Nuclear symmetry energy and Neutron star cooling

Yeunhwan Lim[1]

[1]Daegu University.

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In Collaboration with J.M. Lattimer (SBU),

C.H. Hyun (Daegu), C-H Lee (PNU), and T-S Park (SKKU)

NuSYM13

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Nuclear Symmetry energy

- Large uncertainties in nuclear physics at high density ($\rho > \rho_0$)
- Energy per baryon

$$e(\rho, x) = e(\rho, 1/2) + S_2(\rho)(1-2x)^2 + \cdots$$

- The symmetry energy parameter

$$S_v = S_2(\rho_0), \quad L = 3\rho_0(dS_2/d\rho)_{\rho_0},$$
$$K_{sym} = 9\rho_0^2(d^2S_2/d\rho^2)_{\rho_0}, \quad Q_{sym} = 27\rho_0^3(d^3S_2/d\rho^3)_{\rho_0}$$

- The density dependence of the symmetry energy in nuclear astrophysics

  - The neutronization of matter in core-collapse supernovae

  - The radii and crust thinckness of neutron stars

  - The cooling rate of neutron stars

  - The r-process nucleosynthesis

We can estimate the range of symmetry energy from experiments and theories.

- Nuclear Mass fitting
  Liquid droplet model, Microscopic nuclear force model (Skyrme force model)
- Neutron Skin Thickness
  ^{208}Pb, Sn with RMF and Skyrme
- Dipole Polarizabilities
- Heavy Ion Collisions
- Neutron Matter Theory
  Quantum Monte-Calro, Chiral Lagrangian
- Astrophysical phenomenon
  Neutron stars mass and radius

The optimal points for S_v and L

- S_v and L from nuclear mass constraints

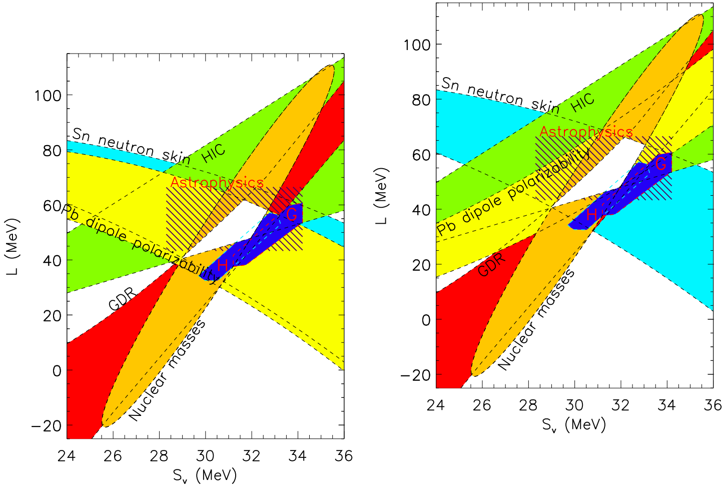


Figure: The allowed region of Sv , L from experiment and observation from J.M.Lattimer and Y. Lim, ApJ. 771, 51 (2013)

Nuclear Force Model - Potential Model

- Skyrme Force Model (EDF)

  - Non relativistic potential model

  - Two body and many body effects (contact force)

$$
\begin{aligned}
v_{ij} =& t_0(1+x_0P_\sigma)\delta(\mathbf{r}_i-\mathbf{r}_j)+\frac{1}{2}t_1(1+x_1P_\sigma)\frac{1}{\hbar^2}\left[\mathbf{p}_{ij}^2\delta(\mathbf{r}_i-\mathbf{r}_j)+\delta(\mathbf{r}_i-\mathbf{r}_j)\mathbf{p}_{ij}^2\right] \\
&+t_2(1+x_2P_\sigma)\frac{1}{\hbar^2}\mathbf{p}_{ij}\cdot\delta(\mathbf{r}_i-\mathbf{r}_j)\mathbf{p}_{ij}+\frac{1}{6}t_3(1+x_3P_\sigma)\rho(\mathbf{r})^\epsilon\delta(\mathbf{r}_i-\mathbf{r}_j) \\
&+\frac{i}{\hbar^2}W_0\mathbf{p}_{ij}\cdot\delta(\mathbf{r}_i-\mathbf{r}_j)(\boldsymbol{\sigma}_i+\boldsymbol{\sigma}_j)\times\mathbf{p}_{ij}\,,
\end{aligned}
\tag{1}
$$

