Direct Approaches from QCD

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Part III of Lecture

Direct Approaches from QCD

- Why are the quark models not sufficient?
  - Hadrons are not few-body systems in a simple Hamiltonian.
  - QCD vacuum does not allow perturbation series to describe physical quantities.
  - Analyses of variety hadron properties require field theoretical methods to describe quark-gluon composite systems.

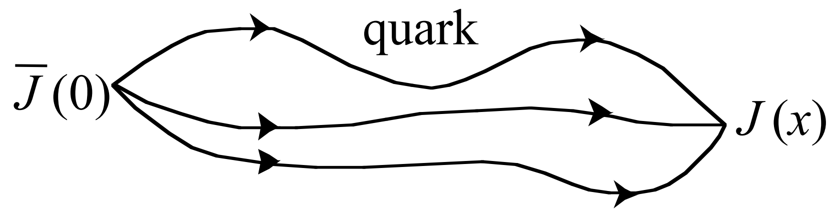
Lattice QCD

- Quark and gluon fields are placed on the lattice.
  - Infinitely many degrees of freedom of QCD are reduced into a finite number of variables on the discretized 4-D lattice.
  - Path integrals of QCD are numerically evaluated on the 4-D Euclidean lattice.
  - The local gauge invariance is maintained by introducing link variables corresponding to the gluon fields.

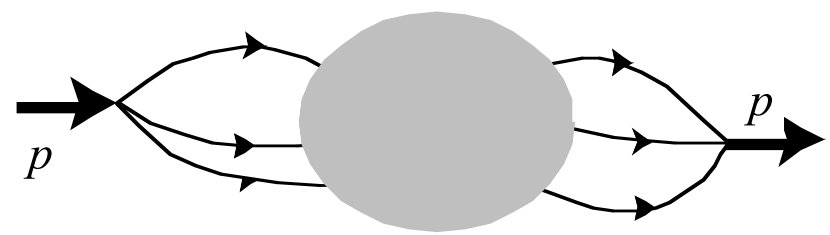
Two Point Correlators

- TPC $\Pi(x)$ contains information of the spectrum

$$\Pi(x) = \langle 0|T(J(x)\bar{J}(0))|0\rangle$$



- $J(x)$: interpolating field operator
  determines quantum numbers
- Fourier transform



$$\Pi(p) \equiv i \int d^4x e^{ip\cdot x} \langle 0|T(J(x)\bar{J}(0))|0\rangle$$

Two Point Correlators

- interpolating field operator

  Mesons

  I =1 vector meson $J_\rho(x) = \bar{q}(x)\gamma^\mu \frac{\vec{\tau}}{2} q(x)$

  I =1 pseudoscalar meson $J_\pi(x) = \bar{q}(x)\gamma^5 \frac{\vec{\tau}}{2} q(x)$

  Baryons

  J =1/2 baryon

$$B(x) = \epsilon_{abc}(u_a^T(x) C\gamma^5 d_b(x)) u_c(x)$$

Baryon operators

- di-quark operators

$$q(x) \xrightarrow{C} q^C(x) = C\bar{q}^T(x), \qquad C = i\gamma^0\gamma^2 = -C^{-1} = -C^T = -C^\dagger$$

$$q^C(t,\vec{x}) = C\bar{q}^T(t,\vec{x}) \xrightarrow{P} -\gamma^0 q^C(t,-\vec{x})$$

$$C = \bar{3},\ I = 0,\ S = 0,\ \text{scalar } 0^+: \qquad \epsilon_{abc}(u_b^T(x)C\gamma^5 d_c(x))$$

$$C = 6,\ I = 1,\ S = 0,\ \text{scalar } 0^+: \qquad (u_b^T(x)C\gamma^5 d_c(x)) + (d \leftrightarrow u)$$

$$C = \bar{3},\ I = 0,\ S = 0,\ \text{pseudoscalar } 0^-: \qquad \epsilon_{abc}(u_b^T(x)Cd_c(x))$$

$$C = 6,\ I = 1,\ S = 0,\ \text{pseudoscalar } 0^-: \qquad (u_b^T(x)Cd_c(x)) + (d \leftrightarrow u)$$

$$J_N^\alpha(x) = \varepsilon_{abc}[(u_a(x)Cd_b(x))(\gamma_5 u_c(x))^\alpha + t(u_a(x)C\gamma_5 d_b(x))u_c^\alpha(x)],$$

Lattice QCD

Path Integral in Field Theory

$$\langle \phi_f, t_f | \phi_i . t_i \rangle = \int [D\phi] \exp\left\{ \frac{i}{\hbar} S[\phi] \right\} = \sum_n \langle \phi_f | n \rangle e^{-iE_n(t_f - t_i)/\hbar} \langle n | \phi_i \rangle$$

$$S[\phi] \equiv \int_{t_i}^{t_f} dt \int d^3x \, \mathcal{L}(\phi, \partial_\mu \phi)$$

Path Integral Form of Partition Function

$$Z = \mathrm{Tr} e^{-i\beta \hat{H}} = \int [Dq] \exp\left(-\frac{1}{\hbar} \int_{t_i}^{t_f} dt \, H(q) \right)$$

$$t_f - t_i = \beta\hbar$$

Lattice QCD

Minkowsky to Euclid

$$\bar{t} = it, \quad \dot{\bar{q}} = -i\dot{q}, \quad \bar{q} = q$$

$$\frac{i}{\hbar} S[q(t)] = -\frac{1}{\hbar} \bar{S}[\bar{q}(\bar{t})] \qquad \bar{S}[\bar{q}(\bar{t})] = \int d\bar{t}\, \bar{L}(\bar{q}, \dot{\bar{q}})$$

$$L(q, \dot{q}) = \frac{m}{2}\dot{q}^2 - V(q) \longrightarrow \bar{L}(\bar{q}, \dot{\bar{q}}) = \frac{m}{2}\dot{\bar{q}}^2 + V(q)$$

$$\mathcal{L}(\phi, \partial_\mu \phi) = \frac{1}{2}\partial_\mu \phi \partial^\mu \phi - \frac{1}{2} m^2 \phi^2 \longrightarrow \bar{\mathcal{L}}(\bar{\phi}, \partial_\mu \bar{\phi}) = \frac{1}{2}\partial_\mu \bar{\phi} \partial_\mu \bar{\phi} + \frac{1}{2} m^2 \bar{\phi}^2$$

$$\partial_\mu \bar{\phi} \partial_\mu \bar{\phi} \equiv (\partial_{\bar{t}} \bar{\phi})^2 + (\vec{\nabla} \bar{\phi})^2$$

Lattice QCD

Euclidean Path Integral in Field Theory (bar and $\hbar$ omitted)

$$\langle \phi_f, t_f = t_i + T | \phi_i . t_i \rangle = \int [D\phi] \exp(-S[\phi]) = \sum_n \langle \phi_f | n \rangle e^{-E_n T} \langle n | \phi_i \rangle$$

$$\xrightarrow{T\to\infty} \quad \langle \phi_f | 0 \rangle e^{-E_0 T} \langle 0 | \phi_i \rangle$$

Green's function

$$\langle \phi_f, t_f = t_i + T | T[\phi(x_1)\phi(x_2)\ldots\phi(x_n)] | \phi_i . t_i \rangle$$

$$\xrightarrow{T\to\infty} \quad \langle \phi_f | 0 \rangle \langle 0 | T[\phi(x_1)\phi(x_2)\ldots\phi(x_n)] | 0 \rangle \langle 0 | \phi_i \rangle e^{-E_0 T}$$

$$= \int [D\phi] \phi(x_1)\phi(x_2)\ldots\phi(x_n)\, e^{-S[\phi]}$$

$$\langle 0 | T[\phi(x_1)\phi(x_2)\ldots\phi(x_n)] | 0 \rangle = \frac{\int [D\phi] \phi(x_1)\phi(x_2)\ldots\phi(x_n)\, e^{-S[\phi]}}{\int [D\phi]\, e^{-S[\phi]}}$$

Lattice QCD

Hadron Masses

$$M(x) \equiv \bar{q}(x)\Gamma q(x)$$

$$\langle 0|T[M(x)M^\dagger(0)]|0\rangle = \frac{\int [Dq][D\bar{q}][DU] M(x)M^\dagger(0)\, e^{-S[q,\bar{q},U]}}{\int [Dq][D\bar{q}][DU]\, e^{-S[q,\bar{q},U]}}$$

$$\sim \text{constant} \times e^{-(E_M - E_0)T} \quad \text{for } x_0 = T \gg 0$$

Hadron Matrix Elements

$$O(y) \equiv \bar{q}(y)\Gamma_O q(y)$$

$$\frac{\int [Dq][D\bar{q}][DU] M(x)O(y)M^\dagger(0)\, e^{-S[q,\bar{q},U]}}{\int [Dq][D\bar{q}][DU]\, M(x)M^\dagger(0)e^{-S[q,\bar{q},U]}} \sim \text{constant} \times \langle M|O|M\rangle$$

$$\text{for } x_0 \gg y_0 \gg 0$$

Lattice QCD

Discretization

$$\phi(x) \longrightarrow \phi(n) \qquad n \equiv (n_1, n_2, n_3, n_4)$$

$$\partial_\mu \phi(x) = \frac{\phi(n+\hat{\mu}) - \phi(n)}{a}, \qquad a\text{: lattice spacing; } \hat{\mu} = (1,0,0,0) \text{ etc}$$

$$\int d^4x = \sum_n a^4 \times$$

scalar field action

$$S_E[\phi] = a^4 \sum_{n,\mu} \frac{(\phi(n+\hat{\mu}) - \phi(n))^2}{2a^2} + \sum_n \frac{m^2 \phi(n)^2}{2}$$

$$Z_E = \int_{-\infty}^{\infty} \cdots \int_{-\infty}^{\infty} \prod_n d\phi(n) e^{-S_E[\phi]}$$

Lattice QCD

Gauge field as link variables

$$A(x) \longrightarrow U(x,y) \equiv \mathrm{P}\exp\left(ig\int_x^y dx_\mu A^\mu\right)$$

parallel transport

$$q^P(x+dx) = q(x) - igA_\mu q(x)dx^\mu \sim e^{-igA_\mu dx^\mu} q(x)$$

$$\rightarrow q^P(y) = U(y,x)q(x)$$

gauge transform

$$q(x) \rightarrow g(x)q(x); \quad q(y) \rightarrow g(y)q(y)$$

$$U(y,x) \rightarrow g(y)U(y,x)g(x)^{-1}$$

$$U_p \equiv U(x, x+dy)\, U(x+dy, x+dy+dx)\, U(x+dx+dy, x+dx)\, U(x+dx, x)$$

$$\sim 1 - a^4 g^2 G_{\mu\nu} G_{\mu\nu}$$

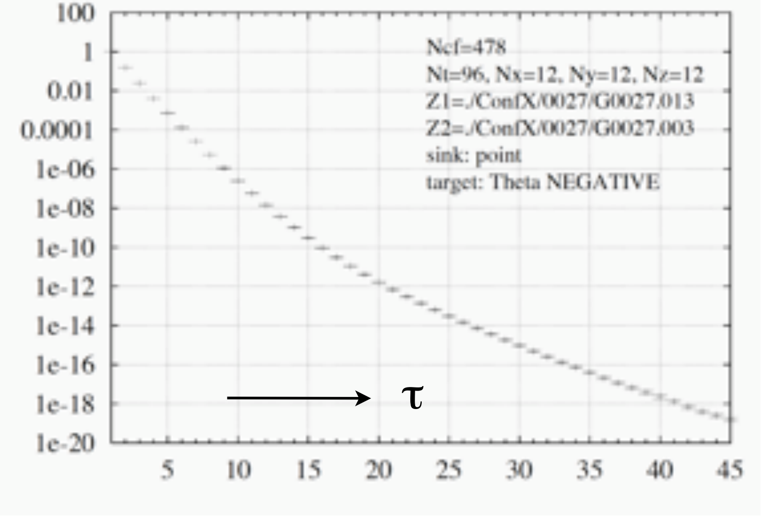
$$S_G = \sum_{\mathrm{p}} \beta \left(1 - \frac{1}{N_c}\mathrm{ReTr}(U_p)\right)$$

Lattice QCD

- Imaginary time two-point correlator

$$\Pi(\vec{p}=0;\tau) = \int d^3x \, \langle 0|T(J(\vec{x},\tau)\bar{J}(0,0))|0\rangle$$

$$= \sum_i \lambda_i^2 e^{-m_i\tau} \xrightarrow{\text{large } \tau} \boxed{\lambda_0^2 e^{-m_0\tau}}$$



