格子QCDによる非摂動現象に寄与するグルーオン成分の研究

S.Gongyo(Kyoto Univ.)
T.Iritani, H.Suganuma (Kyoto.U)

at summer school
17 August 2011

Quantum Chromodynamics (QCD)

- Fundamental theory of strong interaction
- Formulated in terms of Quarks and Gluons

$$L_{QCD} = \bar{q}(i\not{D} - m)q - \frac{1}{2}\mathrm{tr}G^{\mu\nu}G_{\mu\nu}$$

$$\not{D} = D^\mu \gamma_\mu = (\partial^\mu + igA^\mu)\gamma_\mu$$

$$G^{\mu\nu} = \frac{1}{ig}[D^\mu, D^\nu]$$

$$\Downarrow$$

$$\underline{[A^\mu, A^\nu]}$$

Non-Abelian

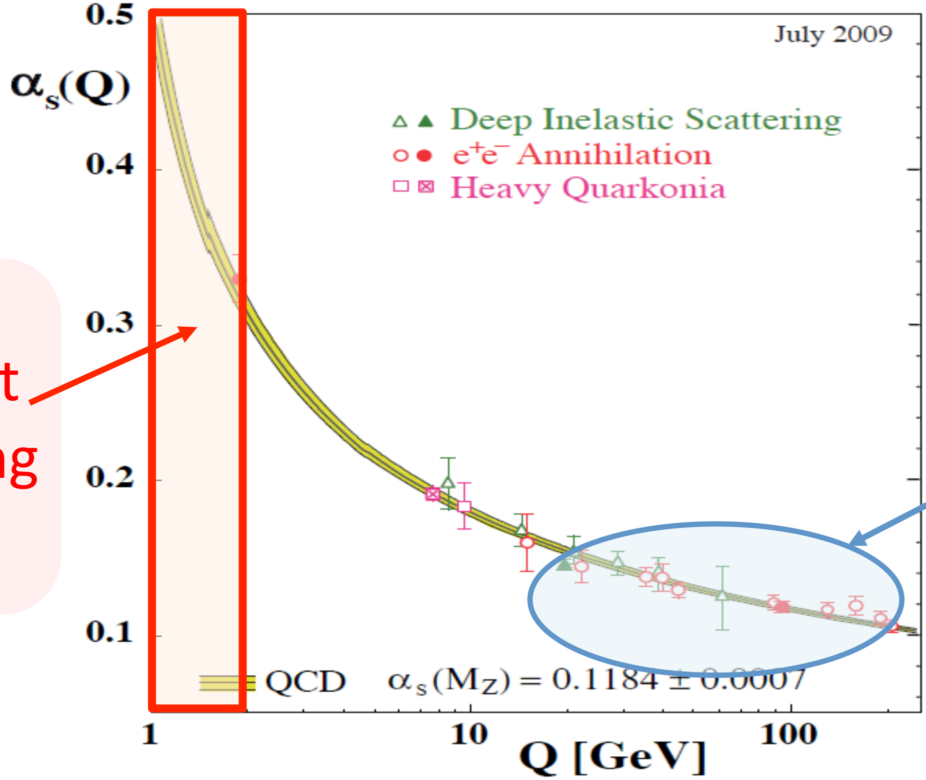