  - 10 parameters can be fitted with nuclear mass data

- Uniform nuclear matter

$$(x_0, x_1, x_2 = -1, x_3, t_0, t_1, t_2, t_3, \epsilon) \quad \longleftrightarrow \quad (m_s^*, m_n^*, B, \rho_0, K, S_v, L, \omega_0)$$

Binding energy : $B = -16\,\mathrm{MeV}$

Saturation density : $n_0 = 0.16\,\mathrm{fm}^{-3}$

Incompressibility : $K = 230 - 260\,\mathrm{MeV}$

Effective mass of nucleons in symmetric nuclear matter : $m_s^* = 0.7M$

Effective mass of nucleons in pure nuclear matter : $m_n^* = 0.6M$

Symmetry energy : $S_v = 30 - 34\,\mathrm{MeV}$

Density derivative of symmetry energy : $L = 30 - 80\,\mathrm{MeV}$

Nuclear surface tension or t_{90-10} : $\omega_0 = 1.15\mathrm{MeV}/\mathrm{fm}^2, \quad t_{90-10} = 2.5 - 2.8\,\mathrm{fm}$.

$$\frac{E}{A} = Cn_0^{2/3}(1+\beta n_0) + \frac{3t_0}{8}n_0 + \frac{t_3}{16}n_0^{\epsilon+1}, \quad C = \frac{3\hbar^2}{10M}\left(\frac{3\pi^2}{2}\right)^{2/3} \tag{2}$$

$$\frac{M_s^*}{M} = \frac{1}{1+\beta n_0}, \quad \beta = \frac{M}{8\hbar^2}(3t_1+t_2) \quad \frac{M_n^*}{M} = \frac{1}{1+\theta n_0}, \quad \theta = \frac{M}{4\hbar^2}(t_1 - t_1 x_1) \tag{3}$$

$$\frac{P(n_0)}{n_0} = 0 = \frac{2}{3}Cn_0^2\left(1+\frac{5}{2}\beta n_0\right) + \frac{3}{8}t_0 n_0 + \frac{t_3}{16}(\epsilon+1)n_0^{\epsilon+1} \tag{4}$$

$$K = 9n^2 \frac{\partial^2 E/A}{\partial n^2}\bigg|_{n=n_0} = -2Cn_0^{2/3} + 10C\beta n_0^{5/3} + \frac{9t_3}{16}\epsilon(\epsilon+1)n_0^{\epsilon+1} \tag{5}$$

$$\begin{aligned} S_v = \frac{n^2}{2}\frac{\partial^2 E/A}{\partial \alpha^2}\bigg|_{\alpha=0} = \frac{5}{9}Cn_0^{2/3} - \frac{5CM}{36\hbar^2}(t_2+3t_1x_1)n_0^{5/3} - \frac{t_3}{24}\left(\frac{1}{2}+x_3\right)n_0^{\epsilon+1} \\ -\frac{t_0}{4}\left(\frac{1}{2}+x_0\right)n_0 \end{aligned} \tag{6}$$

$$\begin{aligned} L_v = 3n_0\frac{\partial S_v}{\partial n}\bigg|_{n=n_0} = \frac{10}{9}Cn_0^{2/3} - \frac{25CM}{36\hbar^2}(t_2+3t_1x_1)n_0^{5/3} \\ -\frac{t_3}{8}(1+\epsilon)\left(\frac{1}{2}+x_3\right)n_0^{\epsilon+1} - \frac{3t_0}{4}\left(\frac{1}{2}+x_0\right)n_0 \end{aligned} \tag{7}$$

$$\omega_0 = n_0 T_0 t_{90-10} I_\omega / I_t, \quad t_{90-10} = \sqrt{\frac{(Q_{nn}+Q_{np})n_0}{2T_0}}\, I_t\,. \tag{8}$$