Lattice QCD spectrum of light hadrons

PACS- CS collaboration, Phys. Rev. D79 034503, 2009

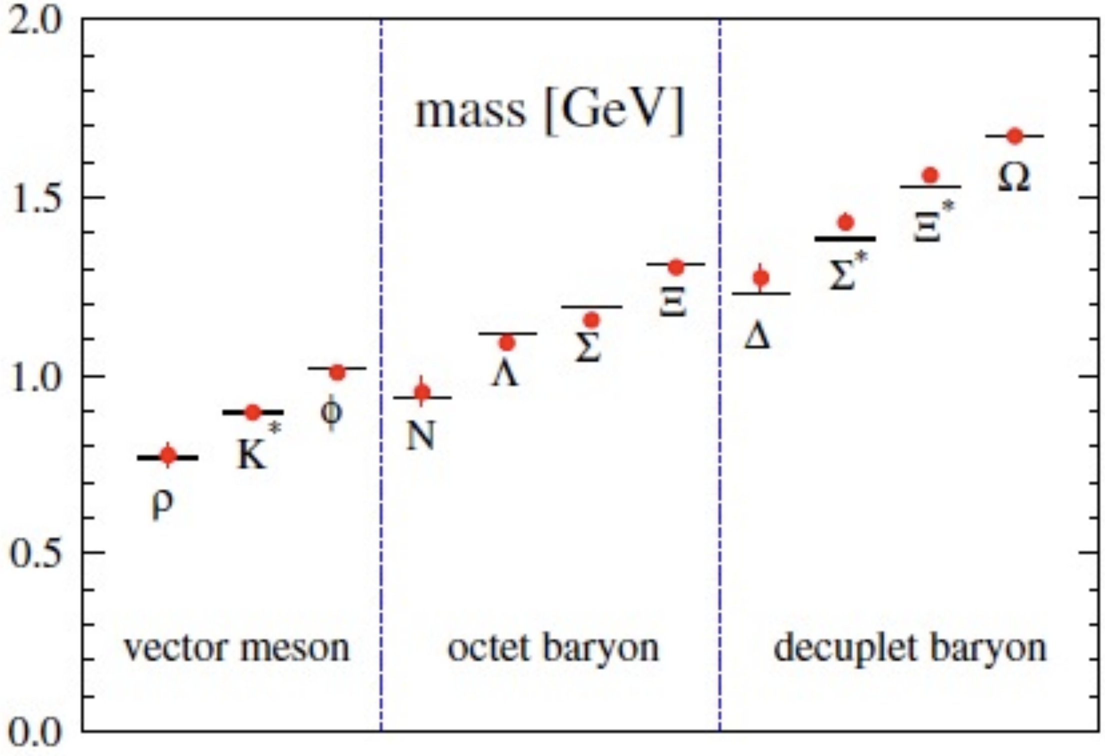
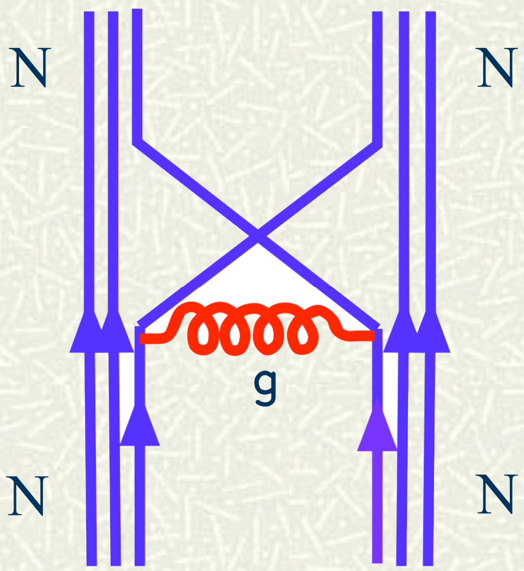


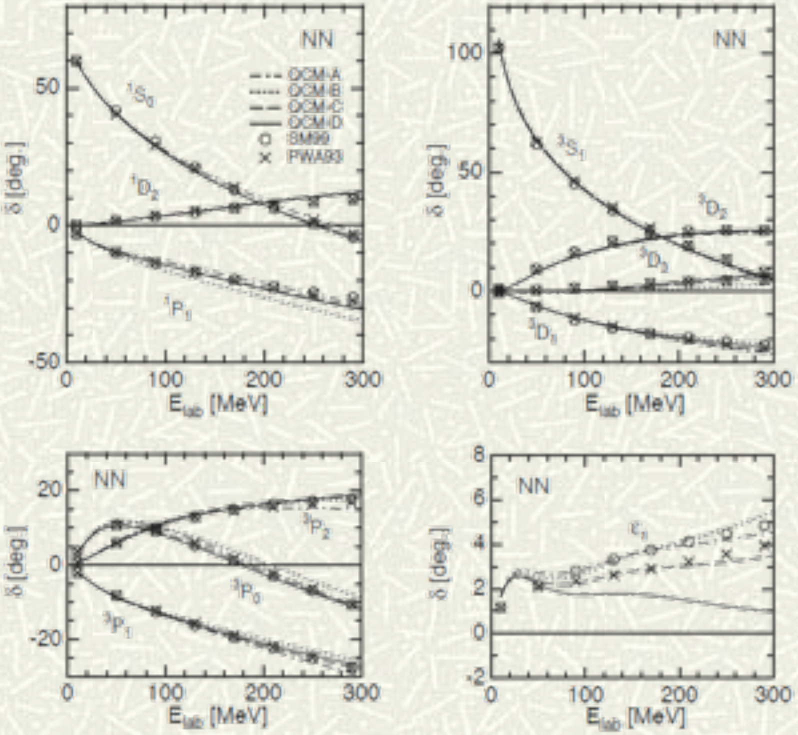
FIG. 24 (color online). Light hadron spectrum extrapolated to the physical point using m_π, m_K and m_Ω as input. Horizontal bars denote the experimental values.

Baryon-baryon interaction

- **Quark Cluster Model** **(M.O., K.Yazaki, 1980)**

Short-range repulsion due to the quark exchange mechanism based on the NR quark model with gluon exchange interaction.

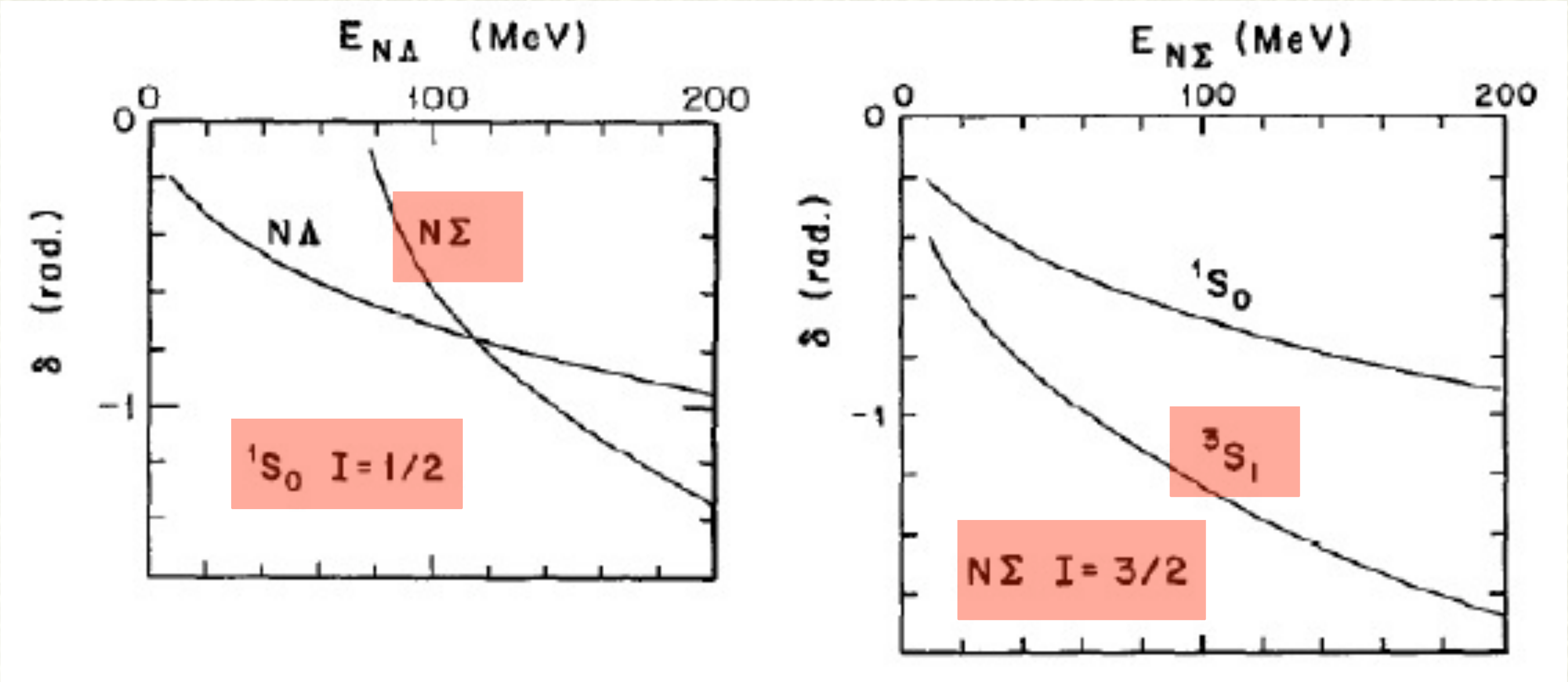




Takeuchi et al. (2000)

Baryon-baryon interaction

- **Strong repulsion in the Pauli forbidden states**



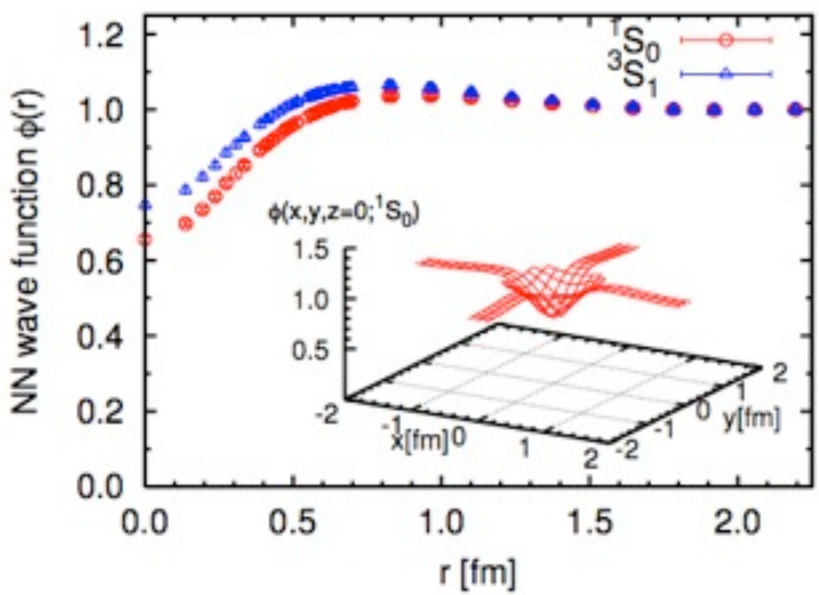
PRL 99, 022001 (2007) PHYSICAL REVIEW LETTERS week ending 13 JULY 2007

