Symmetry Energy: Mixed Messages from Nuclear Fragmentation

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### **Basic Questions:**

- Nuclear Equation of State:
   <ε> of a nucleon in n/p symmetric and asymmetric nuclear matter in equilibrium, at different densities (profiles) & temperatures.
- > Symmetry energy  $< \varepsilon_{sym}$  ( $\rho$ , T, I)>

### Method:

→ For finite nuclei, study limits of nuclear (meta-) stability and disintegration modes, specifically "chemical" modes → isotopic regularities in fragmentation

cluster emission

**Challenges:** Limited scopes of experiments and models produce ambiguities.

**Discussion of examples:** ( <sup>40,48</sup>Ca+<sup>112,124</sup>Sn@35,45 A MeV) **Progress in phenomenological understanding:** expansion dynamics & surface properties of excited nuclei.

### Deducing a Nuclear Equation of State



### **Regularities in Statistical Cluster Emission**



Same cluster (N,Z) emitted from either  $CN_1$  or  $CN_2$ , same  $E^*_{tot}$ 

Emission probability 
$$\infty$$
 formation  
 $\Gamma \propto e^{\Delta S} \simeq e^{Q/T}$  (if  $T \approx const$ )  
 $\Delta S = S_{saddle} - S_{eq} \approx \Delta E^{*th} / T := Q/T$   
 $E_{saddle}^{*th} \approx E_{tot}^{*} - E_{B}(1 - \frac{\rho_{eq}}{\rho_{o}})^{2} - V_{saddle}$   
 $E_{B} = binding$ ,  $V_{saddle} = coll$ , config

$$\frac{\Gamma_{CN_2}(N,Z)}{\Gamma_{CN_1}(N,Z)} \propto \exp\left[\left(\Delta E_B(1-\frac{\rho_{eq}}{\rho_o})^2 + \Delta V_{saddle}\right) / T\right] = \exp\left[\frac{\Delta Q}{T}\right] \qquad \begin{array}{l} \text{Decay widths } \leftrightarrow \Sigma \\ \text{interaction energies} \\ + \text{ EOS (isoEOS)} \end{array}\right]$$

First – order expansion of 
$$E_B$$
 and  $V_{saddle}$ : reference cluster  $(N_0, Z_0)$   
 $V_{saddle}(CN, N, Z) \approx V_{CN0} + \left(\frac{\partial V_{saddle}}{\partial N}\right)_Z \left[N - N_0\right] + \left(\frac{\partial V_{saddle}}{\partial Z}\right)_N \left[Z - Z_0\right]$  and  $E_B(CN, N, Z) = \dots$   
"Isoscaling"

$$\left[ R_{12} \coloneqq \frac{\Gamma_{CN_2}(N,Z)}{\Gamma_{CN_1}(N,Z)} \propto \exp\left[ \left( \frac{\Delta a_{CN}}{T} \right) \cdot N + \left( \frac{\Delta b_{CN}}{T} \right) \cdot Z \right] = e^{\alpha \cdot N + \beta \cdot Z} \right] \xrightarrow{\text{with cluster}}_{\substack{N,Z \\ \text{Parameters}}}$$

### **Complications 1,2**

N,Z

α, β

 $\sim$ 

### **1.** Isospin Dependent Preeq. Dynamics <sup>40,48</sup>Ca+<sup>112</sup>Sn@35A MeV

### Specific experimental setup to measure exclusive n & p with PLFs.



<sup>(</sup>Agnihotri et al., 1998; Schröder et al., 2001)

Fast and asymmetric depletion of system by neutrons compared to protons. Global iso-equilibrium for remnant system approached but not reached. PLFs remember initial A/Z.

Significant rates of fast, nonequilibrium emission of neutrons and protons.

Obvious at sideways angles.



### 2. Dynamic Splitting of PLF\* after Dissipative Rxns

### <sup>48</sup>Ca+<sup>112,124</sup>Sn, E/A = 45 MeV, CECIL Expt. @ LNS Catania



Galilei invariant cross sectionsa) for heavier PLF remnantsb) for lighter remnants (IMFs).

