



J.B. Elliot *et al.*, PRL **88** (042701) 2002
E.A. Guggenheim, J. Chem. Phys. 13, 253 (1945)

Critical Values and Thermodynamics Quantities

m_s	T_c (MeV)	ρ_c (fm^{-3})	P_c (MeV/fm^3)	$P_c/\rho_c T_c$	ΔH (MeV)
0.065	12.12 ± 0.39	0.070 ± 0.006	0.211 ± 0.001	0.25 ± 0.02	31.50 ± 1.01
0.115	12.51 ± 0.35	0.066 ± 0.005	0.209 ± 0.001	0.25 ± 0.02	32.53 ± 0.90
0.165	13.11 ± 0.30	0.064 ± 0.004	0.232 ± 0.001	0.27 ± 0.02	31.46 ± 0.71
0.215	13.39 ± 0.21	0.061 ± 0.002	0.258 ± 0.002	0.31 ± 0.01	32.13 ± 0.50

- T_c observed to increase when increasing m_s
- Critical density ρ_c decrease while pressure P_c increase with m_s
- Critical compressibility factor very close to those of real gases and increase with m_s.
- *ΔH* values in fair agreement with the ones extracted from a recent quantum calculation (*A. Carbone et al.*, PRC **83** (024308) 2011)

Summary

- Phase diagrams have been constructed.

- Temperature, density, pressure as well as critical values observed to depend on source asymmetry.

- Scaling of T, ρ and P to their critical values displayed universality.

- Critical exponent β and critical compressibility factor found to belong to the liquid-gas universality class.

- Present results provide a means to establish the proton-fraction dependence of the EOS in systems with large neutron excess.

Work in progress:

- ✧ Use of phase space coalescence model to extract T and ρ of selected systems.
- ✧ Performing high-statistics CoMD calculations to compare with current results.

Acknowledgments

*SJY Research Group

A.B. McIntosh, P. Cammarata, K. Hagel, L. Heilborn, L.W. May, A. Raphelt, G.A. Souliotis, A. Zarrella and S.J. Yennello

* Z. Kohley, H. Zheng and A. Bonasera

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and Robert A. Welch Fundation grant A-1266

THANK YOU FOR YOUR ATTENTION !!!

Back-up Slides

Spinodal decomposition

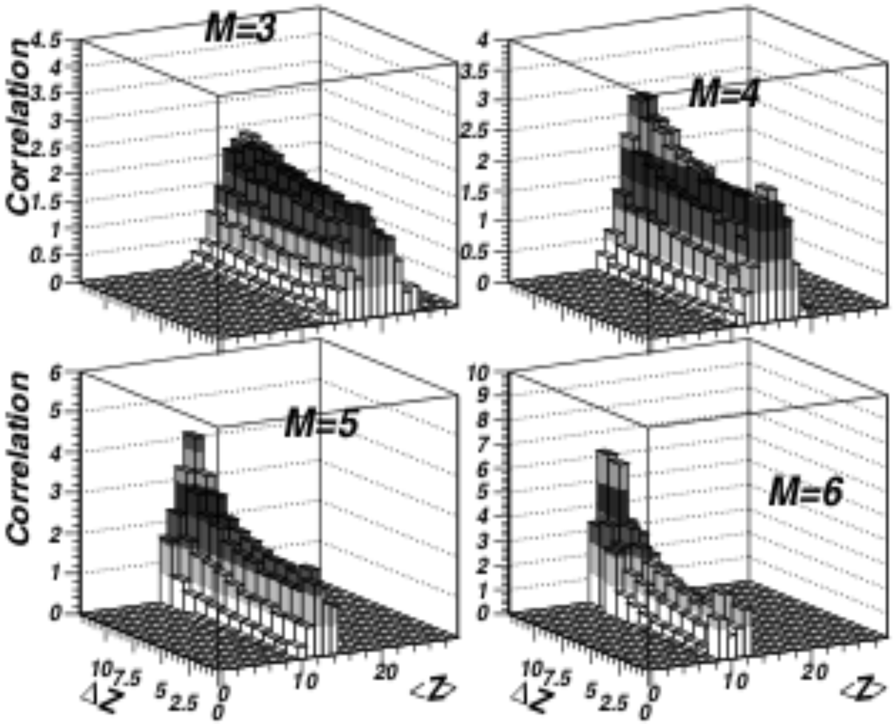


FIG. 4. Experimental higher-order charge correlations for very dissipative "binary" collisions in 32 MeV/nucleon Xe + Sn collisions (see text).

