Quark Model of Hadrons

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PART II of the Lecture

SU(3) Symmetry

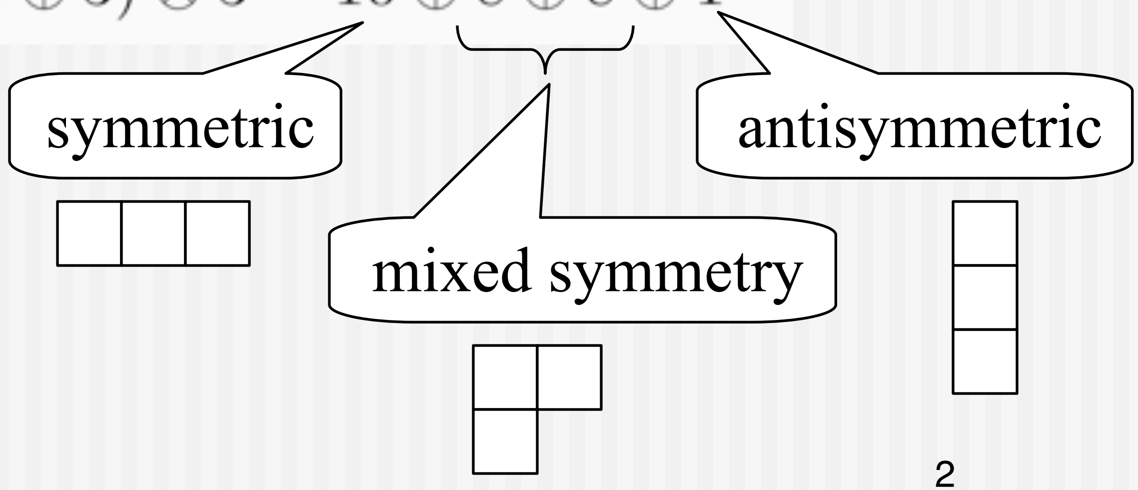
- mesons

$$3 \otimes \bar{3} = 1 \oplus 8$$

- baryons

$$3 \otimes 3 = 6(\text{symmetric}) \oplus \bar{3}(\text{antisymmetric})$$

$$3 \otimes 3 \otimes 3 = (6 \oplus \bar{3}) \otimes 3 = 10 \oplus 8 \oplus 8 \oplus 1$$

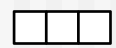


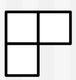
Color and Statistics

- Why do quarks have color?

  ground state baryons

  orbital wave function = symmetic with L=0

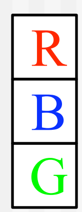
  $SU(3)_f$ x $SU(2)_s$ 

   octet  $S = 1/2$ can be antisymmetric

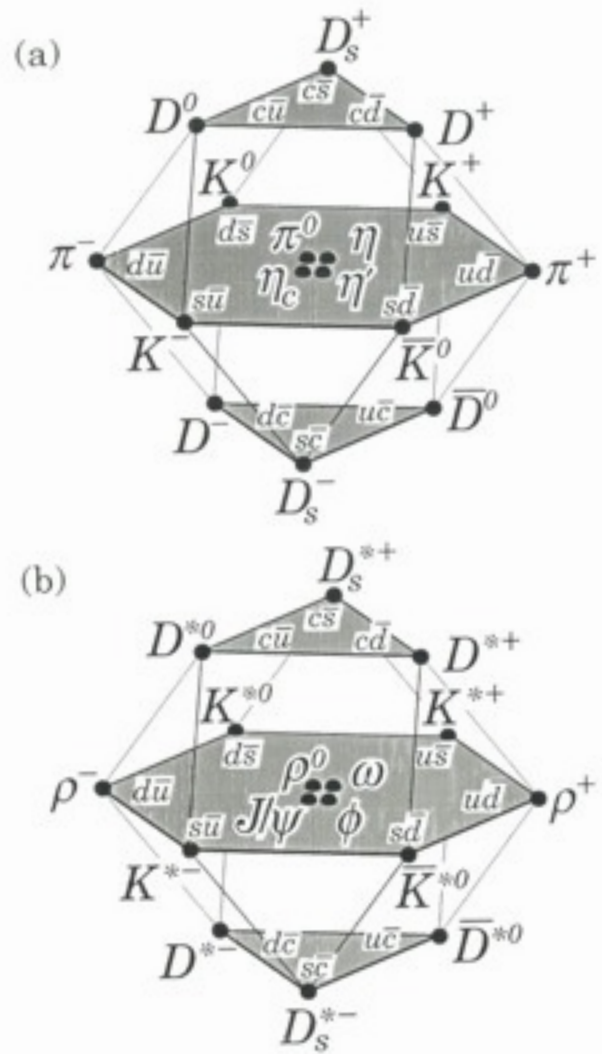
   decuplet  $S = 3/2$ cannot be antisymmetric