Nuclei in Neutron Star outer crust

- LDM, TF, and HF for NS outer crust
  - Easy to deal with because of no neutrons outside nuclei
- Electron contribution and Lattice structure (BCC)
- Up to $\rho \sim 2.0 - 3.0 \times 10^{-4}/\text{fm}^3$ ($\mu_n < 0$)
- Beta equilibrium state for LDM and TF, semi β for HF
- Atomic and mass number are different
- Pressure and energy density are almost same

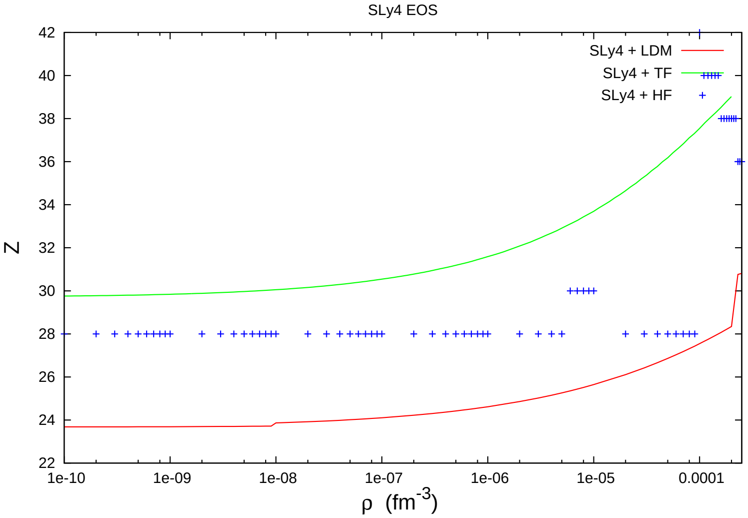


Figure: Atomic number of nuclei in the outer crust of neutron star from SLy4 + LDM, TF, and HF

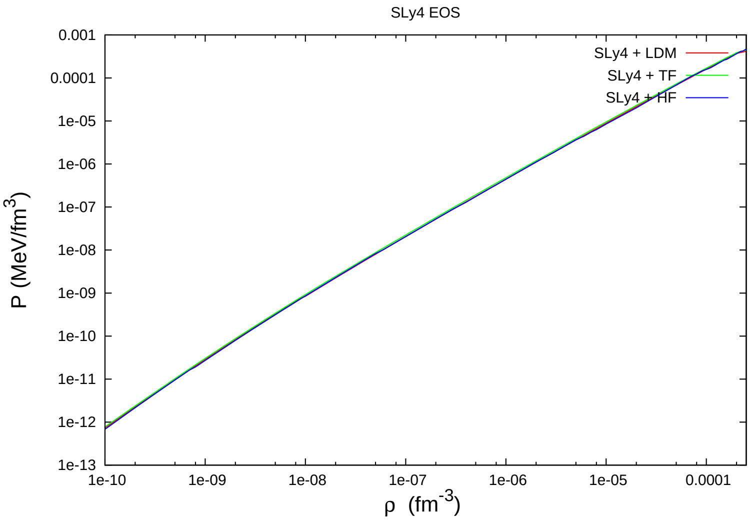


Figure: Pressure in the outer crust of neutron star from SLy4 + LDM, , LDM, and HF

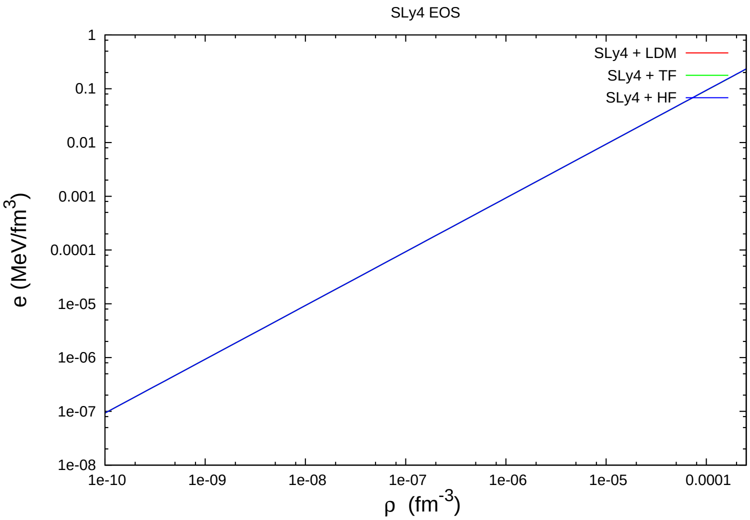


Figure: Energy density in the outer crust of neutron star from SLy4 + LDM, , LDM, and HF

