IAS and Skin Constraints on the Symmetry Energy

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Symmetry Energy in Nuclear Mass Formula

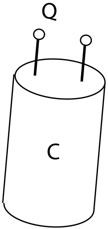
Textbook Bethe-Weizsäcker formula:

$$E = -a_V\, A + a_S\, A^{2/3} + a_C\, \frac{Z^2}{A^{1/3}} + a_a\, \frac{(N-Z)^2}{A} + E_{\rm mic}$$

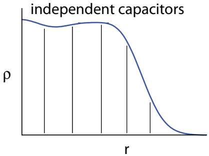
Symmetry energy: charge $n \leftrightarrow p$ symmetry of interactions

Analogy with capacitor:

$$E_a = a_a\, \frac{(N-Z)^2}{A} \equiv \frac{(N-Z)^2}{\frac{A}{a_a}} \Leftrightarrow E = \frac{Q^2}{2C}$$



?Volume Capacitance? $E_a = \frac{(N-Z)^2}{\frac{A}{a_a}} \rightarrow \frac{(N-Z)^2}{\frac{A}{a_a^V} + \frac{A^{2/3}}{a_a^S}}$



Thomas-Fermi (local density) approximation:

$$'C' = \frac{A}{a_a(A)} = \int \frac{\rho\, d\mathbf{r}}{S(\rho)} = \frac{A}{a_a^V}, \quad \text{for } S(\rho) \equiv a_a^V$$

TF breaks in nuclear surface at $\rho < \rho_0/4$ PD&Lee NPA818(2009)36

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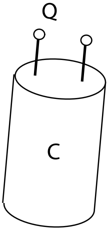

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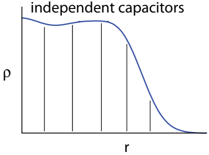

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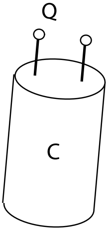

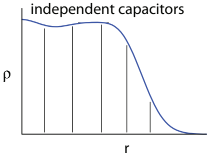

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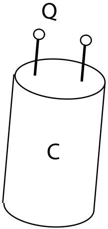

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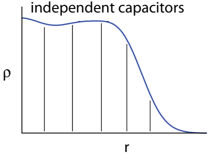

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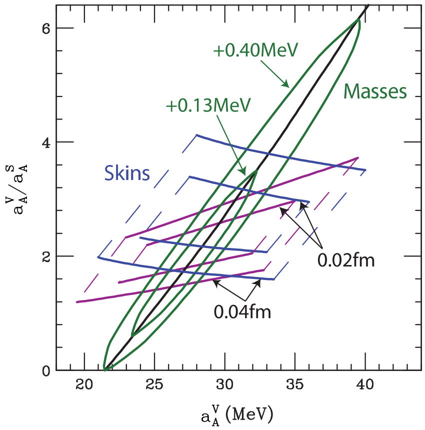
Mass and Skin Fits



Symmetry Energy:

$$E_a = \frac{a_a^V}{A} \frac{(N-Z)^2}{1 + \frac{a_a^V}{a_a^S A^{1/3}}}$$

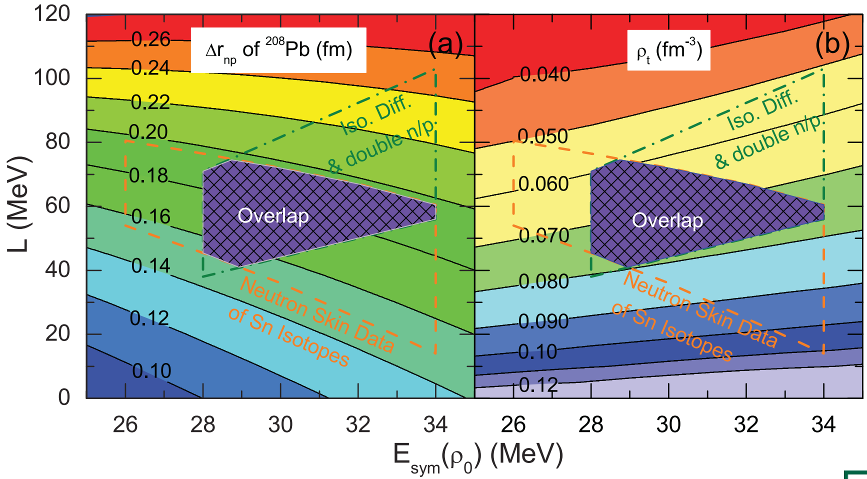
Skin:

$$\Delta r_{np} = \frac{2}{3} \frac{r_{rms}}{A^{1/3}} \frac{a_a}{a_a^S} \left(\frac{N-Z}{A} - Coul \right)$$

PD NPA723(2003)233

$a_a^S \leftrightarrow L$

Fits in L–a_a^V Plane



Lie-Wen Chen *et al* PRC82(10)024321

Charge Invariance

?$a_a(A)$? Conclusions on sym-energy details, following E-formula fits, interrelated with conclusions on other terms in the formula: asymmetry-dependent Coulomb, Wigner & pairing + asymmetry-independent, due to $(N-Z)/A$ - A correlations along stability line [PD NPA727(03)233]!