  ex. Δ^{++} $S_z=3/2 = (u\Uparrow)^3$

- Color wave function of baryons

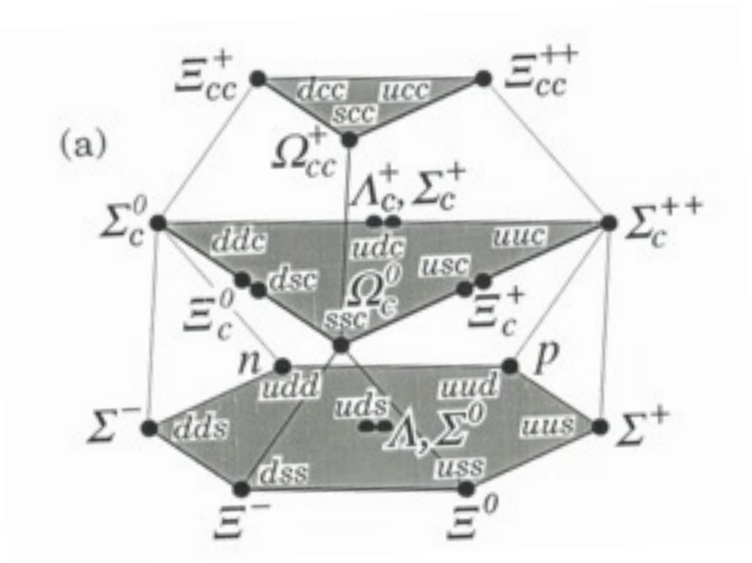
  totally antisymmetric 

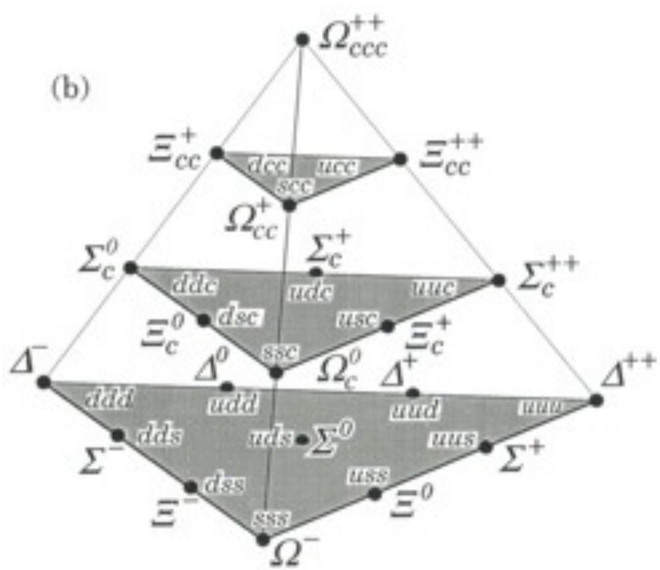
Light meson flavor components



$\frac{1}{\sqrt{2}}(u\bar{s} \pm \bar{s}u)$	$K^+ (K^{+*})$
$\frac{1}{\sqrt{2}}(d\bar{s} \pm \bar{s}d)$	$K^0 (K^{0*})$
$-\frac{1}{\sqrt{2}}(s\bar{u} \pm \bar{u}s)$	$K^- (K^{-*})$
$-\frac{1}{\sqrt{2}}(s\bar{d} \pm \bar{d}s)$	$\bar{K}^0 (\bar{K}^{0*})$
$\frac{1}{\sqrt{2}}(u\bar{d} \pm \bar{d}u)$	$\pi^+ (\rho^+)$
$-\frac{1}{\sqrt{2}}(d\bar{u} \pm \bar{u}d)$	$\pi^- (\rho^-)$
$\frac{1}{2}[(d\bar{d} - u\bar{u}) \pm (\bar{d}d - \bar{u}u)]$	$\pi^0 (\rho^0)$
$\frac{1}{2\sqrt{3}}[(u\bar{u} + d\bar{d} - 2s\bar{s}) \pm (\bar{u}u + \bar{d}d - 2\bar{s}s)]$	$\eta_8^0 (\omega_8^0)$
$\frac{1}{\sqrt{6}}[(u\bar{u} + d\bar{d} + s\bar{s}) \pm (\bar{u}u + \bar{d}d + \bar{s}s)]$	$\eta_1^0 (\omega_1^0)$

Baryon flavor components





SU(3) decuplet

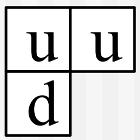
totally symmetric wave functions

SU(3) octet	$\phi_{M,S}$	$\phi_{M,A}$
P	$\frac{1}{\sqrt{6}}[(ud+du)u-2uud]$	$\frac{1}{\sqrt{2}}(ud-du)u$
N	$-\frac{1}{\sqrt{6}}[(ud+du)d-2ddu]$	$\frac{1}{\sqrt{2}}(ud-du)d$
Σ^+	$\frac{1}{\sqrt{6}}[(us+su)u-2uus]$	$\frac{1}{\sqrt{2}}(us-su)u$
Σ^0	$\frac{1}{\sqrt{6}}\left[s\left(\frac{du+ud}{\sqrt{2}}\right)+\left(\frac{dsu+usd}{\sqrt{2}}\right)-2\left(\frac{du+ud}{\sqrt{2}}\right)s\right]$	$\frac{1}{\sqrt{2}}\left[\left(\frac{dsu+usd}{\sqrt{2}}\right)-s\left(\frac{ud+du}{\sqrt{2}}\right)\right]$
Σ^-	$\frac{1}{\sqrt{6}}[(ds+sd)d-2dds]$	$\frac{1}{\sqrt{2}}(ds-sd)d$
Λ^0	$\frac{1}{\sqrt{2}}\left[\frac{dsu-usd}{\sqrt{2}}+\frac{s(du-ud)}{\sqrt{2}}\right]$	$\frac{1}{\sqrt{6}}\left[\frac{s(du-ud)}{\sqrt{2}}+\frac{usd-dsu}{\sqrt{2}}-\frac{2(du-ud)s}{\sqrt{2}}\right]$
Ξ^-	$-\frac{1}{\sqrt{6}}[(ds+sd)s-2ssd]$	$\frac{1}{\sqrt{2}}[(ds-sd)s]$
Ξ^0	$-\frac{1}{\sqrt{6}}[(us+su)s-2ssu]$	$\frac{1}{\sqrt{2}}[(us-su)s]$

