Exotic Hadrons

Makoto Oka

PART IV of Lecture
Exotic hadrons

Long history of exotic hadrons
1977 Jaffe 4-quark states, di-baryons in the MIT bag model

Anything which are not $qq$ or $qqq$ are exotic, as far as they are color singlet.
$qqqq$, $qqqq\bar{q}$ contain extra $qq$
$q\bar{q}g$, $qqqg$, \ldots contain constituent $g$
$gg$, $ggg$, \ldots no quark
Exotic hadrons

Why are exotics interesting?

⭐ QCD does not prohibit exotic hadrons!
⭐ Exotics are more “Colorful”! (Lipkin)

(qq)$_8$ or (qq)$_6$ are allowed only in the multi-quarks.
Exotic hadrons

(1) True exotics

minimal $qq\bar{q}$, $qqqq\bar{q}$, \ldots

ex. $\Theta^+(S=+1)$, $D_s(l=1)$, $Z_c^+(4430)$,

$Z_b^+(10610)$, $Z_b^+(10650)$

(2) Exotic multi-quark components of "normal" hadrons

meson $q\bar{q} + qqq\bar{q}$

baryon $qqq + qqqq\bar{q}$

"Normal" so that no conserved quantum number prohibits the state as $q\bar{q}$ or $qqq$. 

exotic hadrons
Tetraquarks?

Scalar mesons \( J^\pi = 0^+ \)
light meson sector
\[
\begin{align*}
  f_0(\sigma) & \sim 600 \text{ MeV} & \quad f_0' & \sim 980 \text{ MeV} \\
  a_0 & \sim 985 \text{ MeV} & \quad K_0^*(\kappa) & \sim 841 \text{ MeV}
\end{align*}
\]

The mass spectrum does not coincide with the q\(\bar{q}\) states with SU(3) breaking (ideal mixing).

\[
\begin{align*}
  f_0(\sigma) & \sim u\bar{u} + d\bar{d} & \quad f_0' & \sim s\bar{s} \\
  a_0 & \sim u\bar{u} - d\bar{d}, \, u\bar{d} - d\bar{u} & \quad K_0^* & \sim u\bar{s}, \, d\bar{s}, \, s\bar{u}, \, s\bar{d}
\end{align*}
\]

expected spectrum \( m(\sigma) \sim m(a_0) < m(f_0) \)
observed spectrum \( m(\sigma) < m(a_0) \sim m(f_0) \)
Tetraquarks?

4-quark states

\[
\sigma \sim SS = (ud)(\bar{u}\bar{d}) \\
\frac{uu + \bar{DD}}{\sqrt{2}} \sim \frac{(ds)(\bar{d}s) + (su)(\bar{s}\bar{u})}{\sqrt{2}} \\
\frac{uu - \bar{DD}}{\sqrt{2}} \sim \frac{(ds)(\bar{d}s) - (su)(\bar{s}\bar{u})}{\sqrt{2}}
\]

composed of diquarks in flavor 3

\[U = (\bar{d}s)_{S=0,C=3} \quad D = (\bar{s}u)_{S=0,C=3} \quad S = (\bar{u}d)_{S=0,C=3}\]

give the right ordering of the spectrum just by strange quark counting

\[m(\sigma) < m(a_0) \sim m(f_0)\]

Black et al. (2000)
Tetraquarks?

M (MeV)

a_2(1320)

a_1(1230)

ρ(770)

π(137)

f_0(1710)

f_0(1500)

K_0^*(1430)

a_0(1450)

f_0(1370)

qq+G

K_0^*

scalar nonets

4q nonets

5FUSBRVBSLT
Heavy Tetraquarks?