g :coupling constant

$q(x)$:Quark Field

$A^\mu(x)$:Gluon Field

→ A variety of Phenomena

Non-perturbative phenomena for QCD



$$\alpha_s(Q) = \frac{g^2(Q)}{4\pi}$$

Strong Coupling
Color Confinement
Chiral Sym. Breaking
⋮

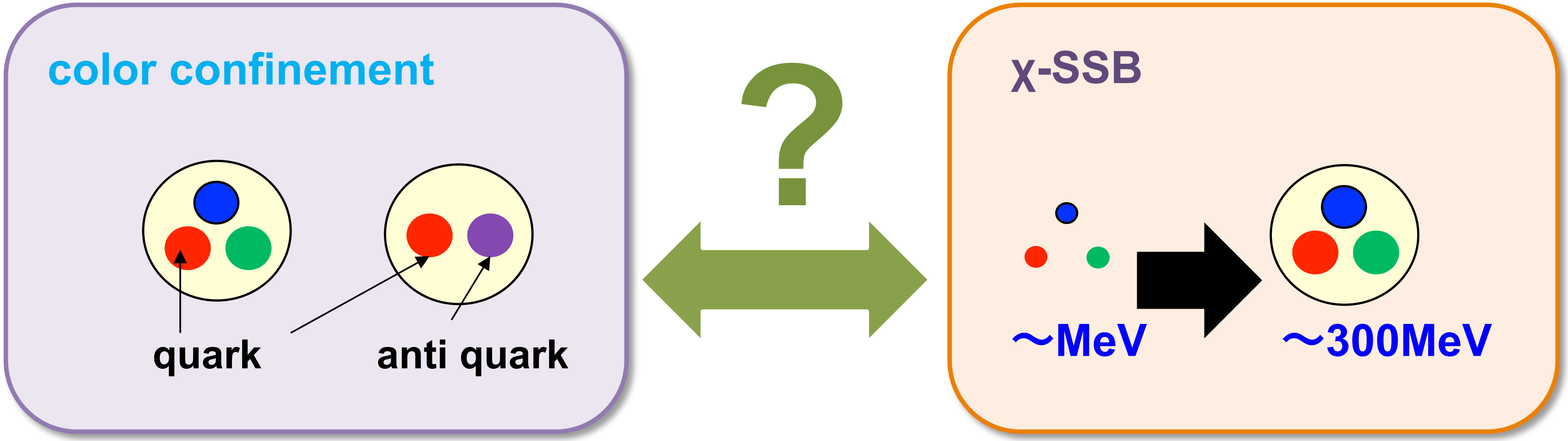
Weak Coupling
Perturbative QCD

From: Particle Data Group

- Color Confinement ⇒ Formation of hadrons
- χ-SSB $< \bar{q}q > \neq 0$ ⇒ hadron mass

Our Questions

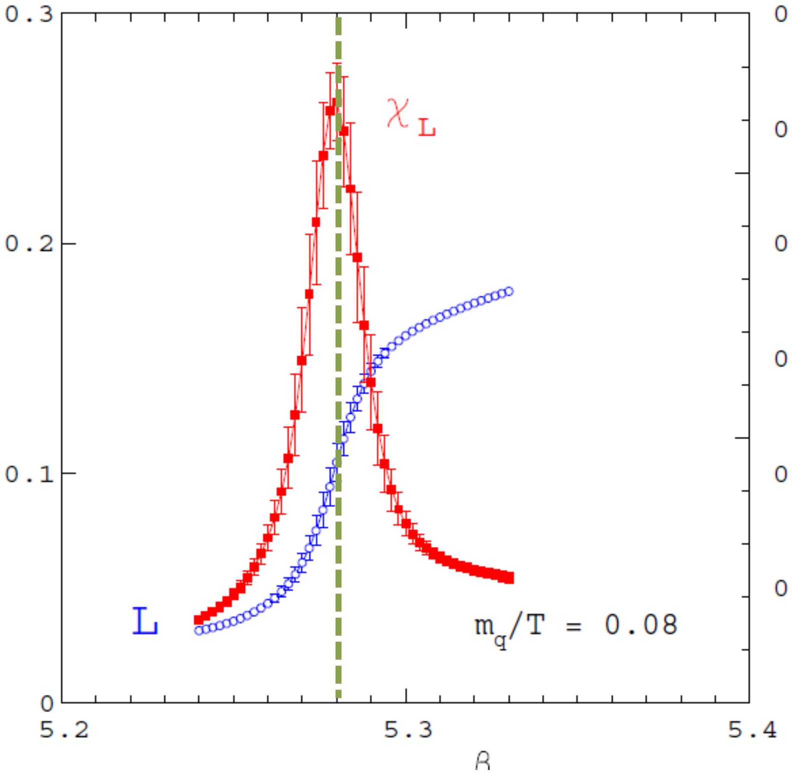
How is the relation between these phenomena?