SU(3) singlet	ϕ_A
Λ_1^0	$\frac{1}{\sqrt{6}}[s(du-ud)+(usd-dsu)+(du-ud)s]$

Proton wave function

- proton S_z=+1/2 = (uud) (⇑⇑⇓)



2-dim representation

$$|S_f\rangle \equiv \sqrt{\frac{2}{3}}uud - \sqrt{\frac{1}{3}}\frac{ud+du}{\sqrt{2}}u = \frac{1}{\sqrt{6}}(2uud - udu - duu)$$

$$|A_f\rangle \equiv \frac{ud-du}{\sqrt{2}}u = \frac{1}{\sqrt{2}}(udu - duu)$$

$$|p\uparrow\rangle = \frac{1}{\sqrt{2}}(|S_f S_s\rangle + |A_f A_s\rangle)$$

$$= \frac{1}{\sqrt{2}}\left[\frac{1}{\sqrt{6}}(2uud - udu - duu)\frac{1}{\sqrt{6}}(2\uparrow\uparrow\downarrow - \uparrow\downarrow\uparrow - \downarrow\uparrow\uparrow) + \frac{1}{\sqrt{2}}(udu - duu)\frac{1}{\sqrt{2}}(\uparrow\downarrow\uparrow - \downarrow\uparrow\uparrow)\right]$$

$$= \frac{2}{\sqrt{18}}(u\uparrow u\uparrow d\downarrow + \text{all perm.}) - \frac{1}{\sqrt{18}}(u\uparrow u\downarrow d\uparrow + \text{all perm.})$$

Quark Model

- Powerful tool to understand hadron spectrum, structures and dynamics.
- With proper dynamical contents, it is applicable to multi-quark systems, such as 2-baryons, pentaquarks.

Quarks in QCD

- QCD Lagrangian

$$\mathcal{L} = \bar{q}(i\not{D} - m)q - \frac{1}{4}\mathrm{Tr}[G_{\mu\nu}G^{\mu\nu}]$$

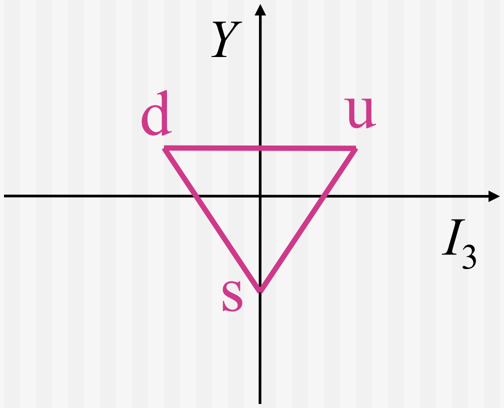
$$D_\mu \equiv \partial_\mu + igA_\mu$$
$$A_\mu \equiv \frac{\lambda^a}{2}A_\mu^a$$
$$G_{\mu\nu} \equiv \partial_\mu A_\nu - \partial_\nu A_\mu + ig[A_\mu, A_\nu]$$