<table>
<thead>
<tr>
<th>state</th>
<th>$M$ (MeV)</th>
<th>$\Gamma$ (MeV)</th>
<th>$J^{PC}$</th>
<th>Seen In</th>
<th>Observed by</th>
</tr>
</thead>
<tbody>
<tr>
<td>$Y_s(2175)$</td>
<td>2175 ± 8</td>
<td>58 ± 26</td>
<td>1$^{--}$</td>
<td>$(e^+e^-)_{ISR}$, $J/\psi \rightarrow Y_s(2175) \rightarrow \phi f_0(980)$</td>
<td>BaBar, BESII, Belle</td>
</tr>
<tr>
<td>$X(3872)$</td>
<td>3871.4 ± 0.6</td>
<td>&lt; 2.3</td>
<td>1$^{++}$</td>
<td>$B \rightarrow KX(3872) \rightarrow \pi^+\pi^- J/\psi, \gamma J/\psi, D\bar{D}^*$</td>
<td>Belle, CDF, D0, BaBar</td>
</tr>
<tr>
<td>$X(3915)$</td>
<td>3914 ± 4</td>
<td>28$^{+12}_{-14}$</td>
<td>2$^{++}$</td>
<td>$\gamma \gamma \rightarrow \omega J/\psi$</td>
<td>Belle</td>
</tr>
<tr>
<td>$Z(3930)$</td>
<td>3929 ± 5</td>
<td>29 ± 10</td>
<td>2$^{++}$</td>
<td>$\gamma \gamma \rightarrow Z(3940) \rightarrow DD$</td>
<td>Belle</td>
</tr>
<tr>
<td>$X(3940)$</td>
<td>3942 ± 9</td>
<td>37 ± 17</td>
<td>0$^+$</td>
<td>$e^+e^- \rightarrow J/\psi X(3940) \rightarrow D\bar{D}^*$ (not $D\bar{D}$ or $\omega J/\psi$)</td>
<td>Belle</td>
</tr>
<tr>
<td>$Y(3940)$</td>
<td>3943 ± 17</td>
<td>87 ± 34</td>
<td>2$^{++}$</td>
<td>$B \rightarrow KY(3940) \rightarrow \omega J/\psi$ (not $D\bar{D}^*$)</td>
<td>Belle, BaBar</td>
</tr>
<tr>
<td>$Y(4008)$</td>
<td>4008$^{+82}_{-49}$</td>
<td>226$^{+97}_{-80}$</td>
<td>1$^{--}$</td>
<td>$(e^+e^-)_{ISR} \rightarrow Y(4008) \rightarrow \pi^+\pi^- J/\psi$</td>
<td>Belle</td>
</tr>
<tr>
<td>$Y(4140)$</td>
<td>4143 ± 3.1</td>
<td>11.7$^{+9.1}_{-6.2}$</td>
<td>1$^{--}$</td>
<td>$B \rightarrow KY(4140) \rightarrow J/\psi\phi$</td>
<td>CDF</td>
</tr>
<tr>
<td>$X(4160)$</td>
<td>4156 ± 29</td>
<td>139$^{+113}_{-65}$</td>
<td>1$^{--}$</td>
<td>$e^+e^- \rightarrow J/\psi X(4160) \rightarrow D^<em>\bar{D}^</em>$ (not $DD$)</td>
<td>Belle</td>
</tr>
<tr>
<td>$Y(4260)$</td>
<td>4264 ± 12</td>
<td>83 ± 22</td>
<td>1$^{--}$</td>
<td>$(e^+e^-)_{ISR} \rightarrow Y(4260) \rightarrow \pi^+\pi^- J/\psi$</td>
<td>BaBar, CLEO, Belle</td>
</tr>
<tr>
<td>$Y(4350)$</td>
<td>4324 ± 24</td>
<td>172 ± 33</td>
<td>1$^{--}$</td>
<td>$(e^+e^-)_{ISR} \rightarrow Y(4350) \rightarrow \pi^+\pi^- \psi'$</td>
<td>BaBar</td>
</tr>
<tr>
<td>$Y(4350)$</td>
<td>4361 ± 13</td>
<td>74 ± 18</td>
<td>1$^{--}$</td>
<td>$(e^+e^-)_{ISR} \rightarrow Y(4350) \rightarrow \pi^+\pi^- \psi'$</td>
<td>Belle</td>
</tr>
<tr>
<td>$Y(4630)$</td>
<td>4634$^{+9.4}_{-10.6}$</td>
<td>92$^{+41}_{-32}$</td>
<td>1$^{--}$</td>
<td>$(e^+e^-)_{ISR} \rightarrow Y(4630) \rightarrow \Lambda_c^+\Lambda_c^-$</td>
<td>Belle</td>
</tr>
<tr>
<td>$Y(4660)$</td>
<td>4664 ± 12</td>
<td>48 ± 15</td>
<td>1$^{--}$</td>
<td>$(e^+e^-)_{ISR} \rightarrow Y(4660) \rightarrow \pi^+\pi^- \psi'$</td>
<td>Belle</td>
</tr>
<tr>
<td>$Z_1(4050)$</td>
<td>4051$^{+24}_{-23}$</td>
<td>82$^{+61}_{-29}$</td>
<td>1$^{--}$</td>
<td>$B \rightarrow KZ_1^\pm(4050) \rightarrow \pi^{\pm}\chi_c$</td>
<td>Belle</td>
</tr>
<tr>
<td>$Z_2(4250)$</td>
<td>4248$^{+185}_{-45}$</td>
<td>177$^{+320}_{-72}$</td>
<td>1$^{--}$</td>
<td>$B \rightarrow KZ_2^\pm(4250) \rightarrow \pi^{\pm}\chi_c$</td>
<td>Belle</td>
</tr>
<tr>
<td>$Z(4430)$</td>
<td>4433 ± 5</td>
<td>45$^{+25}_{-18}$</td>
<td>1$^{--}$</td>
<td>$B \rightarrow KZ(4430) \rightarrow \pi^+\psi'$</td>
<td>Belle</td>
</tr>
<tr>
<td>$Y_b(10890)$</td>
<td>10,890 ± 3</td>
<td>55 ± 9</td>
<td>1$^{--}$</td>
<td>$e^+e^- \rightarrow Y_b \rightarrow \pi^+\pi^- \Upsilon(1, 2, 3S)$</td>
<td>Belle</td>
</tr>
</tbody>
</table>
Heavy Tetraquarks?
Heavy Tetraquarks?