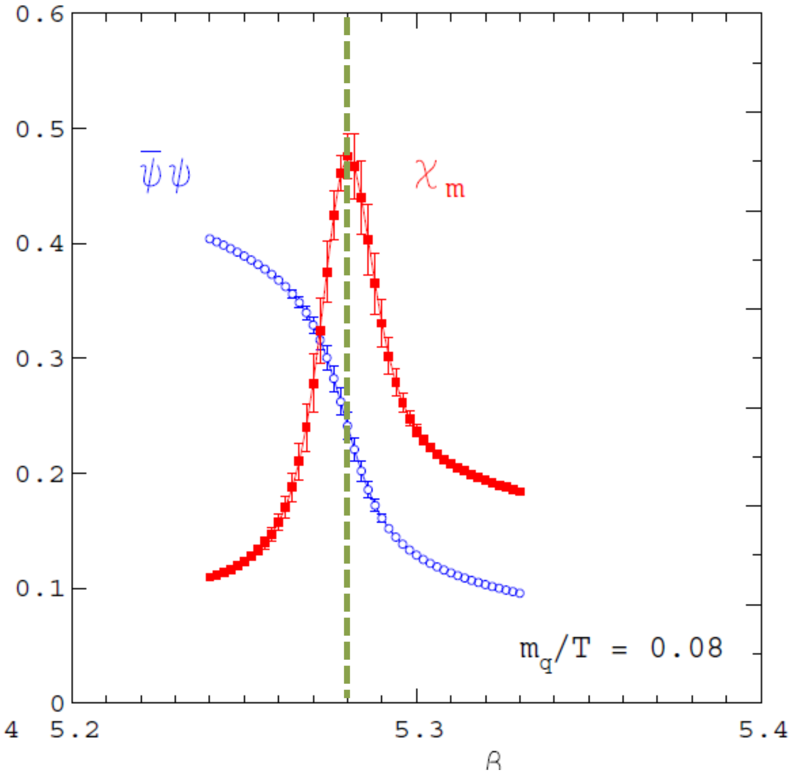
The relation between χ-SSB and Confinement

Finite temperature QCD

F.Karsch, Lect. Notes Phys. (2002)