Best would be to study the symmetry energy in isolation from the rest of E-formula! Absurd?!

Charge invariance to rescue: lowest nuclear states characterized by different isospin values (T, T_z), $T_z = (Z-N)/2$. Nuclear energy scalar in isospin space:

sym energy $$E_a = a_a(A)\,\frac{(N-Z)^2}{A} = 4\,a_a(A)\,\frac{T_z^2}{A}$$

$$\rightarrow\ E_a = 4\,a_a(A)\,\frac{T^2}{A} = 4\,a_a(A)\,\frac{T(T+1)}{A}$$

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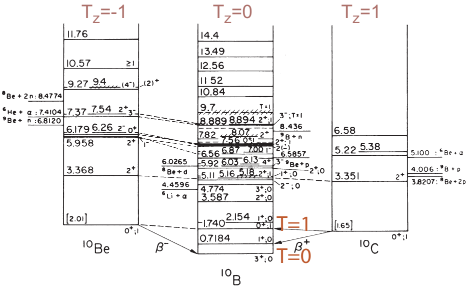
$a_a(A)$ Nucleus-by-Nucleus

$$\rightarrow \; E_a = 4\, a_a(A)\, \frac{T(T+1)}{A}$$

In the ground state T takes on the lowest possible value $T = |T_z| = |N-Z|/2$. Through '+1' most of the Wigner term absorbed.

Formula generalized to the lowest state of a given T (e.g. Jänecke *et al.*, NPA728(03)23).

?Lowest state of a given T: isobaric analogue state (IAS) of some neighboring nucleus ground-state.



Study of changes in the symmetry term possible nucleus by nucleus

PD&Lee arXiv:1307.4130

Queries in the Context of Data

Are expansions valid? Coefficient values??

$$E^*_{\rm IAS} = E_{\rm IAS} - E_{\rm gs} \stackrel{?}{=} \frac{4\, a_a(A)}{A} \, \Delta \big[T(T+1) \big] + \Delta E_{\rm mic}$$

Is the excitation energy linear in the isospin squared??

$$\frac{A}{a_a(A)} \stackrel{?}{=} \frac{A}{a_a^V} + \frac{A^{2/3}}{a_a^S}$$

or

$$a_a^{-1} \stackrel{?}{=} (a_a^V)^{-1} + (a_a^S)^{-1} \, A^{-1/3}$$

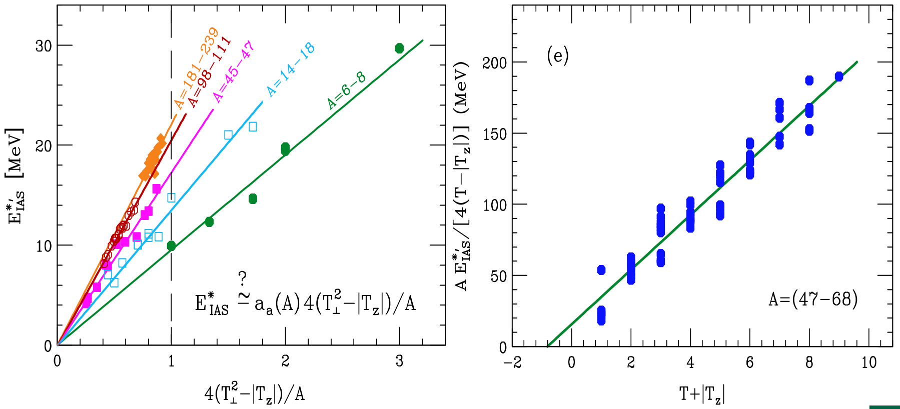
Is the volume-surface separation valid?

⇒ From an a_a^V-a_a^S fit can one learn about a_a^V and L for uniform matter?

