Microscopic calculations of neutron matter

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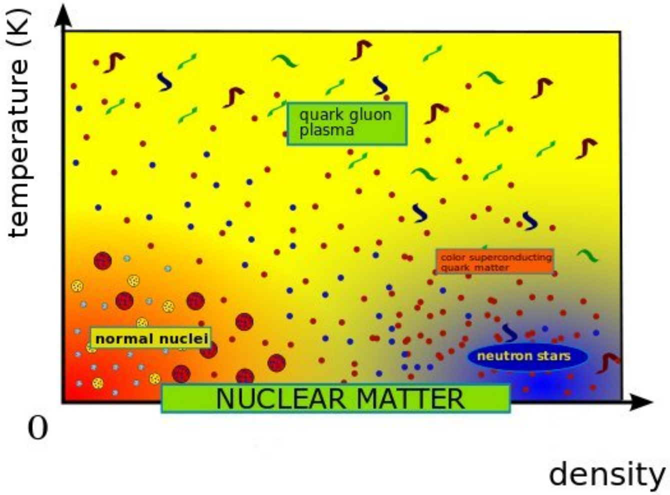
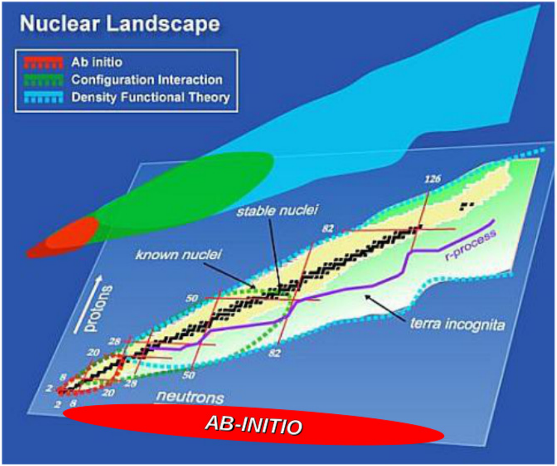
Homogeneous neutron matter

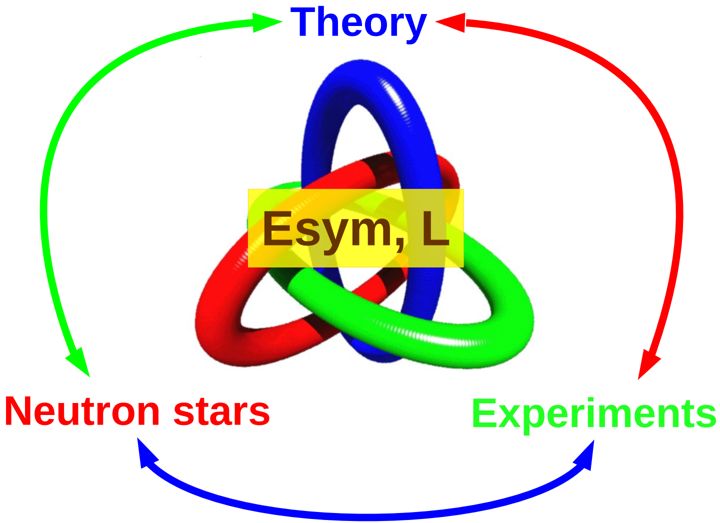


Inhomogeneous neutron matter



W. Nazarewicz – UNEDF

Symmetry energy



Outline

- The model and the method
- **Homogeneous neutron matter**
  - Role of three-neutron force
  - Symmetry energy
  - Neutron star structure
- **Inhomogeneous neutron matter: Skyrme vs ab-initio.**
  - Energy
  - Density and radii
- Conclusions

Nuclear Hamiltonian

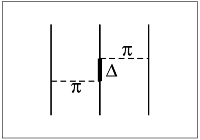
Model: non-relativistic nucleons interacting with an effective nucleon-nucleon force (NN) and three-nucleon interaction (TNI).

