低密度核物質の 非一様構造("Pasta" structure) による3次元結晶

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原子核三者若手夏の学校 原子核パート研究会 @ 白浜荘 '11 8/16

・Menu

Ⅰ. What's "Pasta" ?

Ⅱ. Relativistic Mean Field Theory

Ⅲ. Result (ⅰ) Fixed proton ratio

(ⅱ) β - equilibrium

(ⅲ) Large cell calculation

Ⅳ. Conclusion / Future plan

▪Menu

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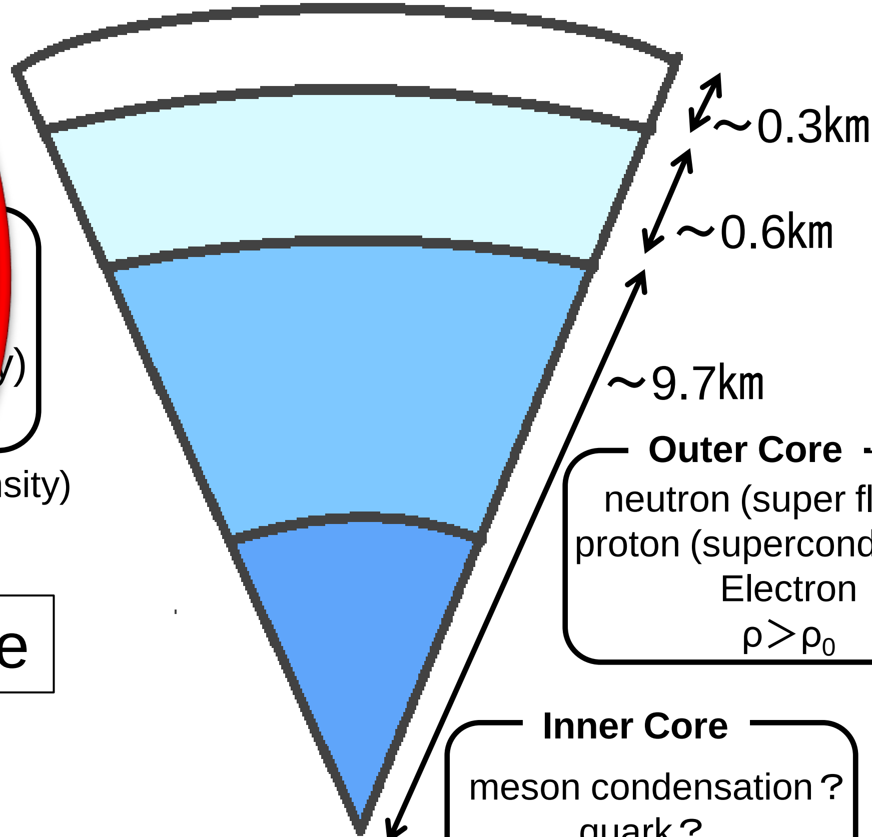
▪Neutron Star

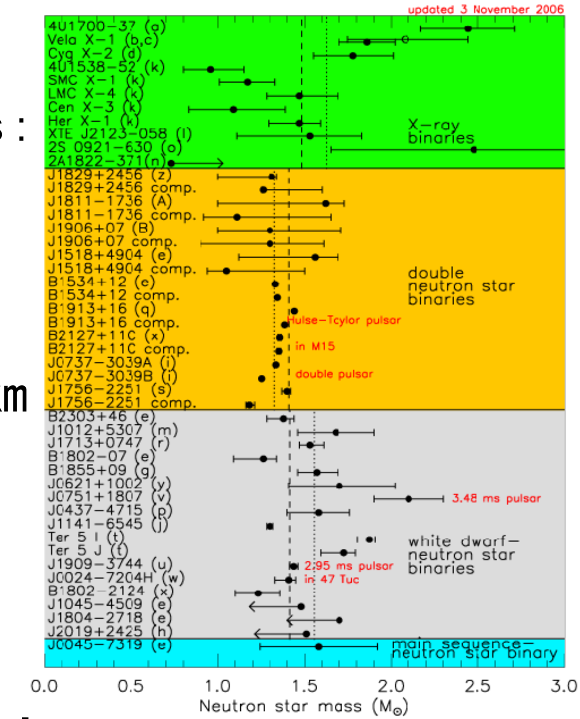
Outer Crust
Nuclei
electron
ρ<ρ_0

Inner Crust
Neutron rich nuclei
electron
neutron(super fluidity)
$\rho \simeq \rho_0$

(ρ_0: normal nucleus density)



Neutron star mass :
$M \sim 1.4 M_\odot$



Outer Core
neutron (super fluidity)
proton (superconductivity)
Electron
ρ>ρ_0

Inner Core
meson condensation？
quark？
$\rho \ggg \rho_0$ ？

Nuclei in lattice

Uniform
Nuclear matter

→ Coexistence of liquid and gas phase

▪Suggestion of Pasta Structure

1.C

Nuclear Physics **A175** (1971) 225—271; © *North-Holland Publishing Co., Amsterdam*

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NEUTRON STAR MATTER

GORDON BAYM †, HANS A. BETHE †† and CHRISTOPHER J. PETHICK †††

Nordita, Copenhagen, Denmark

Received 4 May 1971

which become important here. In fact, it might be more favorable, beyond $u = 0.5$, for the nuclei to "turn inside out", that is, for the neutron gas to exist as a lattice of droplets in a sea of nuclear matter.

VOLUME 50, NUMBER 26 PHYSICAL REVIEW LETTERS 27 JUNE 1983

Structure of Matter below Nuclear Saturation Density

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(Received 5 May 1983)

Progress of Theoretical Physics, Vol. 71, No. 2, February 1984

Shape of Nuclei in the Crust of Neutron Star

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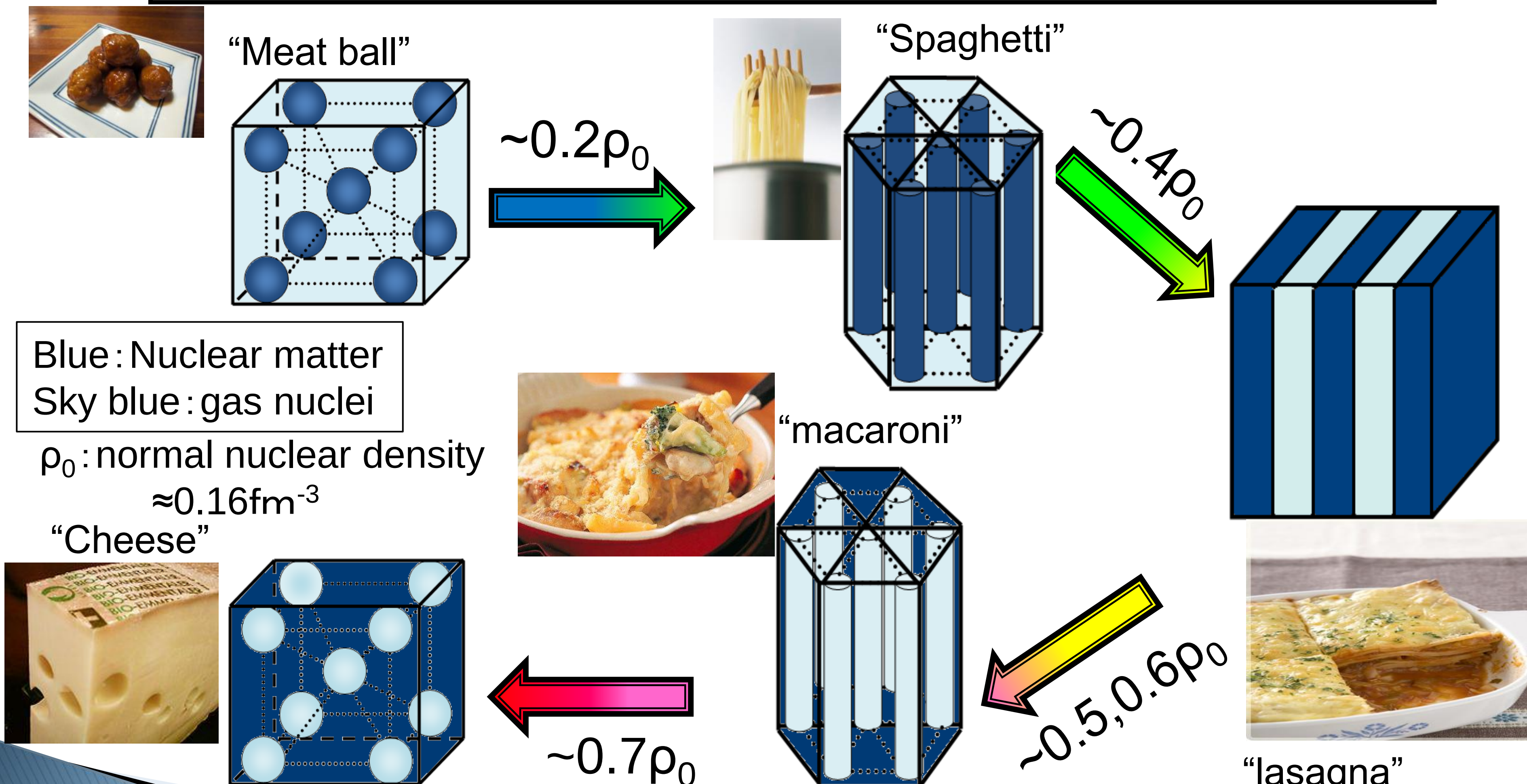
(Received August 30, 1983)

It will be interesting to explore the consequences of these spaghettilike and lasagnalike phases of dense matter. Their physical properties will have to reflect the great departure from isotropy that these phases possess. Neutrino scattering

Pasta Structure = Inhomogeneous structures appear in first phase transition , "mixed phase with structure"



(K.Oyamatsu, Nucl.Phys.**A561**,431(1993))

Pasta Structure = Inhomogeneous structure appear in first phase transition , "mixed phase with structure"

Balance with Surface tension and Coulomb repulsion

Total Energy = E_b(Volume) + E_S(surface) + E_C(Coulomb)

Weizsäcker-Bethe's semi-emperical mass formula

$$\frac{E_C}{A} \propto \frac{Z^2/A^{1/3}}{A} \propto R^2 \Rightarrow \frac{E_C}{A} = aR^2$$