Polyakov Loop <L>

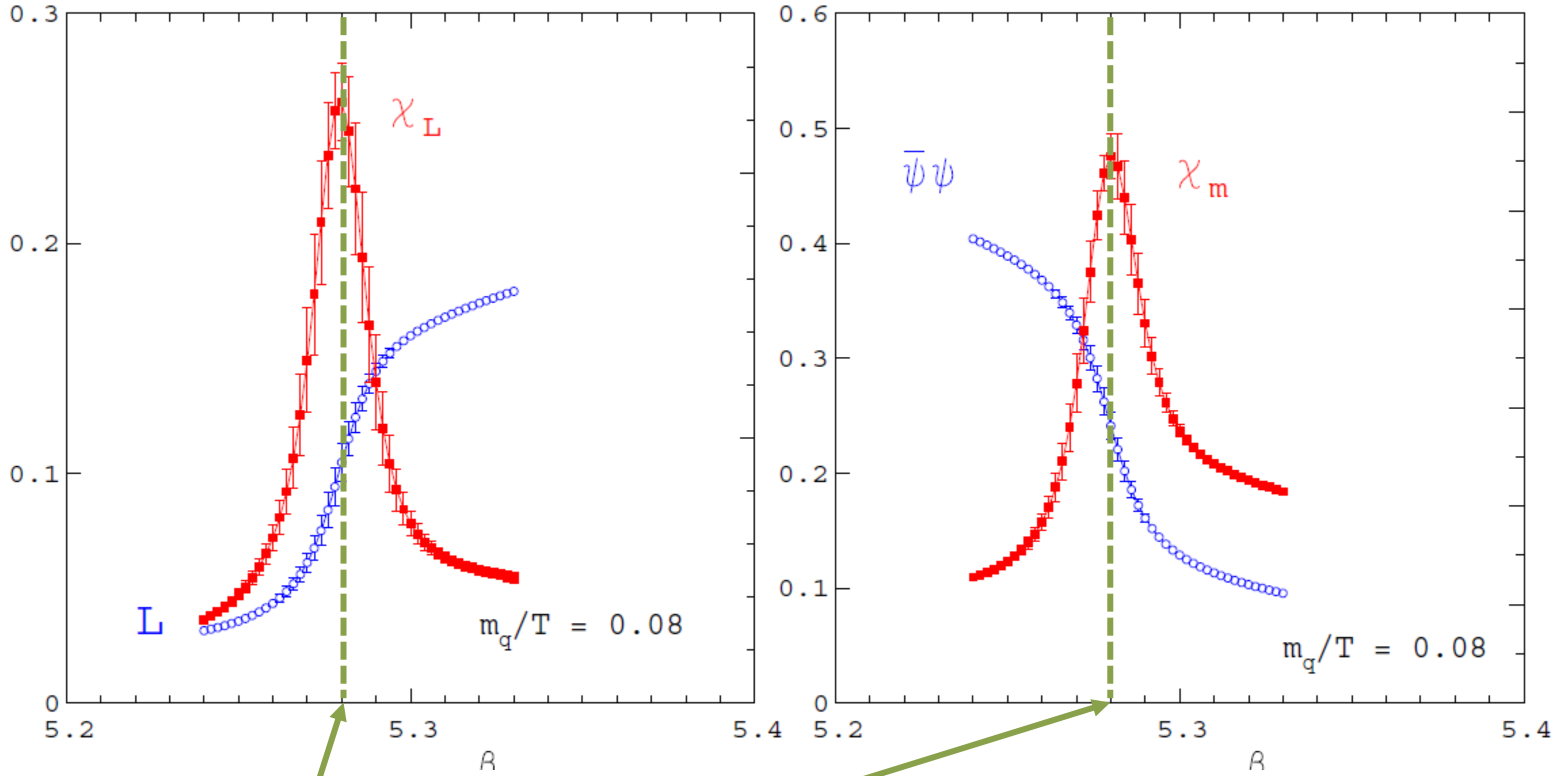


Quark condensate <$\bar{q}q$>

The relation between χ-SSB and Confinement

Finite temperature QCD

F.Karsch, Lect. Notes Phys. (2002)



Polyakov Loop <L>

Quark condensate <$\bar{q}q$>

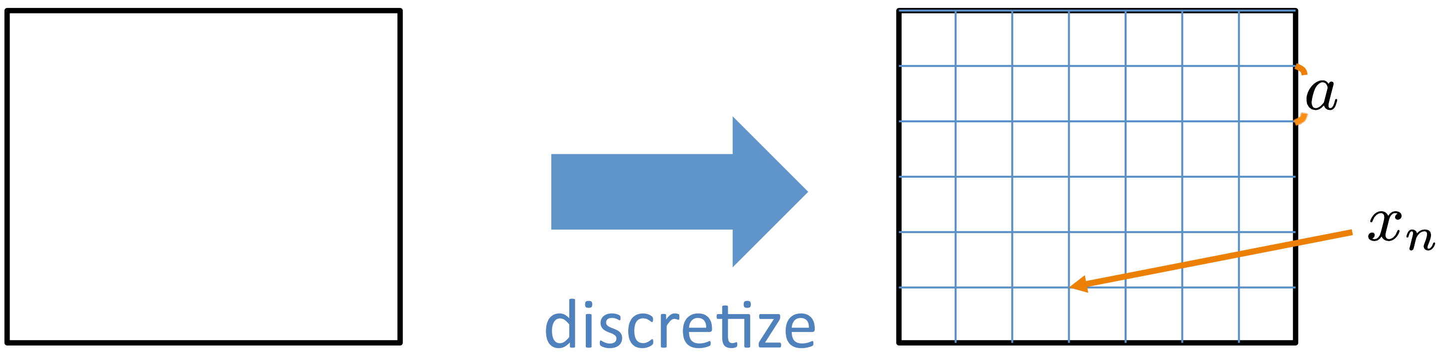
相転移温度の一致 ➡ 閉じ込めとカイラル対称性の破れの関連性を示唆

Formalism of Gauge-Invariant Relevant Gluon Component Determination

S.G, Iritani, Suganuma

Lattice QCD

- Only approach to analyze Non-perturbative phenomena from QCD



Continuum

$$A_\mu(x)$$

Lattice

$$U_\mu(x_n) = e^{iagA_\mu(x_n)}$$

$$Z = \int DA_\mu e^{-S} \Leftarrow$$ Statistical mechanics

Monte Carlo simulation

How is Formalism?