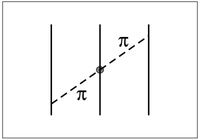
$$H = -\frac{\hbar^2}{2m}\sum_{i=1}^{A}\nabla_i^2 + \sum_{i<j} v_{ij} + \sum_{i<j<k} V_{ijk}$$

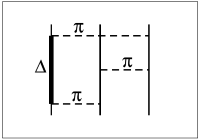
v_{ij} NN (Argonne AV8') fitted on scattering data. Sum of operators:

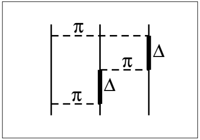
$$v_{ij} = \sum O_{ij}^{p=1,8} v^p(r_{ij})\,, \quad O_{ij}^p = (1, \vec{\sigma}_i \cdot \vec{\sigma}_j, S_{ij}, \vec{L}_{ij} \cdot \vec{S}_{ij}) \times (1, \vec{\tau}_i \cdot \vec{\tau}_j)$$

Urbana–Illinois V_{ijk} models processes like









$+$ short-range correlations (spin/isospin independent).

Quantum Monte Carlo

Evolution of Schrodinger equation in imaginary time **t**:

$$\psi(R,t) = e^{-(H-E_T)t}\psi(R,0)$$

In the limit of $t \to \infty$ it approaches to the lowest energy eigenstate (not orthogonal to $\psi(R,0)$).

Propagation performed by

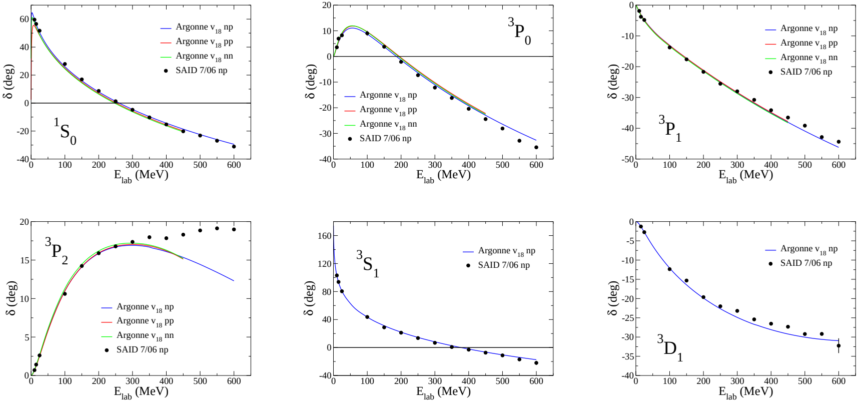
$$\psi(R,t) = \langle R|\psi(t)\rangle = \int dR' G(R,R',t)\psi(R',0)$$

$G(R,R',t)$ is an approximate propagator (small-time limit). We iterate the above integral equation many times in the small time-step limit.
$\rightarrow$ parallel codes and supercomputers.

For a given microscopic Hamiltonian, this method solves the ground–state within a systematic uncertainty of **1–2%** in a **non-perturbative way.**

Nuclear Hamiltonian

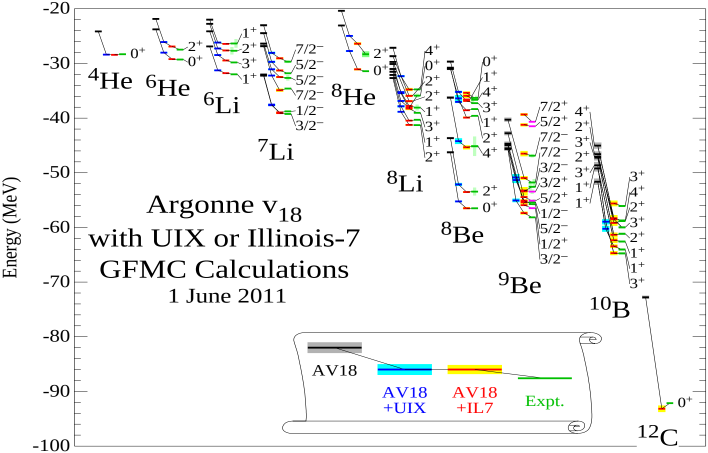
Argonne NN interaction



Wiringa, Stoks, Schiavilla (1995)

Note: at density ρ_0, $k_F \simeq 330$ MeV. Two neutrons have $E_{CM} \simeq 120$ MeV, $E_{LAB} \simeq 240$ MeV. $\rightarrow$ Argonne NN very accurate for densities up to $3\rho_0$.

