

# Topology change, tensor forces and EoS for dense baryonic matter

NuSYM13 –East Lansing

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# “World Class University”/Hanyang University Program 2008-2013

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(other places)

String theorists ...

# THE **MULTIFACETED SKYRMION**

Gerald E Brown  
Mannque Rho  
editors

2010

- Nuclear/hadron
- Condensed matter
- String theory

# Dense baryonic matter

QCD at large density  $n \gg n_0$  (except for  $n \sim \infty$ ) is intractable.

The only available theoretical tool is “large  $N_c$ ” limit (**t Hooft, Witten**).

In that limit, light-quark hadrons can be well described (**Weinberg**) by the constituent quark model (**CQM**).

## Dense baryonic matter

In particular, relevant to nuclear physics, for  $N_c \gg 1$ , the baryon is a skyrmion (**Witten**). Hence the baryon in CQM is equivalent (**Manohar**) to the skyrmion.

One great **good thing** about the skyrmion picture is that it is applicable to single nucleon ( $A=1$ ), nuclei ( $A=$  a few) and nuclear matter ( $A=\infty$ ) with one (single) Lagrangian.

Great **bad things** with large  
 $N_c$  are .....

# Dense baryonic matter

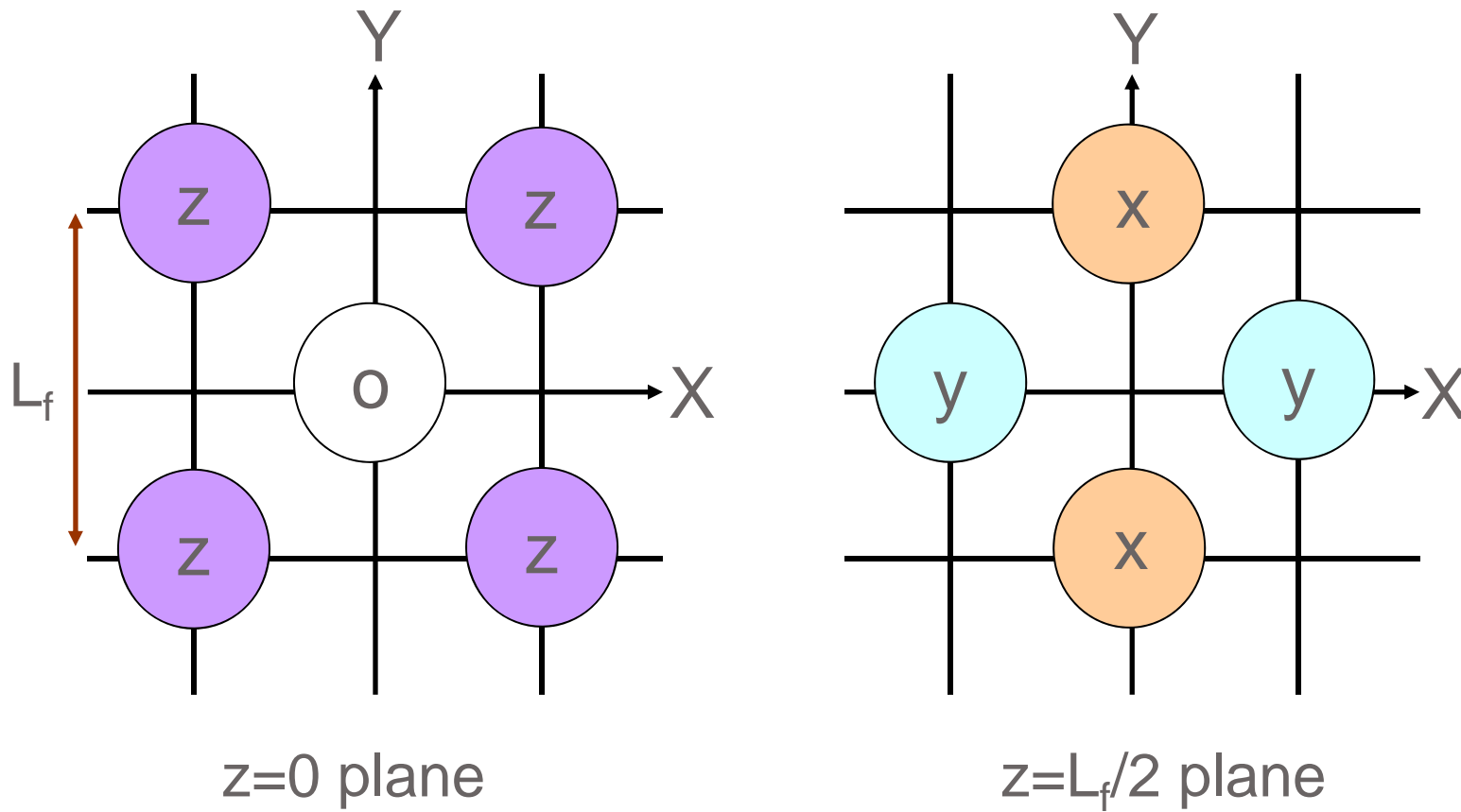
In the large  $N_c$  limit and at large density  $n > n_0$ , the matter is in a crystal form.

Therefore in this limit, we can simulate large density effects by simulating skyrmions on a crystal (Klebanov, Manton).

This is what will be done with the face-centered-cubic configuration (FCC).