Lattice QCD

STEP1. Evaluate Eigenfunctions of $\not{D}^{\mathrm{lat}}_{xy}$

Lattice Dirac Operator

$$\not{D}^{\mathrm{lat}}_{xy} \equiv \frac{1}{2a} \sum_{\mu} \gamma^{\mu} \left[U_{\mu}(x) \delta_{y,x+\hat{\mu}} - U_{-\mu}(x) \delta_{y,x-\hat{\mu}} \right]$$

Eigenvalue, Eigenstate

$$\hat{\not{D}}^{\mathrm{lat}}[U]|n\rangle = \lambda_n |n\rangle$$ i.e.

Eigenfunction

$$\sum_{y} \not{D}^{\mathrm{lat}}_{xy}[U] \bar{\chi}_n(y) = \lambda_n \bar{\chi}_n(x)$$

How is Formalism?

Lattice QCD

STEP1. Evaluate Eigenfunctions of $\not{D}^{\text{lat}}_{xy}$

Lattice Dirac Operator

$$\not{D}^{\text{lat}}_{xy} \equiv \frac{1}{2a} \sum_{\mu} \gamma^{\mu} \left[U_{\mu}(x)\delta_{y,x+\hat{\mu}} - U_{-\mu}(x)\delta_{y,x-\hat{\mu}} \right]$$

Eigenvalue, Eigenstate

$$\hat{\not{D}}^{\text{lat}}[U]|n\rangle = \lambda_n |n\rangle$$ i.e.

Eigenfunction

$$\sum_y \not{D}^{\text{lat}}_{xy}[U]\bar{\chi}_n(y) = \lambda_n \bar{\chi}_n(x)$$

Gauge Trans. $V(x) \in SU(3)$

$$\not{D}^{\text{lat}} \to V(x)\not{D}^{\text{lat}}V(y)^{\dagger}$$

$$\bar{\chi}_n(x) \to V(x)\bar{\chi}_n(x)$$

Lattice QCD

STEP2. Calculate $\langle m|U_\mu|n\rangle$

$$\begin{aligned}\langle m|U_\mu|n\rangle &= \sum_x \langle m|x\rangle\langle x|U_\mu|x+\hat{\mu}\rangle\langle x+\hat{\mu}|n\rangle \\ &= \sum_x \underline{\bar{\chi}_m^\dagger(x)}\,\underline{U_\mu(x)}\,\underline{\bar{\chi}_n(x+\hat{\mu})}\end{aligned}$$

Obtained by
Monte Carlo simulation

Calculated by Step 1

$\Rightarrow$ $\langle m|U_\mu|n\rangle$ is Calculated!

Gauge Trans.

$$\begin{aligned}U_\mu(x) &\to V(x)U_\mu(x)V^\dagger(x+\hat{\mu}) \\ \bar{\chi}_n(x) &\to V(x)\bar{\chi}_n(x)\end{aligned}$$

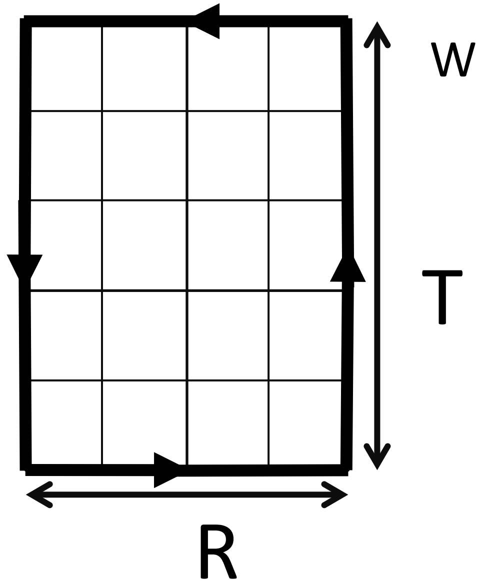
$\Rightarrow$ $\langle m|U_\mu|n\rangle$ is Invariant!
(without phase)