Systems which enter in this region will favorably breakup into nearly equal-sized fragments

A new « classical »thermometer based on proton transverse momentum fluctuations

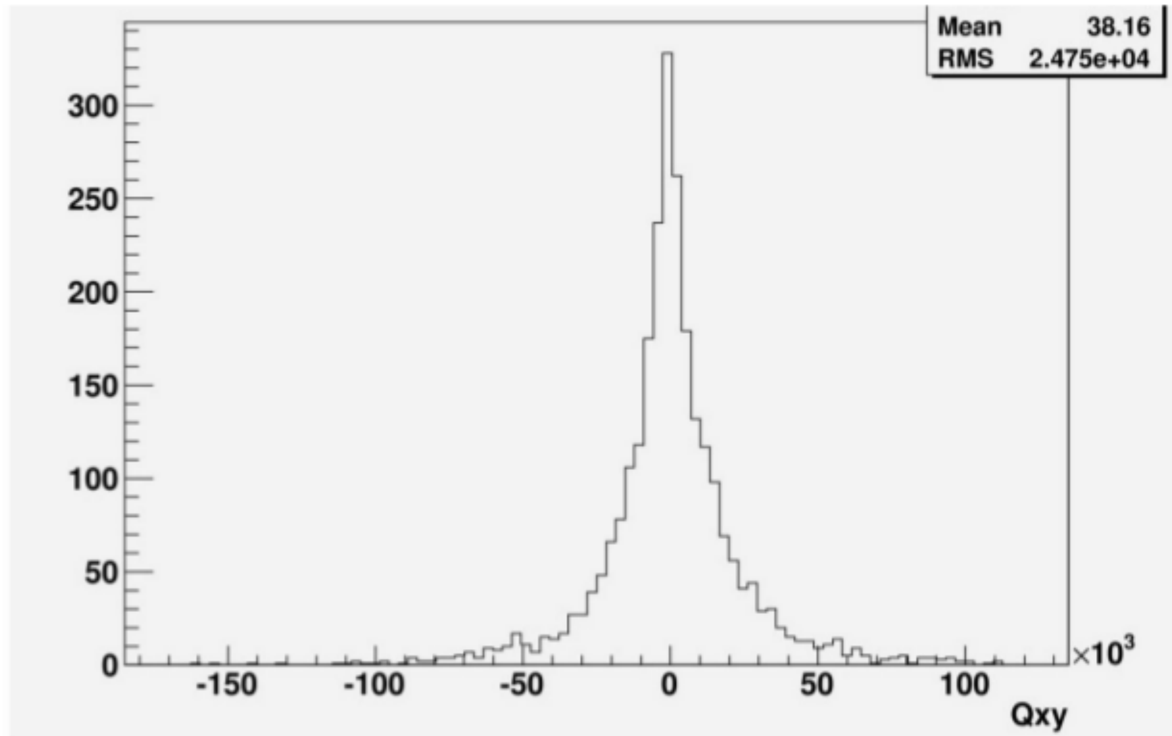
S. Wuenschel, A. Bonasera et al., NPA 843 (2010) 1