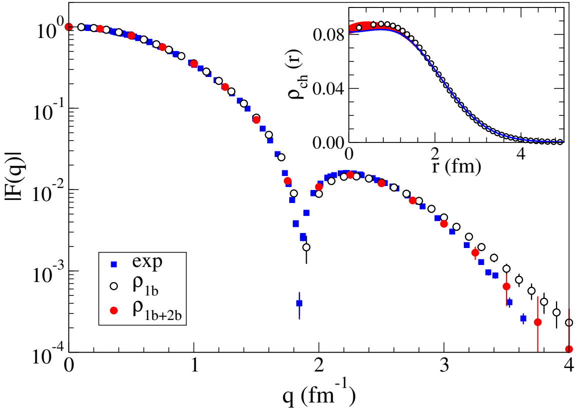
Light nuclei spectrum computed with GFMC



Carlson, Pieper, Wiringa, many papers

Charge form factor of ^{12}C

$$|F(q)| = \langle \psi | \rho_q | \psi \rangle \qquad \rho_q = \sum_i \rho_q(i) + \sum_{i<j} \rho_q(ij)$$



Lovato, Gandolfi, Butler, Carlson, Lusk, Pieper, Schiavilla arXiv:1305.6959

Neutron matter

Assumptions:

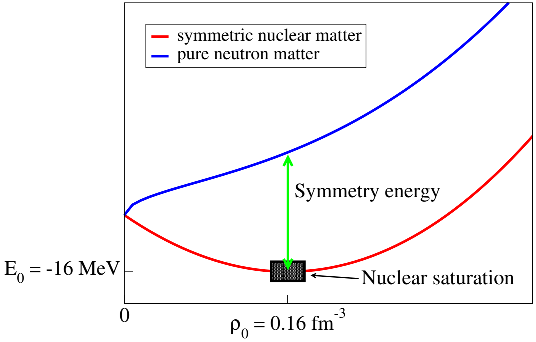
- The two-nucleon interaction reproduces well (elastic) *pp*, *np* and *nn* scattering data up to high energies ($E_{lab} \sim 600$MeV).
- The three-neutron force ($T = 3/2$) very weak in light nuclei, while $T = 1/2$ is the dominant part (but zero in neutron matter). **Difficult to study in light nuclei.**

Nuclear matter EOS:

$$E(\rho, x) = E_{SNM}(\rho) + E_{sym}^{(2)}(\rho)(1 - 2x)^2 + \cdots$$

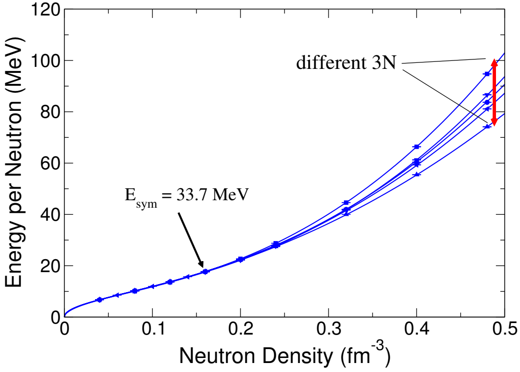
where

$$\rho = \rho_n + \rho_p \,, \quad x = \frac{\rho_p}{\rho}$$



Neutron matter

We consider different forms of three-neutron interaction by only requiring a particular value of E_{sym} at saturation.

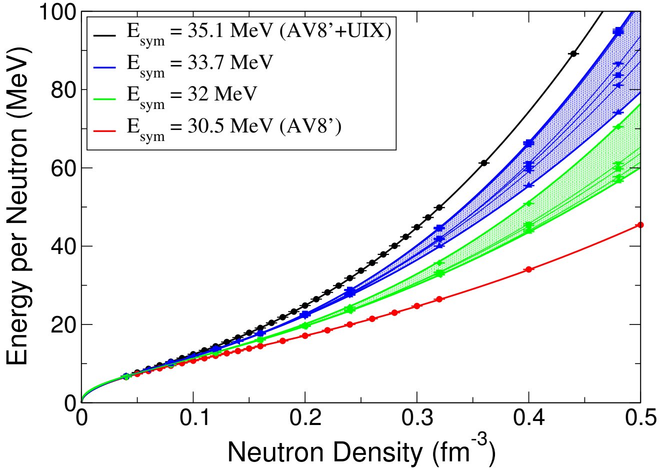


different 3N:

- $V_{2\pi} + \alpha V_R$
- $V_{2\pi} + \alpha V_R^{\mu}$ (several μ)
- $V_{2\pi} + \alpha \tilde{V}_R$
- $V_{3\pi} + \alpha V_R$

Neutron matter and symmetry energy

We then manually tune the neutron matter energy at saturation:

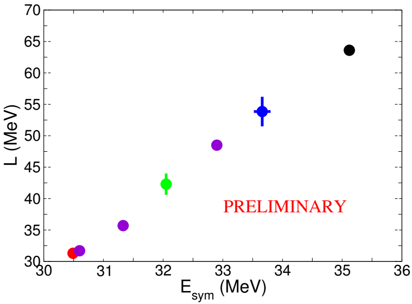


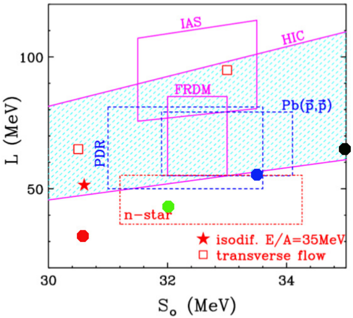
Gandolfi, Carlson, Reddy, (2012).

Neutron matter and symmetry energy

From the EOS, we can fit the symmetry energy around ρ_0 using

$$E_{sym}(\rho) = E_{sym} + \frac{L}{3}\frac{\rho - 0.16}{0.16} + \cdots$$



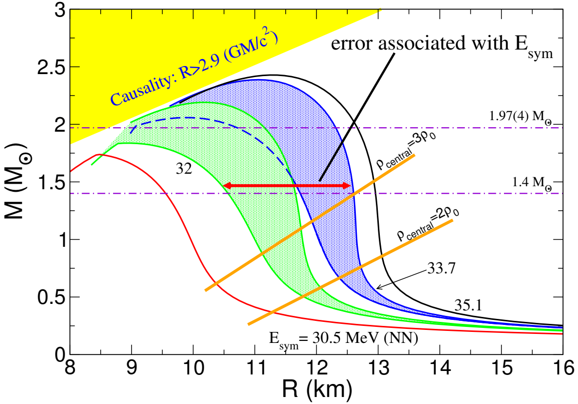


Tsang *et al.*, (2012)

Very weak dependence to the model of 3N force for a given E_{sym}.
Role of NN will be investigated next.

Neutron star structure

EOS used to solve the TOV equations.

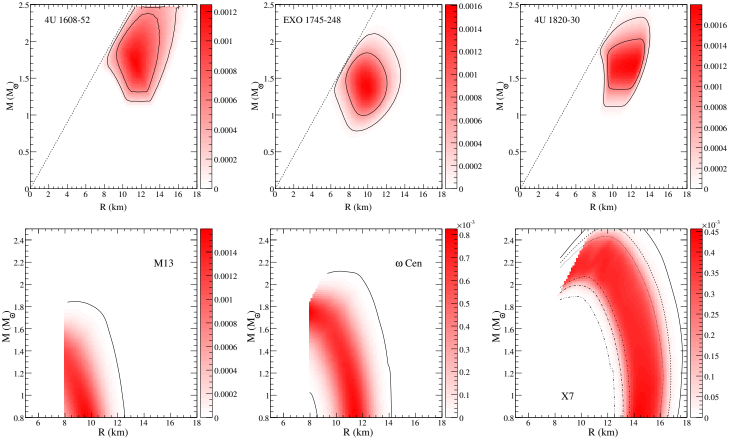


Gandolfi, Carlson, Reddy, PRC 032801, 85 (2012).

Accurate measurement of E_{sym} would put a constraint to the radius of neutron stars, **OR** observation of M and R would constrain E_{sym}!

Neutron stars

Observations of the mass-radius relation are available:



Steiner, Lattimer, Brown, ApJ (2010)

Neutron star matter

We model neutron star matter as

$$E_{NSM} = a\left(\frac{\rho}{\rho_0}\right)^{\alpha} + b\left(\frac{\rho}{\rho_0}\right)^{\beta}, \qquad E_{sym} = a + b + 16, \quad L = 3(a\alpha + b\beta)$$

(form suggested by microscopic simulations),

and a high density model for $\rho > \rho_t$

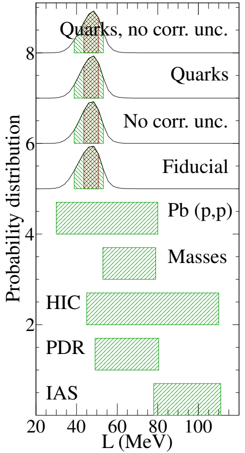
i) two polytropes

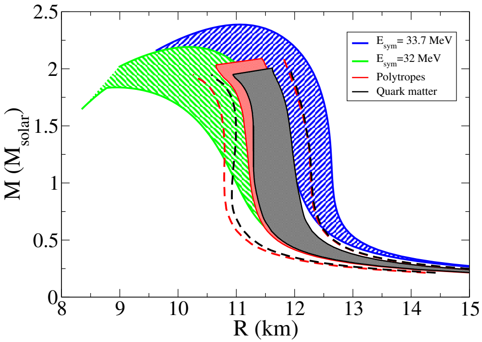
ii) polytrope+quark matter model, Alford et al., ApJ (2005).