Lattice QCD

STEP3. Project with $P \equiv \sum_{n=-N}^{N} |n\rangle\langle n|$

Limit the number

Wilson loop



$$\left\langle \hat{W} \right\rangle = \mathrm{Tr} \prod_{l \in C} U_l$$

$$= \prod_{l \in C} \sum_{n_l} \langle n_l | U_l | n_{l+1} \rangle$$

Projected!

$$\left\langle \hat{W}^P \right\rangle = \prod_{l \in C} \sum_{n_l = -N}^{N} \langle n_l | U_l | n_{l+1} \rangle$$ Gauge Invariant!

$$V(R) = -\lim_{T \to \infty} \frac{1}{T} ln \left\langle \hat{W} \right\rangle \longrightarrow V^P(R) = -\lim_{T \to \infty} \frac{1}{T} ln \left\langle \hat{W}^P \right\rangle$$

Three merits for Gauge-Invariant relevant Gluon Component in terms of Dirac eigen-modes

- Determine the Gauge-Invariant Gluon Component
- Investigate the relation between confinement and χ-SSB via Banks-Casher relation,

$$\langle \bar{q} q \rangle = - \lim_{m \to 0} \lim_{V \to \infty} \frac{\pi}{V} \langle \rho(0) \rangle$$

- Investigate the relation between confinement and Topological charge via Atiyah-Singer Index theorem,

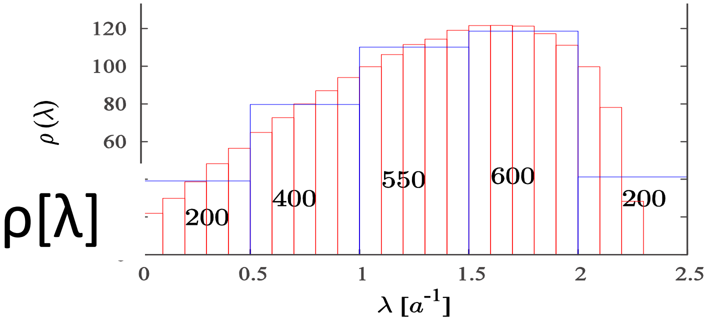
$$Q = \frac{g^2}{16\pi^2} \int d^4x \mathrm{Tr} G_{\mu\nu} \tilde{G}^{\mu\nu} = \nu_R - \nu_L$$

ν_R, ν_L :Right,Left handed zero mode

Numerical result – potential –

$$\langle \bar{q}q \rangle = -\lim_{m\to 0}\lim_{V\to\infty}\frac{\pi}{V}\langle \rho(0) \rangle$$

ρ[λ]



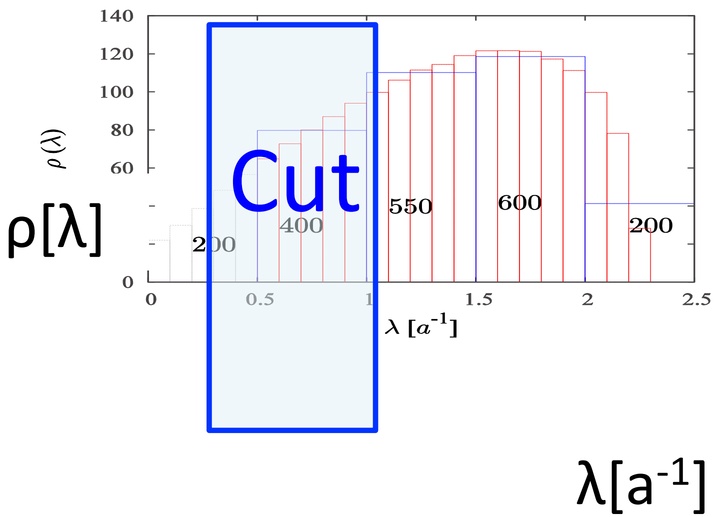
λ[a^{-1}]

$$6^4, \beta = 5.6$$

$$a^{-1} = 0.8[\mathrm{GeV}]$$

$$(a = 0.25[\mathrm{fm}])$$

Numerical result – potential –



$$\langle \bar{q}q \rangle = -\lim_{m\to 0}\lim_{V\to\infty}\frac{\pi}{V}\langle \rho(0)\rangle$$