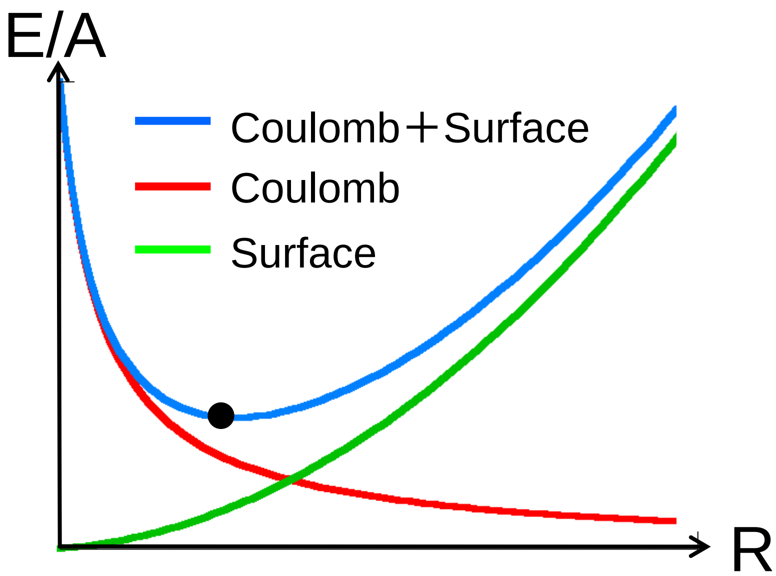
$$(R \propto A^{1/3})$$

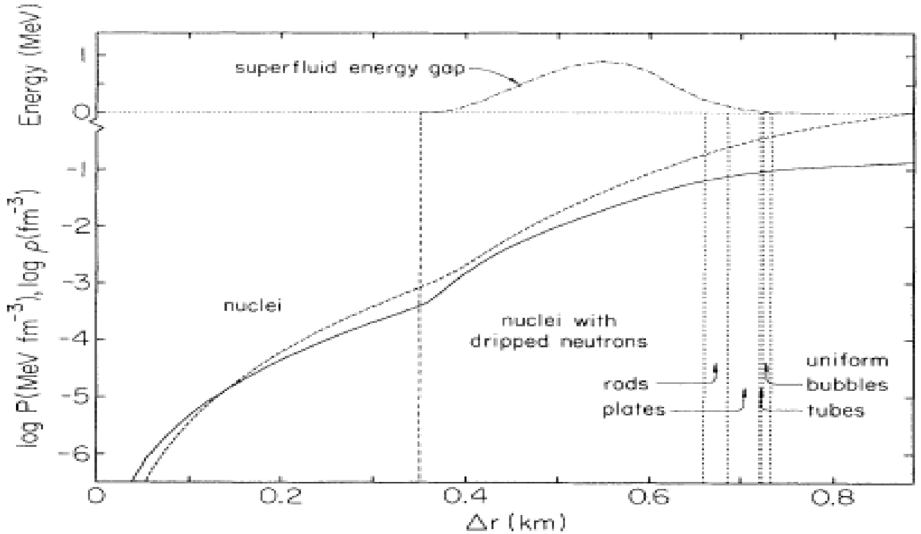
$$\frac{E_S}{A} \propto \frac{A^{2/3}}{A} \propto R^{-1} \Rightarrow \frac{E_S}{A} = bR^{-1}$$

Minimal Energy is

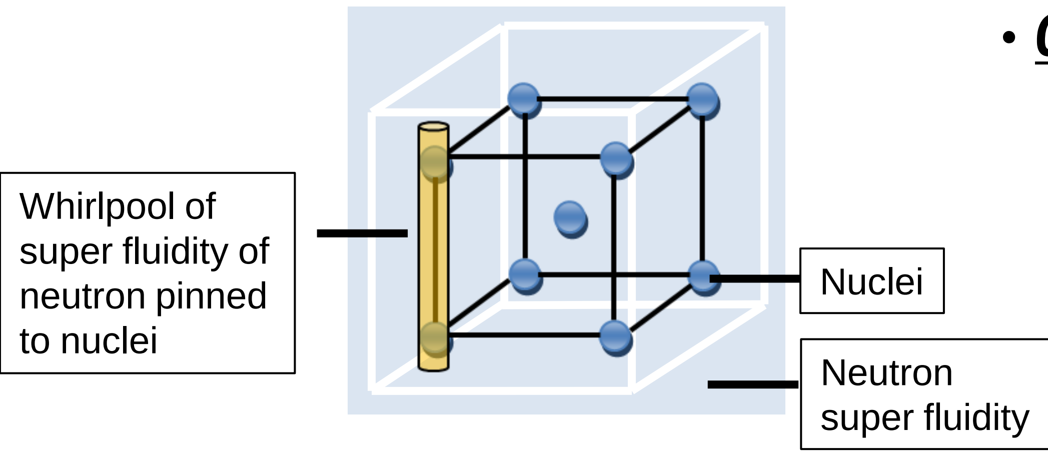
$$\frac{d(E_S/A + E_C/A)}{dR} = 2aR - bR^{-2} = 0$$

$$\therefore \quad 2aR^2 = bR^{-1} \quad 2E_S = E_C$$





- **_Half of Crust' Mass_**
  → ***Influence on EOS of Crust***



- **_Glitch_**
  Rapid increasing of rotational speed of Neutron Star

It may occur from destruction of whirlpool of neutron super fluidity on lattice points?

PHYSICAL REVIEW C 75, 042801(R) (2007)

Impact of nuclear "pasta" on neutrino transport in collapsing stellar cores

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(Received 11 January 2007; published 6 April 2007)

- **_Neutrino differential cross-section in supernova_**

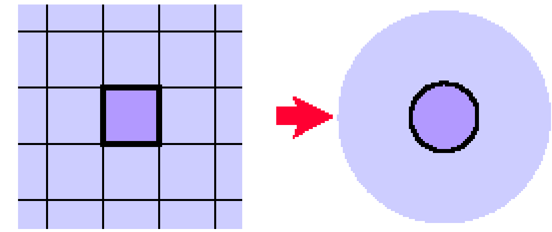
(・Pasta structures of quark-hadron mixed phase in Neutron star core

… etc)

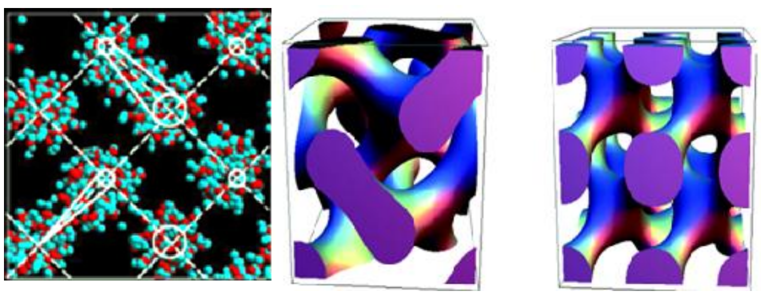
Wigner–Seitz cell approximation

Whole space is divided into equivalent cells.
These cells are imposed geometrical symmetry as follow.
Sphere : 3D, Cylinder : 2D , Slab : 1D

Merit: All calculation is 1D calculation
↓
Fast calculation & Low cost
But … simple structures only appear



- Existence of other structure
  - G.Watanabe, H.Sonoda et.al PRL **103, 121101 (2009)**
  - M.Matsuzaki PRC **73, 028801 (2006)**
  - K.Nakazato, K.Oyamatsu et.al PRL **103, 132501 (2009)**



Gyroid Double-Diamond

- Development of computer performance

Full 3D calculation

▪Menu

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Ⅱ. Relativistic Mean Field Theory

Ⅲ. Result (i) Fixed proton ratio

(ii) β - equilibrium

(iii) Large cell calculation

Ⅳ. Conclusion / Future plan

Relativistic Mean Field Theory

$$L=\bar{\psi}\left(i\gamma^{\mu}\partial_{\mu}-m-g_{\sigma}\sigma-g_{\omega}\gamma^{\mu}\partial_{\mu}-g_{\rho}\gamma^{\mu}\tau^{a}\rho_{\mu}^{a}\right)\psi \quad \text{: Neucleon}$$

$$+\frac{1}{2}\partial_{\mu}\sigma\partial^{\mu}\sigma-\frac{1}{2}m_{\sigma}^{2}\sigma^{2}+\frac{1}{3}bm_{\sigma}(g_{\sigma}\sigma)^{3}+\frac{1}{4}c(g_{\sigma}\sigma)^{3} \quad :\sigma-meson$$

$$-\frac{1}{4}F_{\mu\nu}F^{\mu\nu}+\frac{1}{2}m_{\rho}^{2}\rho_{\mu}^{a}\rho^{a\mu}+\frac{1}{2}\left[\frac{1}{2}\left(\partial^{\mu}\rho^{a\nu}-\partial^{\nu}\rho^{a\mu}\right)\left(\partial_{\mu}\rho_{\nu}^{a}-\partial_{\nu}\rho_{\mu}^{a}\right)\right] \quad :\rho\text{-meson}$$

$$+\frac{1}{2}m_{\omega}^{2}\omega_{\mu}\omega^{\mu}+\frac{1}{2}\left[\frac{1}{2}\left(\partial^{\mu}\omega^{\nu}-\partial^{\nu}\omega^{\mu}\right)\left(\partial_{\mu}\omega_{\nu}-\partial_{\nu}\omega_{\mu}\right)\right] \quad :\omega\text{-meson}$$

mean field approx.

$$\langle\sigma\rangle=\sigma$$

$$\langle\omega^{\mu}\rangle=\delta^{\mu,0}\omega^{\mu}$$

$$\langle\rho^{\mu\nu}\rangle=\delta^{\nu,0}\rho^{\mu\nu}$$

+

- Thomas Fermi approx.
- zero temperature

$$-\nabla^{2}\sigma(\vec{r})+m_{\sigma}^{2}\sigma(\vec{r})=-\frac{dU(\sigma)}{d\sigma}+g_{\sigma N}\left(\rho_{n}^{s}(\vec{r})+\rho_{p}^{s}(\vec{r})\right)$$

$$-\nabla^{2}\omega_{0}(\vec{r})+m_{\omega}^{2}\omega_{0}(\vec{r})=g_{\omega N}\left(\rho_{p}(\vec{r})+\rho_{n}(\vec{r})\right)$$

$$-\nabla^{2}\rho_{0}(\vec{r})+m_{\rho}^{2}\rho_{0}(\vec{r})=g_{\rho N}\left(\rho_{p}(\vec{r})-\rho_{n}(\vec{r})\right)$$

$$\nabla^{2}V(\vec{r})=4\pi e^{2}\left(\rho_{p}(\vec{r})+\rho_{e}(\vec{r})\right)$$

$$\mu_{p}=\mu_{B}-\mu_{e}+V=\upsilon_{p}+g_{\omega N}\omega_{0}+g_{\rho N}\rho_{0}$$

$$\mu_{n}=\mu_{B}=\upsilon_{n}+g_{\omega N}\omega_{0}-g_{\rho N}\rho_{0}$$

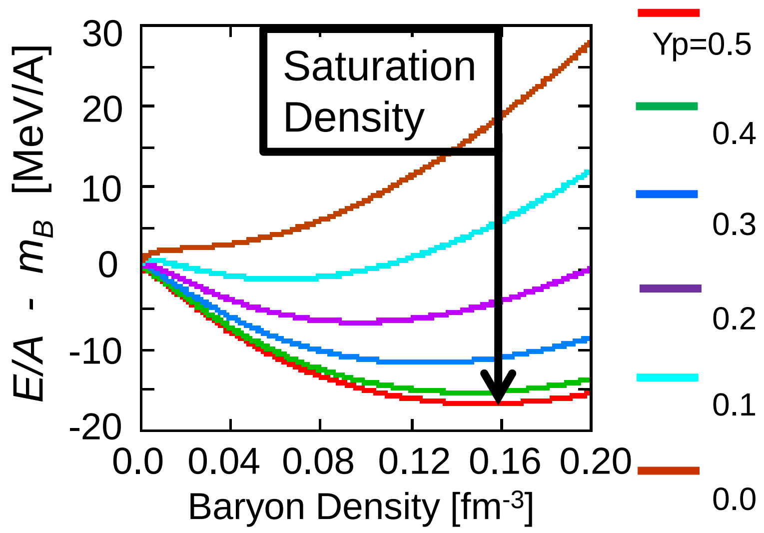
$$\mu_{e}=\left(3\pi^{2}\rho_{e}(\vec{r})\right)^{1/3}+V(\vec{r})$$