# Skyrmion Crystal : FCC

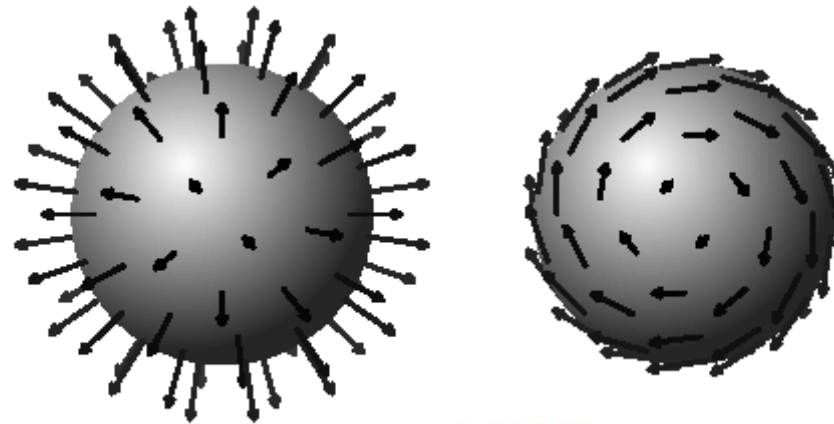
1989, L. Castellejo *et al.* & M. Kugler *et al.*





# Skyrmion-half-skyrmion transition

Skyrmions, hedgehogs, are packed on the crystal, groomed maximally attractive

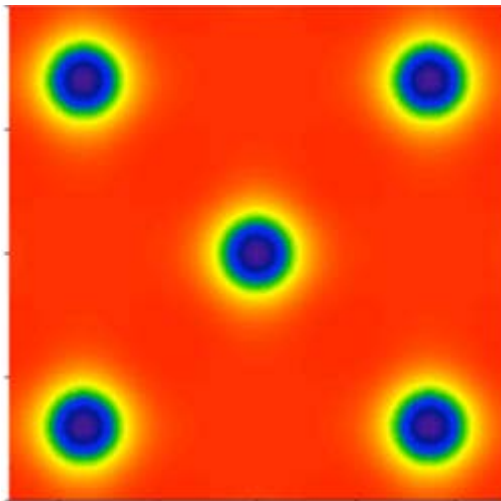


hedgehog



- Skymions in FCC crystals, when squeezed, fractionize into  $\frac{1}{2}$ -skymions in CC at a density denoted  $n_{1/2}$

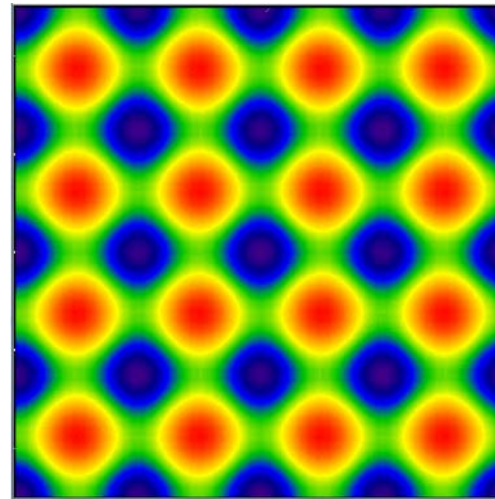
$$U = e^{2i\pi/f_\pi}$$



skyrmions

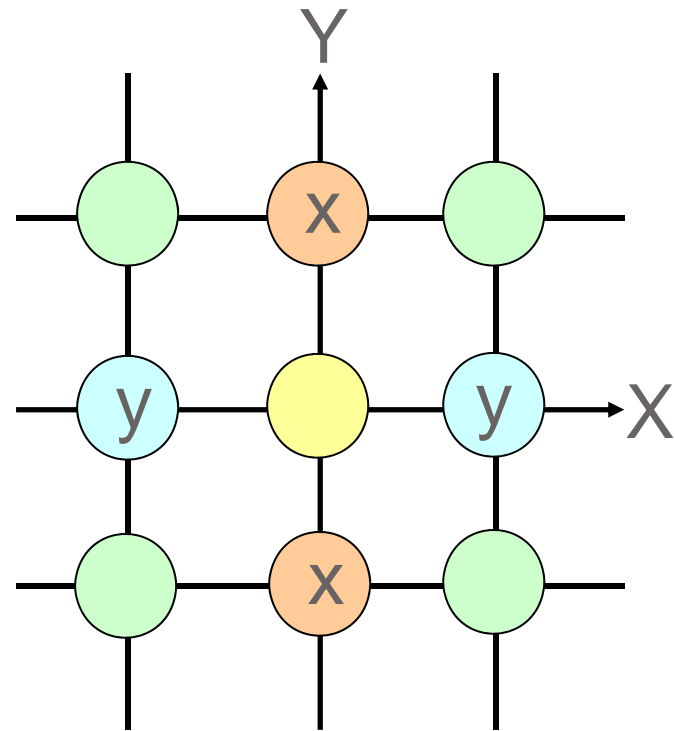
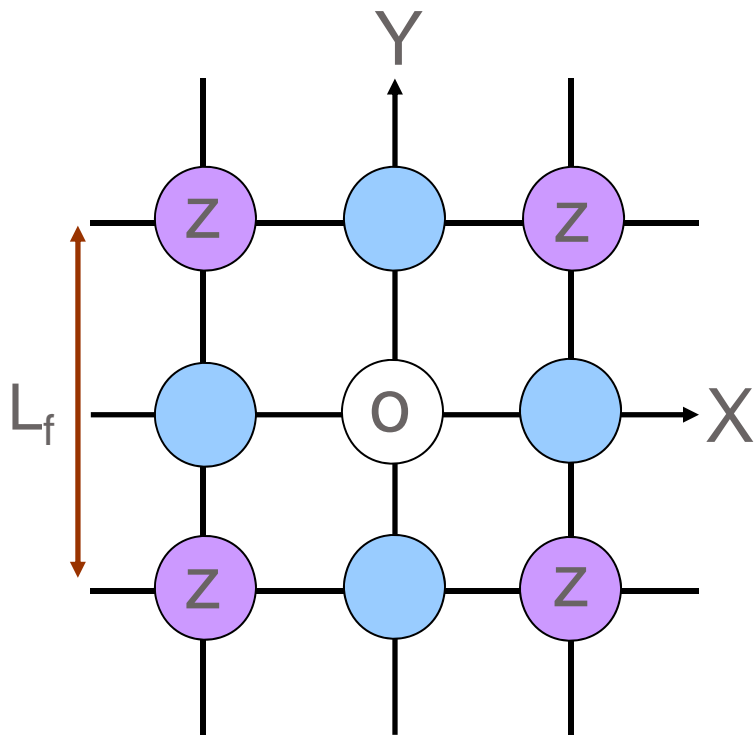