Insight into IAS Analysis

IAS data: Antony *et al.* ADNDT66(97)1

Shell corrections: Koura *et al.* ProTheoPhys113(05)305



Excitation energies to IAS, E^*_{IAS}, for different A, lumped together in narrow regions of A. Is $E_a \propto T(T+1)$? YES

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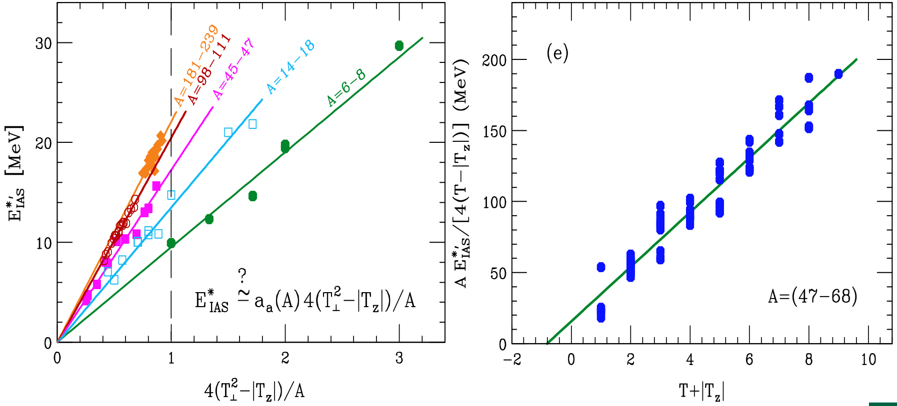

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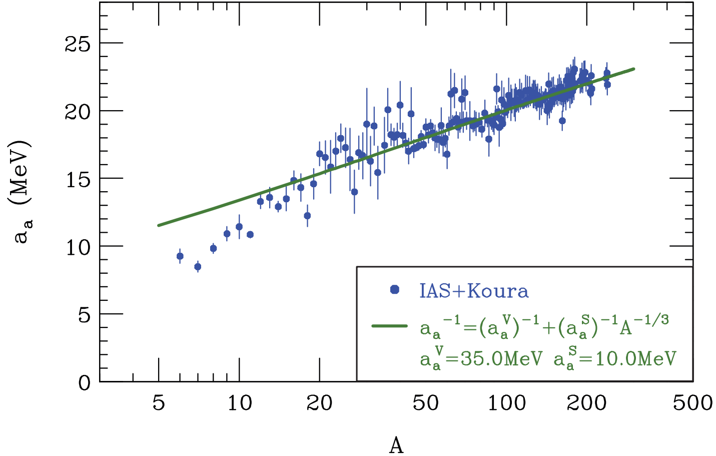
$a_a(A)$ with Shell Corrections

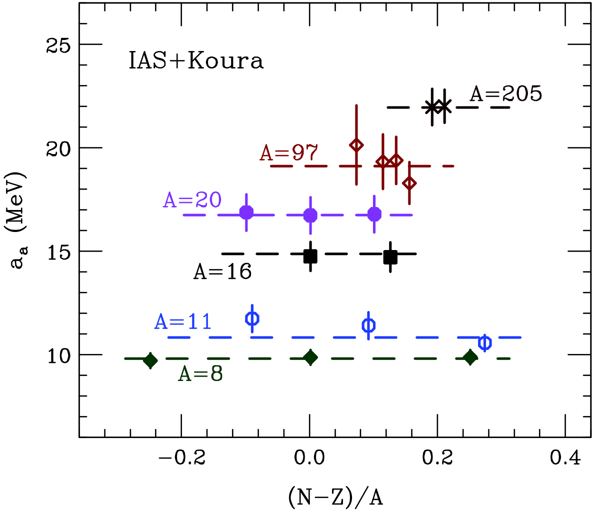
$$a_a(A) = \frac{A}{4} \frac{E^*_{\rm IAS} - \Delta E_{\rm mic}}{\Delta T^2}$$

$E_{\rm mic}$ from: Koura *et al.* ProTheoPhys113(05)305



Heavy nuclei $a_a \sim 22\,\text{MeV}$, light $a_a \sim 10\,\text{MeV}$ (more surface + low ρ)

Z-Dependence of Symmetry Coefficients?

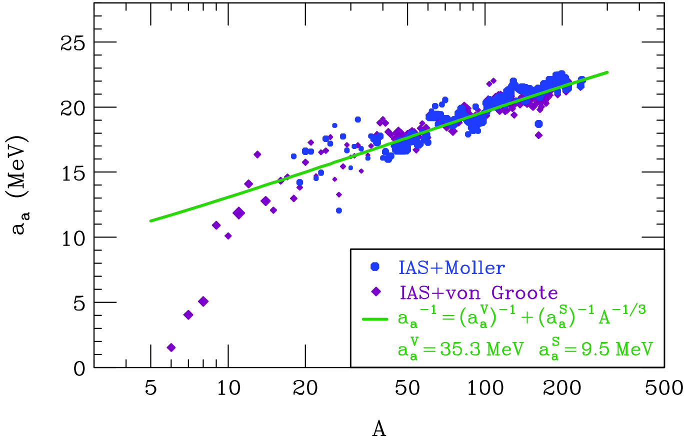


surface
less dominant,
higher
average ρ

surface
more dominant,
lower
average ρ

Symmetry coefficients on a nucleus-by-nucleus basis

Sensitivity to Shell Corrections



Fit to raw data ($A > 30$) in the middle, but:

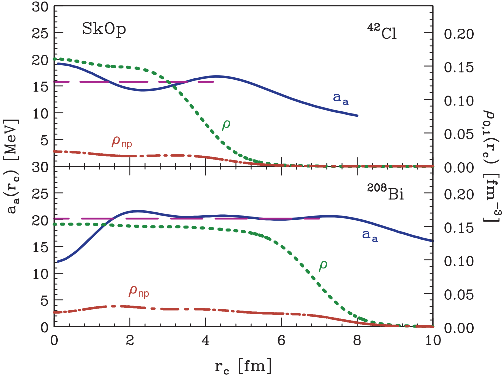
Moller *et al.* fit: $a_a^V = 39.73\,\text{MeV}, \quad a_a^S = 8.48\,\text{MeV}$

von Groote *et al.*: $a_a^V = 31.74\,\text{MeV}, \quad a_a^S = 11.27\,\text{MeV}$

Comparisons to Skyrme-Hartree-Fock

Issues in data-theory comparisons (codes by P.-G. Reinhard):

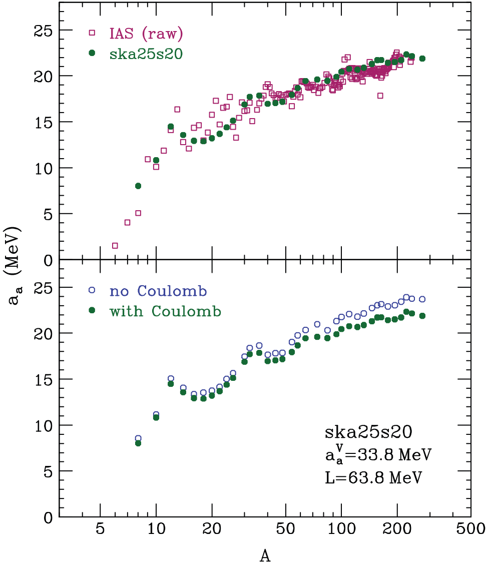
1. No isospin invariance in SHF - impossible to follow the procedure for data
2. Shell corrections not feasible at such scrutiny as for data
3. Coulomb effects.



Solution: Procedure that yields the same results as the energy, in the bulk limit, but is weakly affected by shell effects:

$$\frac{(N-Z)_{r<r_c}}{N-Z} = \frac{C_{r<r_c}}{C} = \frac{a_a}{A\, a_a^V} \int_{r<r_c} \frac{\rho}{S(\rho)}$$

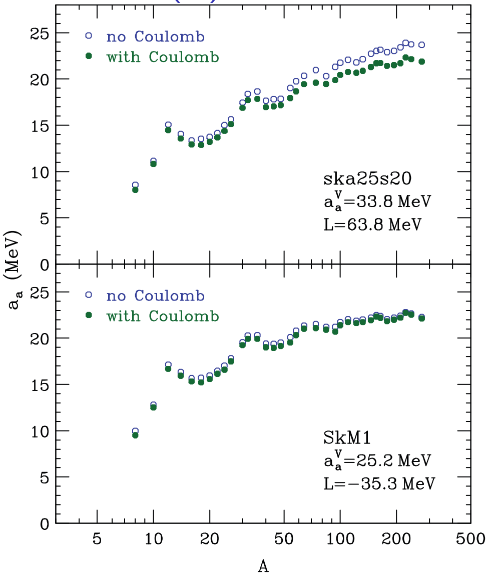
$a_a(A)$ from Mean-Field Calculations



Skyrme-Hartree-Fock theory (codes by P.-G. Reinhard)

Similar behavior with *A* as for IAS

$a_a(A)$ from Different Mean Fields



?Slope L in ρ
$\Leftrightarrow$ slope in A??

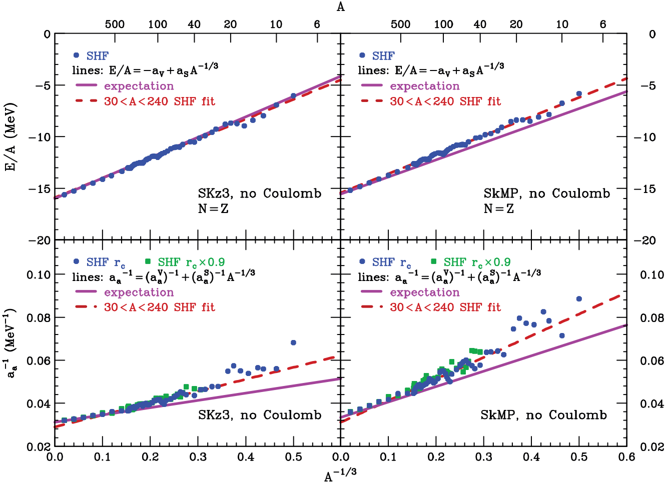
Less impact of the slope L at ρ_0 than expected!