- charged charmonium-like state $Z^\pm(4430)$ observed at the Belle (KEK) from decay of $B^0 \rightarrow KZ \rightarrow \pi^\pm\psi'$
  - Width: 45 MeV!
  - minimal: $cc\bar{u}\bar{d}$
Heavy Tetraquarks?

- Charged bottomium-like state $Z_b^\pm(10610)$, $Z_b^\pm(10650)$ observed at the Belle (KEK) from decay of
Heavy Tetraquarks?

\[ X(3872) \text{ 4-quark state with spin } 1^{++} \]
\[ [cq]_{S=1} + [cq]_{S=0} [cq]_{S=1} \text{ tetra-quark?} \]
\[ \text{or (DD}^\ast \text{bar } + \text{D}^\ast \text{D}^\text{bar}) \text{ molecule?} \]

\[ D_s \text{ mesons: [cq][sq]} \]

Other tetra-quark or molecular candidates include

- Y(4260)
- Y(4360)
- Y(4660)
How shall we determine the number of quarks in hadrons?
Multi-quark components of hadrons

Which hadrons are exotic or do contain exotic multi-quark components?

There are indications that the light scalar mesons, $f_0(600)$, $f_0(980)$, $a_0(980)$, $\kappa(900)$, and/or flavor-singlet negative-parity $\Lambda(1405)$ are multi-quarks.
Multi-quark components of hadrons

Why is $\Lambda(1405)$ likely to be 5q?

$\Lambda(1405)$ $J^\pi = 1/2^-$, flavor singlet

☆ uds $L=1$ orbital excited state with spin $1/2$

   $\Rightarrow J=1/2^-$ and $3/2^-$

☆ udsuu, . . . $L=0$ ground state

(ud)(su)$\bar{u} . .$

s=0 s=0    $S=1/2$  $\Rightarrow J=1/2$ isolated
diquarks

$\Lambda(1520)$ $3/2^-$

The competition between the kinetic energy and the extra quark masses indicates possible mixing of the two Fock components.
Multi-quark components of hadrons

So far, hadrons are regarded as bound states of "valence" quarks defined in the quark model. What does QCD predict?

In QCD, all hadrons, even N(940), contain extra $qar{q}$ as meson clouds and/or sea quarks.

When do we identify the extra flavor-singlet $qar{q}$ (or glue) as "valence" components?

We need a "good" definition of multi-quark-ness.
Multi-quark components of hadrons

Natural approach is to take a set of well-defined quantities, which might be useful for the quark model description.

Ex. overlaps of local operators.

\[ \langle 0 | J_3 | \Lambda \rangle = \lambda \cos \theta \, u(x) \]
\[ \langle 0 | J_5 | \Lambda \rangle = \lambda \sin \theta \, u(x) \]