・Parameters

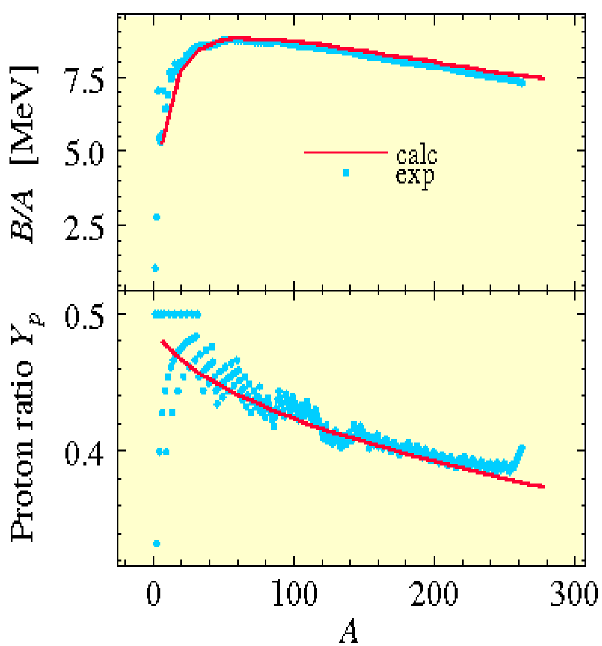
To reproduce the saturation properties of symmetric nuclear matter and the properties of finite nuclei

$$g_{\sigma N}=6.3935 \quad g_{\omega N}=8.7207 \quad g_{\rho N}=4.2696 \quad b=0.008659 \quad c=-0.002421$$

$$m_N=938[\mathrm{MeV}] \quad m_\sigma=400[\mathrm{MeV}] \quad m_\omega=783[\mathrm{MeV}] \quad m_\rho=769[\mathrm{MeV}]$$



PHYSICAL REVIEW C **72, 015802 (2005)**



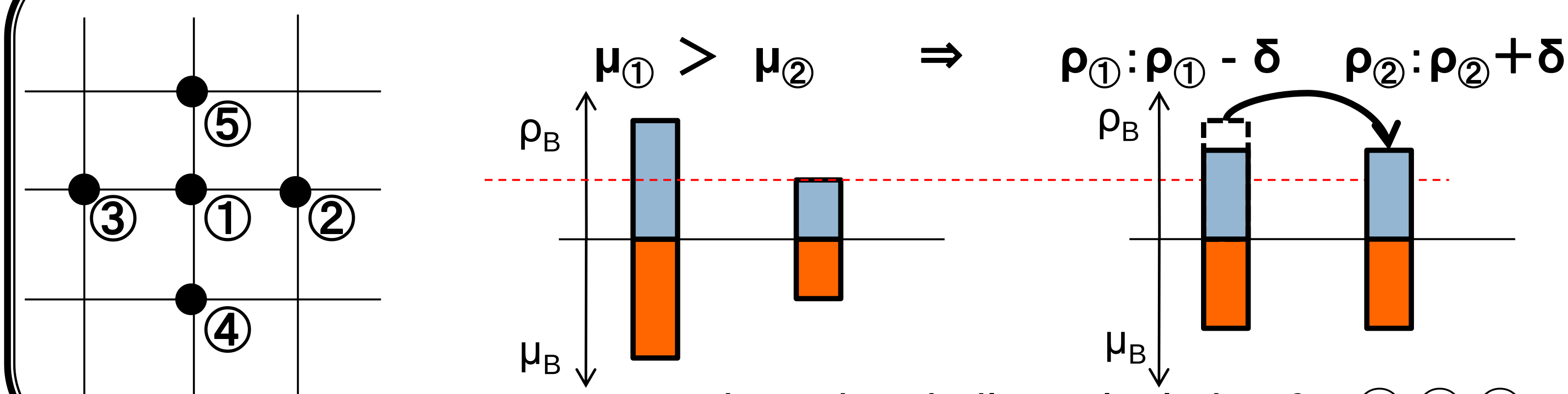
Saturation density : 0.153 [fm^{-3}]

Binding Energy : -16.4 [MeV]

Incompressibility K : 240 [MeV]

・How to solve ・・・?

- Introduce a cubic cell with periodic boundary condition and divide it into grids
- As an initial condition, randomly distribute fermions (n, p, e) over the grid
- We solve coupled differential equations, and simultaneously relax fermions density distributions to attain the uniformity of their chemical potential



Work on the similar calculation for ③,④,⑤

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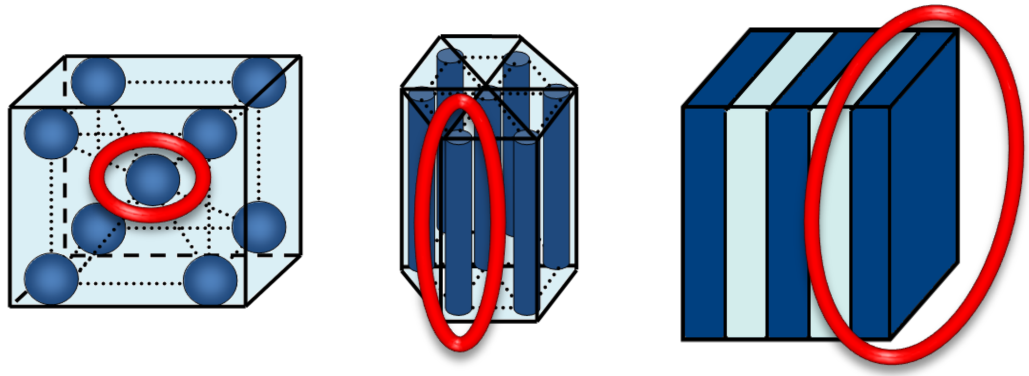
(ii) β - equilibrium
(iii) Large cell calculation

Ⅳ. Conclusion / Future plan

(i) Fixed proton ratio

Basic structure of "Pasta" : Like "atom / molecule" which construct "Crystal"



Basic structure only appears by the calculation using W-S approximation.

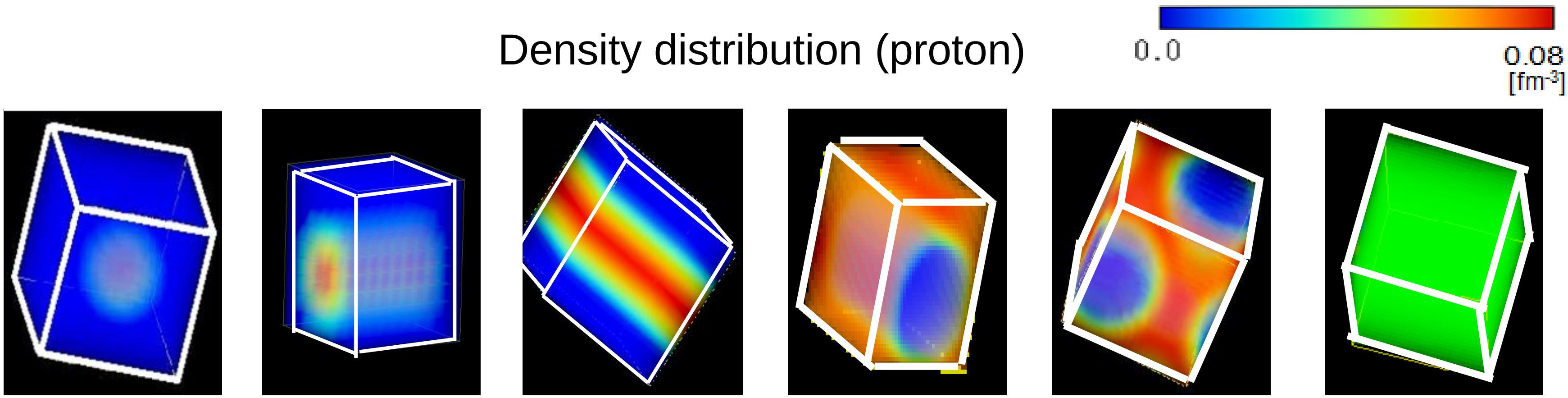
Energy and size of basic structure (R) and cell size (R_w) are calculated in detail.

Compare the Energy and cell size of basic structure with W-S approximation.

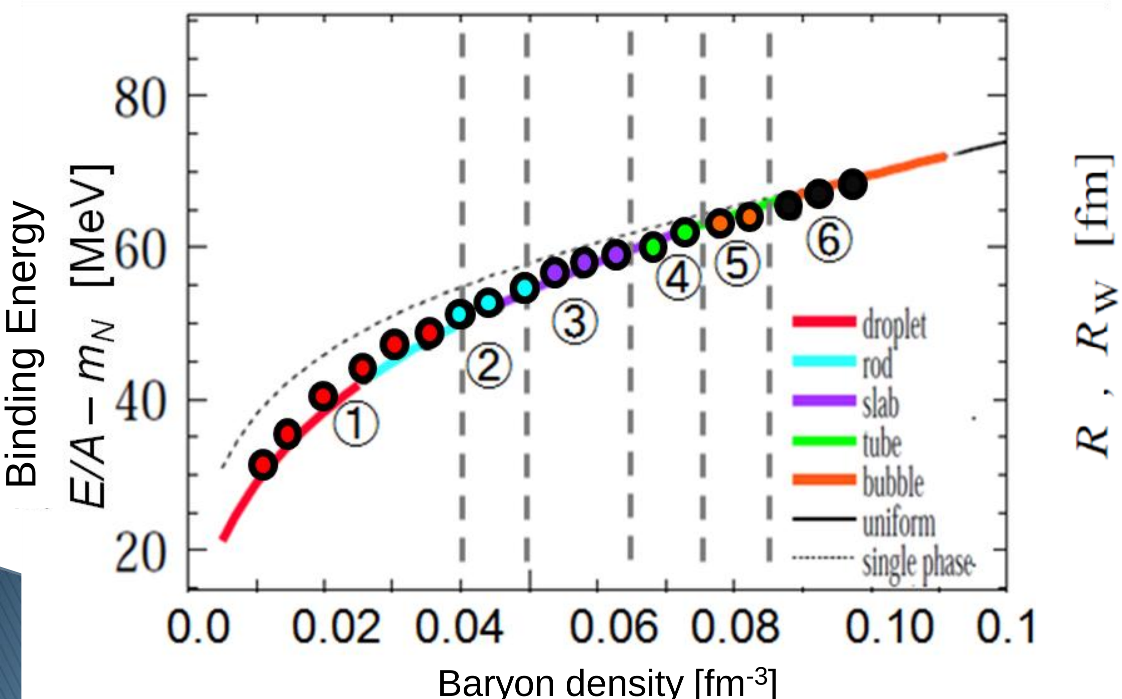
$$R = \begin{cases} R_w \dfrac{\langle \rho_p \rangle^2}{\langle \rho_p^2 \rangle} & (droplet, rod, slab) \\ R_w \left(1 - \dfrac{\langle \rho_p \rangle^2}{\langle \rho_p^2 \rangle} \right) & (tube, bubble) \end{cases}$$