Nuclei in Neutron Star inner crust

- LDM, TF, and HF for NS inner crust
- Electron and neutron contribution and Lattice structure (BCC)
- Up to $\rho \leq 0.5\rho_0$ ($\mu_n > 0$)
- Beta equilibrium state for LDM and TF, semi β for HF
- Nuclear pasta phase might exists
  - Coulomb energy favors geometry
  - easy to deal with in LDM, difficult in TF and HF

$$f_{\rm surf} = \frac{ud\sigma}{r_N}, \qquad f_{\rm Coul} = \frac{4\pi}{5}(\rho_i x_o r_N e)^2 D_d(u) \tag{9}$$

where

$$D_d(u) = \frac{5}{d^2-4}\left[1 - \frac{d}{2}u^{1-2/d} + \frac{d-2}{2}u\right]. \tag{10}$$

$$f_{\rm surf} + f_{\rm Coul} = \beta\mathcal{D}, \quad \mathcal{D} = u\left(\frac{d^2}{D_d(u)}9\right)^{1/3} \tag{11}$$

Nuclear density profile at the inner crust

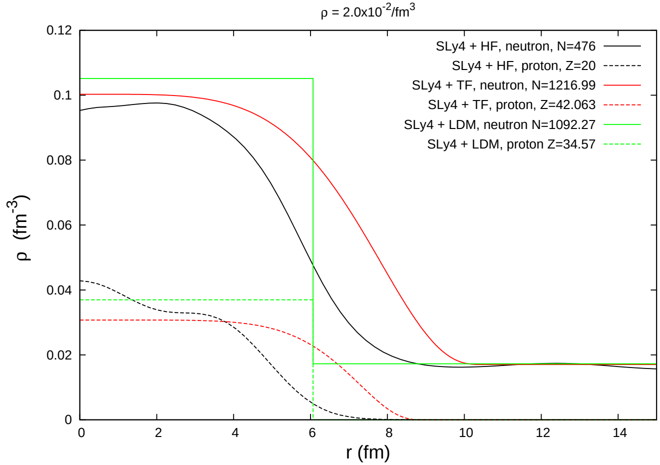


Figure: Nuclear density profile in the inner crust of neutron star at $\rho = 2.0 \times 10^{-2}$/fm^3.