$Q_{xy} = P_x^2 - P_y^2$

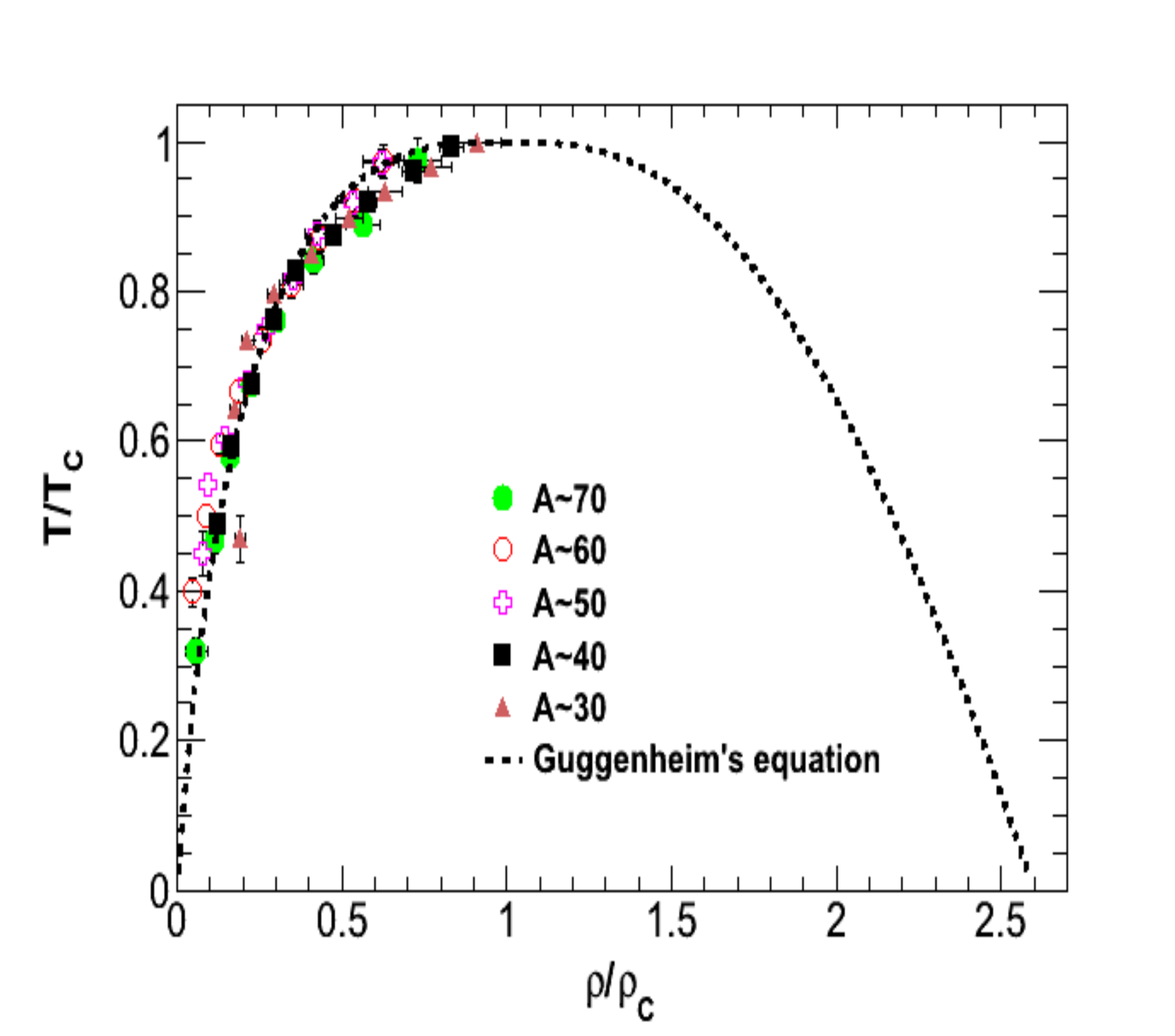
$\sigma_{xy}^2 = \langle Q_{xy}^2 \rangle - \langle Q_{xy} \rangle^2$

$\sigma_{xy} = 2mT$

assuming a classical Maxwell-Boltzmann distribution of the momentum yields at T



Finite size effects



Near the critical point
⇒ Finite size effects exepcted very strong and scaling law might be violated

⇒ β-value is that expected for a liquid-gas phase transition

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Phase diagrams for real gases