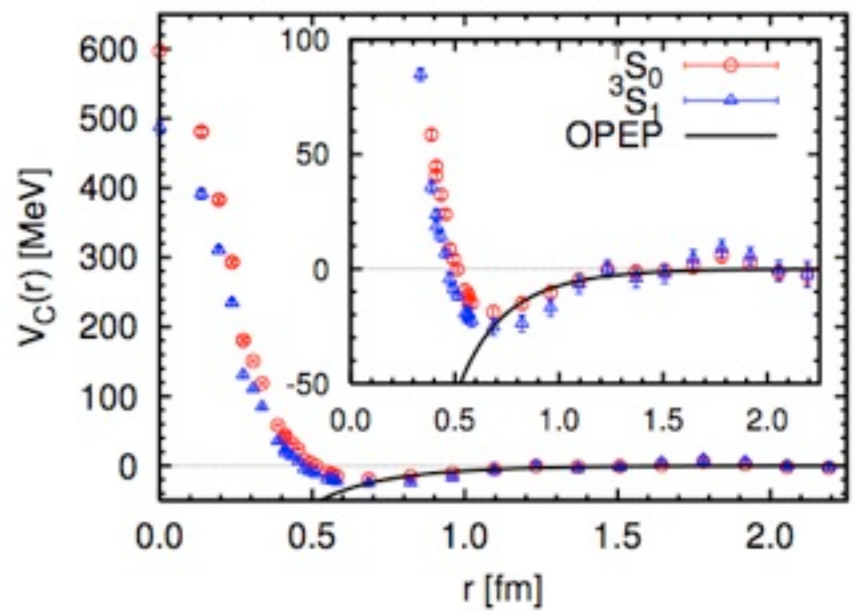
Nuclear Force from Lattice QCD

N. Ishii,[1,2] S. Aoki,[3,4] and T. Hatsuda[2]

$$V_C(r) = E + \frac{1}{2\mu}\frac{\vec{\nabla}^2\phi(r)}{\phi(r)}$$

$$\phi(\vec{r}) \equiv \frac{1}{24}\sum_{\mathcal{R}\in O}\frac{1}{L^3}\sum_{\vec{x}} P^{\tau}_{ij}P^{\sigma}_{\alpha\beta}\left\langle 0 \left| N^i_\alpha(\mathcal{R}[\vec{r}]+\vec{x})N^j_\beta(\vec{x}) \right| \mathrm{NN}\right\rangle$$



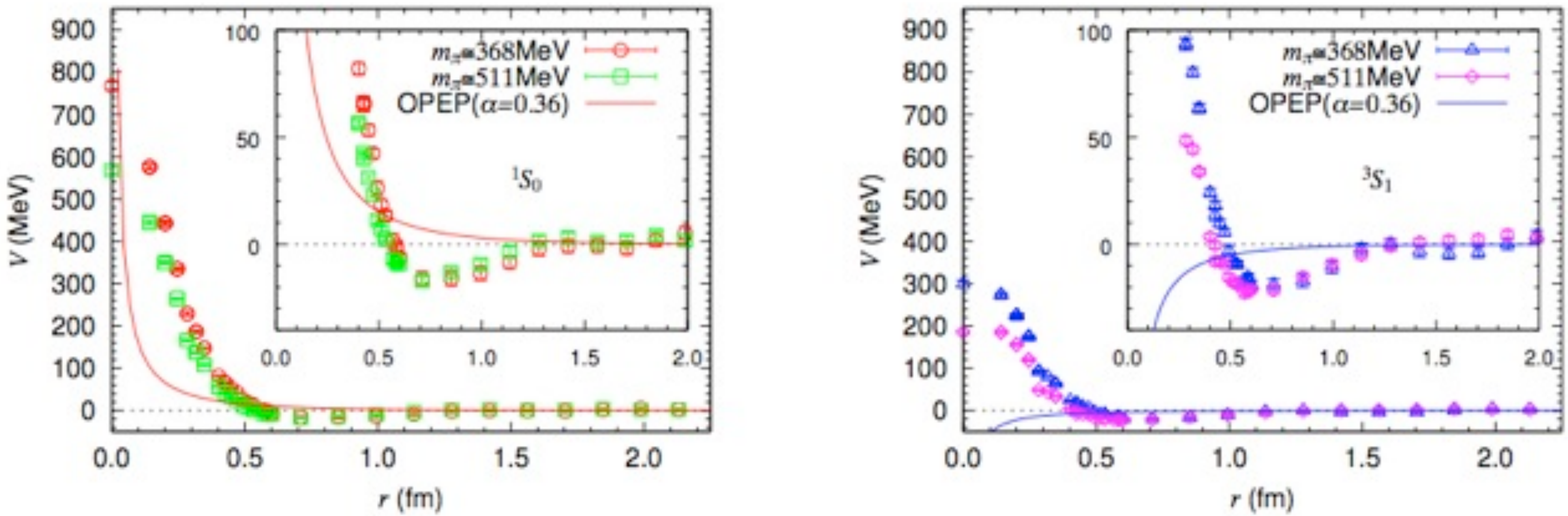


Hyperon-Nucleon Force from Lattice QCD

Hidekatsu Nemura [a,*,1], Noriyoshi Ishii [b], Sinya Aoki [c,d], and Tetsuo Hatsuda

arXiv:0806.1094 [nucl-th]

$p - \Xi^0$ potential



Baryon-baryon interaction

- **Recent development of the BB potential calculation in LQCD**

- In lattice QCD, BB potential can be defined and extracted through 4-point function.

$$W(t-t_0,\vec{r}) = \sum_{\vec{x}} \langle 0|B_i(t,\vec{x}+\vec{r})B_j(t,\vec{x})\,\bar{B}_k(t_0)\bar{B}_l(t_0)|0\rangle$$

at $t-t_0 > t_{sat}$

B.S. amp. $\phi_{E_0}(\vec{r})$

$$V(\vec{r}) = \frac{1}{2\mu}\frac{\nabla^2\phi_{E_0}(\vec{r})}{\phi_{E_0}(\vec{r})} + T_0$$

- Developed by Ishii, Aoki and Hatsuda for NN(2006)

  Phys. Rev. Lett. 99. 022001 (2007) [arXive:nucl-th/0611096]
  arXiv:0909.5585[hep-lat]

- Applied to YN by Nemura, Ishii, Aoki and Hatsuda.

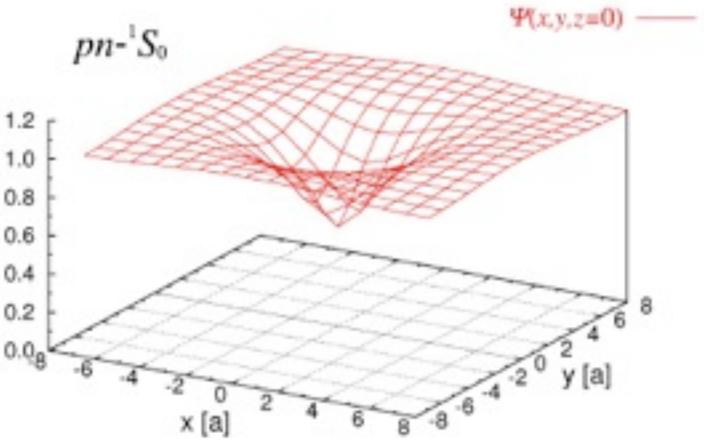
HAL QCD Collaboration

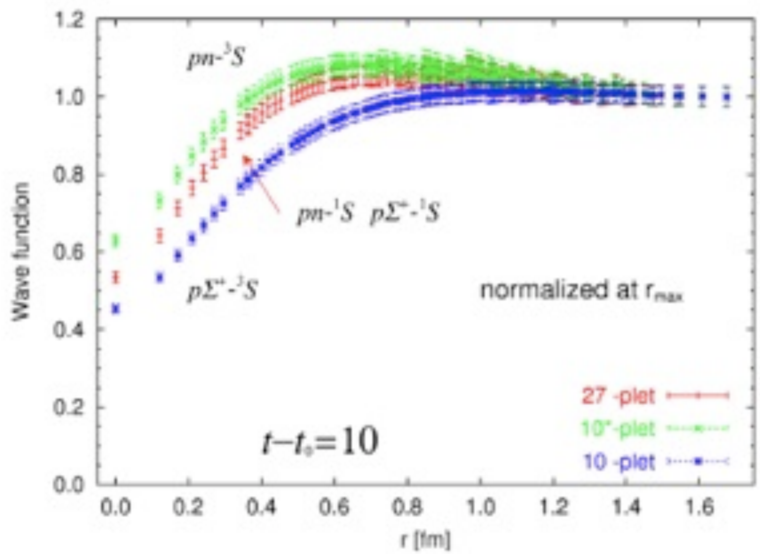
by T. Inoue

Baryon-baryon interaction

- **BS amplitude → potential**

BS amplitude(wave fucntion)





- From these amp. potentials are extracted.