$$U = \xi_L \xi_R^\dagger, \quad \xi_{L,R}$$



Half-skyrmions

# Skyrmion Crystal : cubic half-skyrmion



## “Transition” density $n_{1/2}$

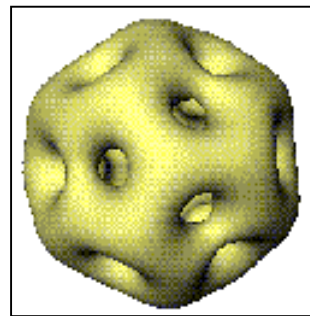
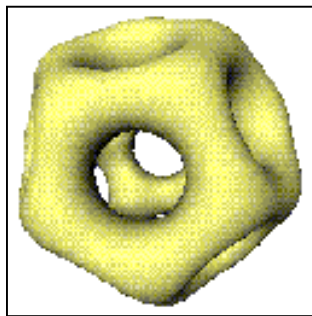
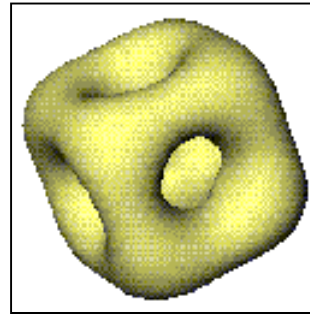
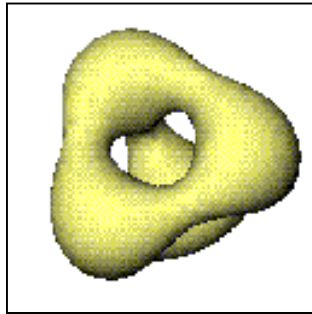
Crude estimate,  $n_{1/2} \approx (1.5-2.0) n_0$ , is reasonable for nuclear physics.

If too low, the theory will break down.

If much higher, then it will be academic, for explicit QCD degrees of freedom need be incorporated.

However, the transition may not belong to Ginzburg-Landau-Wilson paradigm:  
There are no local order parameter fields.

# Multi-Skyrmion system



<http://www.damtp.cam.ac.uk/user/hep/research.html#solitons>

# Symmetry energy

$$E(n, \alpha) = E(n, \alpha = 0) + E_{sym}(n)\alpha^2 + \mathcal{O}(\alpha^4)$$

$$\alpha = (n_n - n_p)/(n_n + n_p)$$

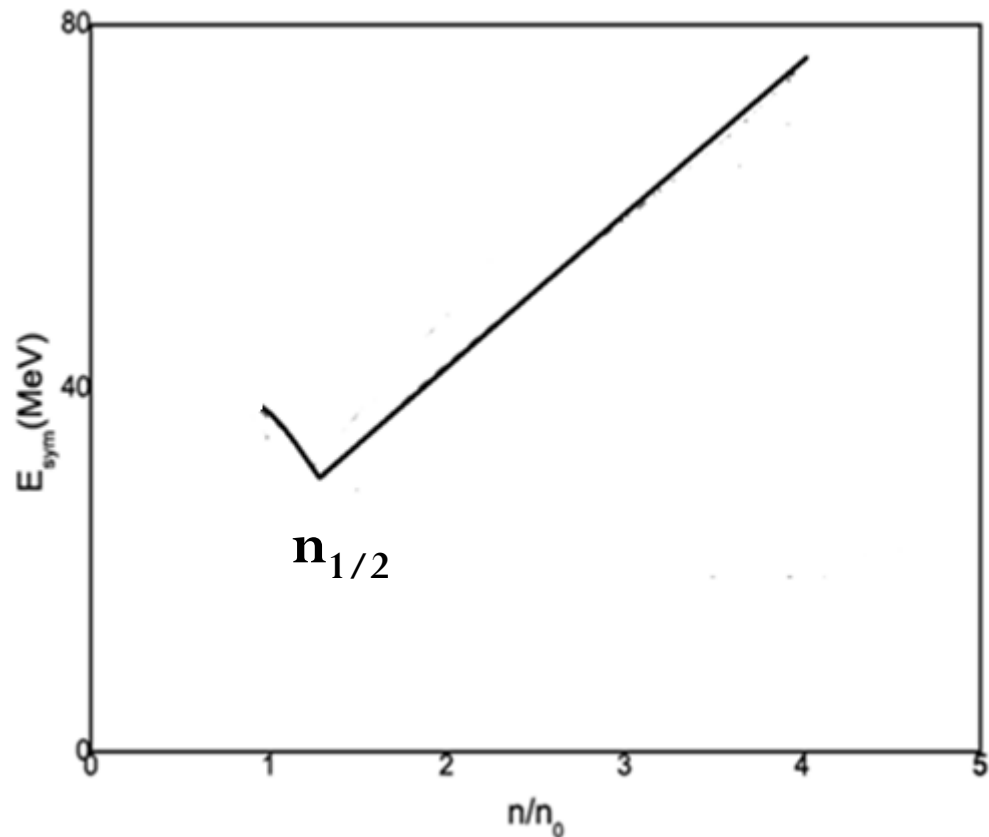
In the large  $N_c$  limit and at large density the skyrmion matter has the symmetry energy  $S$