J. Phys. G: Nucl. Part. Phys. 38 (2011) 113101 Topical Review

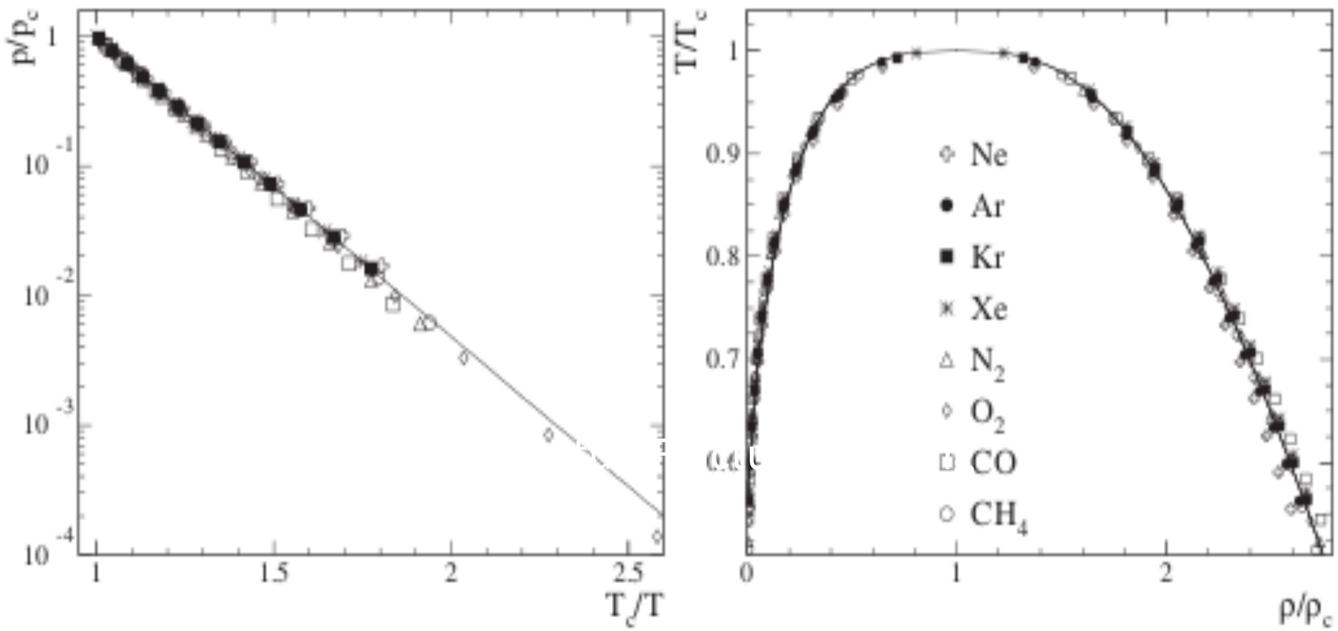
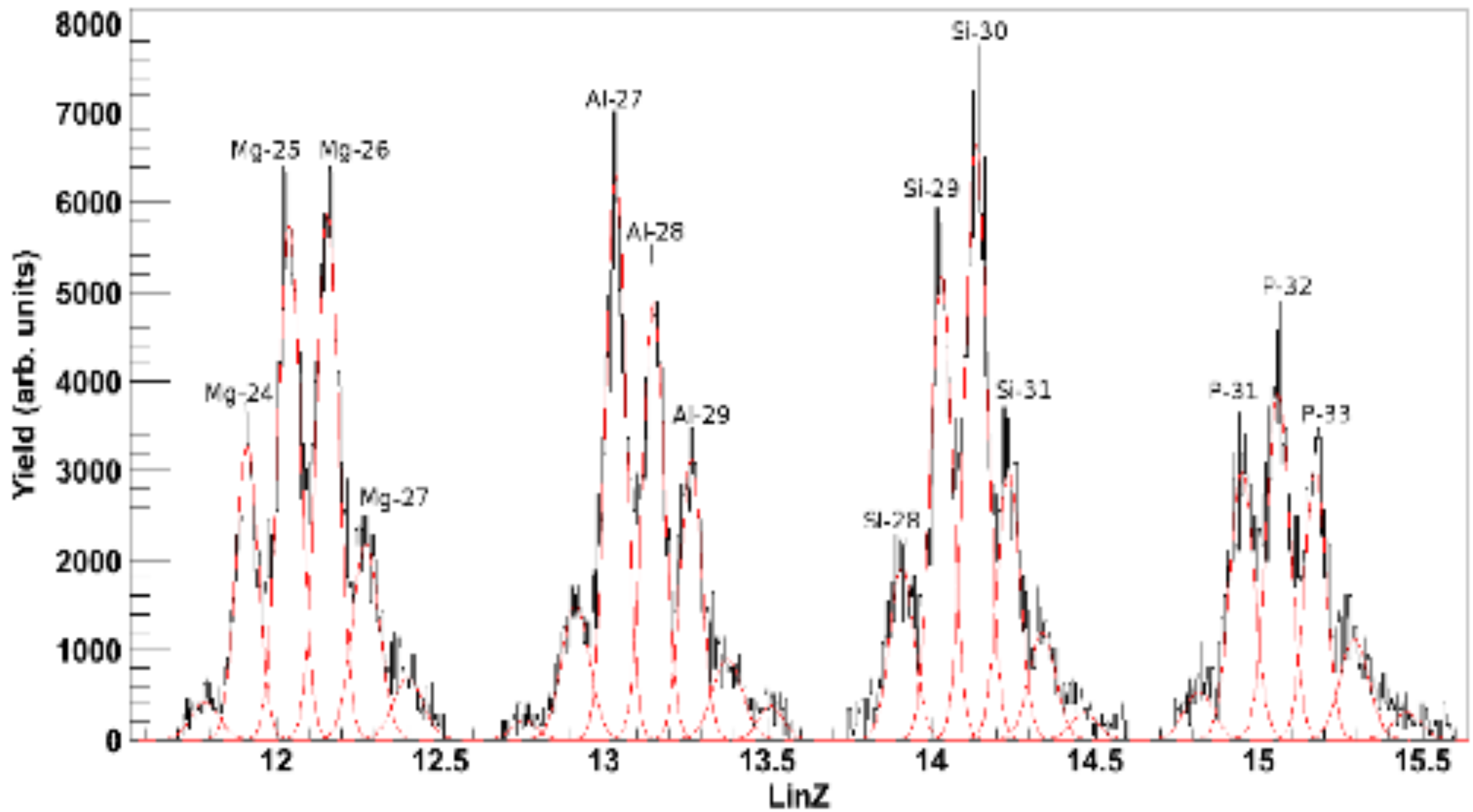