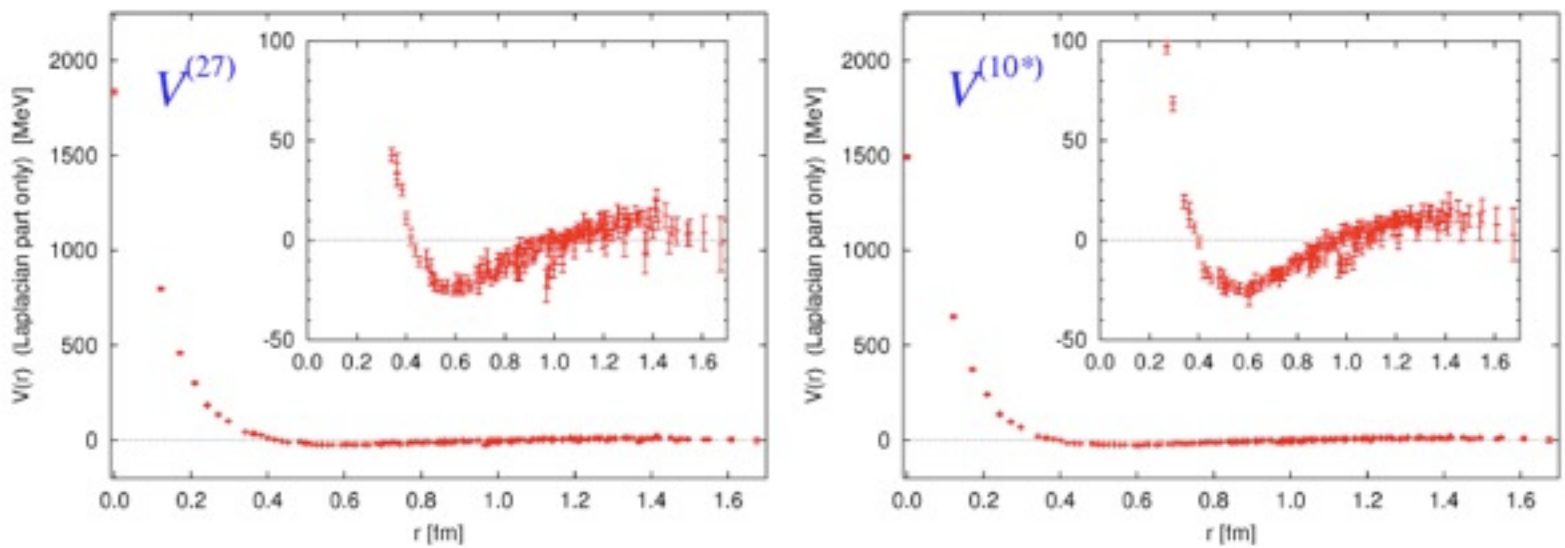
HAL QCD Collaboration
by T. Inoue

Lattice QCD : SU(3) limit

- NN potential (I=1) $V^{(27)}$ and (I=0) $V^{(10*)}$

$V^{(27)}$ and $V^{(10*)}$

HAL QCD Collaboration
by T. Inoue

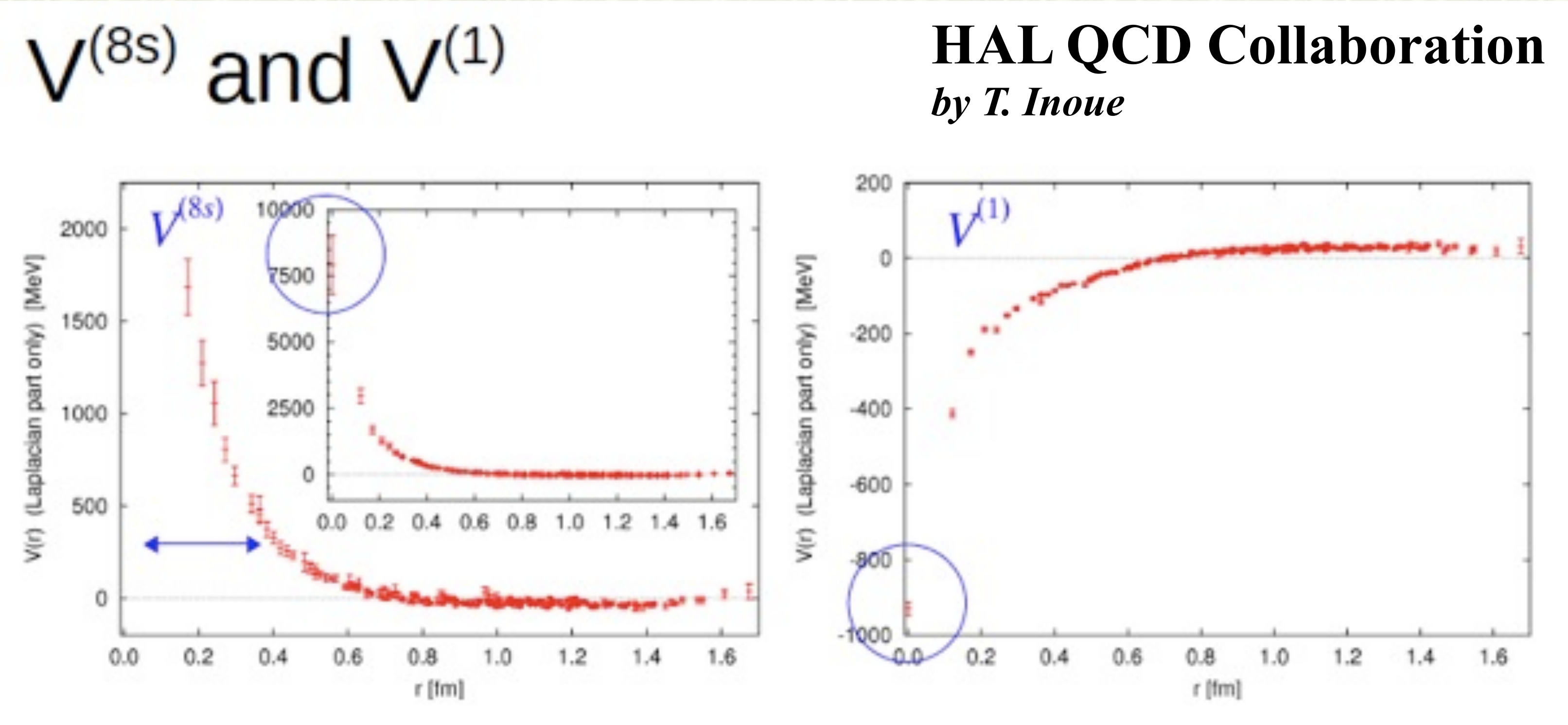


NN singlet even $^1S_0$

NN triplet even $^3S_1$
Deuteron is not 6-quark

Lattice QCD : SU(3) limit

- $V^{(8s)}$ and $V^{(1)}$

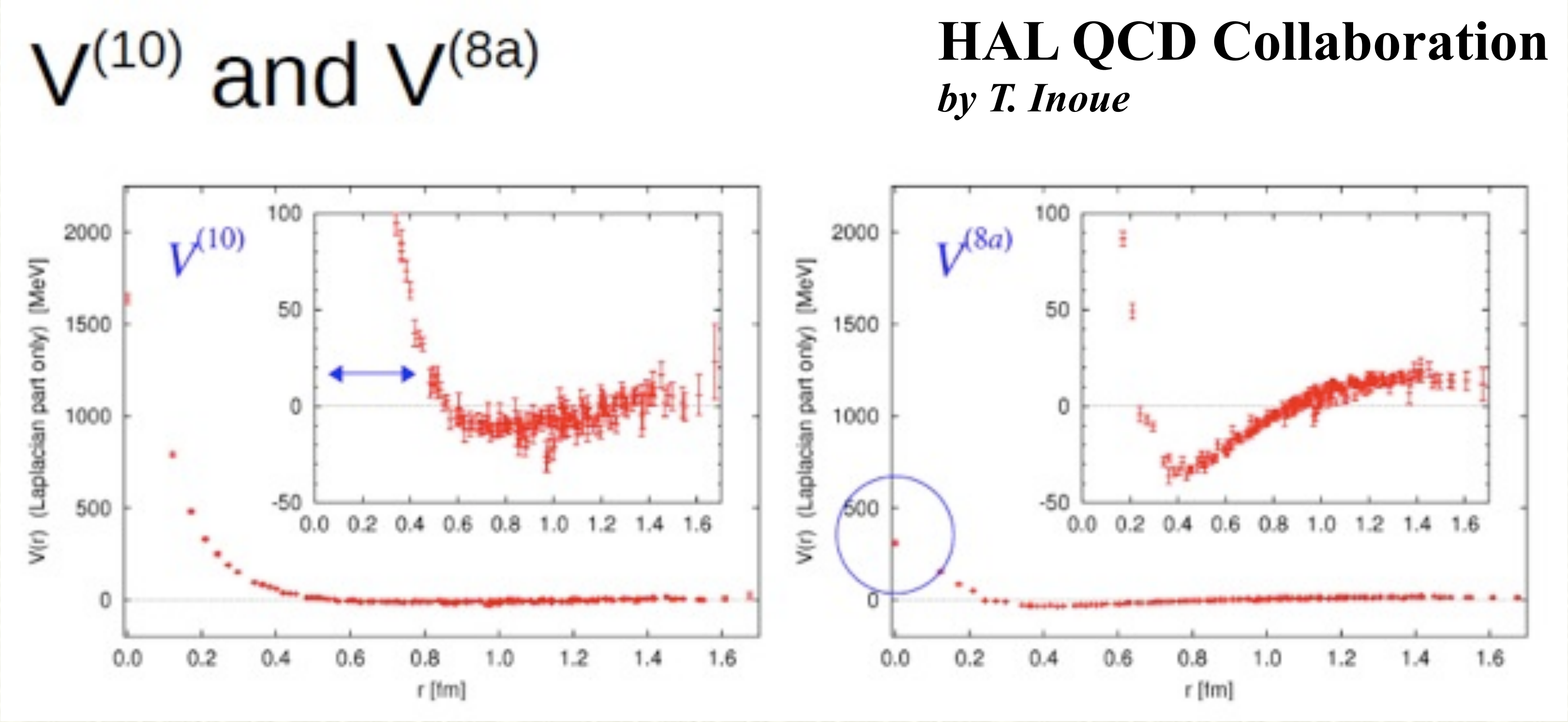


SU(6) pure [51]: Pauli-repulsion
NΣ (I=1/2, J=0) almost forbidden

SU(6) pure [33]
SR attraction

Lattice QCD : SU(3) limit

- $V^{(10)}$ and $V^{(8a)}$



NΣ(I=3/2, J=1) almost Pauli-forbidden

QCD Sum Rules

QCD Sum Rules

Correlation function of composite operators

$$\Pi(p) \equiv i \int d^4x e^{ip\cdot x} \langle 0 | T(J(x)J(0)) | 0 \rangle$$

$J(x)$: Interpolating field (= composite operator)

(1) OPE (Operator Product Expansion) side

$$\Pi(p_E^2) = \sum_n C_n(p_E^2) \langle 0 | O_n(0) | 0 \rangle \qquad O_n \text{ : Local operator}$$

Deep Euclid region $\quad p_E^2 \equiv -p^2 \to \infty$

$\alpha_s(p_E^2) \to 1/\ln(p_E^2) \Rightarrow$ *perturbative*

Nonperturbative effects are taken into account as vacuum condensates

OPE expansion w.r.t. $1/p_E^2$

QCD Sum Rules

Operator Product Expansion for ϕ^4 scalar field theory

$$T[\phi(x)\phi(y)] \xrightarrow{x\to 0} \underbrace{\langle 0|T[\phi(x)\phi(0)]|0\rangle}_{\text{singular part (c-number)}} + \underbrace{:\phi(x)\phi(0):}_{\text{non-singular (normal product)}}$$

Free scalar field

$$\begin{aligned}
-i\Delta(x) &= \langle 0|T[\phi(x)\phi(0)]|0\rangle \\
&= \int \frac{e^{-ip\cdot x}}{p^2 - m^2 + i\epsilon}\frac{d^4p}{(2\pi)^4} \sim 2\pi i \int \frac{1}{2E_p} e^{i\vec{p}\cdot\vec{x} - iE_p t}\frac{d^3\vec{p}}{(2\pi)^3} \\
&\xrightarrow{m\to 0} \frac{1}{(2\pi)^2 2r}\int_0^\infty (e^{ipr} - e^{-ipr})e^{-ipt}dp \sim \frac{i}{(2\pi)^2 x^2}
\end{aligned}$$

QCD Sum Rules

OPE in ϕ^4 theory

$$\mathcal{L} = \frac{1}{2}(\partial_\mu \phi)^2 - \frac{1}{2}m^2\phi^2 - \frac{\lambda}{4!}\phi^4$$

$$-i\Delta(p) = \int \langle 0|T[\phi(x)\phi(0)]|0\rangle e^{ip\cdot x} d^4x \sim C_1(p^2) + C_{\phi^2}(p^2)\langle 0| : \phi^2(0) : |0\rangle + \dots$$

$$C_1(p^2) = \frac{1}{p^2 - m^2} + \frac{\lambda}{2(4\pi)^2}\frac{\Lambda^2}{(p^2-m^2)^2} + \dots \sim \frac{1}{p^2} + \frac{m^2}{p^4}(1 + \frac{\lambda}{32\pi^2}\frac{\Lambda^2}{m^2} + \dots)$$

$$C_{\phi^2}(p^2) = \frac{\lambda}{2}\frac{1}{(p^2-m^2)^2} + \dots \sim \frac{\lambda}{2p^4} + \dots$$

$$\int T[\phi(x)\phi(0)]e^{ip\cdot x}d^4x \sim C_1(p^2) + C_{\phi^2}(p^2) : \phi^2(0) : + \dots$$

QCD Sum Rules

quark propagator

$$iS(x,y) = iS_0(x-y) + g\int d^4z iS_0(x-z)(i\gamma\cdot A(z))iS_0(x-y) + O(g^2)$$

$$\text{fixed point gauge} \quad x\cdot A(x) = 0$$

$$x^\nu G_{\nu\mu} = x^\nu \partial_\nu A_\mu + A_\mu - \partial_\nu(x^\nu A_\mu)$$

$$x \to \alpha x \quad \text{and} \quad \int_0^1 d\alpha$$

$$A_\mu(x) = x^\nu \int_0^1 d\alpha\alpha G_{\nu\mu}(\alpha x) = \frac{1}{2}x_\nu G_{\nu\mu}(0) + \ldots$$

$$iS(x) = \frac{i}{2\pi^2}\frac{1}{(x^2)^2}(\gamma\cdot x) - \frac{1}{12}\langle\bar{q}q\rangle - \frac{1}{192}x^2\langle\bar{q}g\sigma\cdot Gq\rangle + \ldots$$

QCD Sum Rules

Interpolating field operators

Mesons

$$J_\rho(x) = \bar{q}(x)\gamma^\mu \frac{\vec{\tau}}{2} q(x)$$

I=1 vector meson

$$J_\pi(x) = \bar{q}(x)\gamma^5 \frac{\vec{\tau}}{2} q(x)$$

I=1 pseudoscalar meson

Baryons (nucleon)

$$J_N^\alpha(x) = \varepsilon_{abc}[(u_a(x)Cd_b(x))(\gamma_5 u_c(x))^\alpha + t(u_a(x)C\gamma_5 d_b(x))u_c^\alpha(x)],$$

OPE of nucleon correlator (at ***p*** =0)

$$\begin{aligned}
\mathrm{Im}A^{\mathrm{OPE}}(p_0) &= \frac{5+2t+5t^2}{2^{11}\pi^4} p_0^5 \theta(p_0) + \frac{5+2t+5t^2}{2^9\pi^2} p_0 \theta(p_0) \langle \frac{\alpha_s}{\pi} GG \rangle + \\
&\quad \frac{7t^2-2t-5}{12} \frac{1}{2} \delta(p_0) \langle \bar{q}q \rangle^2, \\
\mathrm{Im}B^{\mathrm{OPE}}(p_0) &= -\frac{7t^2-2t-5}{32\pi^2} p_0^2 \theta(p_0) \langle \bar{q}q \rangle + \frac{3(t^2-1)}{32\pi^2} \theta(p_0) \langle \bar{q}g\sigma \cdot Gq \rangle.
\end{aligned}$$