Then one can determine the "mixing angle":

\[
\begin{align*}
\langle J_3(x) \bar{J}_3(0) \rangle & \sim \langle 0 | J_3(x) | \Lambda \rangle \langle \Lambda | \bar{J}_3(0) | 0 \rangle = \lambda^2 \cos^2 \theta \\
\langle J_5(x) \bar{J}_5(0) \rangle & \sim \langle 0 | J_5(x) | \Lambda \rangle \langle \Lambda | \bar{J}_5(0) | 0 \rangle = \lambda^2 \sin^2 \theta \\
\langle J_3(x) \bar{J}_5(0) \rangle & \sim \langle 0 | J_3(x) | \Lambda \rangle \langle \Lambda | \bar{J}_5(0) | 0 \rangle = \lambda^2 \sin 2\theta / 2
\end{align*}
\]
QCD sum rule approach

☆ An approach in QCD sum rule.
  a. 4-quark components of flavor non-singlet scalar mesons, $a_0(I=1; 0^+)$, $K_0(I=1/2; 0^+)$
  b. 5-quark components of $\Lambda$(singlet; $1/2^-$) baryon
  c. 5-quark components of $N(1/2^+)$, $N^*(1/2^-)$ baryons

T. Nakamura, J. Sugiyama, T. Nishikawa, N. Ishii, M.O.

*Phys. Rev. D76 (2007) 114010*
QCD sum rule approach

A set of interpolating fields (singlet $\Lambda$)

\[
J_3 = \epsilon_{abc} \left[ \left( u_a^T C \gamma_5 d_b \right) s_c - \left( u_a^T C d_b \right) \gamma_5 s_c - \left( u_a^T C \gamma_5 \gamma^\mu d_b \right) \gamma_\mu s_c \right] \\
= 2\epsilon_{abc} \left[ \left( u_a^T C \gamma_5 d_b \right) s_c + \left( d_a^T C \gamma_5 s_b \right) u_c + \left( s_a^T C \gamma_5 u_b \right) d_c \right] \\
J_5 = \epsilon_{abc} \epsilon_{def} \epsilon_{efg} \left[ \left( d_a^T C \gamma_5 s_b \right) \left( s_d^T C \gamma_5 u_e \right) \gamma_5 \bar{C} \bar{s}_g \\
+ \left( s_a^T C \gamma_5 u_b \right) \left( u_d^T C \gamma_5 d_e \right) \gamma_5 \bar{C} \bar{u}_g \\
+ \left( u_a^T C \gamma_5 d_b \right) \left( d_d^T C \gamma_5 s_e \right) \gamma_5 \bar{C} \bar{d}_g \right]
\]

How are these operators normalized? Choose a $J_5$ and define a genuine 5-quark operator $J_5'$ so that $J_3$ component of $J_3$ is subtracted.

\[
J_5 = J_5' + \left( -\frac{1}{18} \left( \langle \bar{u}u \rangle + \langle \bar{d}d \rangle + \langle \bar{s}s \rangle \right) J_3 \right)
\]
QCD sum rule approach

then one may determine the operator which couples most strongly to the physical state,

\[
|\Lambda\rangle = \cos \theta |\Lambda_{3q}\rangle + \sin \theta |\Lambda_{5q}\rangle \\
|a_0\rangle = \cos \theta |a_{2q}\rangle + \sin \theta |a_{4q}\rangle
\]

This method is "model independent", but it depends on the choice of the operator.

We may prefer having direct connection to the quark model approaches.
Number of quarks in QCD

☆ Is it legitimate to "count" # of quarks?
Not quite, because there exists no conserved current corresponding to the number of quarks: N(q)+N(\bar{q}).

It depends on the choice of the quark operator. Ex. Bogoliubov transformation changes the definition of the # of quarks.

☆ Are there any observables which distinguish valence and sea quarks.
Can be done in the light-cone frame? i.e. partons?
Number of quarks in QCD

Hadronization in heavy ion collisions: Recombination and fragmentation of partons

meson vs baryon
qq -> 2 $<p_T>$
qqq -> 3 $<p_T>$

Then 4q ? 5q ?
Number of quarks in QCD

Elliptic flow of resonances at RHIC: probing final state interactions and the structure of resonances
C. Nonaka et al. PRC 69 (2004) 031902
Counting valence quarks at RHIC and LHC
L. Maiani et al. PLB 645 (2007) 138
Number of quarks in QCD

Similarly, in DIS and other high energy processes, one may be able to count "valence" quarks.