Experimental Wilczyński contour diagrams for <sup>48</sup>Ca+<sup>112</sup>Sn @E/A=45 MeV. Top: PLF energy vs. angle, Bottom: PLF velocity vs. angle.

Nucleon exchange model (CLAT). Sequential evaporation: GEMINI.



## 2. Dynamic Splitting of PLF\* after Dissipative Rxns

Prompt projectile splitting in proximity (under the influence) of target. Nuclear surface interactions  $\rightarrow$  **aligned asymmetric breakup** 



Evidence for dynamics: 48Ca + 112,124Sn

- 1. Alignment of breakup axis in plane, in direction of flight
- 2. F/B of heavy/light.
- 3. Relative velocity =2x equil. systematics
- 4. Symmetric component (not fission)



### Isoscaling in Dynamic PLF\* Splitting (<sup>48</sup>Ca+<sup>112,124</sup>Sn)

PLFs from 2 dissipative reactions split dynamically. Compare cluster yields  $\rightarrow$  ratios.



Ambiguity due to uncertain reconstruction  $\rightarrow$  Identify cluster origin/mechanism.

## Ambiguity Range for Model Parameter $\rm C_{\rm sym}$

Z <sub>1</sub>	A <sub>1</sub>	Z <sub>2</sub>	A <sub>2</sub>	T(MeV) (±)	C <sub>app</sub>	PLF*s {[ $Z_1$ , $A_1$ ], [ $Z_2$ , $A_2$ ]} split, effective temperature $T$ apparent symmetry energy coefficients $C_{200}$ .		
23	54	22	51	1.7 (0.1)	22.2			
24	55	23	52	2.1 (0.1)	23.8			
23	52	22	49	2.4 (0.1)	24.8	(Fit range 3 isotopes per Z)		
20	49	18	43	2.8 (0.1)	19.1			
21	51	19	45	2.9 (0.1)	20.0	PLF* s {[ $Z_1$ , $A_1$ ], [ $Z_2$ , $A_2$ ]} split, effective temperature $T$ apparent symmetry energy coefficients $C_{app}$ . (Fit range 4 isotopes per Z)		
24	56	23	51	4.5 (0.1)	24.5			
28	46	27	45	5.6 (0.1)	31.7			ergy
Z1	A1	Z2	A2	T(MeV) (±)	C <sub>app</sub> (MeV)			
23	55	22	52	1.47 (0.01)	21.5	α(±)	β(±)	Fit Range
24	56	23	53	1.81 (0.02)	23.3			Isotopes
24	55	23	52	2.14 (0.02)	24.6	0.230 (0.002)	0.122 (0.008)	3
24	54	23	51	2 52 (0 03)	25.8	0.222 (0.002)	0.113 (0.008)	4
24	52	23	49	3.41 (0.04)	27.9	Parameters factors 2-3 smaller		
24	56	22	50	3.95 (0.05)	23.9	than in other reactions.		
23	49	22	46	4.09 (0.05)	29.2	Meaning of apparent "C <sub>sym</sub> "?		
24	55	22	49	4.69 (0.05)	25.2			
24	57	21	48	5.44 (0.06)	23.1	$17-19 \text{ MeV} \leq C_{\text{sym}} \leq 32 \text{ MeV}$		

## Scaling With Ground State Energies (Q<sub>aa</sub>)



### Expansion in Interacting-Fermi Gas Model for CN



Simplest version: harmonic approximation of  $\varepsilon(\rho) \rightarrow$  analytical formulation

$$\varepsilon(\rho) = -\varepsilon_0 + (K/18) \left[ 1 - \left(\frac{\rho}{\rho_0}\right)^2 \right] + \dots$$
 EOS

$$S = 2\sqrt{a \cdot E_{th}^*} \rightarrow E_{th}^* = E_{tot}^* - E_{conf}^*$$

"little-a" = level density parameter  

$$a = a_{Volume} + a_{Surface} = (A\alpha_V + A^{\frac{2}{3}}\alpha_S) \left(\frac{\rho}{\rho_0}\right)^{-\frac{2}{3}}$$

$$\left(\frac{\partial S}{\partial \rho}\right)_{E_{tot}^*} = 0 \quad \rightarrow \quad \begin{cases} \rho_{eq} \\ T \end{cases} \quad \begin{pmatrix} microcan. \\ model \end{pmatrix}$$