QCD Sum Rules

Correlation function of composite operators

(2) Phenomenological side

in terms of the spectral function at $s = p^2 = m^2$

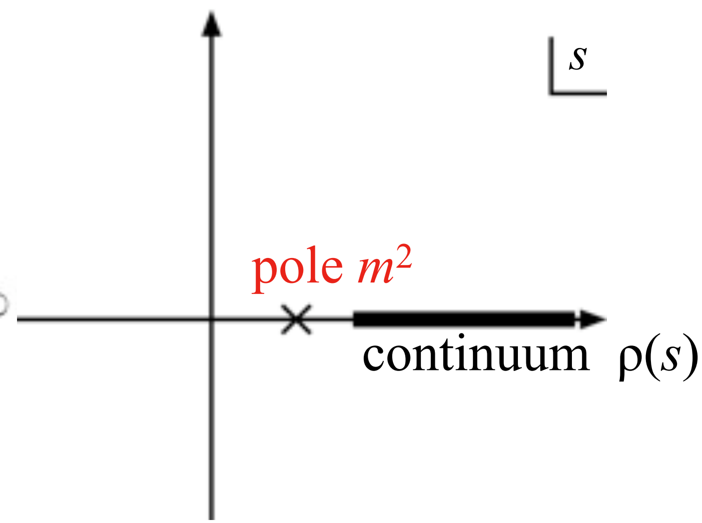
$$\Pi(p^2) = \frac{1}{\pi}\int ds \frac{\rho(s)}{s - p^2}$$

$$\rho(s) = \lambda\delta(s - m^2) + \theta(s - s_0)\rho(s)$$

$\rho(s)$: Spectral function

s_0 : Continuum threshold

$$p_E^2 \equiv -p^2 \to \infty$$



QCD Sum Rules

Correlation function

$$\Pi(p^2) = i \int d^4x e^{ip\cdot x} \langle 0|T[J(x)\bar{J}(0)]|0\rangle$$

$$= i \int d^4x e^{ip\cdot x} \left\{ \theta(x^0) \sum_{m(\vec{q})} \langle 0|J(x)|m(\vec{q})\rangle \langle m(\vec{q})|\bar{J}(0)|0\rangle \right.$$

$$\left. + \theta(-x^0) \sum_{m(\vec{q})} \langle 0|\bar{J}(0)|\bar{m}(\vec{q})\rangle \langle \bar{m}(\vec{q})|J(x)|0\rangle \right\}$$

$$= i \int d^4x e^{ip\cdot x} \sum_m \int \frac{d^3\vec{q}}{(2\pi)^3} \left\{ \theta(x^0) \langle 0|J(x)|m(\vec{q})\rangle \langle m(\vec{q})|\bar{J}(0)|0\rangle \right.$$

$$\left. + \theta(-x^0) \langle 0|\bar{J}(0)|\bar{m}(\vec{q})\rangle \langle \bar{m}(\vec{q})|J(x)|0\rangle \right\}$$

$$= \sum_m \int \frac{d^3\vec{q}}{(2\pi)^3} i \int d^4x e^{ip\cdot x} \left\{ \theta(x^0) e^{-iq\cdot x} \langle 0|J(0)|m\rangle \langle m|\bar{J}(0)|0\rangle \right.$$

$$\left. + \theta(-x^0) e^{iq\cdot x} \langle 0|\bar{J}(0)|\bar{m}\rangle \langle \bar{m}|J(0)|0\rangle \right\}$$

$$= \sum_m |\langle 0|J(0)|m\rangle|^2 \frac{-2E_m}{p^2 - m^2 + i\epsilon} \qquad E_m \equiv \sqrt{\vec{p}^2 + m^2}$$

QCD Sum Rules

$$\rho(p^2) \equiv \mathrm{Im}\Pi(p^2) = \sum_m 2E_m |\langle 0|J(0)|m\rangle|^2 \pi\delta(p^2 - m^2)$$

Dispersion relation

$$\Pi(p^2) = \frac{1}{\pi}\int_0^\infty \frac{\rho(s)}{s-p^2} ds$$

Subtraction

$$\frac{1}{s-p^2} = \frac{1}{s} + \frac{p^2}{s(s-p^2)}$$

$$\Pi(p^2) = \frac{p^2}{\pi}\int_0^\infty \frac{\rho(s)}{s(s-p^2)} ds + \Pi(0)$$

$$\Pi(p^2) = \frac{p^{2n}}{\pi}\int_0^\infty \frac{\rho(s)}{s^{2n}(s-p^2)} ds + \sum_{k=0}^{n-1} a_k p^{2k}$$

QCD Sum Rules

(3) Analyticity of the correlator
dispersion relation

$$\Pi(p^2) = \frac{1}{\pi}\int_0^\infty ds \frac{\mathrm{Im}\Pi(s)}{s-p^2}$$

QCD duality threshold s_0

$$\mathrm{Im}\Pi^{\mathrm{OPE}}(s) = \mathrm{Im}\Pi^{\mathrm{PH}}(s) \quad \text{for } s > s_0$$

$$\int_0^{s_0} \frac{\mathrm{Im}\Pi^{\mathrm{OPE}}(s)}{s-p^2} ds = \int_0^{s_0} \frac{\mathrm{Im}\Pi^{\mathrm{PH}}(s)}{s-p^2} ds$$

QCD Sum Rules

(4) To improve:

Borel transformation M^2

$$\Pi(p^2 = -p_E^2) \to \mathcal{B}_{M^2}\Pi \equiv \tilde{\Pi}(M^2) = \lim_{p_E^2, n\to\infty, M^2 \equiv p_E^2/n = \text{finite}} \frac{(p_E^2)^{n+1}}{n!}\left(-\frac{d}{dp_E^2}\right)^n \Pi(p_E^2)$$

Borel sum rule for the imaginary part of Π ($s = p_E^2$)

$$\mathcal{B}_{M^2}\int_0^{s_0}\frac{\text{Im}\Pi(s)}{s+p_E^2}ds = \int_0^{s_0} e^{-s/M^2}\text{Im}\Pi(s)ds$$

$$\int_0^{s_0} e^{-s/M^2}\text{Im}\Pi^{\text{OPE}}(s)ds = \int_0^{s_0} e^{-s/M^2}\text{Im}\Pi^{\text{PH}}(s)ds$$

QCD Sum Rules

Borel transform

$$\tilde{\Pi}(M^2) = \lim_{\substack{-p^2, n \to \infty \\ M^2 = -p^2/n}} \frac{(-p^2)^{n+1}}{n!} \left(-\frac{d}{dp^2}\right)^n \Pi(p^2)$$

$$\frac{1}{(-p^2)^n} \longrightarrow \frac{1}{(n-1)!(M^2)^{n-1}}$$

$$\frac{1}{(-p^2+m^2)^n} \longrightarrow \frac{1}{(n-1)!(M^2)^{n-1}} e^{-m^2/M^2}$$

$$\int ds \frac{\rho(s)}{-p^2+s} \longrightarrow \int ds\, e^{-s/M^2} \rho(s)$$

QCD Sum Rules

Sum rule for the ρ meson

L.J. Reinders et al., *Hadron properties from QCD sum rules* 55

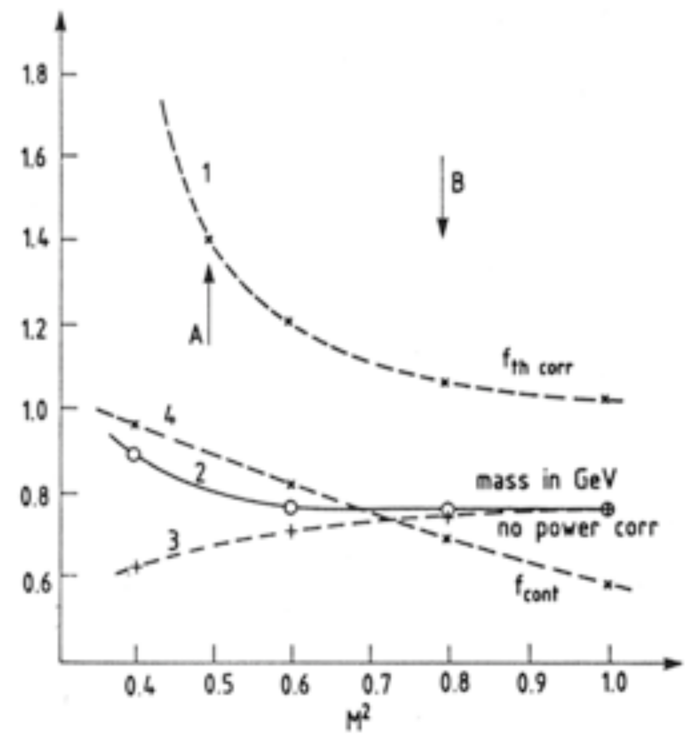


Fig. 11. The ρ meson mass with and without power corrections. The continuum threshold $s_0 = 1.5\,\mathrm{GeV}^2$. Also shown are the functions f_{cont} and $f_{th\,corr}$ defined in the text. The region between the arrows A and B is considered to be reliable for determining the resonance parameters. Figure adopted from [1].

QCD Sum Rules

- Parity projection for a $J = 1/2$ Baryon

$$
\begin{aligned}
J(t,\vec{x}) &\rightarrow +\gamma^0 J(t,-\vec{x}) \\
\gamma^5 J(t,\vec{x}) &\rightarrow +\gamma^5\gamma^0 J(t,-\vec{x}) = -\gamma^0\gamma^5 J(t,-\vec{x})
\end{aligned}
$$

$$
\begin{aligned}
\langle 0|J(x)|B^+(\vec{p})\rangle &= \lambda_+ u_+(\vec{p})e^{-ip\cdot x} \\
\langle 0|J(x)|B^-(\vec{p})\rangle &= \lambda_- \gamma^5 u_-(\vec{p})e^{-ip\cdot x}
\end{aligned}
$$

$$
\begin{aligned}
\Pi_T(q) &= \int d^4x\, e^{iq\cdot x} i\langle 0|T(J(x)\bar{J}(0))|0\rangle \\
&= -\int dm_+ \frac{\rho^+(m_+)}{\not{q} - m_+} + \int dm_- \gamma^5 \frac{\rho^-(m_-)}{\not{q} - m_-}\gamma^5 \\
&= -\int dm_+ \frac{\rho^+(m_+)}{\not{q} - m_+} - \int dm_- \frac{\rho^-(m_-)}{\not{q} + m_-}
\end{aligned}
$$

QCD Sum Rules

- Time ordered correlation function

D. Jido, N. Kodama, M.Oka, PR D54 (1996)

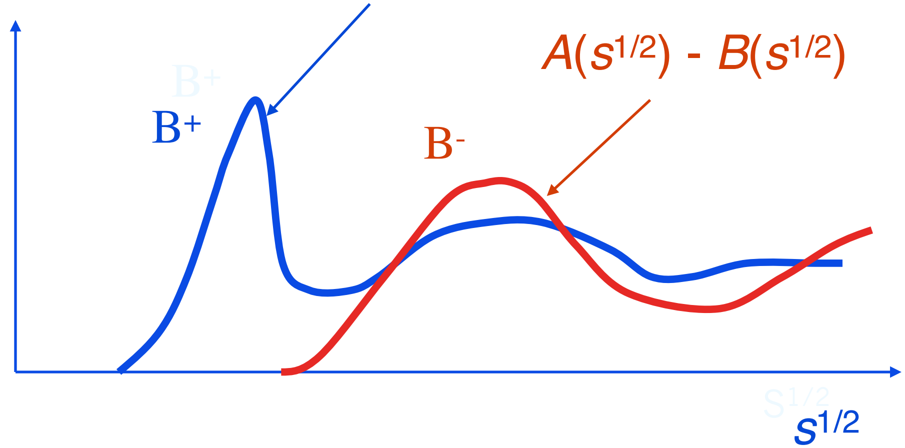
$$\Pi(p) = i \int d^4x e^{ip\cdot x}\theta(x_0)\langle 0|J_B(x)\bar{J}_B(0)|0\rangle$$

Rest frame: $p = (p_0, 0)$

$$\begin{aligned} \mathrm{Im}\,\Pi(p_0) &= (\lambda_+)^2\frac{\gamma_0+1}{2}\delta(p_0-m_+) + (\lambda_-)^2\frac{\gamma_0-1}{2}\delta(p_0-m_-) \\ &\quad + \cdots \text{(continuum)} \\ &\equiv \gamma_0 A(p_0) + B(p_0). \end{aligned}$$