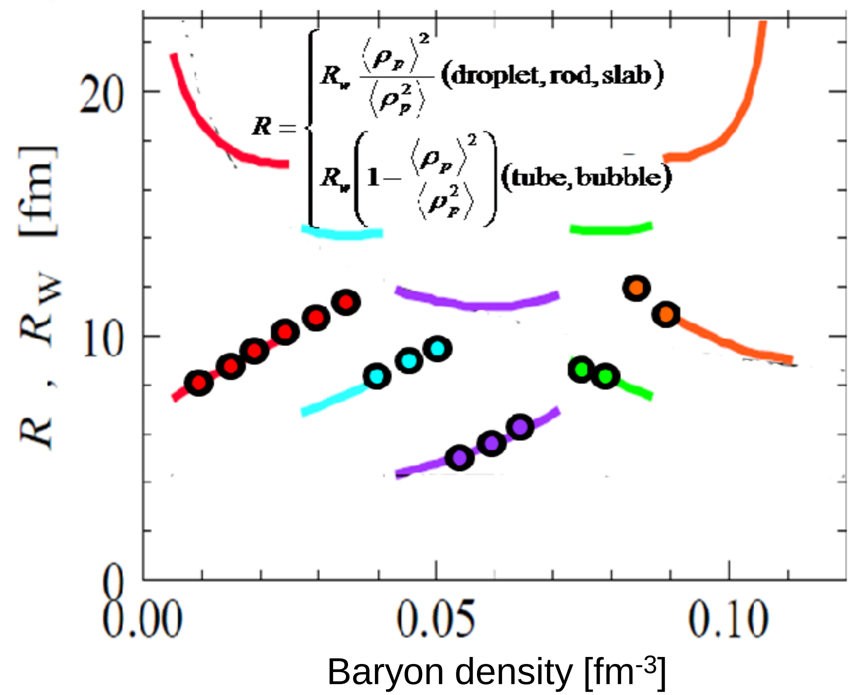
(i) Fixed proton ratio [Yp(=(A-Z)/A)=0.5]

Density distribution (proton)

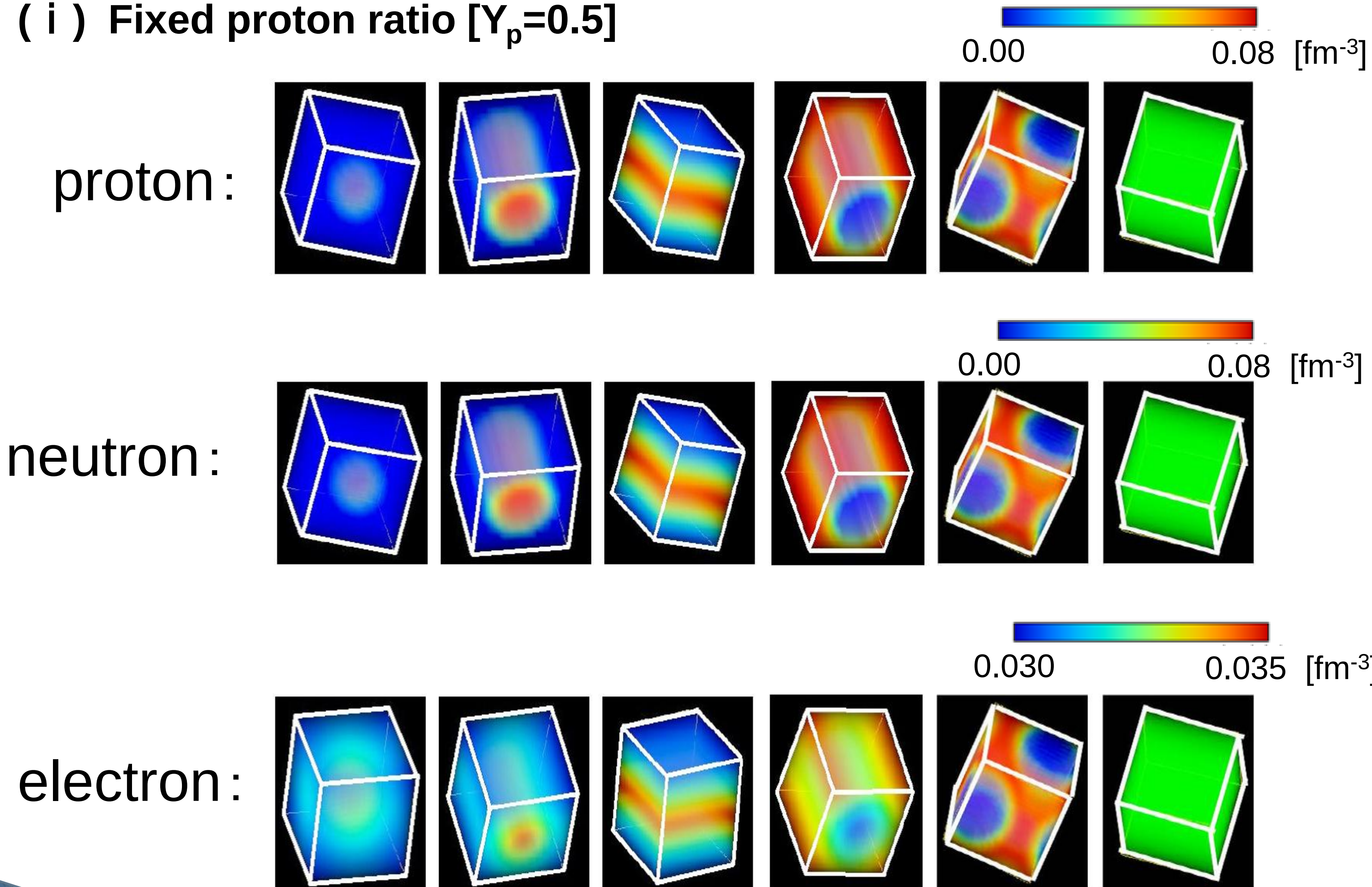


①0.020fm^{-3} ②0.045fm^{-3} ③0.050fm^{-3} ④0.070fm^{-3} ⑤0.080fm^{-3} ⑥0.100fm^{-3}





(i) Fixed proton ratio [Y_p=0.5]



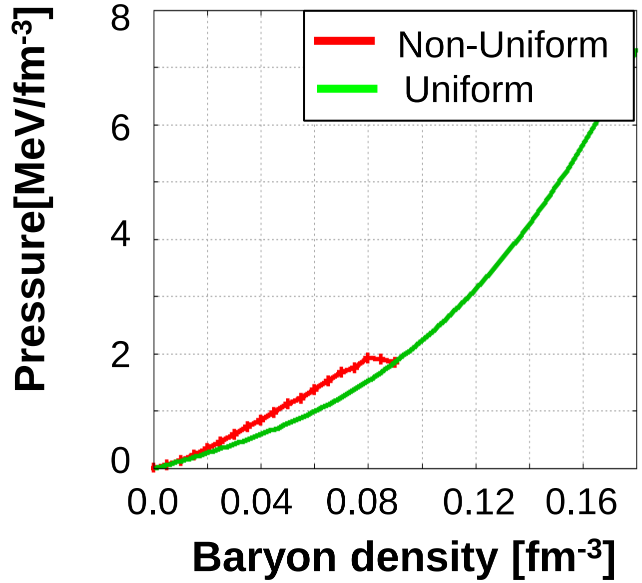
▪Mechanism of clusterization

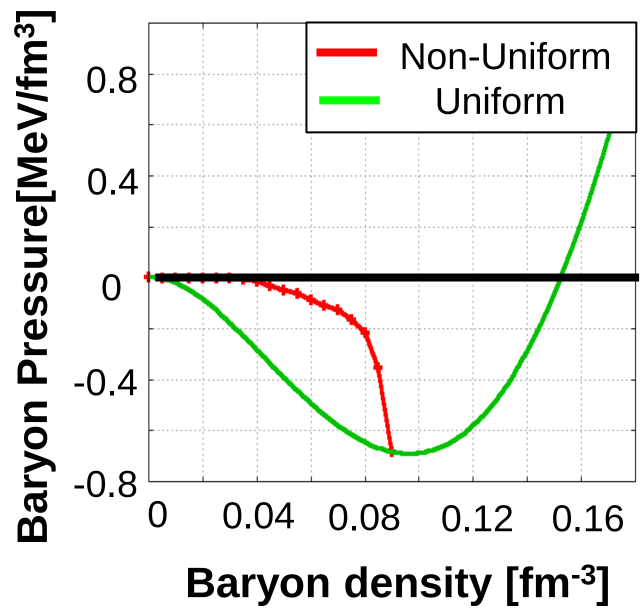
◦ Total Pressure :
positive by electron partial pressure

◦ Baryon partial pressure :
negative in $\rho_B < \rho_0$
→ unstable
→ clusterization (pasta structure)

◦ $\rho_0 < \rho_B$: Uniform matter
← Energy loss of Coulomb repulsion and Surface tension

(ρ_0: normal nucleus density)





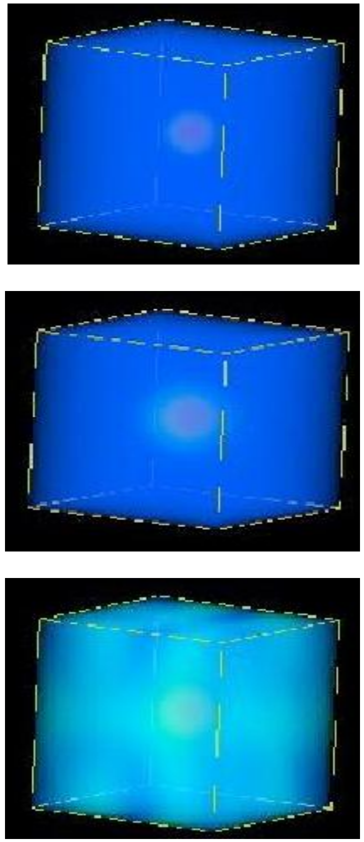
(ii) β-equilibrium (Crust of neutron star)

Full Calculation

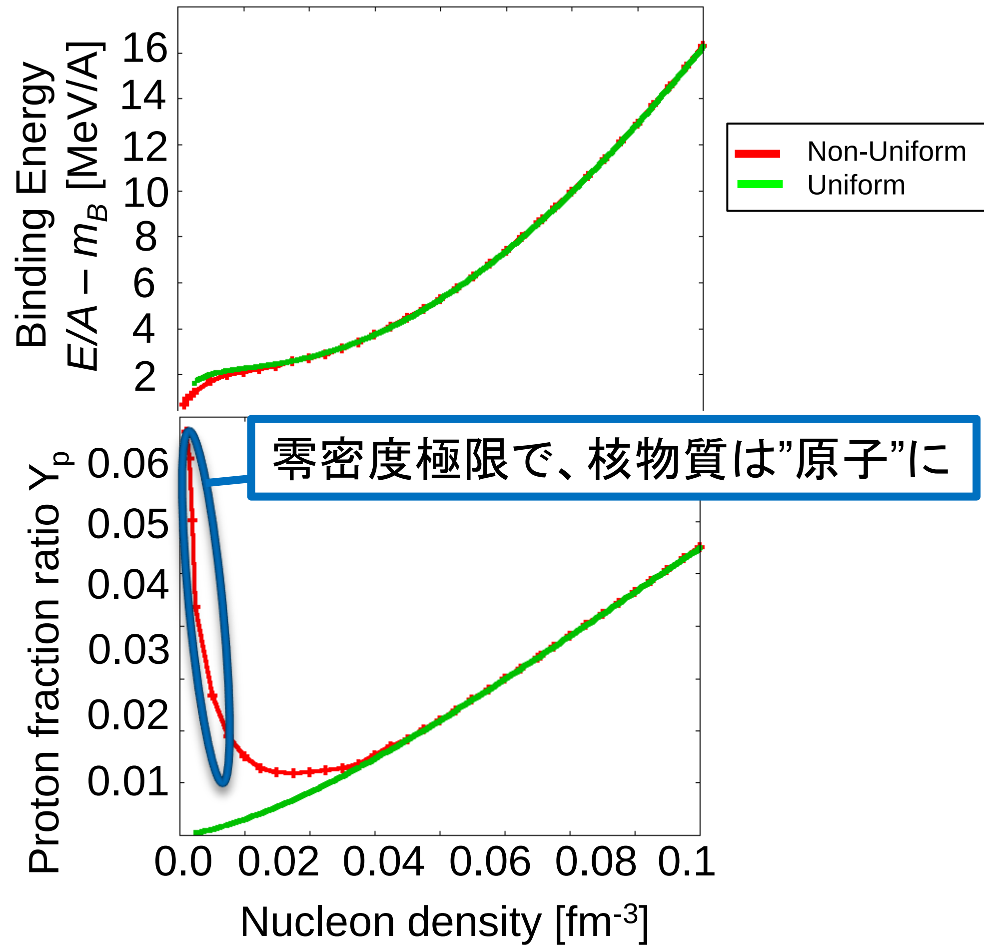
・proton

・neutron

・electron



nucleon density : 0.03[fm^{-3}]