Experimental **a** systematics  $\rightarrow$  model relates  $E^* \leftrightarrow Density \leftrightarrow T$ 

More sophisticated CN model

## Isoscaling with an Interacting Fermi Gas Model

Studied schematic model isoscaling dependence on T for  $\rho = \rho_0$ . and for g.s. I dependence. Ad hoc ansatz for surface symmetry energy  $(\rho,T) \rightarrow$  Essentially similar isoscaling plots.

$$\begin{bmatrix} E_v(\rho) = \left[ B_0 + \frac{K}{18} \left( 1 - \rho/\rho_0 \right)^2 \right] \cdot \left( 1 - \kappa_v I^2 \right) A \\ E_s(T) = \left[ \alpha_s(T) - T \frac{d\alpha_s(T)}{dT} \right] \cdot \left( 1 - \kappa_s I^2 \right) A^{2/3} ??$$



40  $E_{sym}(\rho) = C_0 \cdot (\rho/\rho_0)$ Symmetry Energy (MeV) 30 20 10 A =185 0.0 0.5 1.0 1.5

Need better model surface

### Surface Vaporization



$$\begin{split} & \text{Tõke & Schröder, PRC subm. (2011), PLB subm. (2013)} \\ & S_{conf}\left(E_{conf},I\right) \coloneqq 2 \cdot \sqrt{a_{conf}} \cdot \left(E_{tot} - E_{conf}\right) & \begin{array}{c} Configuration \\ entropy \\ a_{conf} \coloneqq \alpha_0 \cdot \rho_0^{2/3} \cdot R(I) \int \rho^{1/3}(\vec{r}) d^3r & \begin{array}{c} Level \ density \\ parameter \\ \rho(r) \colon Matter \ density, \ R(I) \colon iso - asymmetry \ factor \\ & E_{conf}\left(\rho,I\right) \coloneqq c_V \cdot \left(1 - \frac{\rho}{\rho_0}\right)^2 + c_I \cdot \left(\frac{\rho}{\rho_0}\right) \cdot I^2 & \begin{array}{c} Harmonic \\ approximation \\ \end{array} \\ & \overline{\delta^2 S \ge 0 \rightarrow \ instability \rightarrow "sudden" \ decay} \end{split}$$

#### \* New nuclear modes \*

- Up to  $E^*/A \approx 4.5$  MeV expansion, surface diff
- T levels off  $T \approx 5.6 \text{ MeV}$  (like experiment),
- C negative → instability
- n-rich surface parts vaporize first, ejected.
- Residues n-poorer but hotter →
- With I<sup>2</sup> "Distillative Boiling"

## Summary & Conclusions

- Generic character of isotopic regularities (isoscaling), sensitivity to sum of EOS and interaction terms. Ratios obscure some physics.
- Significant ambiguity due to poor identification of parent isotopes ("resolution effects").
- Isoscaling observed also for competing mechanisms (dynamic splitting).
- Ground state masses explain (several) isoscaling phenomena.
- Progress in thermodynamics of finite nuclei (expansion, surface, caloric).
- Development of phenomenological method to interpret nuclear decay modes (boiling, distillative vaporization, cluster emission barriers).

Experimental/theoretical challenges:

- Characterize reaction mechanism.
- Determine pre-equilibrium modes (mean field vs. scattering).
- Determine equilibrium conditions of CN emitting clusters.
- > Need more specific (exclusive) experiments with high statistics.
- > More direct measurements of isospin sensitive observables (incl. n's).
- Realistic modeling of many primary reaction and secondary decay features simultaneously.

# THANK YOU !

### Instability Modes of Finite Nuclei

A=100, A1=A2=50 K=220 MeV 1.5 0.5



Entropy S of a two-phase system (1,2) vs. total energy,  $E_{tot}/A$ , and energy asymmetry,  $(E_1-E_2)/E_{tot.}$ .