⬇

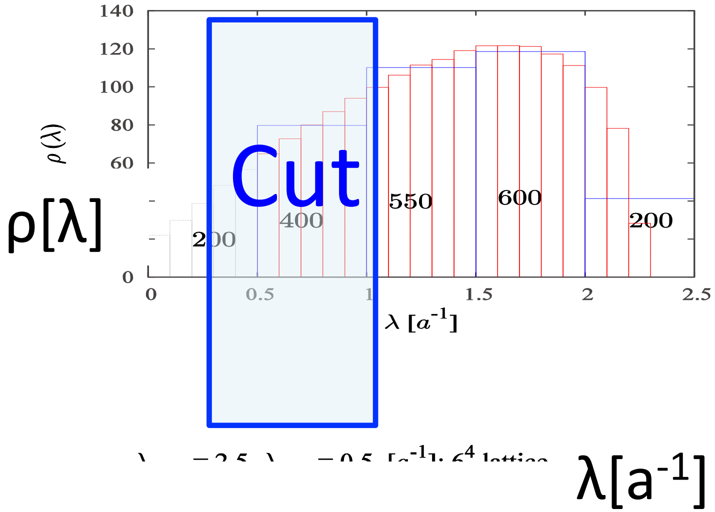
Reduced

$$6^4, \beta = 5.6$$

$$a^{-1} = 0.8[\mathrm{GeV}]$$

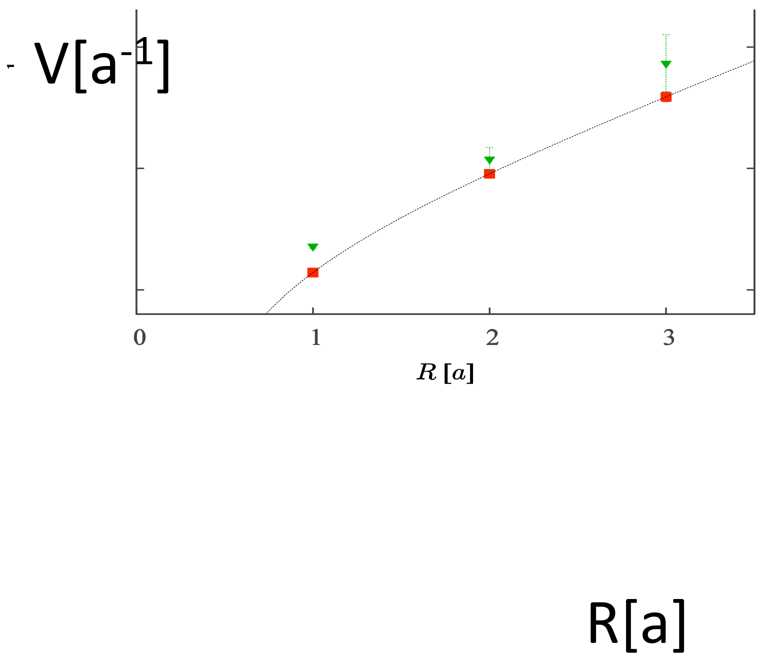
$$(a = 0.25[\mathrm{fm}])$$

Numerical result – potential –



$$\langle \bar{q}q \rangle = -\lim_{m\to 0}\lim_{V\to\infty}\frac{\pi}{V}\langle \rho(0)\rangle$$

Reduced



Almost unchanged

$$6^4, \beta = 5.6$$

$$a^{-1} = 0.8[\mathrm{GeV}]$$

$$(a = 0.25[\mathrm{fm}])$$

Summary and Future work

Summary

- We formulate the method to extract Relevant Gauge-Invariant Gluon Component for QCD
- Confinement and χ-SSB are not related directly through Dirac eigen-modes. (preliminary)

Future work

- We study the relation between confinement and χ-SSB or topological charge
  (lattice spacing, dynamical quark, finite temperature, etc)