By changing ρ_t and the high density model we can understand systematic errors in E_{NSM} parametrization.

We also add a correction to account for the proton fraction present in neutron stars.

Neutron star observations





$32 < E_{sym} < 34$ MeV

$43 < L < 52$ MeV

Steiner, Gandolfi (2012).

Now take a breath, and let's change topic.

Neutron drops

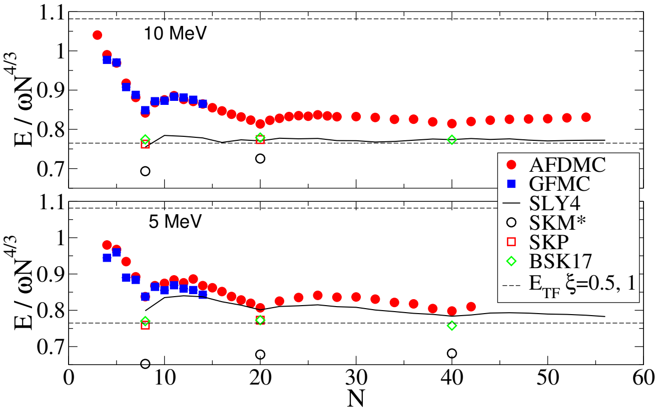
Why study neutron drops?
Are they nothing more than a pure simple toy model?

- Provide an easy model of inhomogeneous neutron matter
- Model neutron-rich nuclei
- Calibrate Skyrme models for neutron-rich systems (useful to check $\nabla\rho$ terms in different geometries)

$$H = -\frac{\hbar^2}{2m}\sum_{i=1}^{A}\nabla_i^2 + \sum_{i<j} v_{ij} + \sum_{i<j<k} V_{ijk} + \sum_i V_{ext}(r_i)$$

Neutron drops, harmonic oscillator well

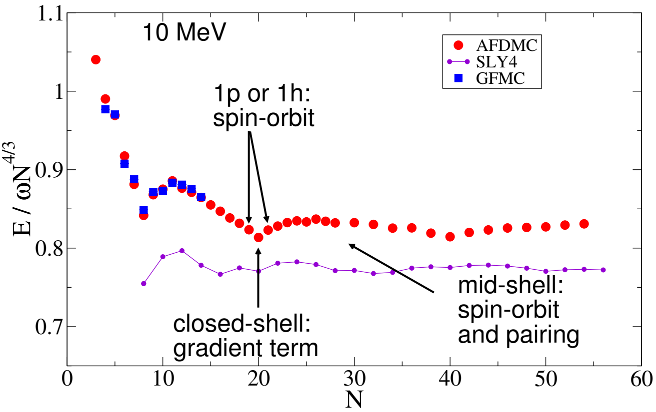
External well: harmonic oscillator with $\hbar\omega$=5, 10 MeV.



Skyrme systematically overbind neutron drops.

Neutron drops, harmonic oscillator well

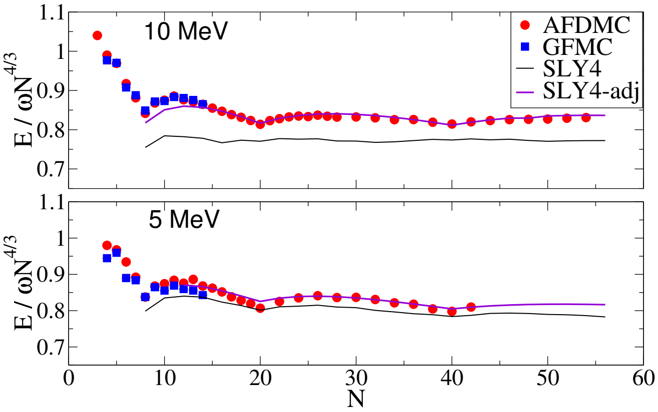
Fixing Skyrme force:



The correction is very similar in all the Skyrme forces we considered.