$$S \equiv E_{sym}(n) \approx 1/8\mathcal{I} + \dots$$

*Moment of inertia*

$E_{sym}$  has a cusp at  $n_{1/2}$

The symmetry energy is  $O(1/N_c)$  but robust in the skyrmion descriptom



# “vacuum” properties

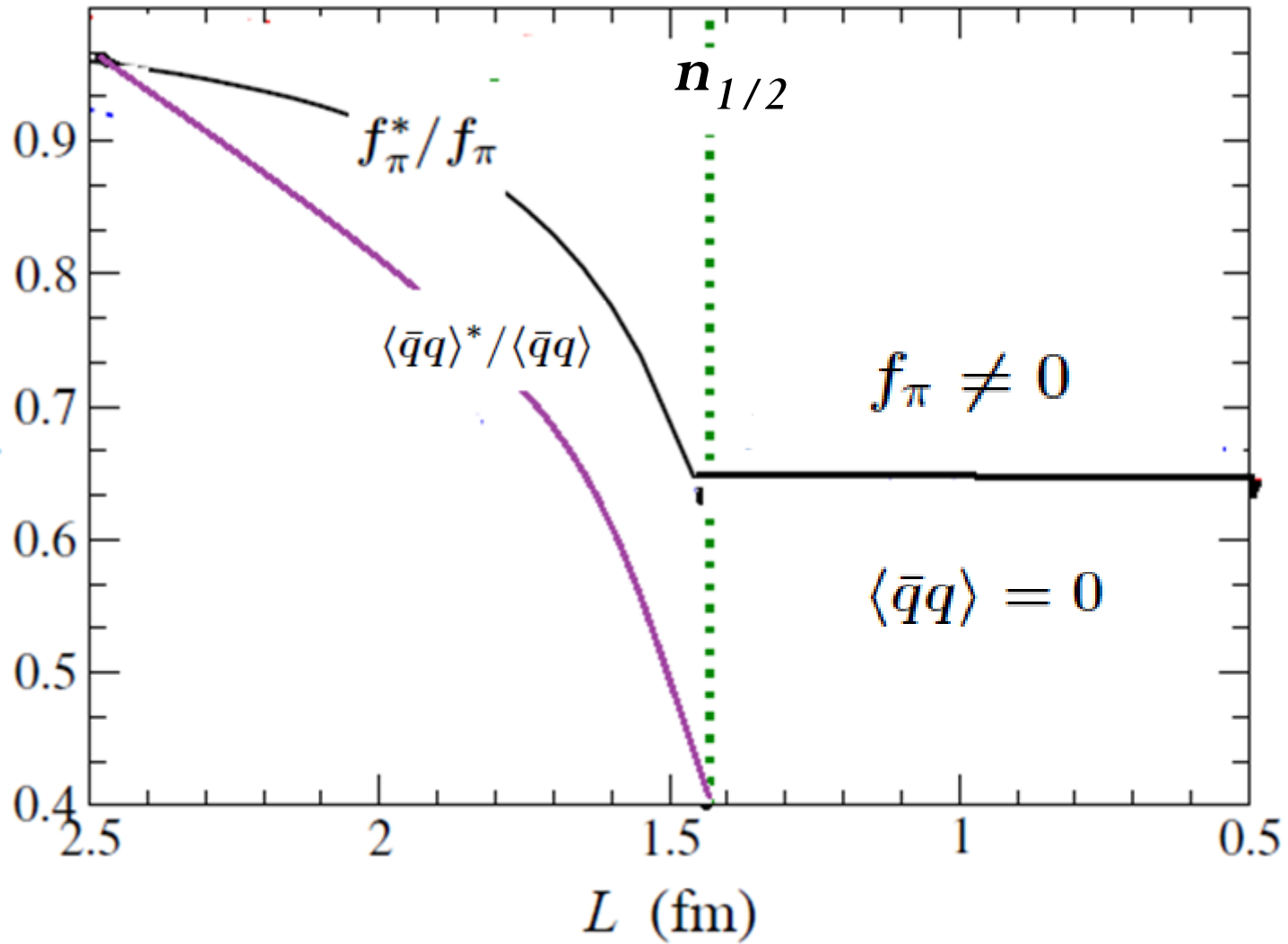
How to understand the cusp ?

To understand the cusp, we have to know how density affects the pion decay constant ( $f_\pi$ ) & quark condensate ( $\sigma \sim \langle \bar{q}q \rangle$ ), the vacuum property.