QCD Sum Rules

Spectral function



A : chiral even terms
B : chiral odd terms

B+ - B- mass difference
is induced by nonzero $B(p^0)$

Borel Sum Rules for m_+ and m_-

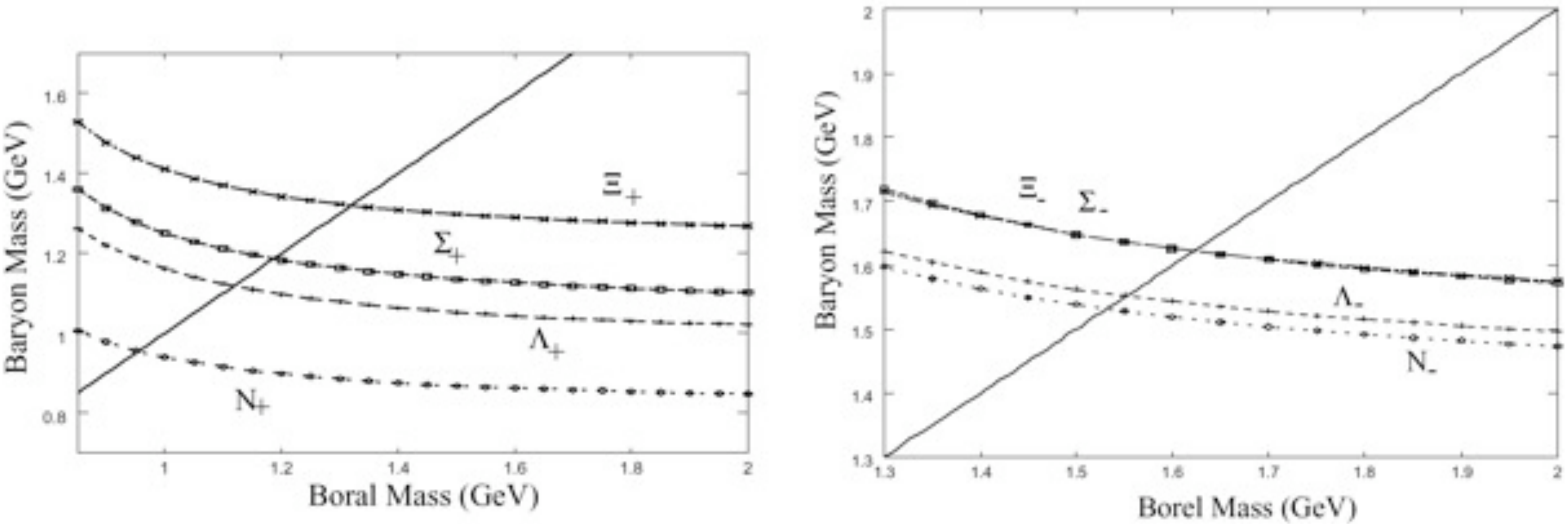
$$\int_0^{s_+} [A^{\rm OPE}(p_0) + B^{\rm OPE}(p_0)] \exp\left[-\frac{p_0^2}{M^2}\right] dp_0 = (\lambda_+)^2 \exp\left[-\frac{m_+^2}{M^2}\right],$$

$$\int_0^{s_-} [A^{\rm OPE}(p_0) - B^{\rm OPE}(p_0)] \exp\left[-\frac{p_0^2}{M^2}\right] dp_0 = (\lambda_-)^2 \exp\left[-\frac{m_-^2}{M^2}\right],$$

QCD Sum Rules

Baryon Masses from QCD Sum Rules

D. Jido, M. Oka, A. Hosaka, NP A629 (1998)



Unit: GeV

Baryon	N_+	Λ_+	Σ_+	Ξ_+	N_-	Λ_-	Σ_-	Ξ_-	Λ_{S-}	Λ_{S+}
Sum rule	0.94	1.12	1.21	1.32	1.54	1.55	1.63	1.63	1.31	2.94
Exp.	0.94	1.12	1.19	1.32	1.535	1.67	1.62	—	1.405	—

Bayesian Inference of Spectral Function in the QCD Sum Rules

P. Gubler, M.O., Prog. Theor. Phys., 124 (2010) 995,

arXiv: 1005.2459v1

P. Gubler, K. Morita and M.O., Phys. Rev. Lett., in print,

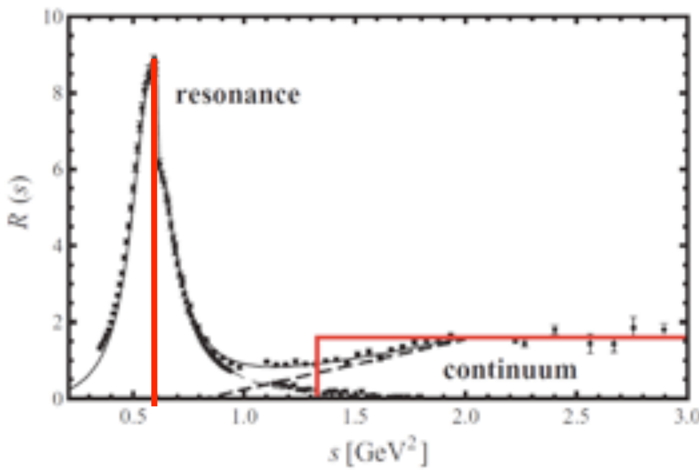
arXiv:1104.4436 [hep-ph].

K. Ohtani, P. Gubler, M.O., To be published, arXiv:1104.5577 [hep-ph]

K. Suzuki, P. Gubler, K. Morita and M.O., inpreparation.

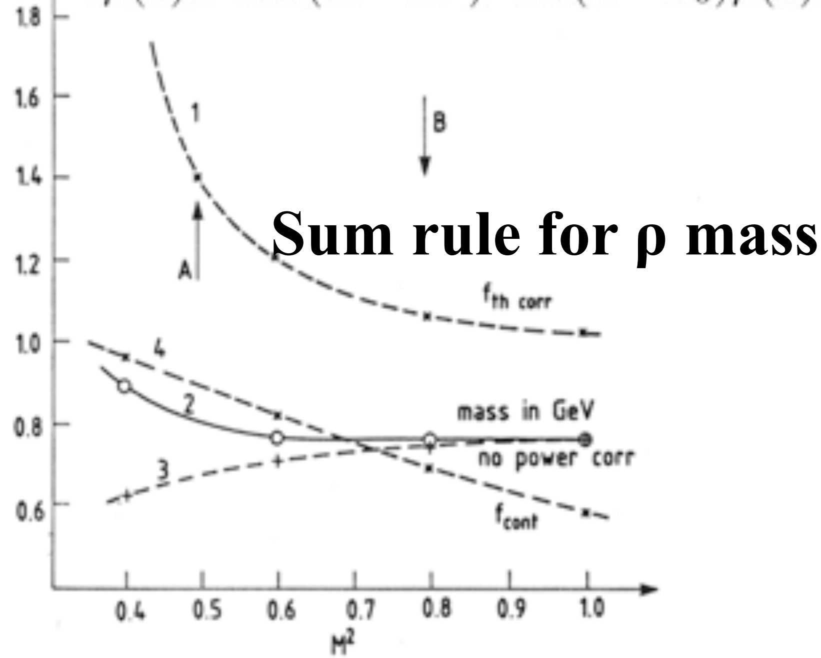
Bayesian Inference of Spectral Function in the QCD Sum Rules

One of the first and most successful QCD sum rule so far is for the ρ meson mass, where the "pole + continuum" ansatz works well.



Y. Kwon, M. Procura, and W. Weise, PRC 78 (2008) 055203.

$$\rho(s) = \lambda\delta(s - m^2) + \theta(s - s_0)\rho(s)$$



by Reinders, Rubinstein, Yazaki

Bayesian Inference of Spectral Function in the QCD Sum Rules

- **We use the Bayesian inference theory to obtain the most probable spectral function that satisfies the sum rule.**
- **The same method, Maximum Entropy Method (MEM) was formulated to obtain the spectral function from the lattice QCD data.**

  Asakawa, Hatsuda, Nakahara, Prog.Part.Nucl.Phys.46 (2001) 459-508.

Bayesian Inference of Spectral Function in the QCD Sum Rules

The Borel sum rule is reduced to a mathematical problem to invert the integral relation:

$$G(M) = \int_0^\infty d\omega\, K(M,\omega)\rho(\omega) \qquad \rho(\omega) \geq 0$$

$$K(M,\omega) = \frac{2\omega}{M^2} e^{-\omega^2/M^2}$$

insensitive to ω=0

***G*(*M*) is given by the QCD OPE, and ρ(ω) is estimated.**

A similar problem in the lattice QCD:

by Asakawa, Hatsuda, Nakahara

$$G(\tau) = \sum_{\vec{x}} \langle 0|O(\vec{x},\tau)O^\dagger(0,0)|0\rangle = \int_0^\infty d\omega K(\tau,\omega)\rho(\omega)$$

$$K(\tau,\omega) = e^{-\omega\tau}$$

Maximum Entropy Method

Bayes' Theorem

$$P[\rho, G|I] = P[\rho|G, I]P[G|I] = P[G|\rho, I]P[\rho|I]$$

$$\longrightarrow \quad P[\rho|G, I] = P[G|\rho, I]P[\rho|I]/P[G|I]$$

P[A|B] denotes the conditional probability of A given B. ρ=ρ(ω) is the spectral function to be estimated, G=G(M) is the OPE result, and I is the other general condition for the spectral function, in particular its positivity. P[G| ρ, I] is the <u>likelihood function</u> and P[ρ| I] is called <u>prior probability</u>.

To obtain the most probable spectral function, we find the maximum of P[ρ|G, I].

Maximum Entropy Method

Likelihood function

We assume the Gaussian distribution, similarly to the χ^2 fitting.

$$P[G|\rho, I] = Z_L^{-1} e^{-L[\rho]}$$

$$L[\rho] = \frac{1}{2(M_{\rm max} - M_{\rm min})} \int_{M_{\rm min}}^{M_{\rm max}} dM \, \frac{[G(M) - G_{\rm OPE}(M)]^2}{\sigma^2(M)}$$

$$G(M) = \int_0^\infty d\omega \, K(M,\omega)\rho(\omega)$$

Prior probability

$$P[\rho|I] = Z_s^{-1} e^{\alpha S[\rho]}$$

$$S[\rho] = \int_0^\infty d\omega \Big[\rho(\omega) - m(\omega) - \rho(\omega) \log\Big(\frac{\rho(\omega)}{m(\omega)}\Big)\Big]$$

Shannon-Jaynes entropy

$m(\omega)$: default model, which maximize the entropy.

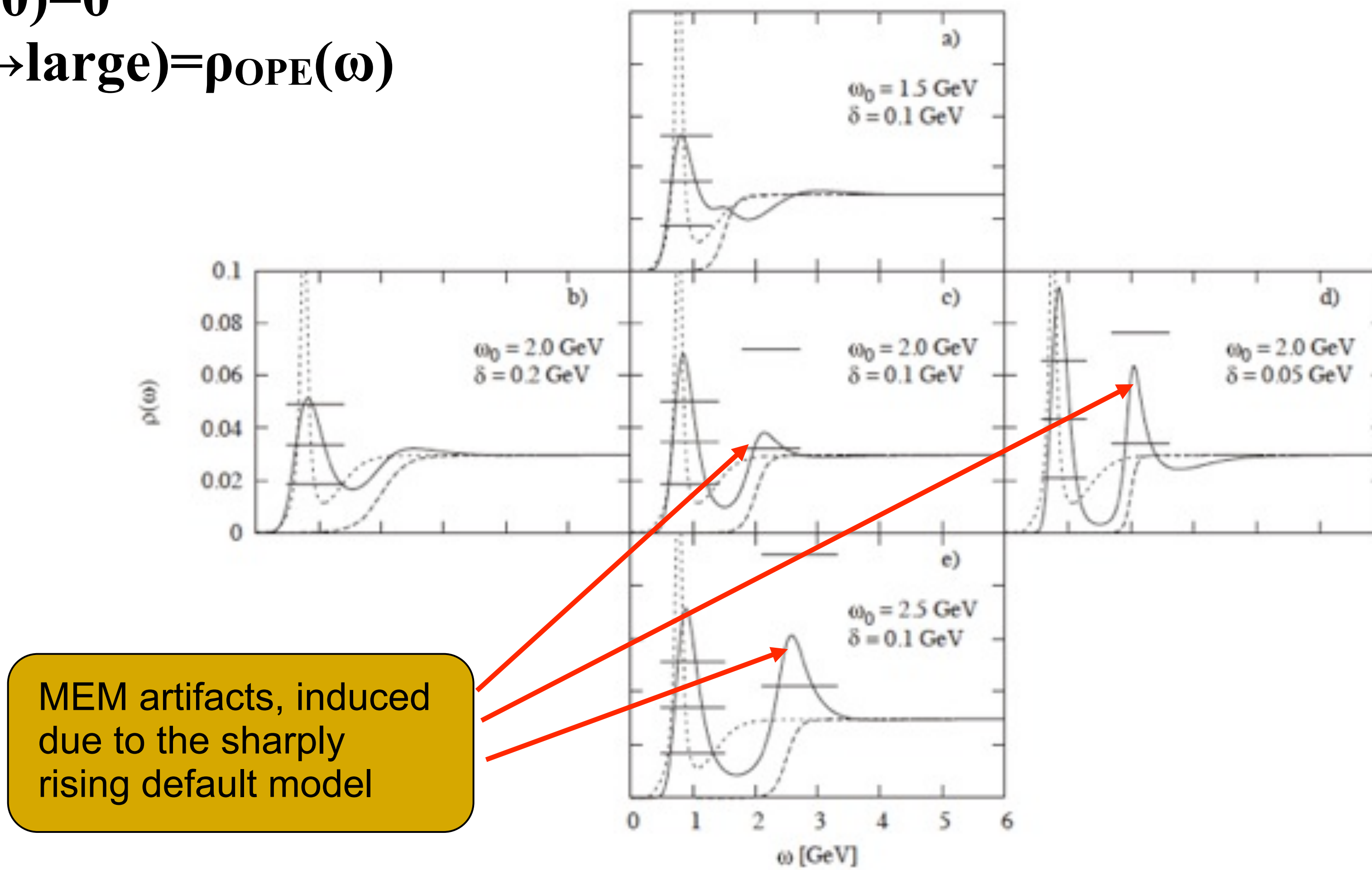
Default model

$$m(\omega) = (1 + \frac{\alpha_s}{\pi})\frac{1}{1+e^{(\omega_0-\omega)/\delta}}$$

conditions required

$m(\omega=0)=0$

$m(\omega\rightarrow\text{large})=\rho_{OPE}(\omega)$



P. Gubler, M.O., ArXiv: 1005.2459v1

Application to charmonium at finite temperature

- Prediction of "J/ψ Suppression by Quark-Gluon Plasma Formation"

T. Matsui and H. Satz, Phys. Lett. B 178, 416 (1986).

…

- During the last 10 years, a picture has emerged from studies using lattice QCD (and MEM), where J/ψ survives above T_C.