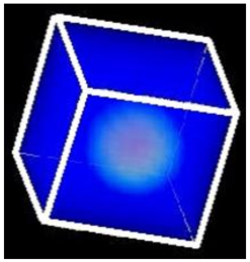


Only Sphere shape appears

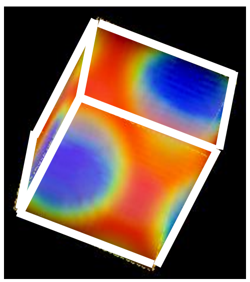
→ Similar result with Wigner-Seitz cell approximation

(iii) Crystal structure

Calculation in small cell:
- Without symmetry
- Periodic boundary condition



→ Simple Cubic



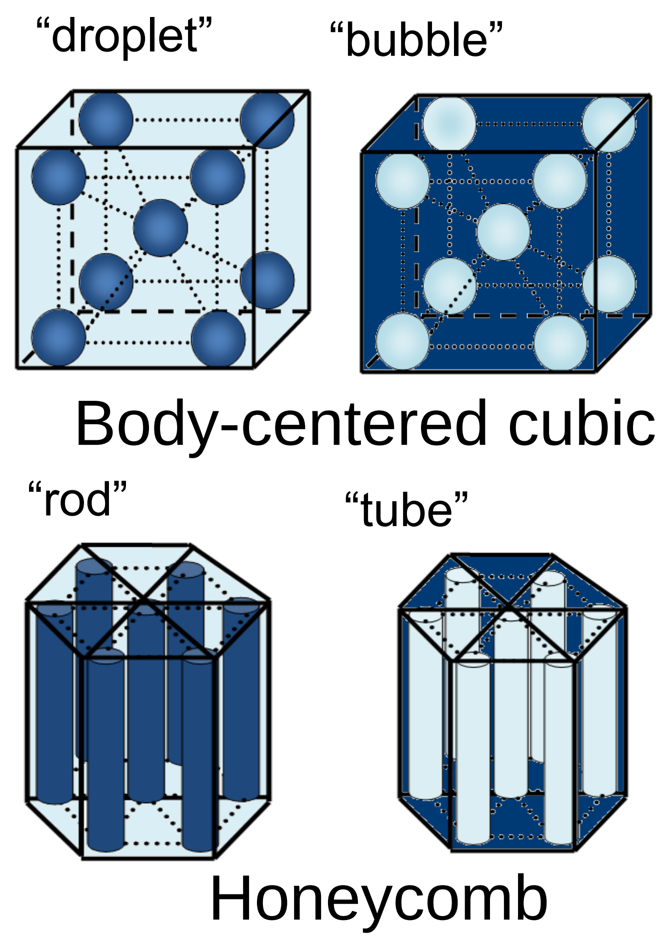
→ Body-centered cubic

Limitation for crystal structure by cell size

⇩

Calculation in Large cell

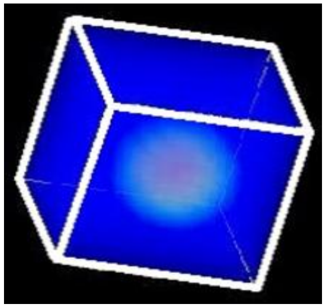
Pasta crystal structure



(K.Oyamatsu, Nucl.Phys.**A561**,431(1993))

(iii) Crystal structure (Y_p=0.5)

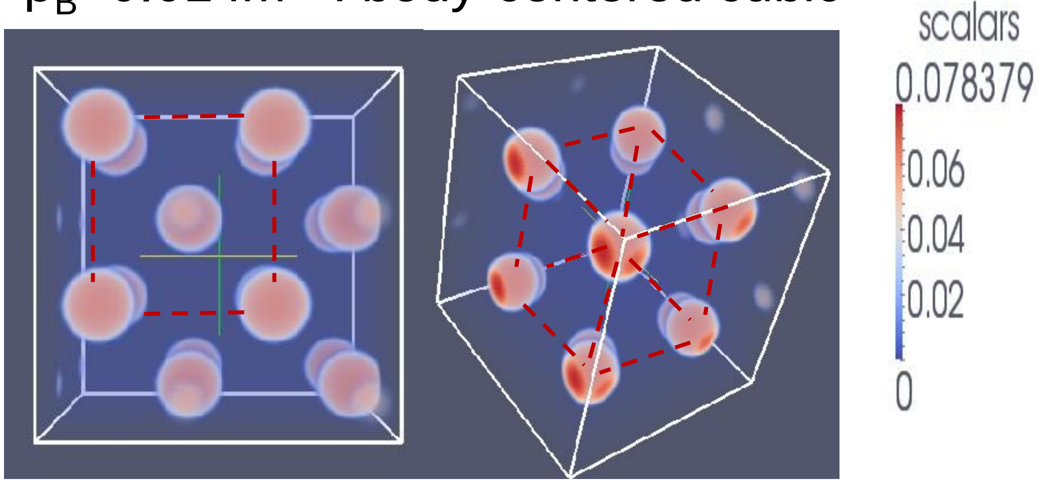
droplet



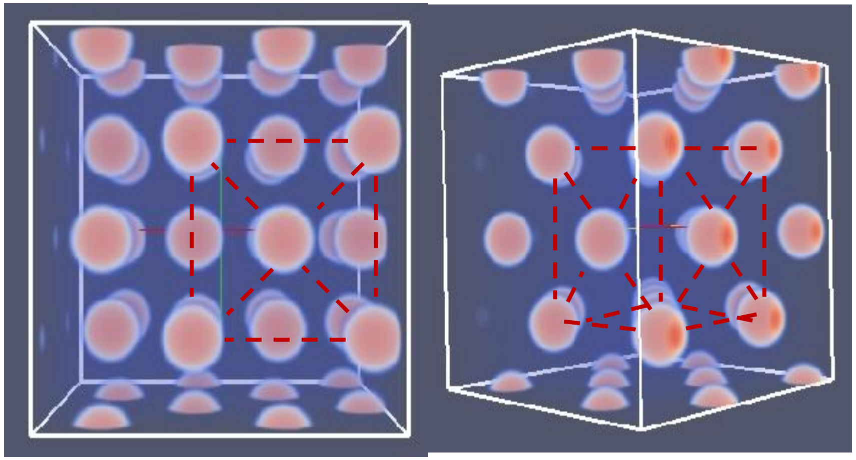
“simple” or “bcc” or “” or “fcc” or ?

“bcc” → “fcc”
(change by baryon density)

ρ_B=0.01 fm^{-3} : body-centered cubic

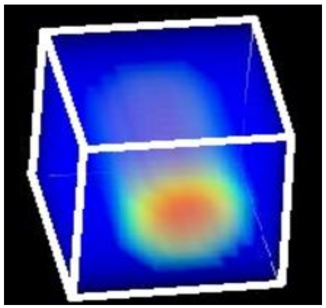


ρ_B=0.015 fm^{-3} : face-centered cubic



(iii) Crystal structure (Y_p = 0.5)

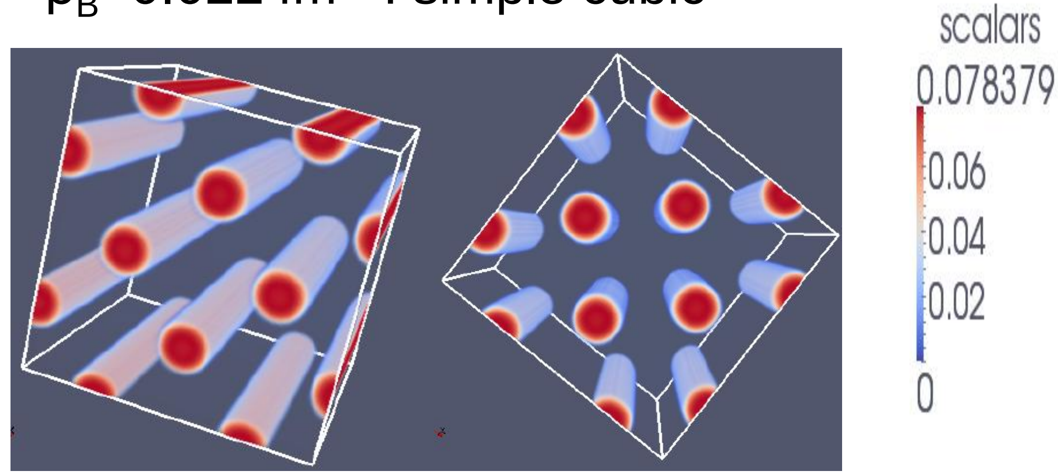
rod



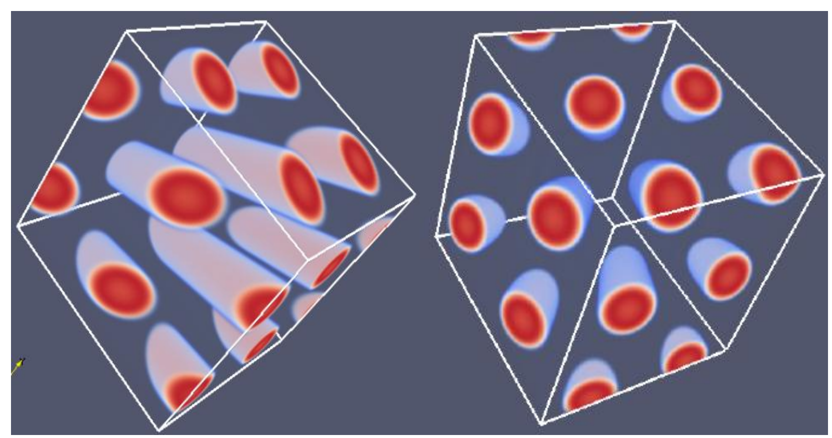
"Simple" or
"Honey-cum" or ･･･?