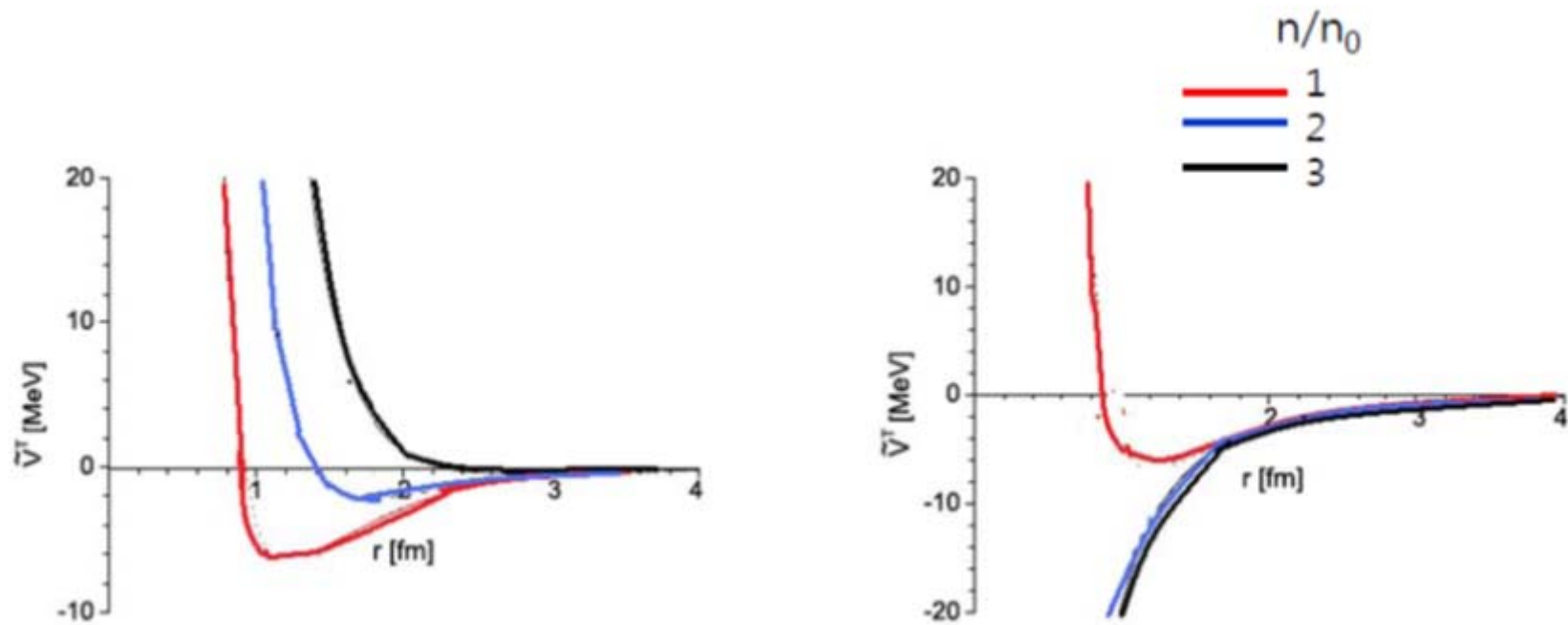
These *basic* quantities come out of fluctuations on top of background given by the skyrmion crystal.



# New phase diagram



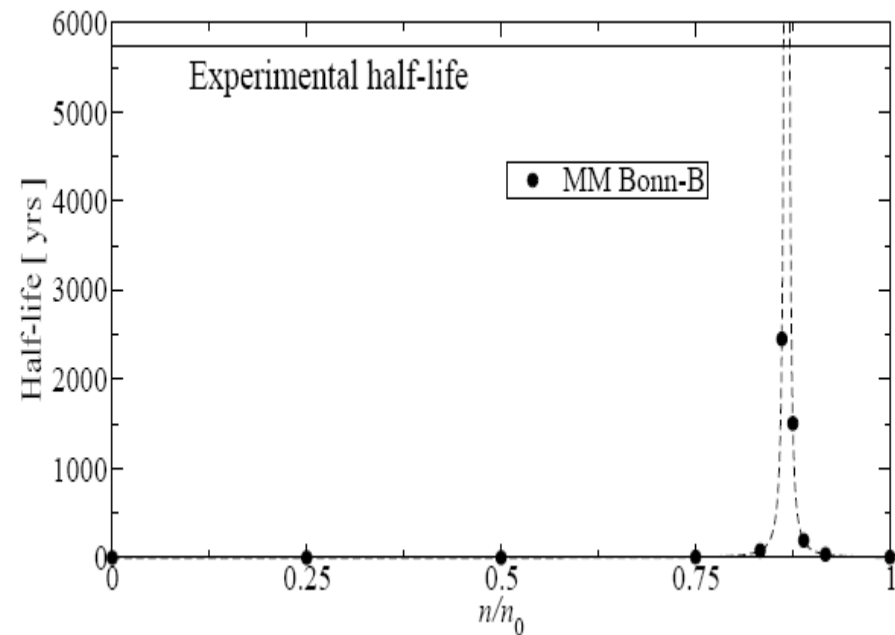
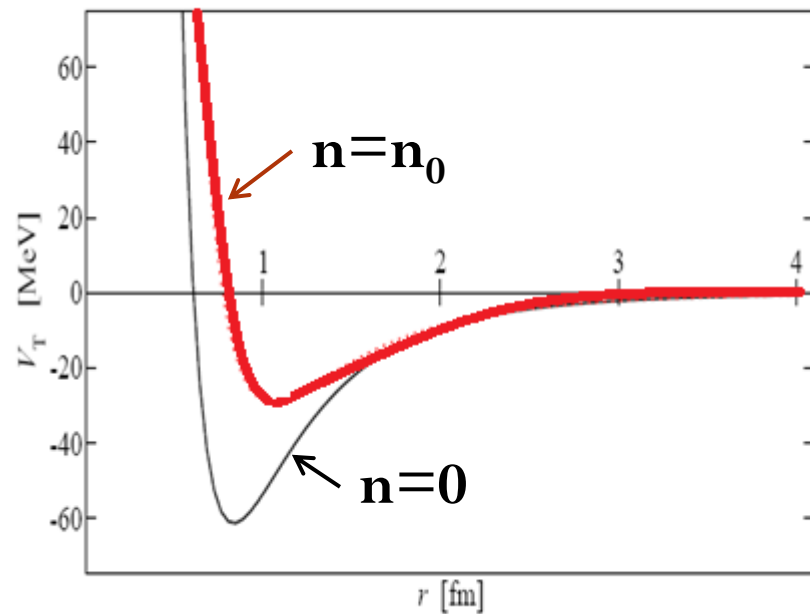
# Drastic effect on tensor forces