Crust thickness

- Transition density from energy difference

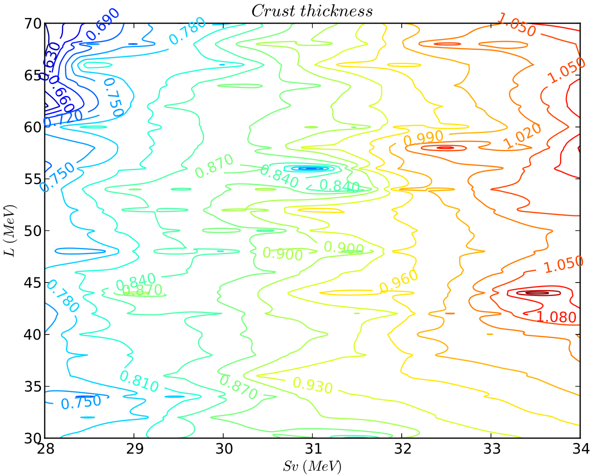


Figure: Contour plot of crust thickness of $1.4M_{\odot}$ for S_v and L

Crust thickness

- Transition density from thermodynamic instability

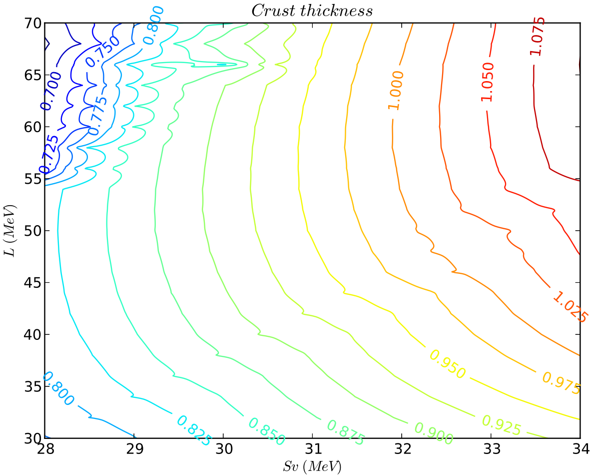


Figure: Contour plot of crust thickness of $1.4M_{\odot}$ for S_v and L

The relativistic equations of thermal evolution

- Diffusion equation

$$\frac{1}{4\pi r^2 e^{2\Phi}}\sqrt{1-\frac{2Gm}{c^2 r}}\frac{\partial}{\partial r}(e^{2\Phi}L_r) = -Q_\nu - \frac{C_v}{e^\Phi}\frac{\partial T}{\partial t} \quad (12a)$$

$$\frac{L_r}{4\pi r^2} = -\kappa\sqrt{1-\frac{2Gm}{c^2 r}}e^{-\Phi}\frac{\partial}{\partial r}(Te^\Phi) \quad (12b)$$

- Q_ν : Neutrino emission rate : $Q_\nu = Q_\nu(T, \rho_n, \rho_p)$
- C_v : Heat capacity (specific heat) : $C_v = C_v(T, \rho_n, \rho_p)$
- κ : Thermal conductivity : $\kappa = \kappa(T, \rho_n, \rho_p)$
- e^Φ : General relativistic metric function : $e^\Phi = \sqrt{1-\frac{2GM}{rc^2}}$
- L (T) is defined on even (odd) grid : L_{2i}, T_{2i+1}
- Two boundary conditions : $L_0 = 0$, $T_s = T_s(T_b)$
- Henyey method is used to find new temperature

$$T_i^{n+1} = T_i^n + \Delta t\frac{dT_i^n}{dt} \quad \rightarrow \quad T_i^{n+1} = T_i^n + \Delta t\frac{dT_i^{n+1}}{dt}$$

Equation of state

- Lots of nuclear force models are available
- EOS based on RMF : TM1, FSU Gold, NL3, . . .
- EOS based on variational principle : APR
- EOS based on EDF : SLy4 (0, $\cdots$, 10)
- Phenomenological model : PAL
- EOS should explain the maximum mass of neutron stars greater than $2.02M_\odot$

	ρ_0 (fm^{-3})	B (MeV)	K (MeV)	S_v (MeV)	L (MeV)
SLy4	0.159	15.97	229.91	32.0	45.94
APR	0.16	16.0	266	32.6	60
TMA	0.147	16.0	318	30.68	90
TM1	0.145	16.3	281	36.8	111
NL3	0.148	16.2	271	37.3	118
EDF0	0.156	16.11	229.7	32.4	42.33
EDF1	0.156	16.11	229.7	29.0	40.5
EDF2	0.156	16.11	229.7	28.0	30.0
EDF3	0.156	16.11	229.7	32.7	61.9
EDF4	0.156	16.11	229.7	29.0	61.9

Neutron stars' mass and radii

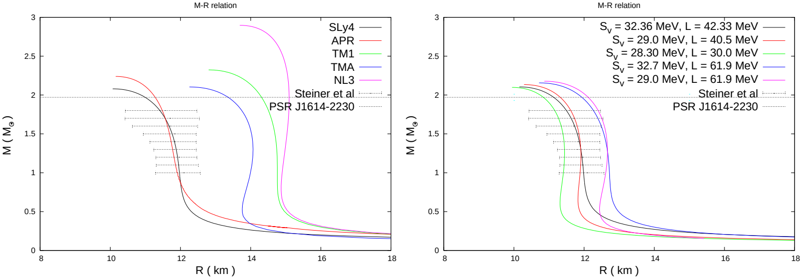


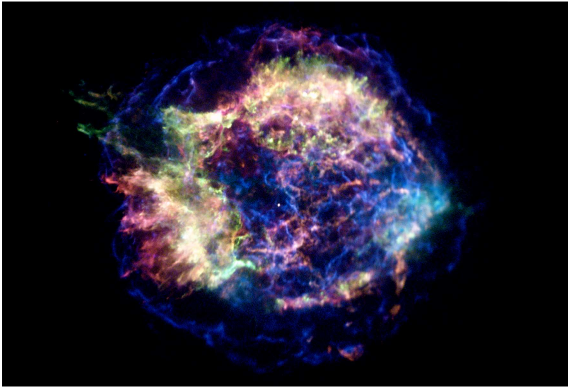
Figure: Mass and radius relation for neutron stars