- quark

B =1/3, C =3

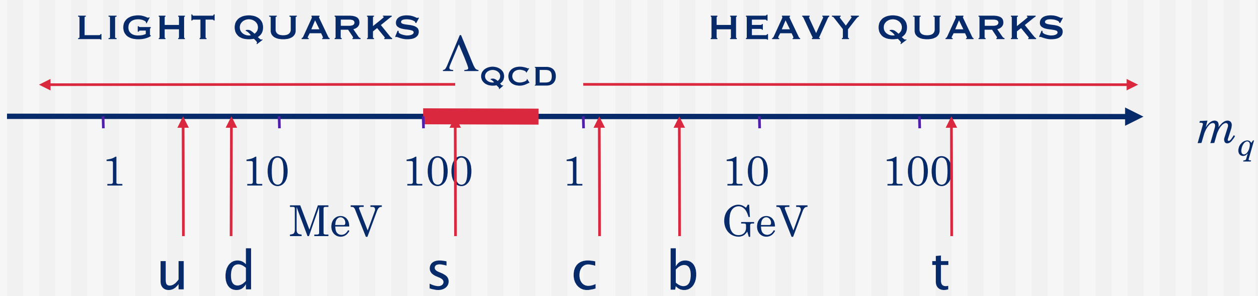
(u,d) : I =1/2, S =0, Y =1/3

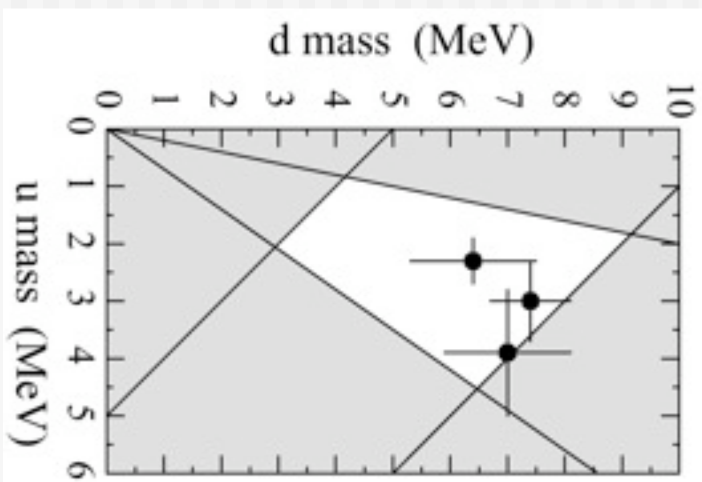
s : I =0, S = -1, Y = - 2/3

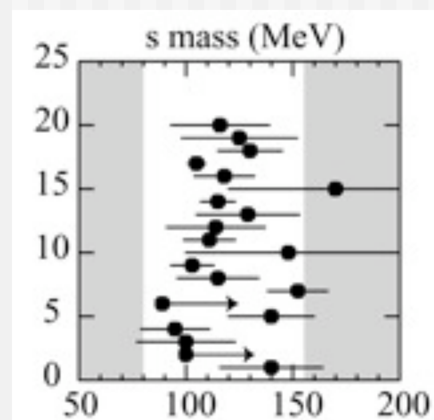


Quarks in QCD

- Quark masses and scale of QCD







Constituent Quark

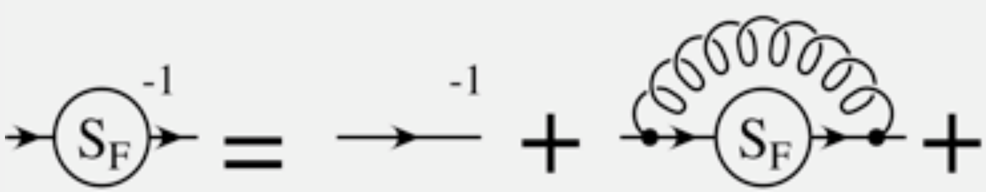
- Dyson-Schwinger equation

$$
\begin{aligned}
S_F^{-1}(p) &= S_{F0}^{-1}(p) - \Sigma(p) \\
& S_{F0}^{-1} = \not{p} - m \\
& S_F^{-1} = A(p)\not{p} - \boxed{B(p)}
\end{aligned}
$$

→ effective mass generated

dynamical chiral symmetry breaking

dressed quark propagator   gluon



Constituent Quark

- Conserved currents are not renormalized.

  I , Y , C charges do not change.