Figure 2. The reduced pressure as a function of the inverse of the reduced temperature (left) and the reduced temperature as a function of reduced density (right), in the manner of Guggenhiem [23, 24]. Data for eight different fluids fall on the same curve. The data for the fluids were obtained from the NIST database [16]. The pressure–temperature curve (left) is from [24] and the temperature–density curve (right) is from [23].

Experiment (PID)



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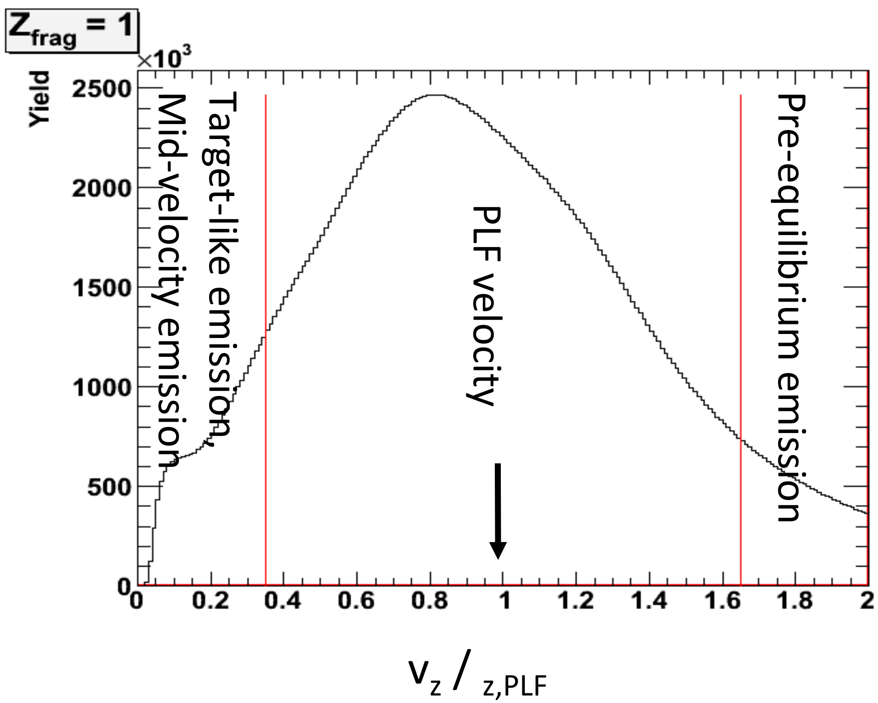
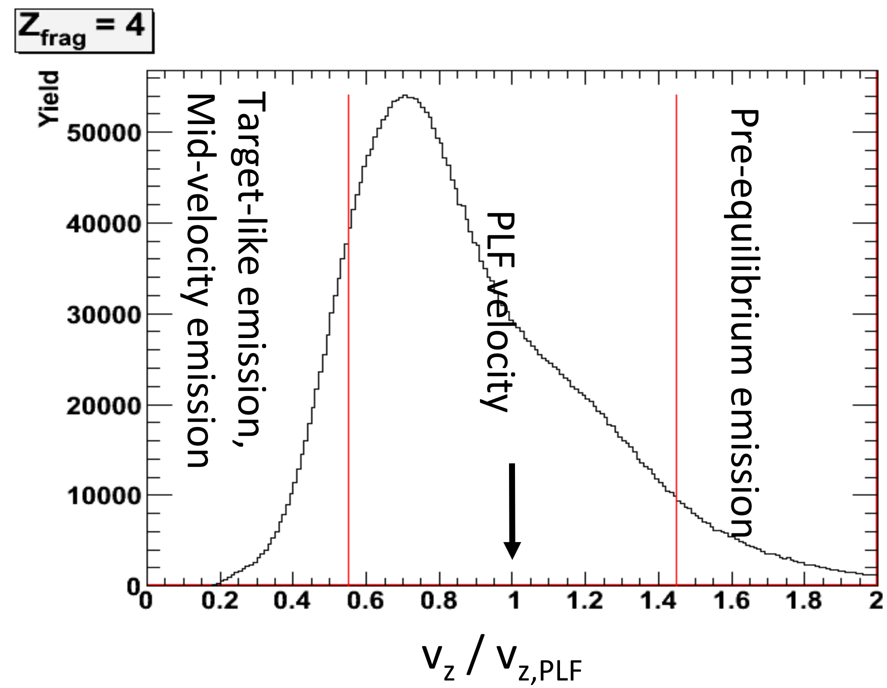
Source Selection 1: Velocity Cut