??Difficulty for L determination??

Model-Independent Large-A Expansion??

Symbols: results of spherical no-Coulomb SHF calcs

⇒ Lines: volume-surface decomposition - expectation vs fit



→Symmetric matter energy f/sample Skyrmes

∼ Works

→Symmetry coefficient

∼ Not. . .

Expectations from half-∞ matter.

Can $S(\rho)$ Be Constrained??!

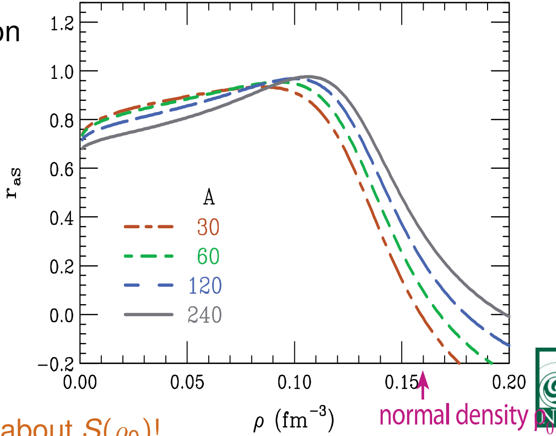
Pearson correlation coefficient

$$r_{XY} = \frac{\langle (X - \langle X \rangle)(Y - \langle Y \rangle) \rangle}{\sqrt{\langle (X - \langle X \rangle)^2 \rangle \langle (Y - \langle Y \rangle)^2 \rangle}}$$

$|r| \sim 1$ - strong correlation
$r \sim 0$ - no correlation

$X \equiv a_a(A)$
$Y \equiv S(\rho)$

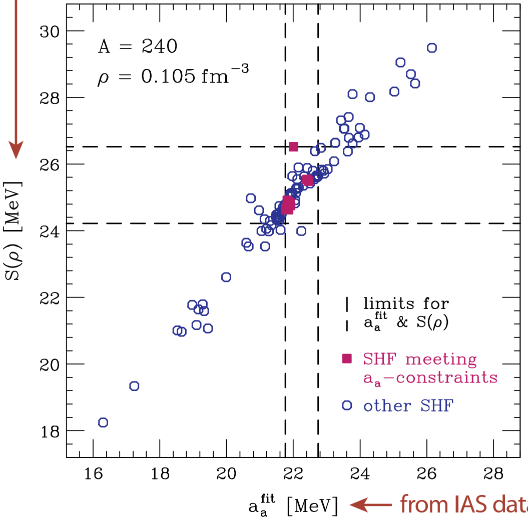
Ensemble of Skyrmes



Nearly no information about $S(\rho_0)$!

Symmetry-Energy Correlations When Strong

uniform matter



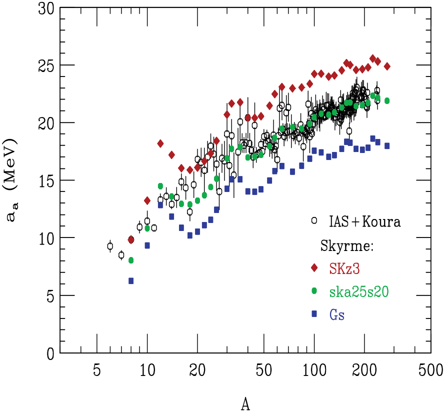
← from IAS data

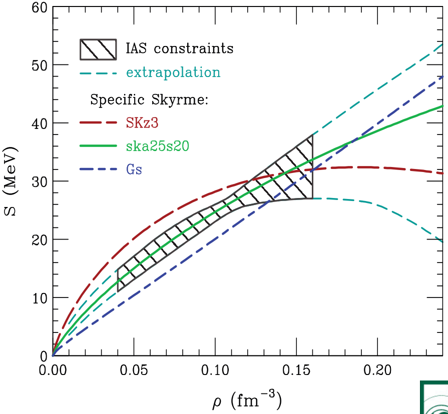
NO $S(\rho) \approx a_a$!