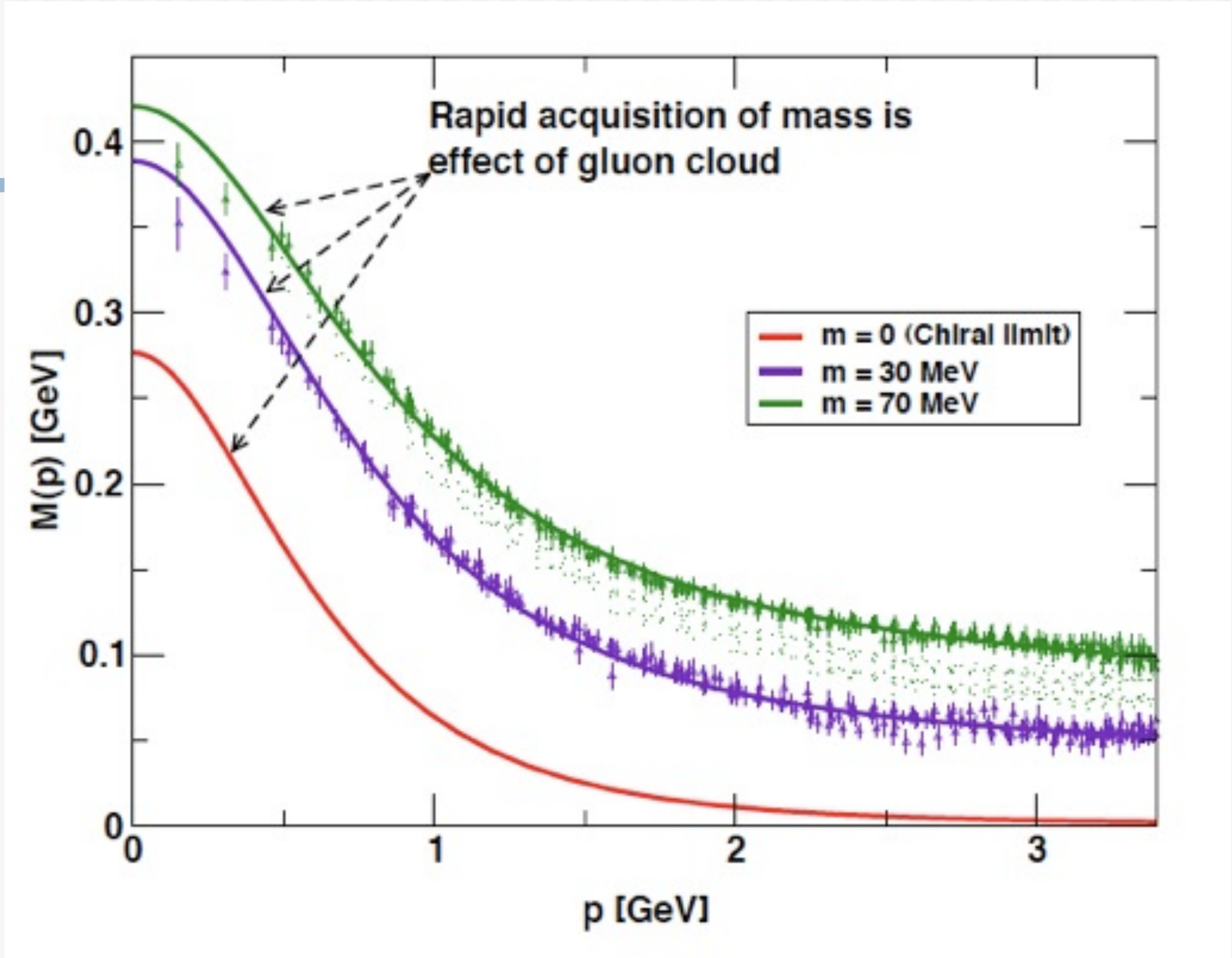
- Constituent quark mass

  $m_q \approx 300$ MeV (u, d)

  $m_s \approx 500$ MeV (s)

- Residual interactions are weak.

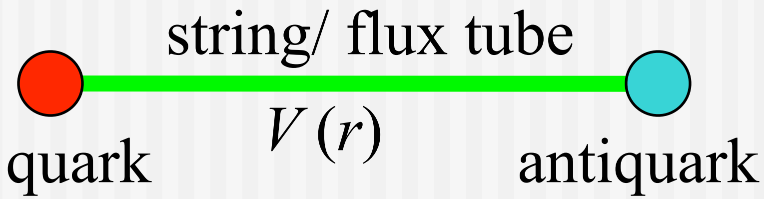
  except confinement



M. S. Bhagwat and P. C. Tandy(2006)

Color Confinement

- color singlet-ness of hadrons



- Light quarks connected by string

$H = p + \sigma r$ with $J = pr$ fixed

Virial theorem $E(J) = 2\sqrt{\sigma J}$

or $m_J^2 = 4\sigma J$ (Regge trajectory)