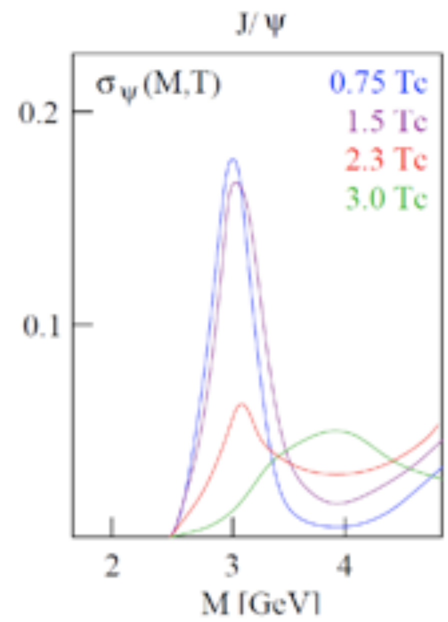
M. Asakawa and T. Hatsuda, Phys. Rev. Lett. 92 012001 (2004).

S. Datta *et al*, Phys. Rev. D69, 094507 (2004).

T. Umeda *et al*, Eur. Phys. J. C39S1, 9 (2005).

A. Jakovác *et al*, Phys. Rev. D75, 014506 (2007).

…



(schematic)

taken from
H. Satz, Nucl.Part.Phys. **32**, 25 (2006).

The charmonium sum rules at T=0

The sum rule:

$$M(\nu) = \int_0^\infty e^{-\nu t} \rho(4m_c^2 t) dt \qquad (\nu \equiv \frac{M^2}{4m_c^2})$$

$$M(\nu) = A(\nu)\left[1 + a(\nu)\alpha_s(\nu) + b(\nu)\frac{\langle \frac{\alpha_s}{\pi} G^2 \rangle}{m_c^4} + d(\nu)\frac{\langle g^3 G^3 \rangle}{m_c^6}\right]$$

perturbative term including α_s correction

Non-perturbative corrections including condensates up to dim 6

Developed and analyzed in:

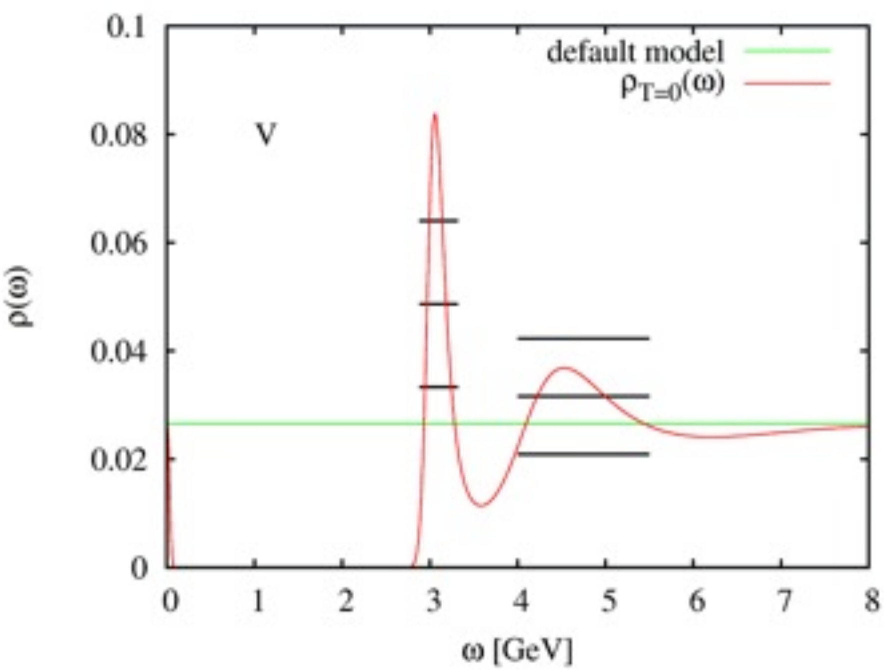
M.A. Shifman, A.I. Vainshtein and V.I. Zakharov, Nucl. Phys. B147, 385 (1979); B147, 448 (1979).

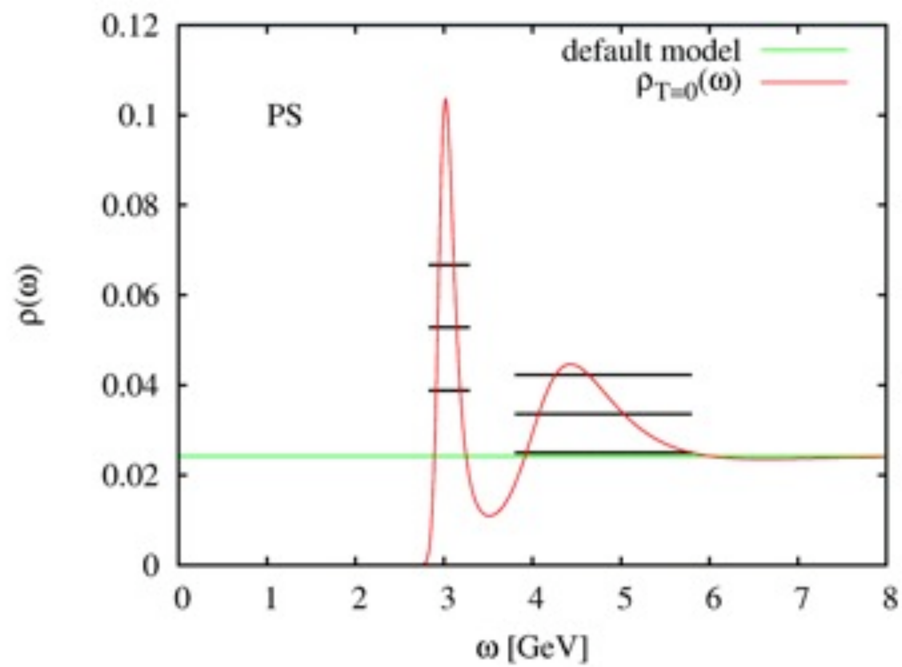
L.J. Reinders, H.R. Rubinstein and S. Yazaki, Nucl. Phys. B 186, 109 (1981).

R.A. Bertlmann, Nucl. Phys. B 204, 387 (1982).

J. Marrow, J. Parker and G. Shaw, Z. Phys. C 37, 103 (1987).

MEM Analysis at T=0





$m_{J/\psi}$=3.06 GeV

$m_{\eta c}$=3.02 GeV

Here, the following values were used:

$$\langle \frac{\alpha_s}{\pi} G^2 \rangle = 0.012 \pm 0.0036 \text{ GeV}^4$$

$$m_c = 1.277 \pm 0.026 \text{ GeV}$$

Experiment:

$m_{J/\psi}$= 3.10 GeV

$m_{\eta c}$= 2.98 GeV

The charmonium sum rules at finite T

The application of QCD sum rules has been developed in:

T.Hatsuda, Y.Koike and S.H. Lee, Nucl. Phys. B 394, 221 (1993).

$$M(\nu) = \int_0^\infty e^{-\nu t} \rho(4m_c^2 t) dt \qquad (\nu \equiv \frac{M^2}{4m_c^2})$$

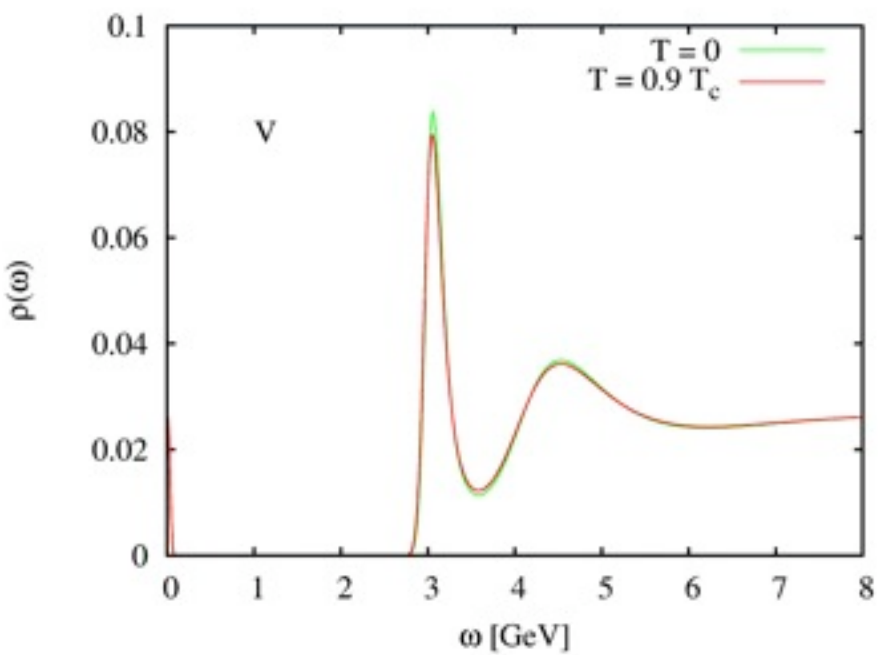
$$M(\nu) = A(\nu)\Big[1 + a(\nu)\alpha_s(\nu) + b(\nu)\frac{\langle\frac{\alpha_s}{\pi}G^2\rangle_T}{m_c^4} + c_n(\nu)\frac{\langle\frac{\alpha_s}{\pi}G^2\rangle_{T,2}}{m_c^4} + d(\nu)\frac{\langle g^3 G^3\rangle_T}{m_c^6}\Big]$$

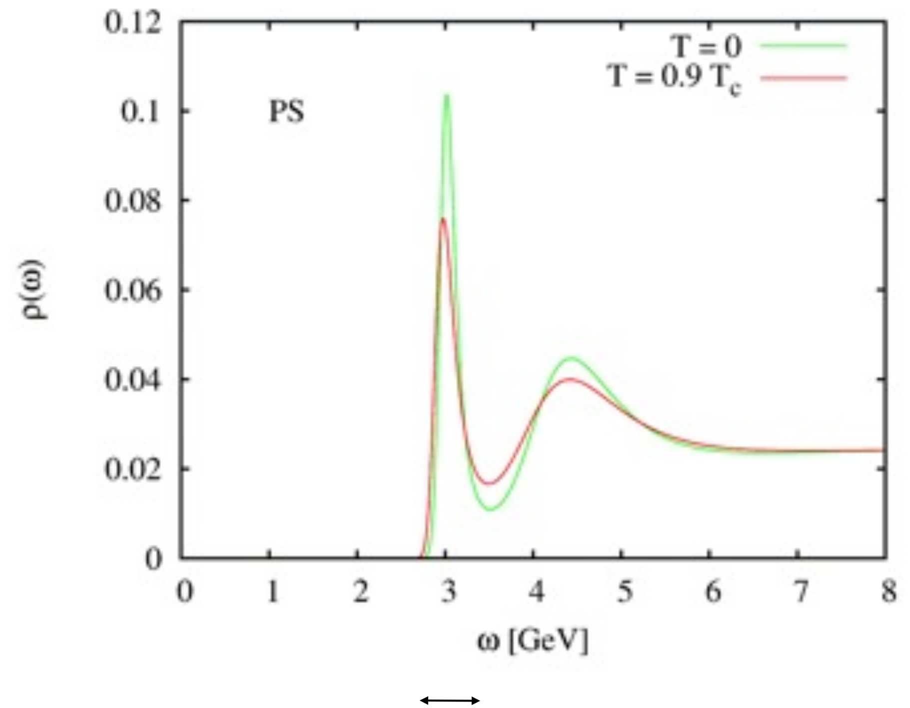
depend on T

$$\langle\frac{\alpha_s}{\pi}G^2\rangle_T = \langle\frac{\alpha_s}{\pi}G^2\rangle \text{vac.} - \frac{8}{11}(\epsilon - 3p)$$

$$\langle\frac{\alpha_s}{\pi}G^2\rangle_{T,2} = -\frac{\alpha_s(T)}{\pi}(\epsilon + p)$$