Vehicle: phenomenological mean-field theory

Constraints on Symmetry Energy $S(\rho)$

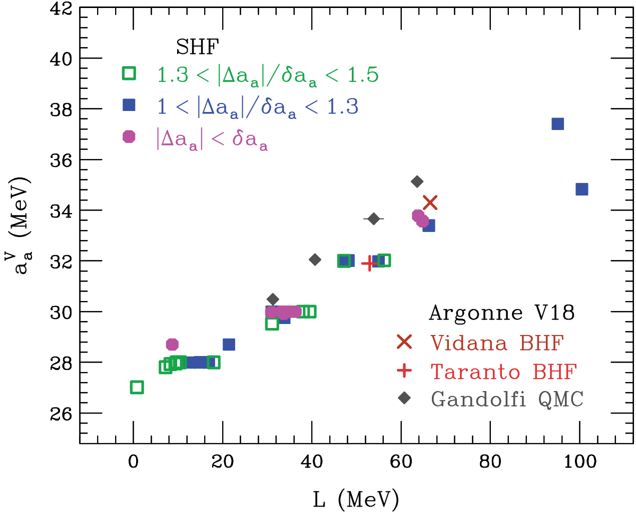
Demand that Skyrme approximates IAS results at $A > 30$ produces a constraint area for $S(\rho)$:





?Slope constraint??

Correlation at ρ_0



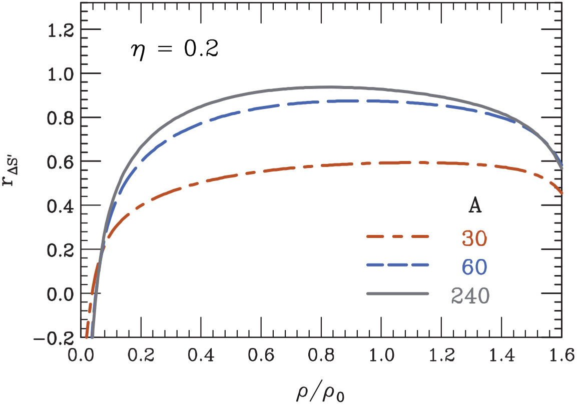
?Slope constraint??

Asymmetry Skin & Energy Stiffness

Pearson coef of
$\Delta r_{np} = r_n^{\text{rms}} - r_p^{\text{rms}}$
& stiffness of S

$$\gamma(\rho) := \frac{\rho}{S} \frac{\mathrm{d}\, S}{\mathrm{d}\, \rho}$$

f/different A
at fixed
$\eta = (N - Z)/A$

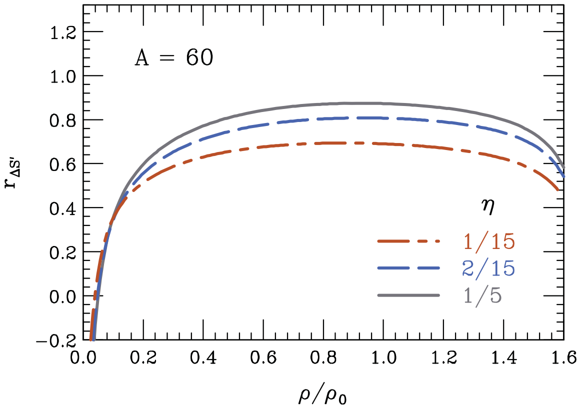


Asymmetry Skin & Energy Stiffness

Pearson coef of Δr_{np} & stiffness

$$\gamma(\rho) := \frac{\rho}{S}\frac{\mathrm{d}S}{\mathrm{d}\rho}$$

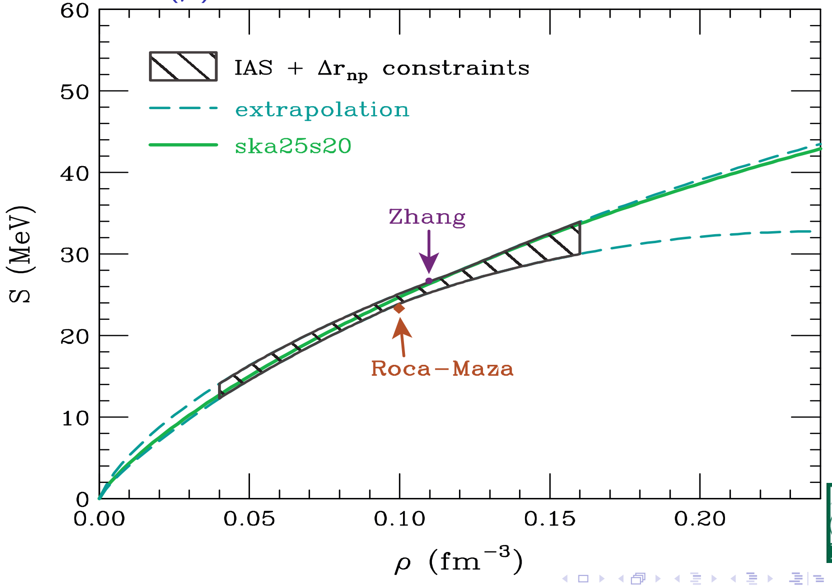
f/different $\eta = (N - Z)/A$ at fixed A