Without      With  
 $\frac{1}{2}$ -skyrmion phase

# C14 dating probes scaling for $n < n_{1/2}$

J.W. Holt et al, PRL **100**, 062501 (08)



**But tensor forces  $> n_{1/2}$  remain unexplored.**

Suppose symmetry energy is dominated by tensor forces

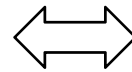
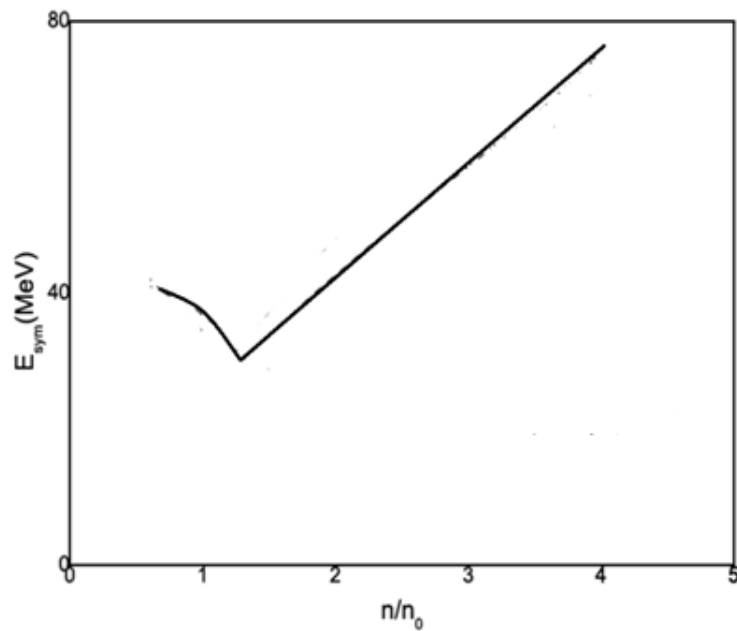
Closure approximation gives

$$E_{sym} \propto \frac{|V_T|^2}{\bar{E}}$$

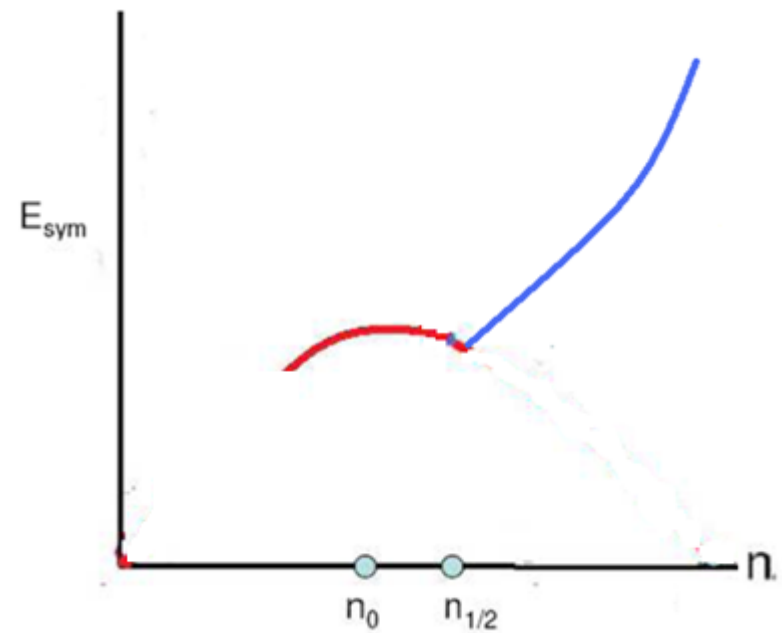
$|V_T|$  drops up to  $n_{1/2}$ , then starts going up for a constant  $\bar{E}$ . Thus the cusp.

# Topology change $\rightarrow$ change in tensor forces

## $\frac{1}{2}$ -skyrmion

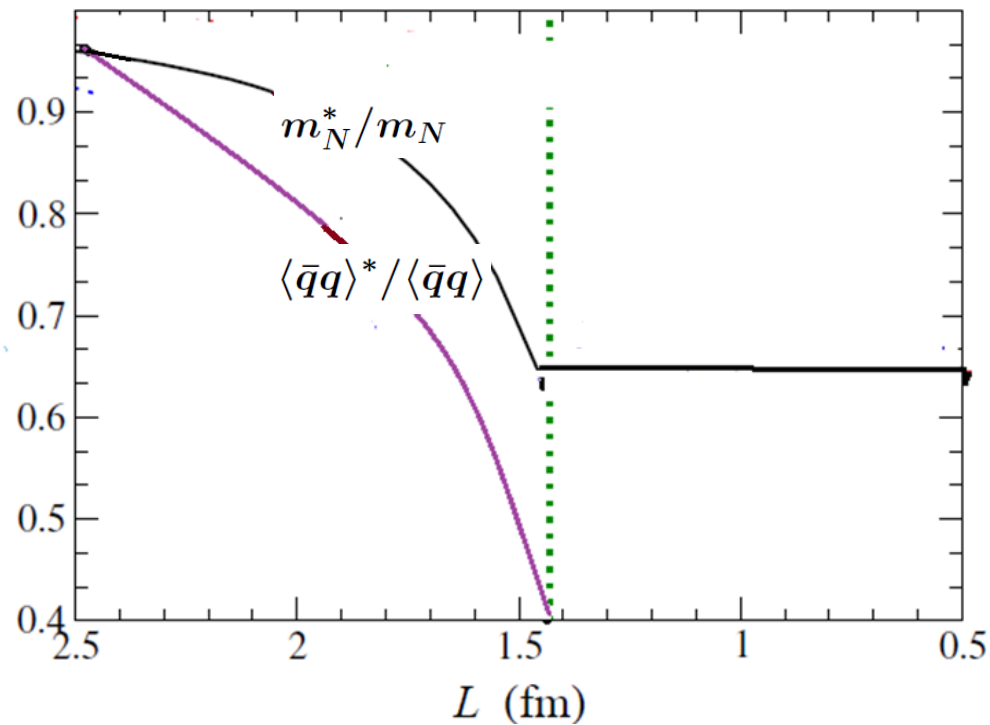


## Scaling tensor force



# Nucleon mass

In the large  $N_c$  limit,  $m_N \propto \text{const.} f_\pi$ .