S is given relative to entropy,  $S_{uniform}$ , of the uniform system (1+2).

Contours of the equilibrium matter density vs. isoasymmetry, (N - Z)/A and  $E_{tot}/A$ . Slid lines: 1) ground-state energy

- 2) boundary of meta-stability domain
- 3) boundary of domain of positive heat capacity

4) boundary of domain stable against uniform expansion.

Further theoretical model extension: Saddle and barrier energies  $\rightarrow$  J.Tõke



### Isoscaling in Dynamic PLF\* Splitting (<sup>48</sup>Ca+<sup>112,124</sup>Sn)



Ambiguity due to uncertain reconstruction  $\rightarrow$  Identify cluster origin/mechanism.

- Need reaction model to simulate simultaneous observables.
- □ Need realistic model to relate  $\{\alpha, \beta\} \leftarrow \rightarrow C_{sym}(\rho)$

Extend statistical CN model

### Hurdles for Studies of the Symmetry Energy

- Competing reaction mechanisms produce similar isotopic phenomena
- Lack of equilibration in most reaction systems
- Non-uniform symmetry energy (bulk vs. surface)
- Poor definition/reconstruction of equilibrated system
- Indirect determination of isospin dependencies (ratios)
- Unsystematic "spot" studies, variation of reaction parameters ( $E_{cm}$ , L,..)
- Secondary evaporation effects/"side feeding"



Related observations v.Harrach/Specht, Fuchs et al., Wilczyński et al., de Filippo et al.

## Early Studies



summary of early Russian work



## Nuclear Stability Criteria (Gibbs)

### Meta-Stability:

 $(\mathbf{S} \rightarrow \mathbf{S}_{\max}) \rightarrow (\mathbf{S} - \mathbf{S}_{\max}) \approx \delta^2 \mathbf{S} < \mathbf{0} \rightarrow \text{driving force}$ 

### Normal Modes



2 δU 1

Thermal Stability Isochoric heat capacity Energy fluctuations

Isothermal compressibility

Mechanical Stability

Density fluctuations

 $\delta^2 S = -C_V \left(\frac{\delta T}{T}\right)^2 < 0$  $C_V = (\partial U / \partial T)_V > 0$  $\sigma_F^2 \approx T^2 C_V$ 

 $\delta^2 S = -\frac{1}{\kappa_{\tau}} \frac{\left(\delta T\right)^2}{T \cdot V} < 0$ 

 $\sigma_{\rho}^2 \approx \bar{\rho}^2 (T/V) \kappa_T$ 

 $\kappa_{T} = -(1/V)(\partial V/\partial p) > 0$ 



Chemical Stability Chemical potential

Chemical fluctuatio

$$\delta^{2}S = -\sum_{i,j} \left( \frac{\partial}{\partial N_{j}} \frac{\mu_{i}}{T} \right) \delta N_{i} \delta N_{j} < 0$$
  
$$\mu_{i} = \left( \frac{\partial U}{\partial N_{i}} \right)_{S,V,N_{j}} > 0$$
  
ons  
$$\sigma_{N}^{2} \approx T \left( \frac{\partial \mu}{\partial N} \right)_{V,T}^{-1}$$



### N & Z Dependence of Statistical Cluster Emission

$$\begin{array}{c}
\rho_{o} & \longrightarrow & \rho_{eq} \\
S = S_{eq}(CN) & S_{saddle}(Res + Frag)
\end{array}$$

Fixed cluster (N,Z) emitted from either  $CN_1(I_1)$  or  $CN_2(I_2)$ , same  $E^*$ 