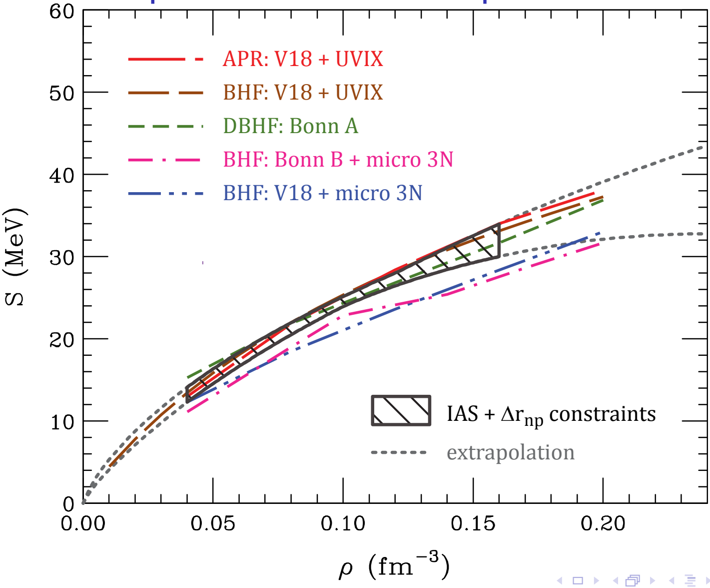
Asymmetry Skins from Measurements

Nucleus	Reference	Data Source	$\Delta\, r_{np}$ [fm]	$\Delta\, r_{np}^{GF}$ [fm]
^{48}Ca	Friedman [92]	pionic atoms	0.13 ± 0.06	
	Gils et al. [93]	elastic α scattering	0.175 ± 0.050	
	Ray [94]	elastic $\vec{p}$ scattering	0.229 ± 0.050	
	Clark et al. [95]	elastic p scattering	0.103 ± 0.040	
	Shlomo et al. [96]	elastic p scattering	0.10 ± 0.03	
	Gibbs et al. [97]	elastic π scattering	0.11 ± 0.04	
		combined results	0.129 ± 0.053⊠	0.215 ± 0.012
•				
^{207}Pb	Starodubsky et al. [99]	elastic p scattering	0.186 ± 0.041	0.175 ± 0.023
^{208}Pb	Starodubsky et al. [99]	elastic p scattering	0.197 ± 0.042	
	Ray [94]	elastic $\vec{p}$ scattering	0.16 ± 0.05	
	Clark et al. [95]	elastic p scattering	0.119 ± 0.045	
	Zenihiro et al. [98]	elastic p scattering	0.211 ± 0.063	
	Friedman [92]	elastic π^+ scattering	0.11 ± 0.06	
	Friedman [92]	pionic atoms	0.15 ± 0.08	
		combined results	0.159 ± 0.041⊠	0.179 ± 0.023

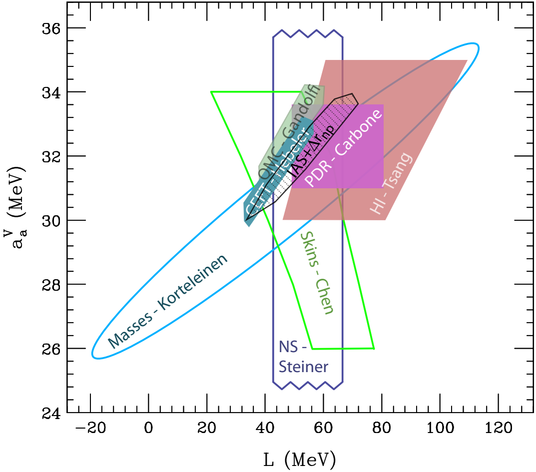
$S(\rho)$ from Combined Constraints



Comparison to Microscopic Calculations



Symmetry Energy at ρ_0?



$$S(\rho) = a_a^V + \frac{L}{3}\,\frac{\rho-\rho_0}{\rho+0} + \dots$$

Conclusions

- Symmetry-energy term weakens as nuclear mass number decreases: from $a_a \sim 23$ Mev to $a_a \sim 9$ MeV for $A \lesssim 8$.
- For $A \gtrsim 25$, $a_a(A)$ may be fitted with $a_a^{-1} = (a_a^V)^{-1} + (a_a^S)^{-1} A^{-1/3}$, where $a_a^V \approx 35$ MeV and $a_a^S \approx 10$ MeV.
- Weakening of the symmetry term can be tied to the weakening of $S(\rho)$ in uniform matter, with the fall of ρ.
- Including skin sizes, significant, $\lesssim \pm 1.0$ MeV, constraints on $S(\rho)$ at densities $\rho =$(0.04-0.13) fm^{-3}.
- Around ρ_0: *strongly correlated* $a_a^V = (30.2\text{–}33.7)$ MeV and $L = (35\text{–}70)$ MeV.

To do: Dedicated Skyrme interactions.