"Simple" → "Honey-cum"
(change by baryon density)

ρ_B=0.022 fm^{-3} : simple cubic

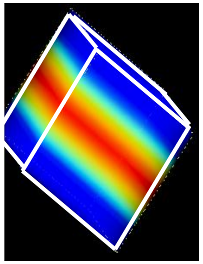


ρ_B=0.028 fm^{-3} : Honey-cum

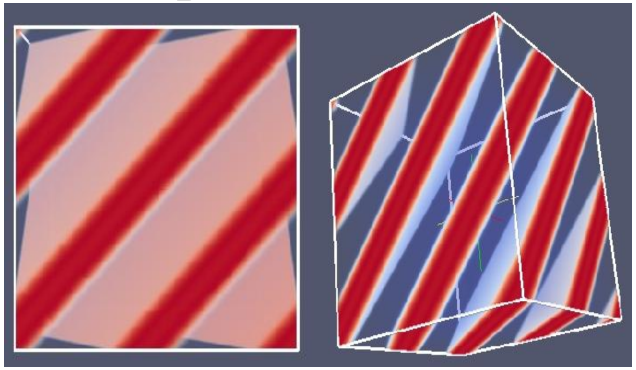


(iii) Crystal structure

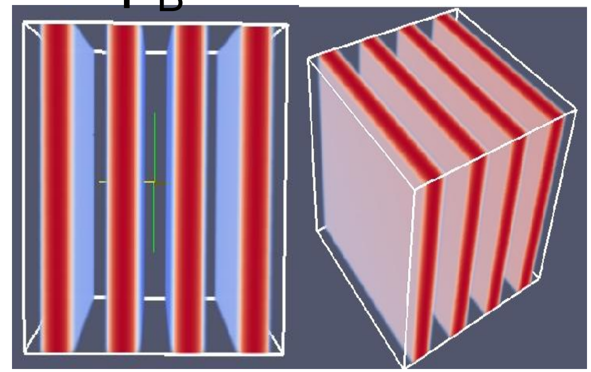
slab

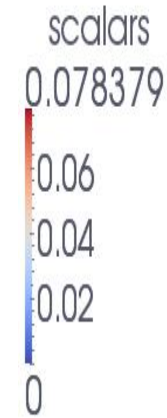


ρ_B=0.05 fm^{-3}

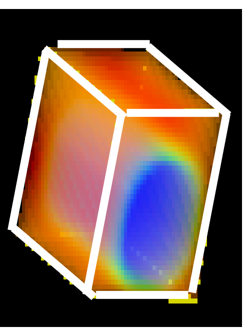


ρ_B=0.06 fm^{-3}





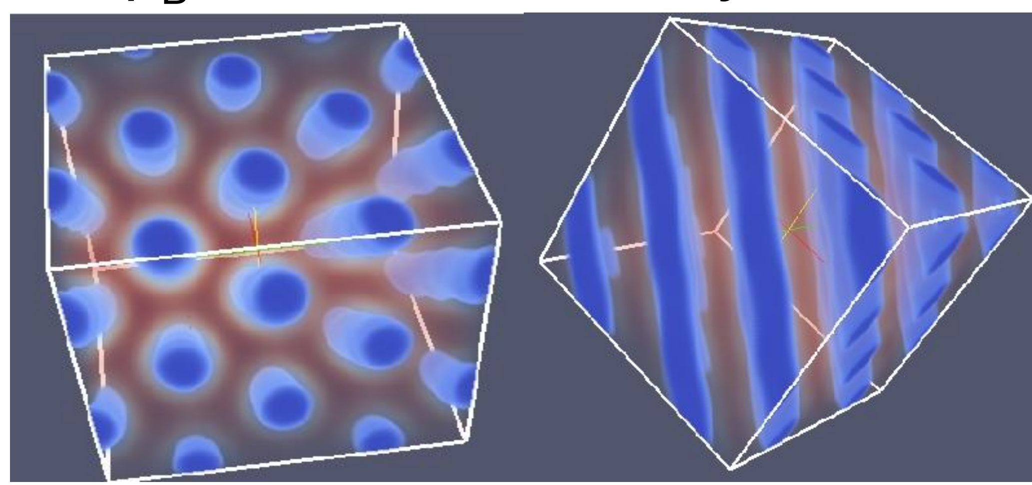
tube



“Simple” or
“Honeycomb” or
・・・?

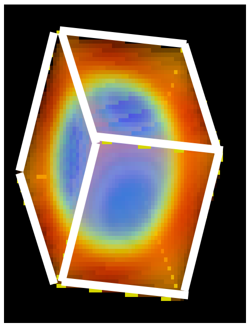
“Honeycomb”

ρ_B=0.075 fm^{-3} : Honey-cum



(iii) Crystal structure

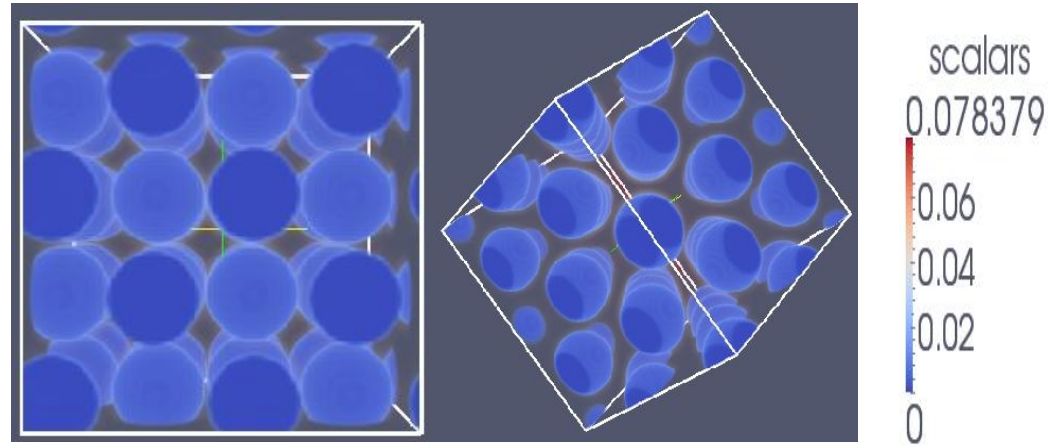
<u>bubble</u>



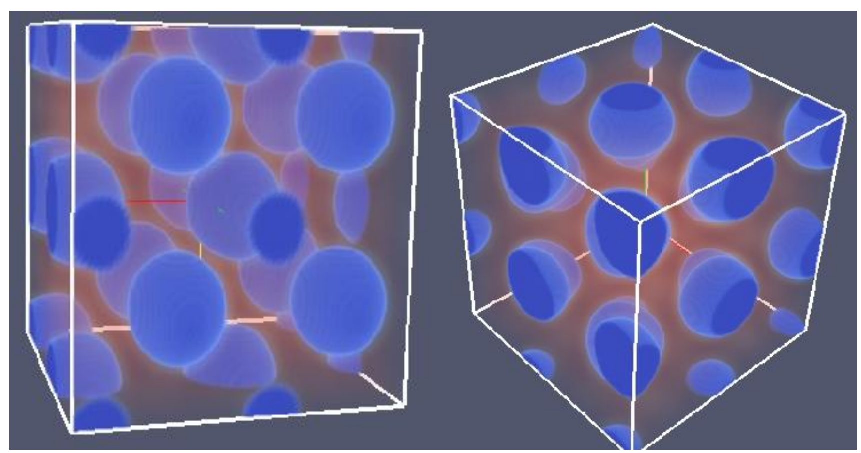
"Simple" or "fcc" or "bcc" or ･･･?

"bcc" → "fcc"
(change by baryon density)

ρ_B=0.085 fm^{-3} : face-centered cubic



ρ_B=0.09fm^{-3} : body-centered cubic

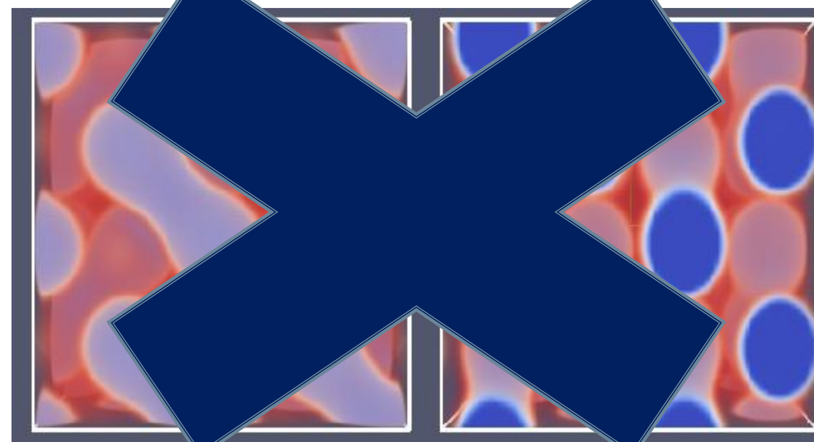


(iii) Crystal structure

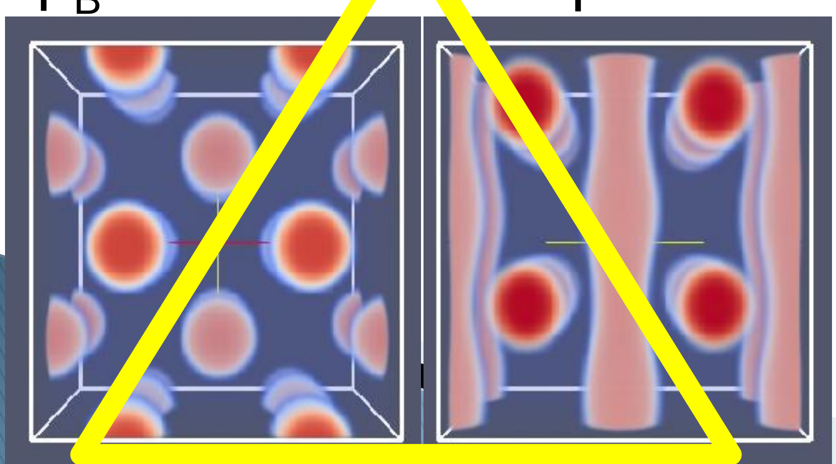
Complex Pasta structure

ρ_B=0.015 fm^{-3} : dumbell

ρ_B=0.09 fm^{-3}



ρ_B=0.025 fm^{-3} : droplet & rod



小さいセルでの計算との比較
：　0.14MeV低い
大きいセルでの比較：　未実行

プログラムミス(？)

Ⅳ. Conclusion / Future

We demonstrate 3D calculation of non-uniform low-density nuclear matter based on Relativistic mean field theory and Thomas-Fermi approximation.

- For fixed proton ratio calculation, we perform same cell size calculation of W-S approximation and get almost same pasta structures and baryon density dependence of binding energy.

- For β-equilibrium calculation, only sphere shape appears. It is similar result with W-S cell approximation.

- We perform large size cell calculation in which some basic structure appear.

Ⅳ. Conclusion / Future

- Expansion of cell size
  - What crystal structure is the most stable state?
  - EOS for various proton fraction ratio

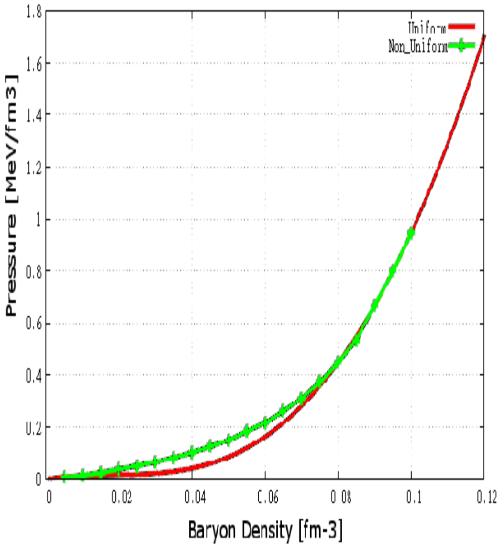
- Extension to high density and finite temperature nuclear matter

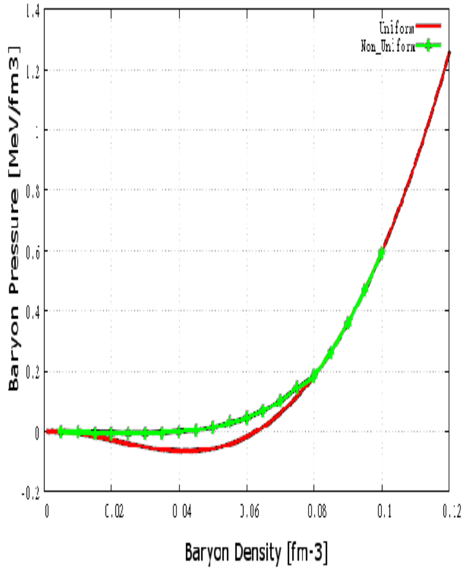
- Comparing with QMD calculation result for supernova compression process and local minimum states of our calculation.