Physics from EOS

- Nuclear matter in the core of neutron stars
  Easy to calculate (Relativistic, Non-relativistic)
- Three approaches for crust : Liquid droplet, Thomas Fermi, Hartree-Fock
  Among the many EOSs, APR, SLy series are probably the best
- Composition of constituents : Protons, neutrons, electrons
  On and off direct URCA process (proton fraction)
- Boundaries of inner crust and out crust, (neutron drip)
  Boundaries of URCA process
- Atomic number in case of crust
  Need to calculate Q_ν, C_V, and κ
- Effective masses for proton and neutron
  Effective masses are involved in formulae for Q_ν, C_V, and κ
- Volume fraction of heavy nuclei

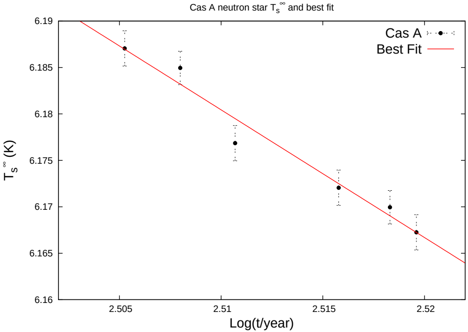
Cas A Neutron Star

- Casiopea A supernova remnant



- First light : Chandra observations (1999)
- Distance : $\sim$ 3.4 kpc
- Age : 330$\pm$ 10 years

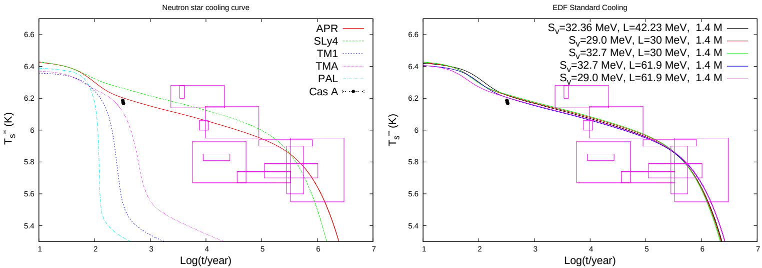
- Temperature profile of CAS A neutron star



$$\frac{d \ln T_s^\infty}{d \ln t} = -1.375 \quad \text{(best fit)} \quad \text{vs} \quad -0.07 \sim -0.15 \quad \text{(standard cooling)}$$

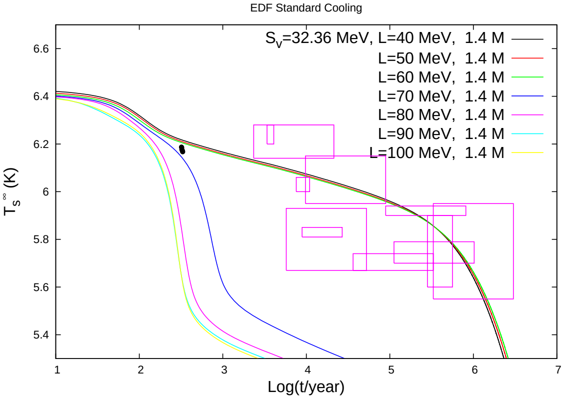
NS Cooling (Standard)

- Standard cooling : DU, MU, NB, No superfluidity, No bose condensation



- Direct URCA process turned on in case of TM1, TMA and PAL
- Standard cooling process cannot explain the fast cooling in Cas A
- S_v and L effect on thermal relaxation time