Note: bulk term of Skyrme fit neutron matter.

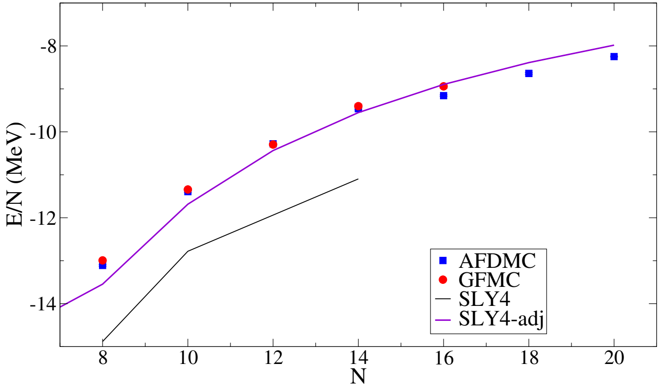
We add the **missing repulsion** by adjusting the gradient term $G_d[\nabla\rho_n]^2$, the pairing and spin-orbit terms.



Gandolfi, Carlson, Pieper (2011).

Neutron drops, adjusted Skyrme force

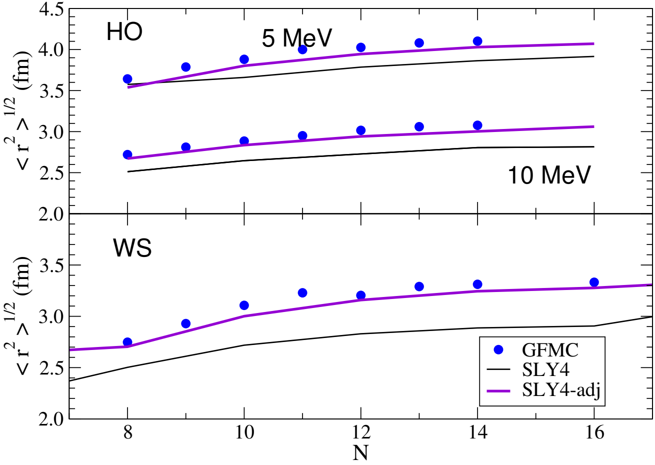
Neutrons in the Wood-Saxon well are also better reproduced by the adjusted SLY4.



Gandolfi, Carlson, Pieper (2011).

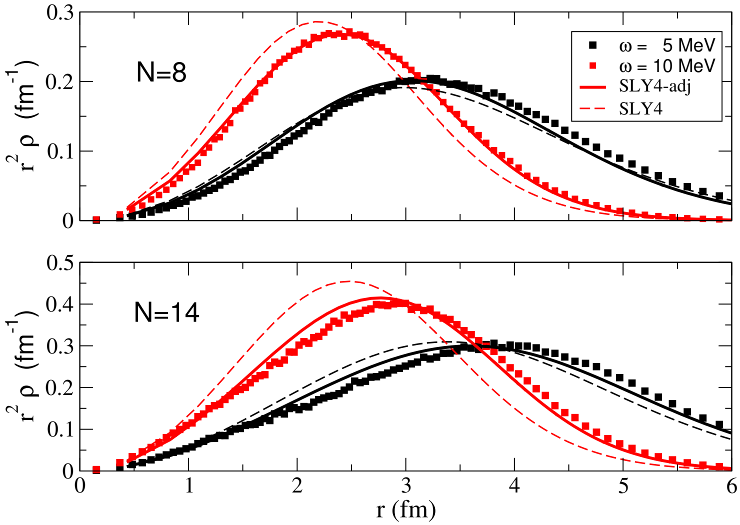
Neutron drops: radii

Correction to radii using the adjusted-SLY4.



Gandolfi, Carlson, Pieper (2011).

Neutron radial density:



Gandolfi, Carlson, Pieper (2011).

Gradient term

Where is the gradient term important?

Just few examples:

- Medium large neutron-rich nuclei
- Phases in the crust of neutron stars
- Isospin-asymmetry energy of nuclear matter

QMC methods useful to study nuclear systems in a coherent framework:

- Light nuclei spectra and form factors well in agreement with experimental data. Calculation of neutron matter EOS now possible with the same accuracy.
- Three-neutron force is the bridge between E_{sym} and neutron star structure.
- Neutron star observations becoming competitive with experiments.
- Properties of confined neutrons useful to test iso-vector terms of nuclear energy density functionals

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Thank you!