PD&Lee arXiv:1307.4130

PHY-1068571

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- Symmetry-energy term weakens as nuclear mass number decreases: from $a_a \sim 23$ Mev to $a_a \sim 9$ MeV for $A \lesssim 8$.
- For $A \gtrsim 25$, $a_a(A)$ may be fitted with $a_a^{-1} = (a_a^V)^{-1} + (a_a^S)^{-1} A^{-1/3}$, where $a_a^V \approx 35$ MeV and $a_a^S \approx 10$ MeV.
- Weakening of the symmetry term can be tied to the weakening of $S(\rho)$ in uniform matter, with the fall of ρ.
- Including skin sizes, significant, $\lesssim \pm 1.0$ MeV, constraints on $S(\rho)$ at densities $\rho =$(0.04-0.13) fm^{-3}.
- Around ρ_0: *strongly correlated* $a_a^V = (30.2\text{–}33.7)$ MeV and $L = (35\text{–}70)$ MeV.

To do: Dedicated Skyrme interactions.

PD&Lee arXiv:1307.4130

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NSCL

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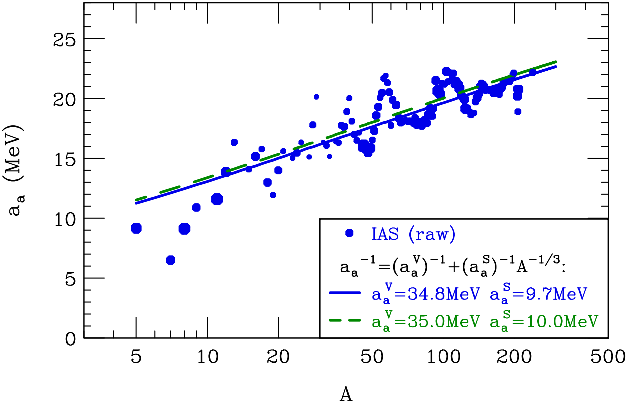
$a_a(A)$ without Shell Corrections

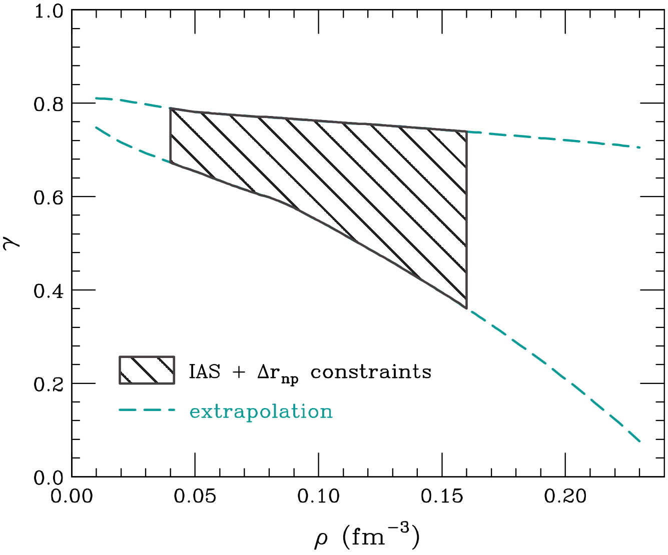
$$a_a(A) = \frac{A}{4}\,\frac{E^*_{\text{IAS}}}{\Delta T^2}$$

IAS data: Antony *et al.* ADNDT66(97)1



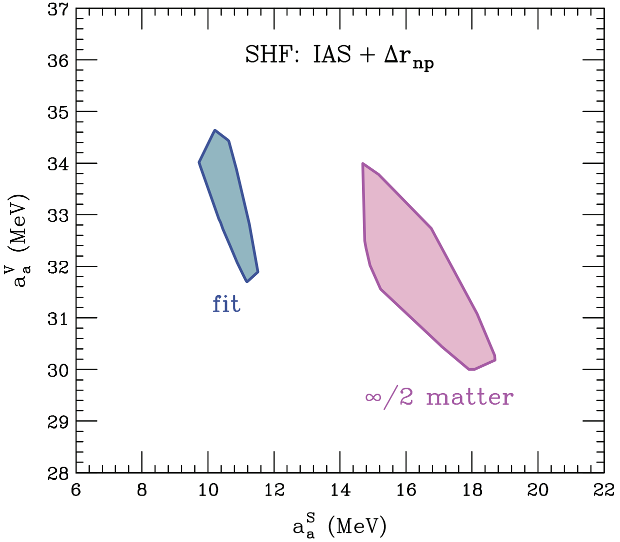
Lines: fits to $a_a(A)$ assuming *volume-surface competition* analogous to that for E_1. ??Fundamental knowledge??

Stiffness of the Symmetry Energy



$S \propto \rho^{\gamma}$

Robustness of Macroscopic Description?



Constraints at ρ_0