The nucleon mass has two components:

$$m_N = m_0 + f(\langle \bar{q}q \rangle) \rightarrow m_0 \quad \text{for} \quad \langle \bar{q}q \rangle \rightarrow 0$$

## Where does $m_N$ come from ????

Here the chiral invariant  $m_0 \approx (0.7-0.8)m_N$  has no connection to the spontaneous breaking of chiral symmetry (**Nambu, Goldstone ...**). Given that quark masses are tiny, where does the bulk of the proton mass come from?

*Mystery of “mass without mass”*

*Nucleon in the phase  $n > n_{1/2}$  is a weird object with the zero quark condensate and a chiral invariant mass. Not the nucleon in RMP!!*

# Large $N_c$ QCD hints at chiral bag...

D.B. Kaplan arXiv: 1306.5818

NJL

$$\frac{\delta}{\delta\Phi(x)} \int d^4x \left( N_c \lambda^2 \text{Tr} \Phi^\dagger \Phi - \ln \langle \det(\mathcal{D} + m) \rangle_\Phi \right. \\ \left. - \ln \langle \det G_{ax,by} \rangle_\Phi \right) = 0 .$$

Chiral bag/skyrmion



# So where could the nucleon mass come from ?

Conjectures:

- ❖ Intrinsic property of QCD due to confinement (e.g., “parity-doubled model” of **De Tar and Kunihiro**)
- ❖ Emergent phenomenon due to strong nuclear many-body correlations associated with topology change
- ❖ Or ???

*Fundamental problem for nuclear physics*

# Phenomenological consequences

- ❖ **Compact stars;**

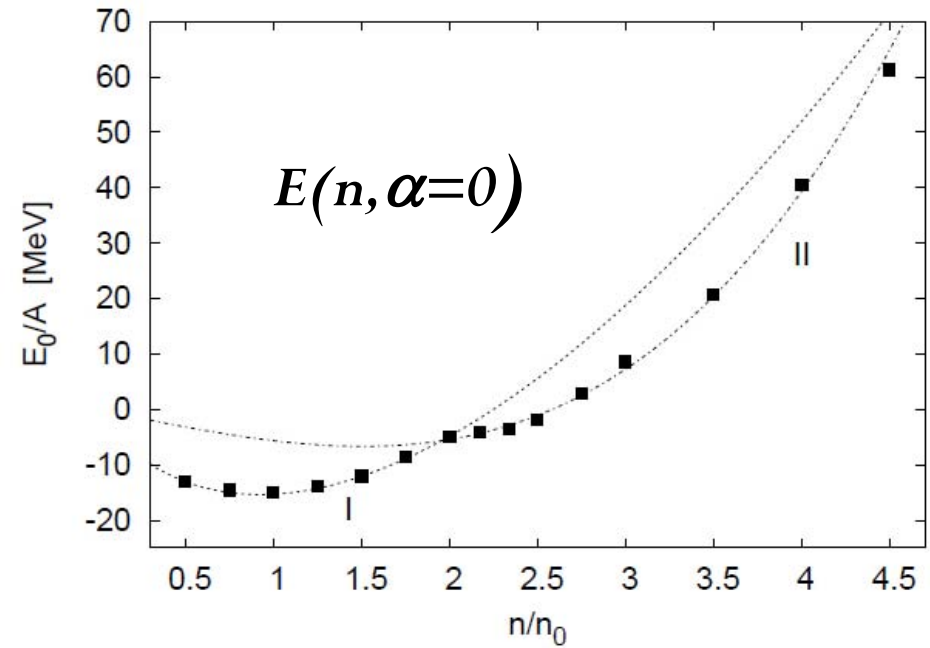
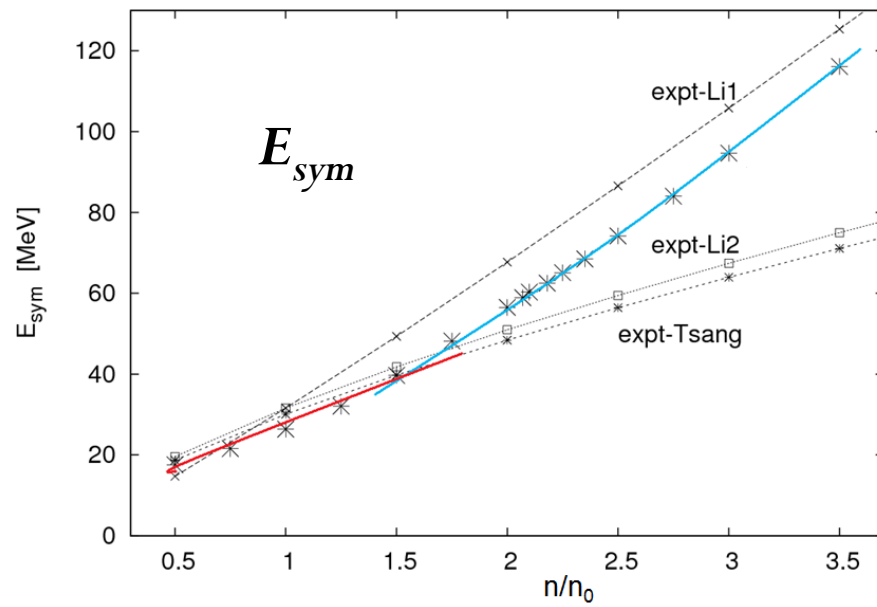
Dong, Kuo, Lee, Machleidt, Rho, Phys. Rev. C87, 054332 (13)