Emission probability 
$$\infty$$
 formation  
 $\Gamma \propto e^{\Delta S} \simeq e^{Q/T}$  (if  $T \approx const$ )  
 $\Delta S = S_{saddle} - S_{eq} \approx \Delta E^{*th} / T := Q/T$   
 $E_{saddle}^{*th} = E_{tot}^{*} - E_{Bind} (1 - \frac{\rho_{eq}}{\rho_{o}})^{2} - V_{saddle}$ 

$$\frac{\Gamma_{CN_{2}}(N,Z)}{\Gamma_{CN_{1}}(N,Z)} \propto \exp\left[\left(V_{saddle_{1}} - V_{saddle_{2}}\right)/T\right] = \exp\left[\Delta Q/T\right] \quad Decay \ Widths$$

$$\Delta Q = \Delta V_{saddle}\left[\rho, E_{tot}^{*}, I\right] \quad same \ cluster, \ different \ CN_{1}, CN_{2}$$

$$V_{saddle}(CN, N, Z) \approx V_{CN0} + \left(\frac{\partial V_{saddle}}{\partial N}\right)_{Z}\left[N - N_{0}\right] + \left(\frac{\partial V_{saddle}}{\partial Z}\right)_{N}\left[Z - Z_{0}\right] \quad \begin{array}{c} \text{Reference} \\ \text{cluster} (N_{0}, Z_{0}) \end{array}$$

$$V_{saddle}(CN, N, Z) \approx Q_{CN} + a_{CN}N + b_{CN}Z, \ factors \ depend \ on \ \rho$$

$$R_{12} \coloneqq \frac{\Gamma_{CN_2}(N,Z)}{\Gamma_{CN_1}(N,Z)} \propto \exp\left[\left(\frac{\Delta a_{CN}}{T}\right) \cdot N + \left(\frac{\Delta b_{CN}}{T}\right) \cdot Z\right] = e^{\alpha \cdot N + \beta \cdot Z}$$

"Isoscaling" with cluster N,Z

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### Interacting Fermi Gas Model for CN



Simplest version: harmonic approximation of  $\varepsilon(\rho) \rightarrow$  analytical formulation

$$\varepsilon(\rho) = -\varepsilon_0 + (K/18) \left[ 1 - (\rho/\rho_0)^2 \right]$$
 EOS

$$S = 2\sqrt{a \cdot E_{th}^*} \to E_{th}^* = E_{tot}^* - E_{conf}^* \quad \text{Entropy}$$
$$\left(\frac{\partial S}{\partial \rho}\right)_{E_{tot}^*} = 0 \quad \to \begin{cases} \rho_{eq} \\ T \end{cases}$$

Experimental **a** systematics  $\rightarrow$  model relates E\*  $\leftarrow \rightarrow$  Density  $\leftarrow \rightarrow$  T

Coulomb Radius  $r_c = 1.16 \cdot (\rho/\rho_o)^{-1/3} fm$ Level Density Parameter  $a = a_{Volume} + a_{Surface} = (A\alpha_V + A^{\frac{2}{3}}\alpha_S)(\rho/\rho_o)^{-\frac{2}{3}}$ 

### **Discovery: Spinodal Surface Vaporization**



\* New nuclear modes \*

n-rich surface parts vaporize first, Residues n-poorer but hotter. With *I*<sup>2</sup> "Distillative Boiling"

Tõke & Schröder, PRC subm. (2011), PLB subm. (2013) **Configuration**  $S_{conf}(E_{conf},I) \coloneqq 2 \cdot \sqrt{a_{conf} \cdot (E_{tot} - E_{conf})}$ entropy Level density  $a_{conf} \coloneqq \alpha_0 \cdot \rho_0^{2/3} \cdot R(I) \int \rho^{1/3}(\vec{r}) d^3r$ parameter  $\rho(r) \coloneqq C(R_{1/2}, d) \cdot \rho_0 \cdot \left| 1 - erf\left(\frac{r - R_{1/2}}{\sqrt{2}d}\right) \right| \text{ Matter density}$  $R(I) \approx \left\lceil 1 - \frac{1}{9}I^2 \right\rceil = \left\lceil 1 - \frac{1}{9}\left(\frac{N-Z}{A}\right)^2 \right\rceil$ Asymmetry factor  $E_{conf}(\rho, I) := c_V \cdot \left(1 - \frac{\rho}{\rho_0}\right)^2 + c_I \cdot \left(\frac{\rho}{\rho_0}\right) \cdot I^2 \quad Harmonic \\ approximation$  $\delta^2 S \ge 0 \rightarrow$  instability  $\rightarrow$  "sudden" decay *Finite – range calculations*:

More general Skyrme interactions (K = 220 MeV)  $\varepsilon^{EOS}(\rho) \coloneqq \rho \cdot \left[ a \cdot \left( \frac{\rho}{\rho} \right) + \frac{b}{c} \cdot \left( \frac{\rho}{\rho} \right)^{\sigma} \right] = a = -62.43 MeV$ b = 70.75 MeV

$$\mathcal{E}_{\text{int}}^{\text{Los}}(\rho) \coloneqq \rho \cdot \left[ a \cdot \left[ \frac{\gamma}{\rho_0} \right]^+ \frac{1}{\sigma + 1} \cdot \left[ \frac{\gamma}{\rho_0} \right] \right] \qquad b = 70.75 \text{Me}$$
$$\sigma = 2.0$$

 $E_{conf}^{EOS} = R_{norm} \int \varepsilon_{int}^{EOS} \left( \rho \left( \vec{r} - \vec{r}' \right) \right) \cdot \exp\{-\frac{\left( \vec{r} - \vec{r}' \right)^2}{2\lambda^2} \right\} d^3r d^3r' \quad Folding$ 



Surface entropy increases  $\alpha$  and  $|\beta|$  (More matter in surface) But decreases T  $\rightarrow$  cancellation of effects.

EOS of asymmetric nuclear matter

E(
$$\rho, \delta$$
)  $\approx$  E( $\rho, \delta = 0$ ) + E<sub>sym</sub>( $\rho$ ) $\delta^2$ ,  $\delta = \frac{\rho_n - \rho_p}{\rho_n + \rho_p}$   
etry energy E<sub>sym</sub>( $\rho$ ) = E<sub>sym</sub>( $\rho_0$ ) +  $\frac{L}{3} \left( \frac{\rho - \rho_0}{\rho_0} \right) + \frac{K_{sym}}{18} \left( \frac{\rho - \rho_0}{\rho_0} \right)^2$ 

Symme

Slope

 $L = 3\rho_0 \frac{\partial E_{sym}(\rho)}{\partial \rho}$ theoretical values -50 to 200 MeV  $K_{sym} = 9\rho_0^2 \frac{\partial^2 E_{sym}(\rho)}{r^2}$ theoretical values -700 to 466 MeV Curvature

 $E_{sym}(\rho_0) \approx 30 \text{ MeV}$ 

Symmetry energy coefficient

Nuclear matter Incompressibility  $K(\delta) = K_0 + K_{asy} \delta^2$ ,  $K_{asy} \approx K_{sym} - 6L$ experimental values  $K_0 \sim 230-240$  MeV,  $K_{sym} \sim 566 \pm 1350-139 \pm 1617$  MeV

EOS: Total energy of a nucleon in nuclear matter is density dependent  $(\rho_0 = 0.17 \text{ n/fm}^3)$ 

n-p asymmetry energy is uncertain







### THE CHIMERA DETECTOR

### Laboratori del Sud, Catania/Italy

### **CHIMERA** characteristic features

Experimental Method	$\Delta E-E$ → Charge $\Delta E-E$ E-TOF → Velocity, Mass Pulse shape Method → LCP				
Basic element	Si (300µm) + CsI(Tl) telescope				
Primary experimental observables	TOF $\delta t \leq 1 \text{ ns}$ Kinetic energy, velocity $\delta E/E$ Light charged particles $\approx 2\%$ Heavy ions $\leq 1\%$				
Total solid angle $\Delta\Omega/4\pi$	94%				
Granularity	1192 modules				
Angular range	<b>1°&lt; θ &lt; 176°</b>				
Detection threshold	<0.5 MeV/A for H.I. ≈ 1 MeV/A for LCP				





JuSym 13

## Angular Alignment and Coplanarity



Preferred orientation of deformed prescission PLF: lighter IMF backwards (towards TLF) → Minimizing energy

Orientation of the PLF scission axis  $\Theta_{\text{Tilt}} \approx$  $90^{\circ}\pm 25^{\circ}$ . → Coplanarity

30

**Distribution of Tilt** 

90

120

Tilting Angle  $\Theta_{tilt}$  (deg.)

150

180