The charmonium spectral function at finite T

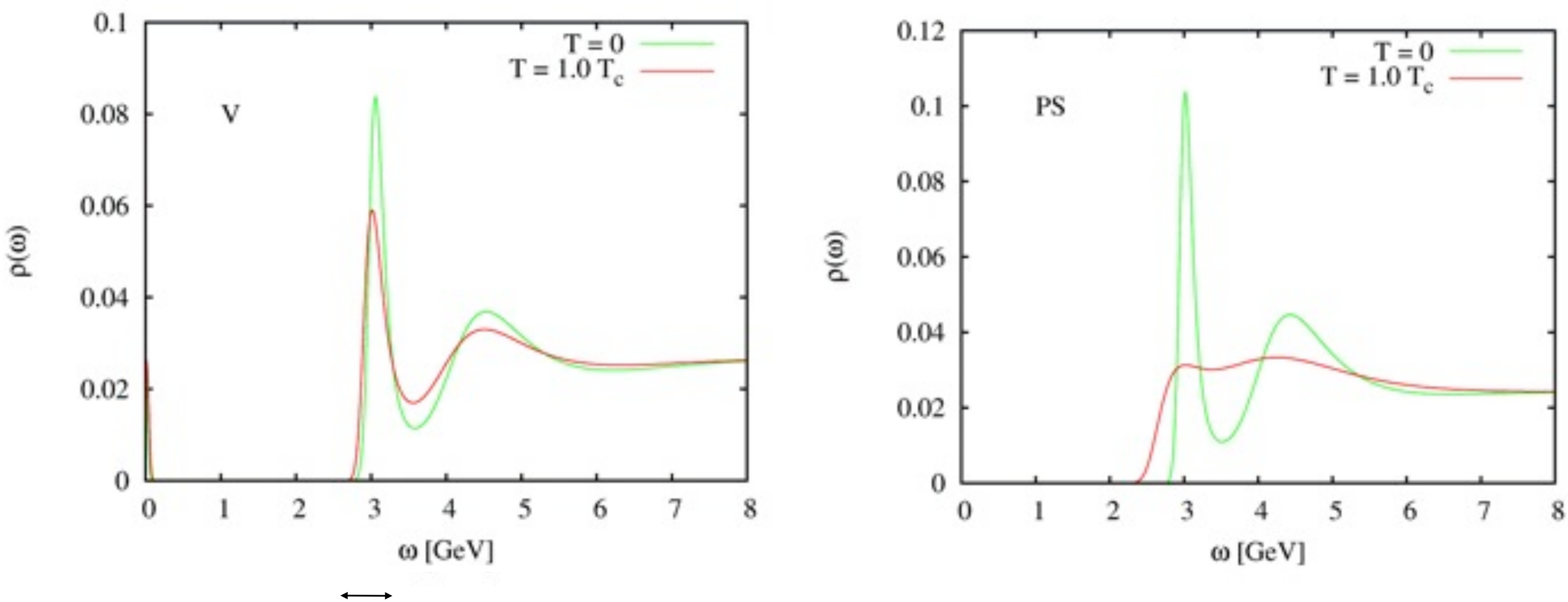




negative shift of ~40 MeV,

consistent with the analysis of Morita and Lee.

The charmonium spectral function at finite T

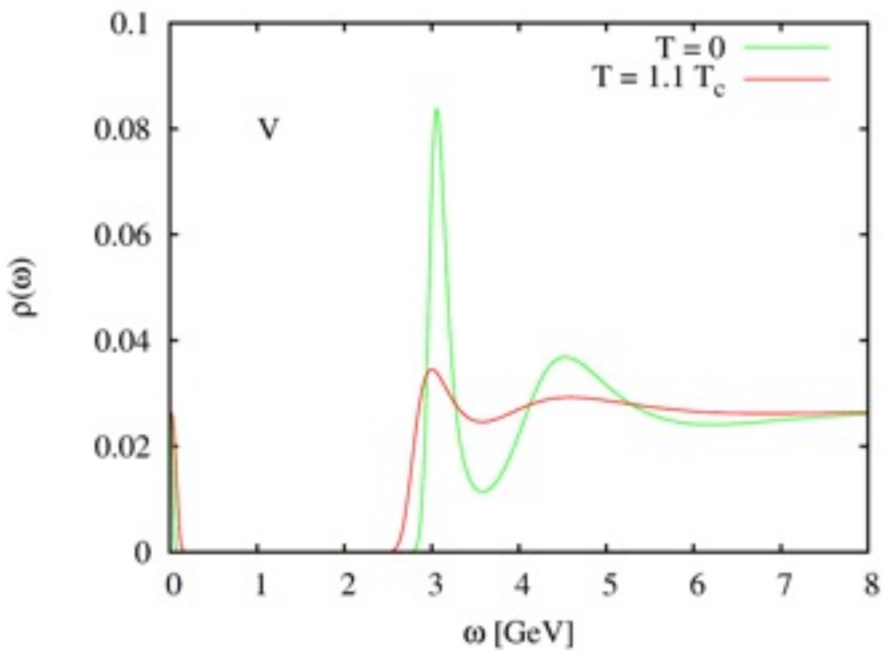


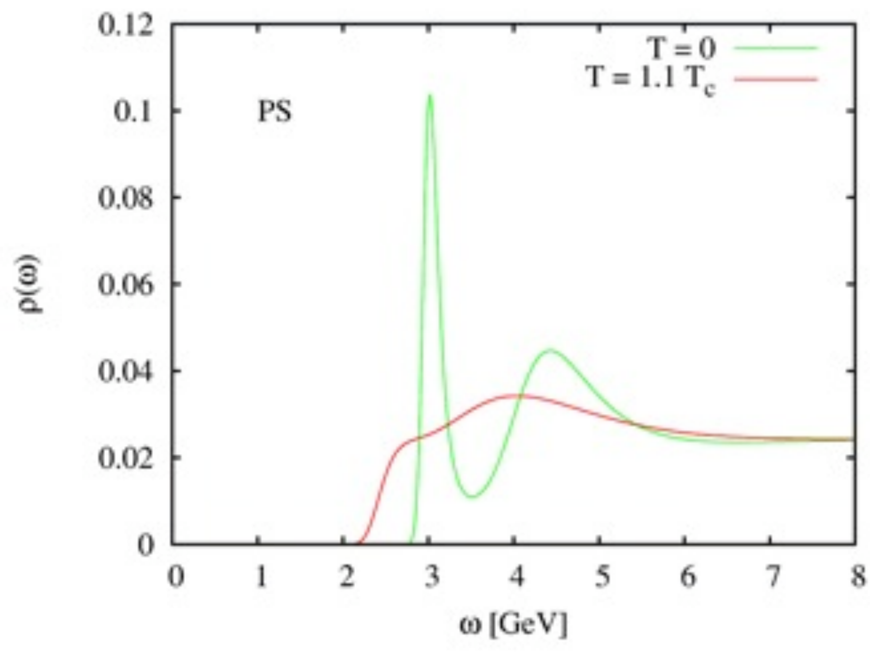
negative shift of ~50 MeV,

consistent with the analysis of Morita and Lee.

Melting

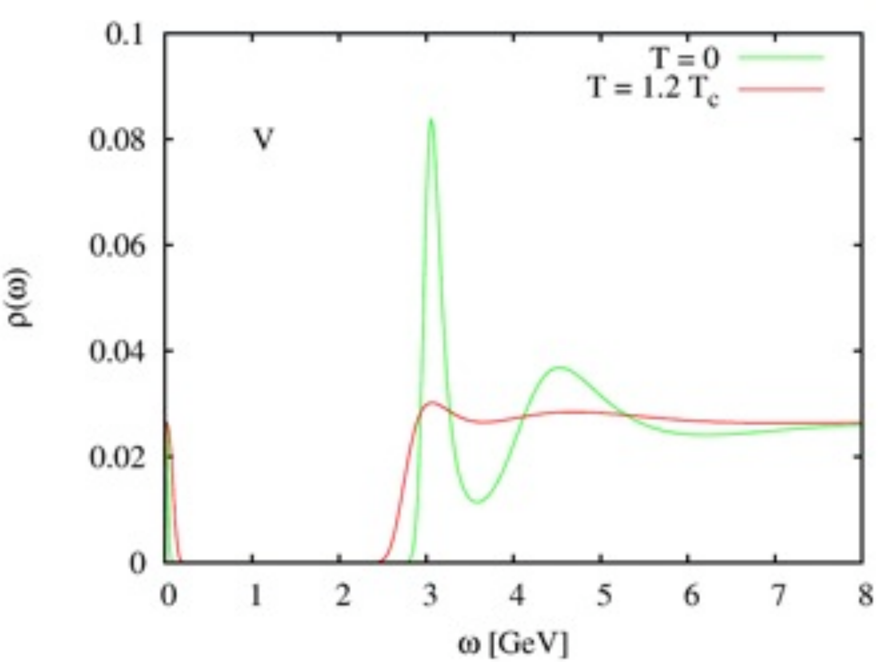
The charmonium spectral function at finite T

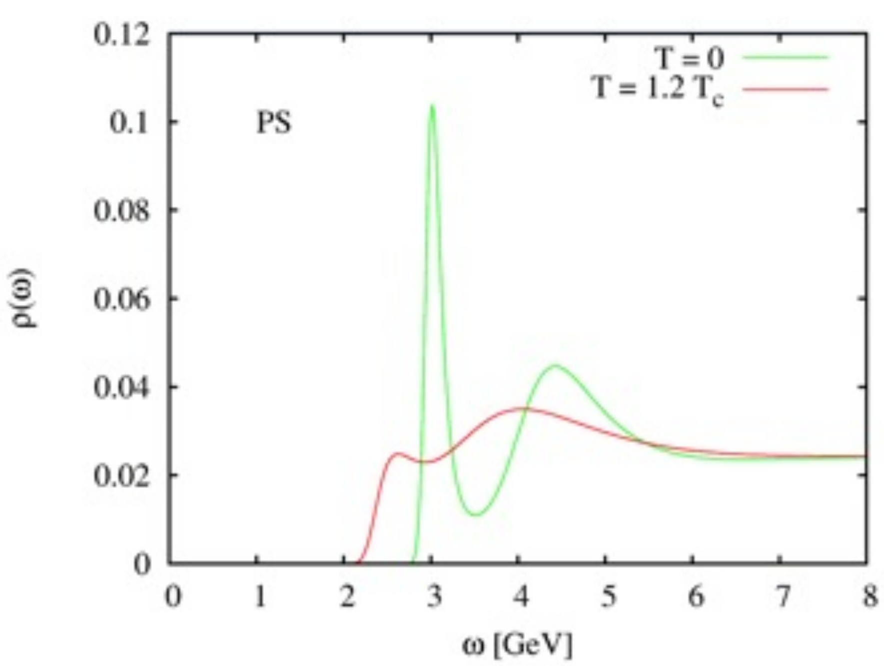




Melting

The charmonium spectral function at finite T





Both J/ψ and η_c have melted completely.

Conclusions

- We have shown that MEM can be applied to QCD sum rules
- The "pole + continuum" ansatz is not a necessity
- We could observe the melting of the S-wave charmonia using finite temperature QCD sum rules and MEM
- Both η_c and J/ψ seem to melt between T ~ 1.0 T_C and T ~ 1.1 $T_{C,}$, which is below the values obtained in lattice QCD