- Standard cooling with EDF



Nuclear Superfluidity

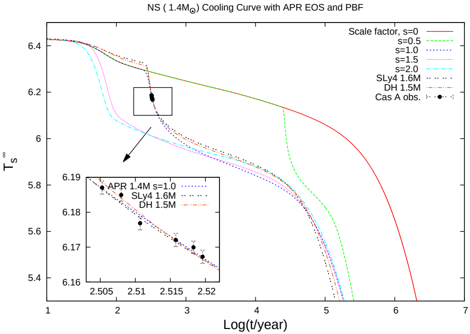
- Nuclear ground state can be achived with Cooper pairing
  ex) Even-Even nuclei is more stable than even-odd, odd-odd
- Nuclear superfluidity is uncertain at high densities ($\rho \sim \rho_0$).
  $^3P_2$ N, $^1S_0$ P for higher densities (Neutron star core)
  $^1S_0$ N for lower densities (Neutron star crust)
- Nuclear superfluidity gives reduction factor for neutrino emission, heat capacity, and thermal conductivity.
  $Q_{DU} \to Q_{DU}R_{DU}$, $Q_{Mod,U} \to Q_{DU}R_{Mod,U}$, $Q_B \to Q_B R_B$
  $C_{n,p} \to C_{n,p}R_{n,p}$, $C_n^{\rm crust} \to C_n^{\rm crust}R_n$, $\kappa_n^{\rm core} \to \kappa_n^{\rm core}R_{\kappa,n}$
- It opens neutrino emission process from Cooper Pair Breaking and Formation (PBF).

$$Q_{PBF} = 1.17 \times 10^{21} \frac{m_n^*}{m_n} \frac{p_F(n_n)}{m_n c} T_9^7 \mathcal{N}_\nu \mathcal{R}(T/T_{c,n})\,\mathrm{erg\,cm^{-3}\,s^{-1}}$$

$$(\text{cf.} \quad Q_{\rm mod,n} = 8.55 \times 10^{21} \left(\frac{m_n^*}{m_n}\right)^3 \left(\frac{m_p^*}{m_p}\right) \left(\frac{n_p}{n_0}\right)^{1/3} T_9^8 \alpha_n \beta_n \mathcal{R}_{\rm mod,n} \mathrm{erg\ cm^{-3} s^{-1}})$$

NS Cooling (Enhanced)

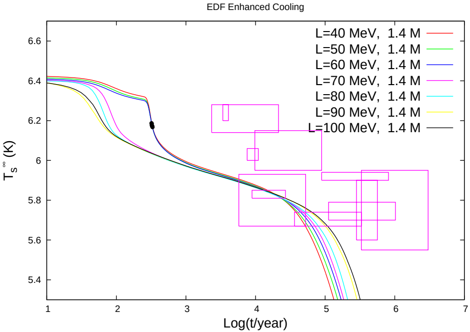
- Enhanced cooling : Pair Breaking and Formation (PBF)



- Observation of Cas A NS indicates the existence of nuclear superfluidity
- Different EOSs (APR, SLy4) may give same slope but different critical temperature for nuclear matter
- EOSs good for Cas A cooling are passing SLB criteria area.

NS Cooling (Enhanced)

- Enhanced cooling with EDF

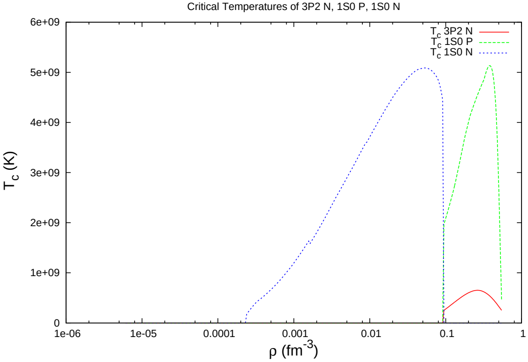


- Large L's give low T_c's

	$L = 40$ MeV	$L = 50$ MeV	$L = 60$ MeV
T_c ($^3P_2$)	6.76×10^8 K	6.42×10^8 K	6.23×10^8 K

Critical Temperature

- Fast cooling in Cas A NS can be explained with nuclear superfluidity and PBF
- T_c for $3P_2$ is $5 \sim 7 \times 10^8$K.
- The range of S_v and L might be confirmed from Cas A cooling rate.



Conclusion

- It's not likely that DURCA happened in CAS A NS
- mass of CAS A NS $> 1.4M_{\odot} \rightarrow L < 70$ MeV
- Superfluidity gives the critical temperature of $^3P_2$
- Need to use flexible RMF (eg, FSU-GOLD) model to compare with EDF