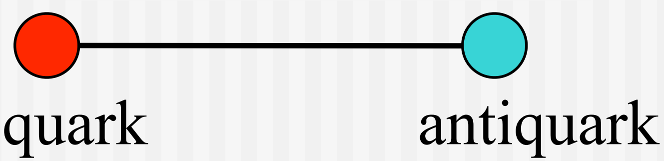
Color Confinement

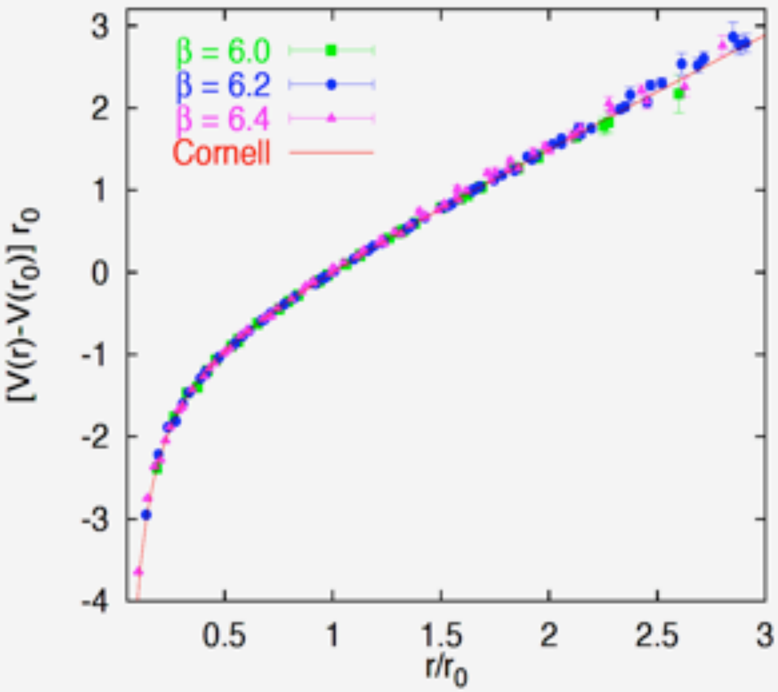
- heavy quark : quarkonium

  Lattice QCD: Wilson loop

- Cornell potential

$$V(r) = -\frac{e}{r} + \sigma r$$





quenched LQCD

r_0: Sommer scale

G.S. Bali / Phys. Rep. 343 (2001) 1

Color Confinement

- Casimir scaling

G.S. Bali / Phys. Rep. 343 (2001) 1

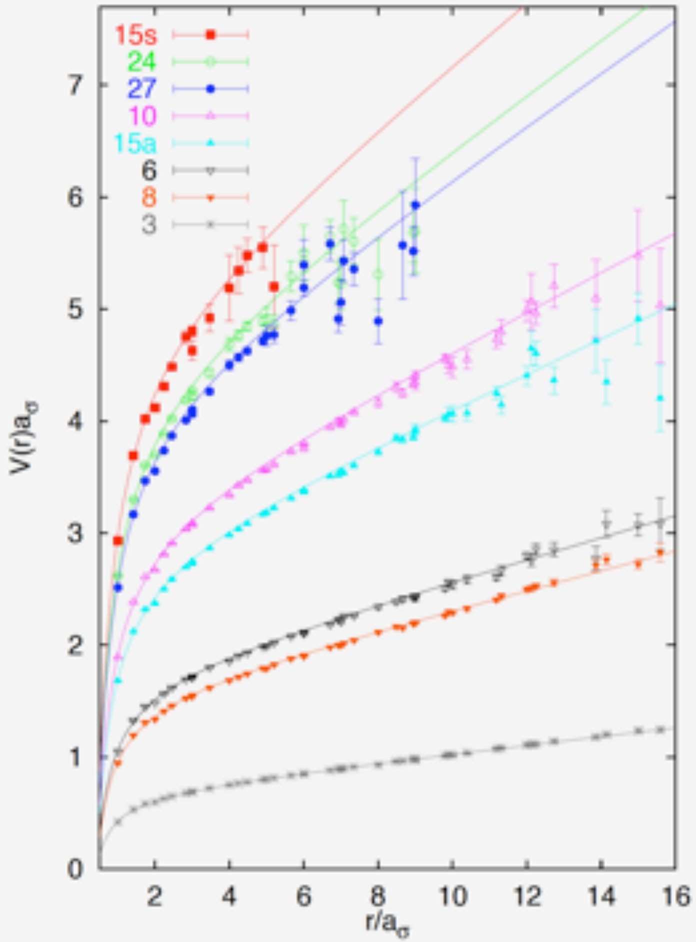
$$V(r) = \sum_a (T_1^a T_2^a) v(r)$$

$$\sum_a (T_1^a T_2^a) = \frac{1}{2}[\sum_a (T_1^a + T_2^a)^2 - \sum_a (T_1^a)^2 + (T_2^a)^2]$$

$3\text{x}\bar{3}=1 \quad - 4/3$

$3\text{x}3=\bar{3} \quad - 2/3$

$8\text{x}8=1 \quad -3$



Color Confinement

- Confinement potential

$$V(r_{12}) = -\sum_a (\lambda_1^a \lambda_2^a)\, a\, r_{12}$$

string tension $\sigma = \frac{16}{3} a \sim 1\mathrm{GeV/fm}$

  - confine colored subsystem
  - no confinement between color singlet objects
- Lorentz property?
  - Lorentz scalar or vector?
    relativistic effects? ex. spin-obit interaction

Charmonium and bottomium

Potential model approach

$$H = 2m_Q + \frac{\vec{p}^2}{m_Q} + S(r) + V(r)$$

$$S(r) = \sigma r + b$$

$$V(r) = -\frac{4}{3}\frac{\alpha_s(r)}{r} \simeq -\frac{4}{3}\frac{\alpha_s^0}{r}(1 - \exp(-(r/R_c)^\kappa)$$

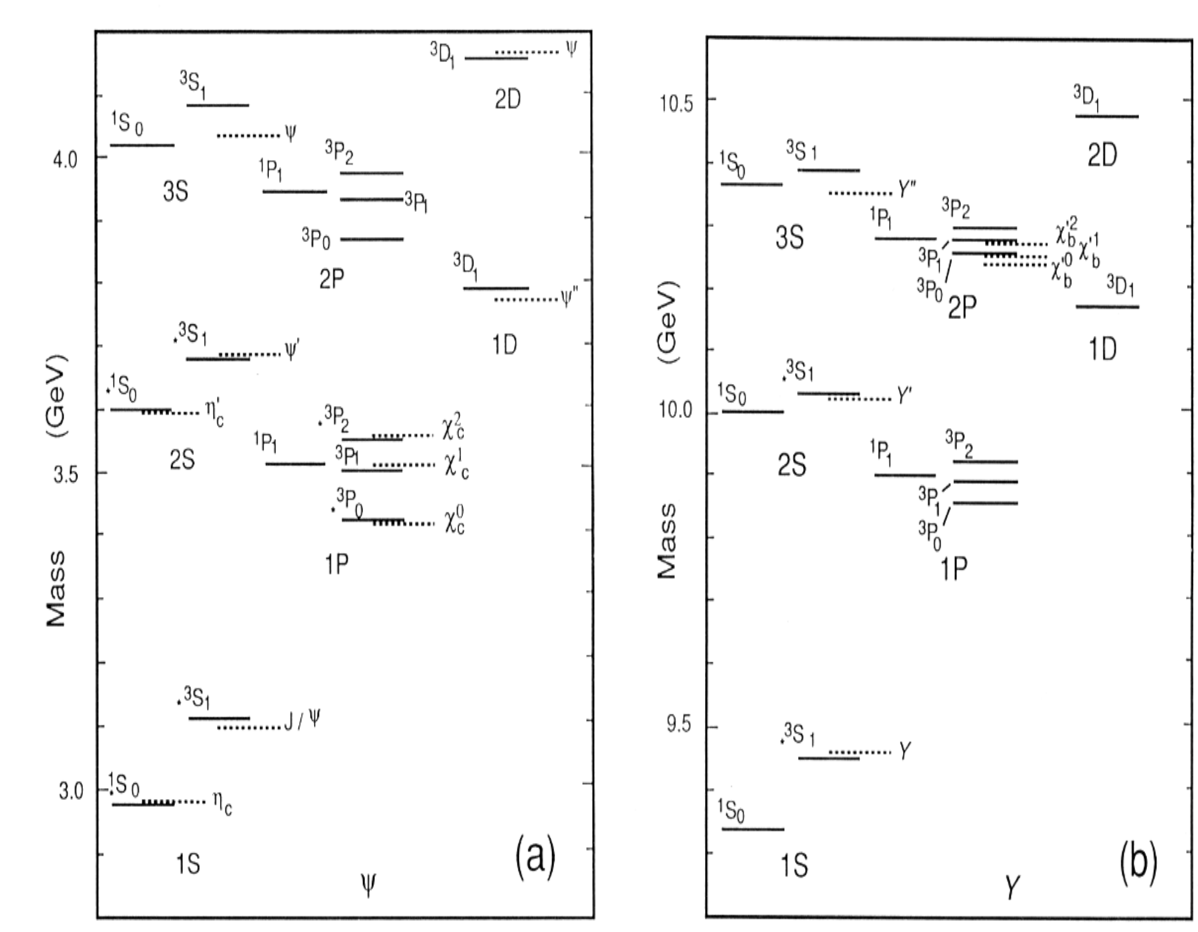
R_c and κ fit to the running coupling constant