- Remove particles that do not belong (on average) to a statistically emitting QP source
  - => Compare laboratory velocity of each particle to that of the heaviest charged particle measured in the event.





$$Z = 1: \quad 0.35 \leq {}^{v_z}/_{v_{z,PLF}} \leq 1.65$$

$$Z = 2: \quad 0.40 \leq {}^{v_z}/_{v_{z,PLF}} \leq 1.60$$

$$Z \geq 3: \quad 0.55 \leq {}^{v_z}/_{v_{z,PLF}} \leq 1.45$$

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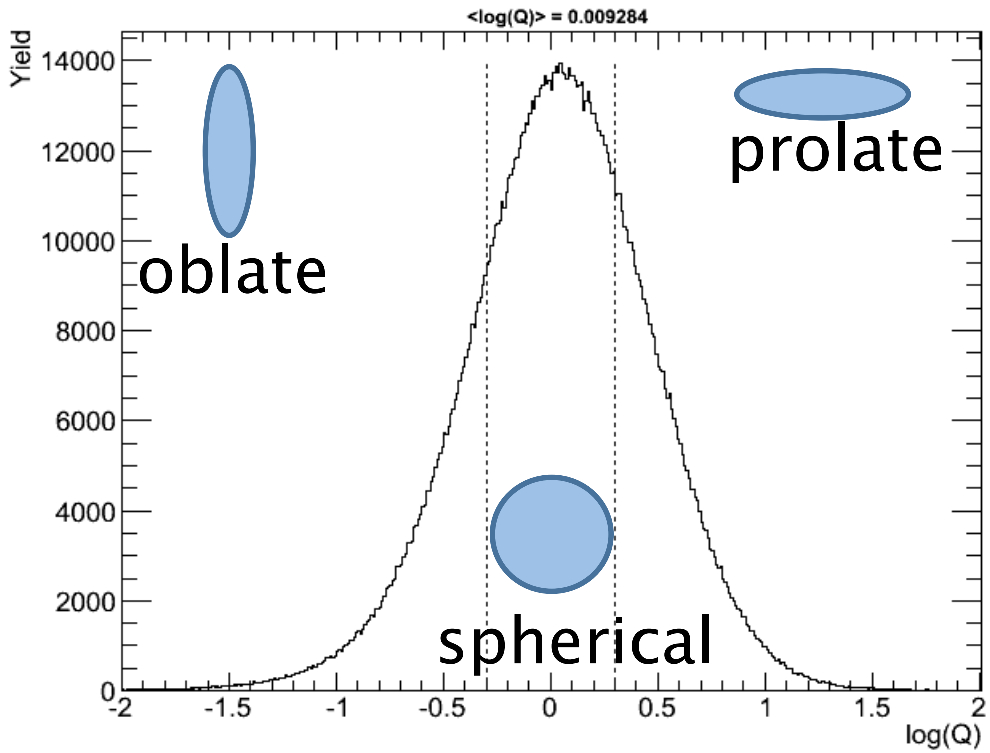
Source Selection 3: Velocity Cut

- Sphericity cut: Select events with near-zero average momentum quadrupole.



(Velocity cut and Mass cut imposed)

$$Q = \frac{\sum p_{z,i}^{2}}{\frac{1}{2}\sum p_{T,i}^{2}}$$

$$-0.3 \leq \log Q \leq 0.3$$

Select events with near-zero average momentum quadrupole.