- ❖ **Shell evolution à la T. Otsuka et al**

Project associated with RAON/IBS

$$E(n, \alpha) = E(n, \alpha = 0) + E_{sym}(n)\alpha^2 + \mathcal{O}(\alpha^4)$$

Dong, Kuo, Lee, Machleidt, Rho 2013

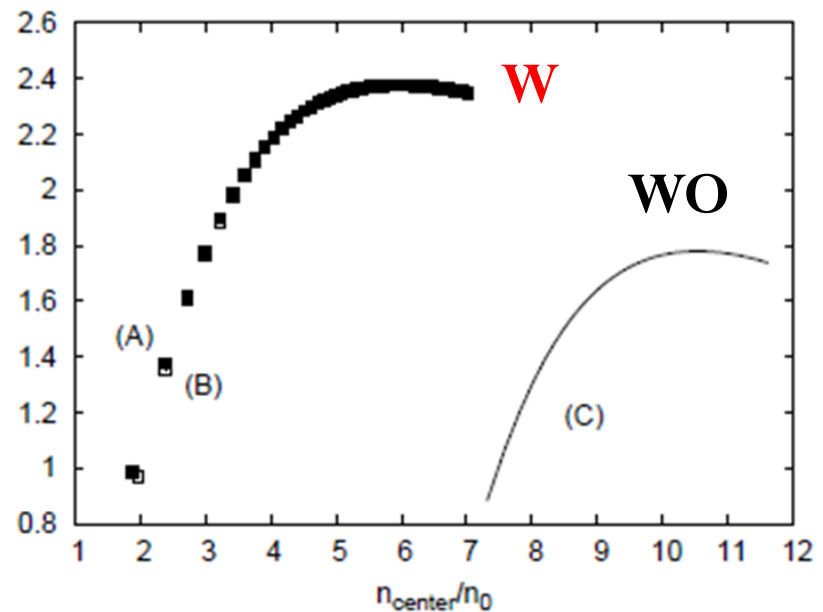
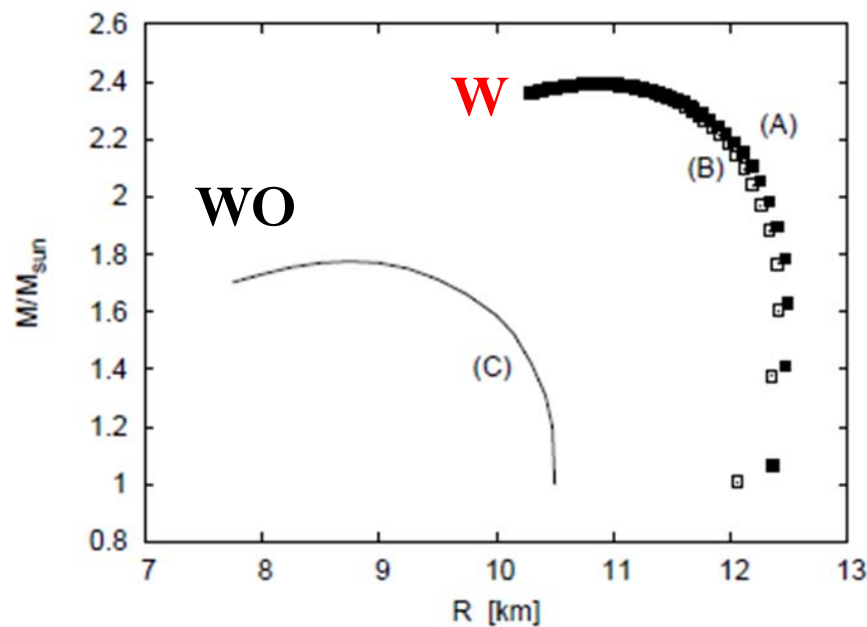


# Star properties predicted with (W) and without (WO) 1/2-skyrmions

$$E_{\text{sym}} \approx 27 \text{ MeV}, L \approx 64 \text{ MeV}$$

$$M_{\text{max}} \approx 2.4 M_{\text{sun}}, R \approx 11 \text{ km},$$

$$n_{\text{cent}} \approx 5.5 n_0 < n_{\text{causality}}$$



# “Renormalization persistency “ of tensor forces

T. Otsuka et al: “*The bare or intrinsic tensor forces are UNRENORMALIZED by short-range correlations and core polarization in the single-particle shell evolution*”

Evolution of single-particle energy

$$\Delta\epsilon_p(j) = \frac{1}{2}(V_{jj'}^{T=0} + V_{jj'}^{T=1})n_n(j')$$

Monopole matrix element

$$V_{j,j'}^T = \frac{\sum_J(2J+1)\langle jj'|V|jj'\rangle_{JT}}{\sum_J(2J+1)}.$$

# What to measure ?

❖ The *tell-tale* slope change of  $E_{sym}$  at  $\sim n_{1/2}$   
Dong, Kuo, Lee, Machleidt, Rho, Phys. Rev. C87, 054332 (13)

❖ *Precision* control of the monopole matrix element in exotic nuclei

Project associated with RAON/IBS ??



**Tantalizing hint at the origin  
of the proton mass !!**

**Thanks for your attention!**