S.N. Mukherjee, et al., Phys. Rep. 231 (1993)

Charmonium and bottomium

S.N. Mukherjee, et al., Phys. Rep. 231 (1993)

$$\sigma = 0.90\text{GeV/fm}$$
$$b = -0.030\text{GeV}$$
$$\alpha_0 = 0.732$$
$$R_c = 0.5\text{GeV}^{-1}$$
$$\kappa = 0.582$$



Baryon Spectrum

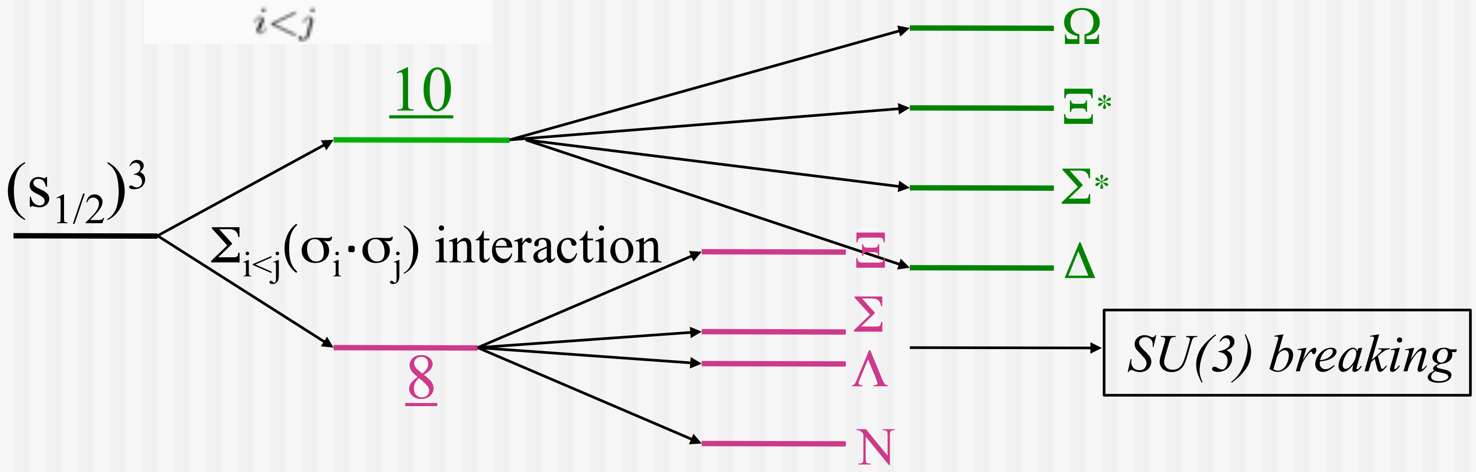
- Single particle motion

  $(s_{1/2})^3$ $J = 1/2$ $\underline{8}$

  $J = 3/2$ $\underline{10}$

  hyperfine interaction

  $$\Delta_{ss} \sum_{i<j} \vec{\sigma}_i \cdot \vec{\sigma}_j$$



Baryon Spectrum

- HF interaction in Baryon
  - N-Δ mass splitting (300 MeV) ↔ Δ_{ss} ~ 50 MeV
  - Λ–Σ mass splitting (~77 MeV) from SU(3) breaking

$$\Sigma_{\rm HF} = \Delta_{ss}\{\vec{\sigma}_u\cdot\vec{\sigma}_d + \xi\times\vec{\sigma}_s\cdot(\vec{\sigma}_d+\vec{\sigma}_u)\}$$

Δ_{ss} → 50 MeV

Λ $(ud)_{I=0,S=0}\,s$ 50MeV x [(–3) + 0 * ξ]

Σ $(ud)_{I=1,S=1}\,s$ 50MeV x [1 + (–4) * ξ]

ξ - factor: s-u, s-d HF interaction is weaker than u-d.

for ξ = 3/5 → Σ – Λ = (8/15) x150 MeV = 80 MeV

Origin of (σ·σ) Interaction

One gluon exchange (OgE) or color-magnetic (CM) interaction

Breit-Fermi, DeRujula-Georgi-Glashow

$$\alpha_s \lambda_i \frac{\vec{\sigma}_i \cdot \vec{q}}{m_i} \frac{1}{q^2} \lambda_j \frac{\vec{\sigma}_j \cdot \vec{q}}{m_j} \simeq \frac{\alpha_s}{m_i m_j} (\lambda_i \cdot \lambda_j)(\vec{\sigma}_i \cdot \vec{\sigma}_j) \delta(\vec{r}_{ij})$$