Parton distribution = valence + sea

Cannot measure the pdf of resonances: $f_0/a_0$, $\Lambda$ etc.
Exotic hadron search by fragmentation functions

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2Institute of Particle and Nuclear Studies, High Energy Accelerator Research Organization (KEK)
1-1, Ooho, Tsukuba, Ibaraki, 305-0801, Japan
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1-1, Ooho, Tsukuba, Ibaraki, 305-0801, Japan
(Dated: August 4, 2007)

We propose that fragmentation functions should be used for searching exotic hadrons by finding differences between favored and disfavored functions. As an example, fragmentation functions of the scalar meson $f_0(980)$ are investigated. We found that various models such as quark-antiquark and tetraquark states are distinguished by noting second moments and functional forms of the fragmentation functions. By a global analysis of $f_0(980)$ production data in electron-positron annihilation, its fragmentation functions and their uncertainties are determined. However, the data are not accurate enough to judge its internal structure at this stage. If precise data are taken in future, its configuration should be determined. We could investigate other exotic hadrons in the same way by their fragmentation functions.

Number of quarks in QCD

- Fragmentation functions
  - contain non-perturbative information on hadronization
  - determined by a global analysis of $e^+e^- \rightarrow h+X$ experimental data

- Similar behavior as PDFs
  - Favored FF - valence quarks
    - a constituent of produced hadrons
    - peaked at medium to large $z$
  - Disfavored FF - sea quarks
    - peaked at small $z$
### Number of quarks in QCD

Expectations from various possible structure of exotic hadrons

<table>
<thead>
<tr>
<th>Type</th>
<th>Configuration</th>
<th>Second moments</th>
<th>Peak positions</th>
</tr>
</thead>
<tbody>
<tr>
<td>Nonstrange $q\bar{q}$</td>
<td>$(u\bar{u} + d\bar{d})/\sqrt{2}$</td>
<td>$M_s &lt; M_u &lt; M_g$</td>
<td>$z_u^{\text{max}} &gt; z_s^{\text{max}}$</td>
</tr>
<tr>
<td>Strange $q\bar{q}$</td>
<td>$s\bar{s}$</td>
<td>$M_u &lt; M_s &lt; M_g$</td>
<td>$z_u^{\text{max}} &lt; z_s^{\text{max}}$</td>
</tr>
<tr>
<td>Tetraquark (or $K\bar{K}$)</td>
<td>$(u\bar{u}s\bar{s} + d\bar{d}s\bar{s})/\sqrt{2}$</td>
<td>$M_u \sim M_s &lt; M_g$</td>
<td>$z_u^{\text{max}} \sim z_s^{\text{max}}$</td>
</tr>
<tr>
<td>Glueball</td>
<td>$g\bar{g}$</td>
<td>$M_u \sim M_s &lt; M_g$</td>
<td>$z_u^{\text{max}} \sim z_s^{\text{max}}$</td>
</tr>
</tbody>
</table>

**s\bar{s} picture for $f_0(980)$**
- $u$ (disfavored)
- $s$ (favored)
- $g$

**Tetraquark picture for $f_0(980)$**
- $u, s$ (favored)
- $g$

*exotic hadrons*
Number of quarks in QCD

$\chi^2/d.o.f. = 0.907$

Total Number of data: 23

**Tetra-quark** configuration

- favored FF: u and s quarks
- Peak at large-z ($z \sim 0.85$)
- $z_u^{\text{max}} \sim z_s^{\text{max}}$
  - or

**SS** configuration

- $M_u < M_s$
- $(M_u/M_s = 0.43 \pm 6.73)$

Large uncertainty

Need further precise data

*exotic hadrons*

2nd moments

- $M_u = 0.0012 \pm 0.0107$
- $M_s = 0.0027 \pm 0.0183$
- $M_g = 0.0090 \pm 0.0046$
Number of quarks in QCD

- We propose a plausible way of searching exotic hadrons using the fragmentation functions in high energy collisions.
- The analysis reveals the quark-gluon structure of excited (exotic) hadrons.
  
  The favored & disfavored FFs show similar properties as valence & sea quark distributions: peak position: $z^{\text{max}}$
  The 2\textsuperscript{nd} moments of FFs are compared with the order counting of perturbative production processes.

Applied to the global analysis of FFs of the $f_0(980)$ production. Indicating tetra-quark and/or $s\bar{s}$ configuration
Large uncertainty of the current production data does not allow to distinguish them.
Conclusion

- Exotics (including dibaryons) may provide critical information in understanding hadrons from QCD, in particular, on
  - mechanism of confinement
  - perturbative vs non-perturbative dynamics
  - symmetry and broken symmetry

- We need to establish "multiquark-ness" in terms of QCD.
- Fragmentation functions in high energy production processes may be useful in determining number of valence quarks and flavor compositions of hadrons.