END

Back Up

Yp=0.1





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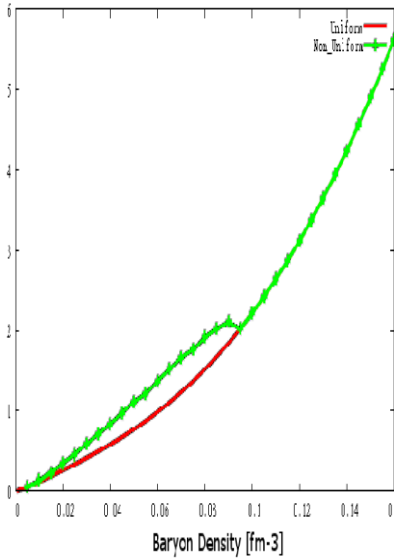

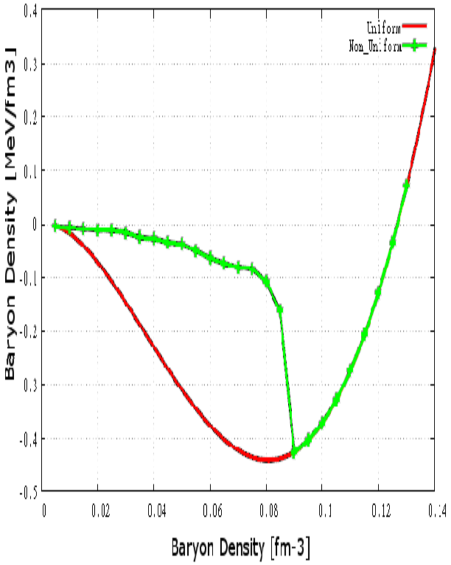

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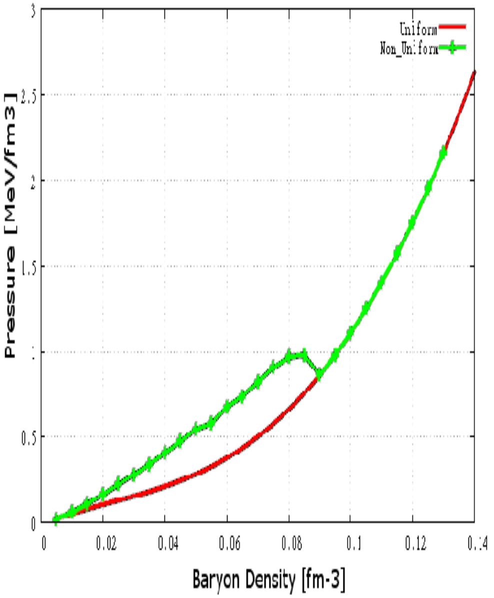
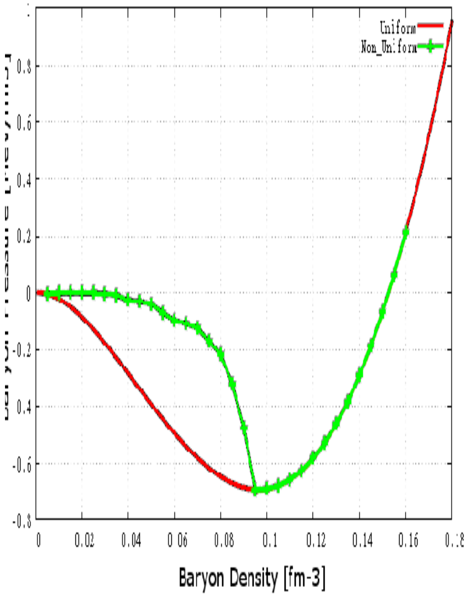
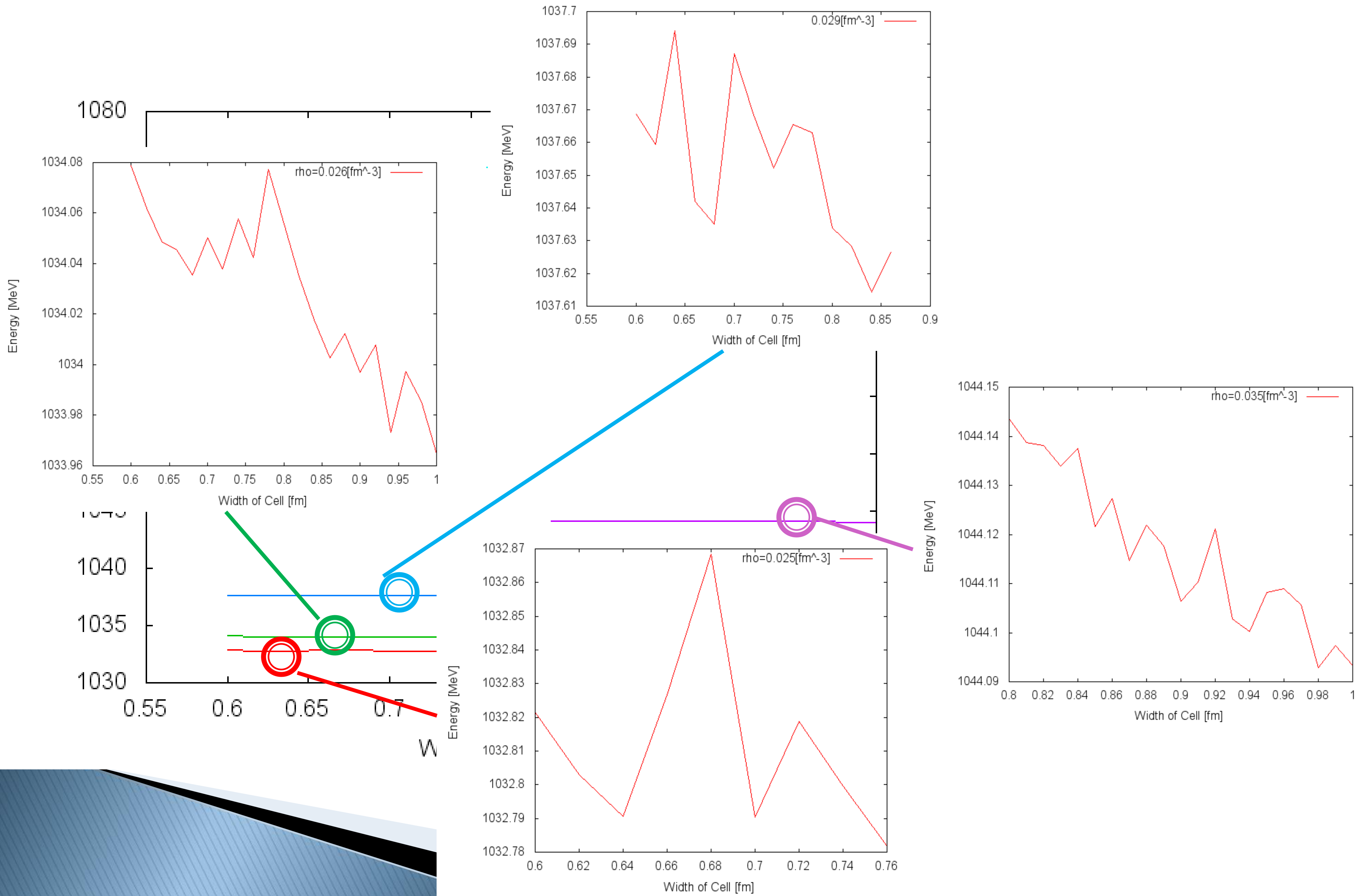
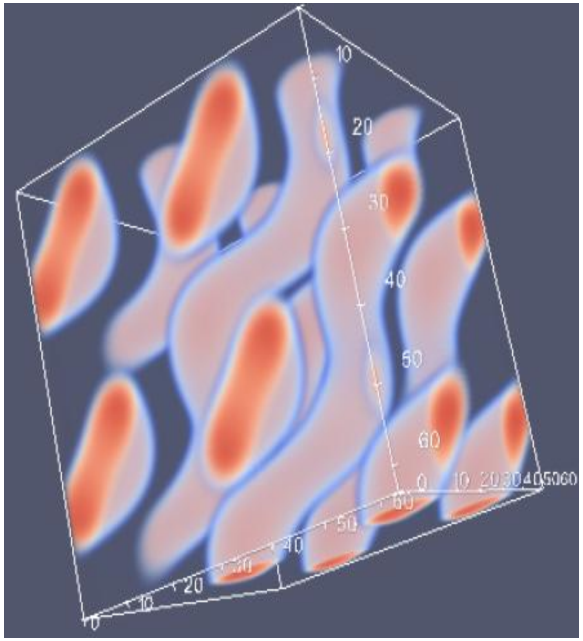