SU(3) breaking $m_u/m_s \sim 3/5$

$$\Sigma_{\mathrm{CM}} = -\Delta_{\mathrm{CM}} \Sigma_{i<j} \xi_{ij} (\lambda_i^c \cdot \lambda_j^c)(\vec{\sigma}_i \cdot \vec{\sigma}_j)$$

N-Δ mass splitting (300 MeV) ↔ $\Delta_{\mathrm{CM}} \sim 18.75$ MeV

Origin of (σ·σ) Interaction

- Baryon masses $m_q \sim 360$ MeV $m_s \sim 540$ MeV

  $M_N = 3\, m_q + \langle V_{cm} \rangle_N = 360\text{x}3 - 150 \approx 930$ MeV

  $M_\Delta = 3\, m_q + \langle V_{cm} \rangle_\Delta = 360\text{x}3 + 150 \approx 1230$ MeV

  $M_{\Lambda,\Sigma} = 2\, m_q + m_s + \langle V_{cm} \rangle_{\Lambda,\Sigma}$

  $= 360\text{x}2 + 540 - 90 \approx 1170$ MeV

- *H dibaryon* : $S = -2$, $B = 2$

  $M_H = 4\, m_q + 2\, m_s + \langle V_{cm} \rangle_H$

  $= 360\text{x}4 + 540\text{x}2 - 450 \approx 2070$ MeV

  ΛΛ threshold 2230 MeV

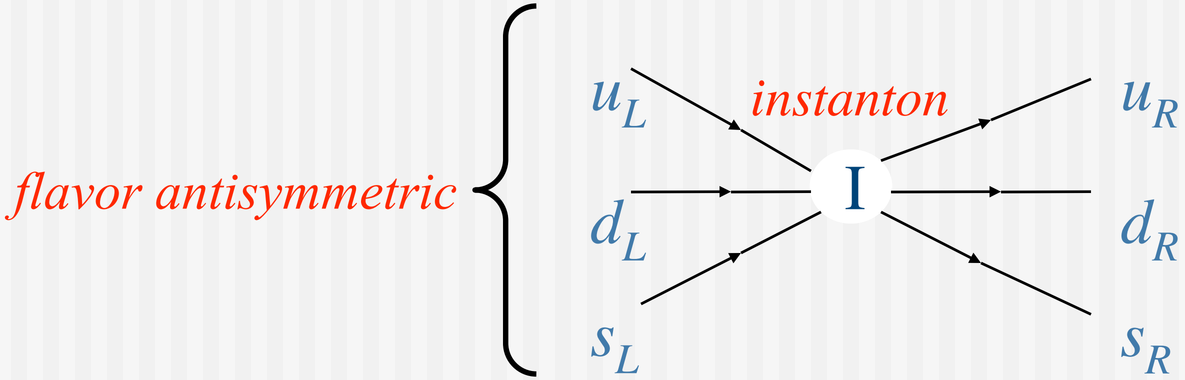
  20-year searches were not successful.

Instanton Induced Interaction

(2) Instanton-induced-interaction (III)

aka Kobayashi-Maskawa-'t Hooft (KMT)

instanton-light-quark couplings



Instanton Induced Interaction

Instanton-induced-interaction (III)

flavor antisymmetric u-d-s 3-body repulsion

flavor antisymmetric 2-body attraction

$$V^{(3)}_{\rm III} = V^{(3)} \sum_{(ijk)} \mathcal{A}^f \left[1 - \frac{1}{7}\left(\boldsymbol{\sigma}_i \cdot \boldsymbol{\sigma}_j + \boldsymbol{\sigma}_j \cdot \boldsymbol{\sigma}_k + \boldsymbol{\sigma}_k \cdot \boldsymbol{\sigma}_i\right)\right] \delta(\boldsymbol{r}_{ij})\delta(\boldsymbol{r}_{jk})$$

$$V^{(2)}_{\rm III} = V^{(2)} \sum_{i<j} \mathcal{A}^f \left[1 - \frac{1}{5}(\boldsymbol{\sigma}_i \cdot \boldsymbol{\sigma}_j)\right] \delta(\boldsymbol{r}_{ij})$$

$= V_{ij}^{(2)}\,(2/5)\,(1 - \sigma_i . \sigma_j)\,\delta\,(r_{ij})$ in the baryon

spin-spin interaction

proportional to $1/m_i m_j$

Shuryak-Rosner (1989)
Takeuchi-Oka (1989)

New instanton picture

- III (2-body) $\Sigma_{\mathrm{III}} = \Delta_{\mathrm{III}}\Sigma_{i<j}\mathcal{A}^{f}_{ij}\xi_{ij}[1 - \frac{1}{5}(\vec{\sigma}_i \cdot \vec{\sigma}_j)]$

N-Δ mass splitting (300 MeV) ↔ $\Delta_{\mathrm{III}} \sim 125$ MeV

- III (3-body)

3-body repulsion *flavor singlet* (*u-d-s*) u d s

for H dibaryon $M_{\mathrm{H}} > m_{\Lambda\Lambda}$ *threshold*

flavor singlet u u / d d / s s