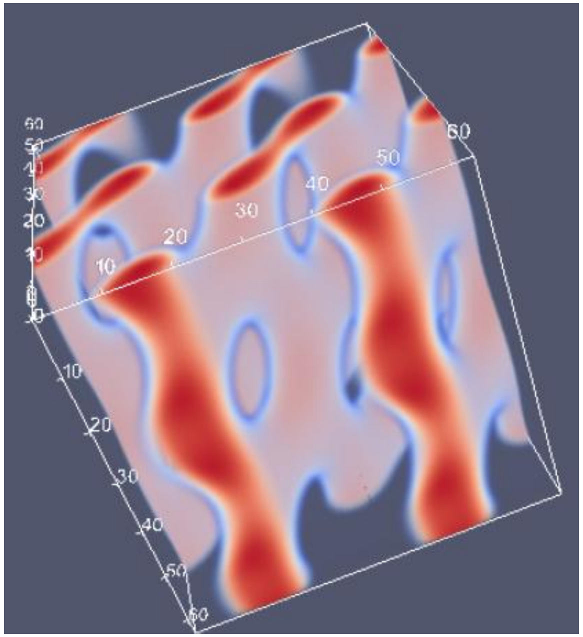


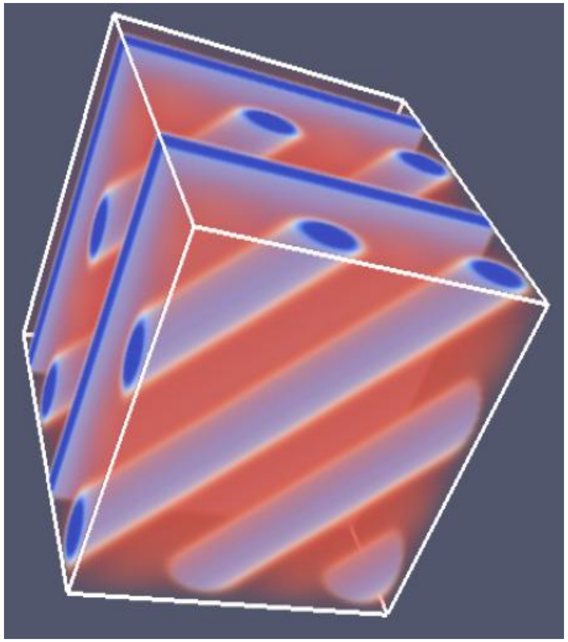
・グリッド幅とエネルギー



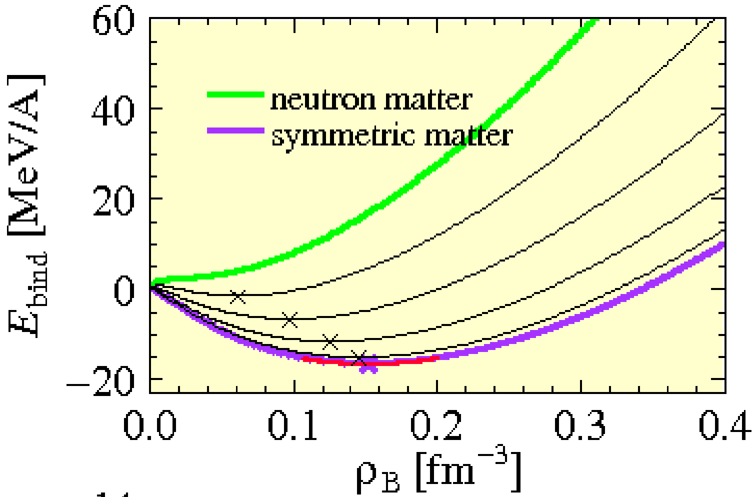
Complex Pasta structure

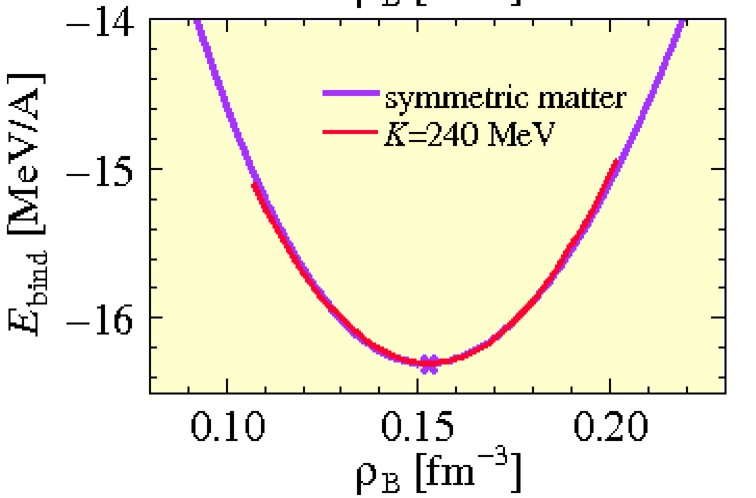






核物質の結合エネルギー





核物質の結合エネルギーのモデルの1例。
密度ρ_0(約0.16fm^{-3})の対称核物質
がもっとも安定。

核物質の固さ(incompressibility)

$$K = p_F^2 \frac{d^2\varepsilon}{dp_F^2} = 9\rho^2 \frac{d^2\varepsilon}{d\rho^2} = 9\frac{dP}{d\rho}$$

は重要な量だが、まだ決まっていない。

図の赤線は$K = 240$ MeVの２次曲線。

パスタ構造　=　一次相転移に伴う物質の非一様構造
「構造を持った混合相」

$$\text{Total Energy} = (\text{bulk}) + (\text{Surface}) + (\text{Coulomb})$$

クーロン斥力と表面張力の釣合いによる規則的な構造

$$\frac{E_C}{A} \propto \frac{Z^2/R}{A} \propto R^2 \quad \Rightarrow \frac{E_C}{A} = aR^2$$

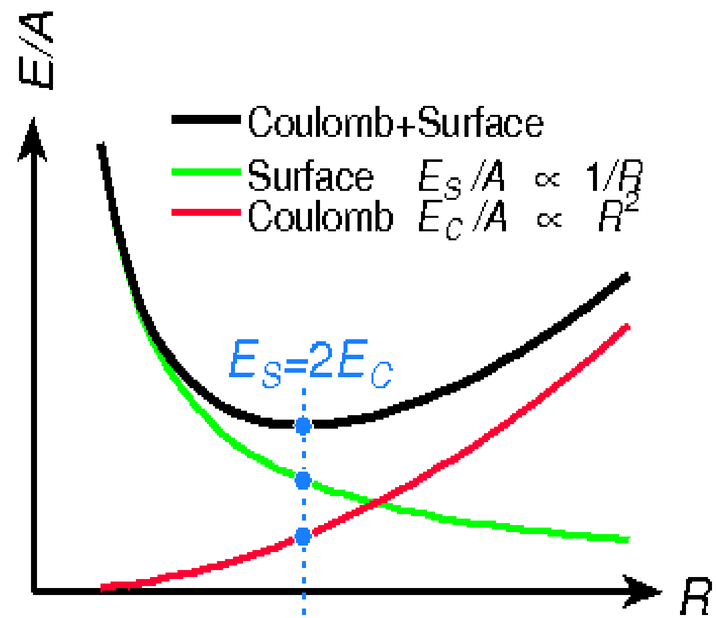
$$\frac{E_S}{A} \propto \frac{R^2}{R^3} \propto R^{-1} \quad \Rightarrow \frac{E_S}{A} = bR^{-1}$$

$$\frac{d((E_C + E_S)/A)}{dR} = 0 \quad \text{(エネルギー最少)}$$

$$\frac{d(aR^2 + bR^{-1})}{dR} = 2aR - bR^{-2} = 0$$

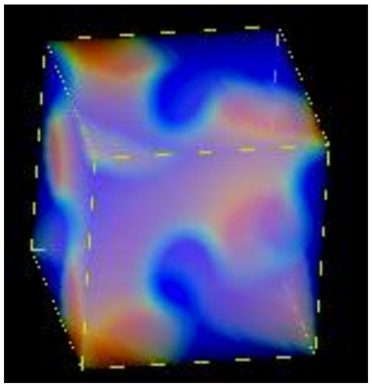
$$2aR^2 = bR^{-1}$$

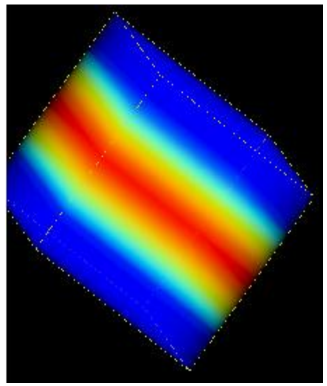
$$2E_C = E_S \quad \text{(釣り合い条件)}$$



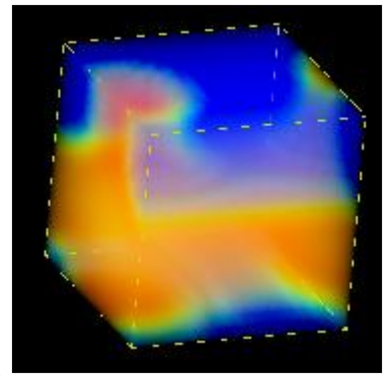
N_{Grid}=50×50×50, グリッド幅=0.8fm の場合

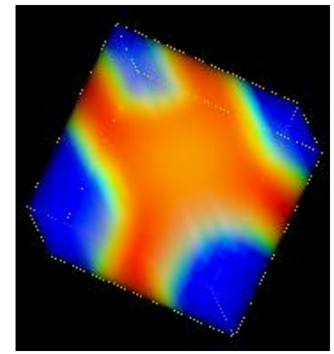
$\rho = 0.3\rho_0$



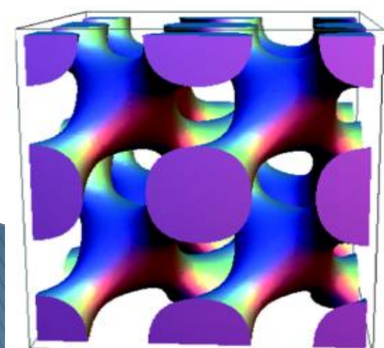


$\rho = 0.4\rho_0$

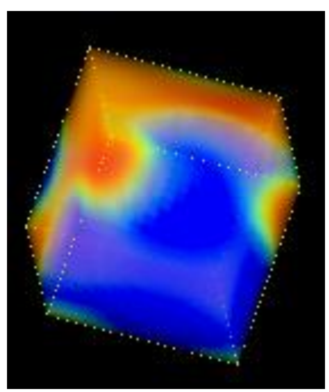


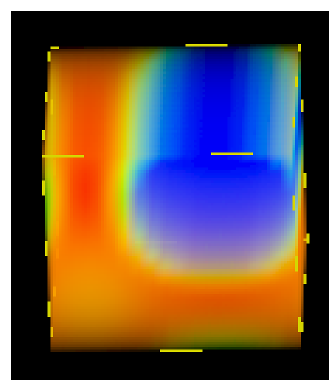


?



$\rho = 0.5\rho_0$





左:ローカルミニマム

右:基底状態

どの近似でもパスタ構造は現れる

(1)Liquid-drop model (Macroscopic)

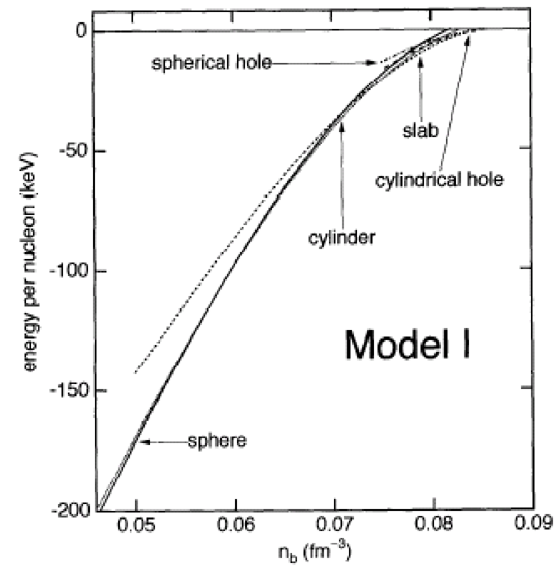
- Ravenhall,Pethick&Willson,PRL 50,2066(1983)
- Hashomoto,Seki&Yamada,PTP 71,320 (1984)
- Lorentz,Ravenhall&Pethick,PRL 70,379 (1993)

(2)Thomas-Fermi (Semi-classical)

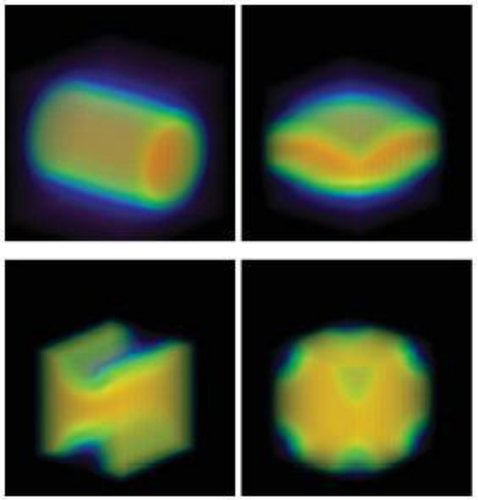
- Williams&Koonin, NPA 435, 844 (1985)
- Oyamatsu, NPA 561 ,431 (1993)
- Sumiyoshi&Oyamatsu&Toki ,NPA 595, 323 (1995)

(3)Hartree-Fock (Quantum)

- Magierski&Heenen, PRC 65, 045804 (2002)
- Gögelein & Müther, PRC 76, 024312 (2007)
- Newton & Stone, PRC 79, 055801 (2009)

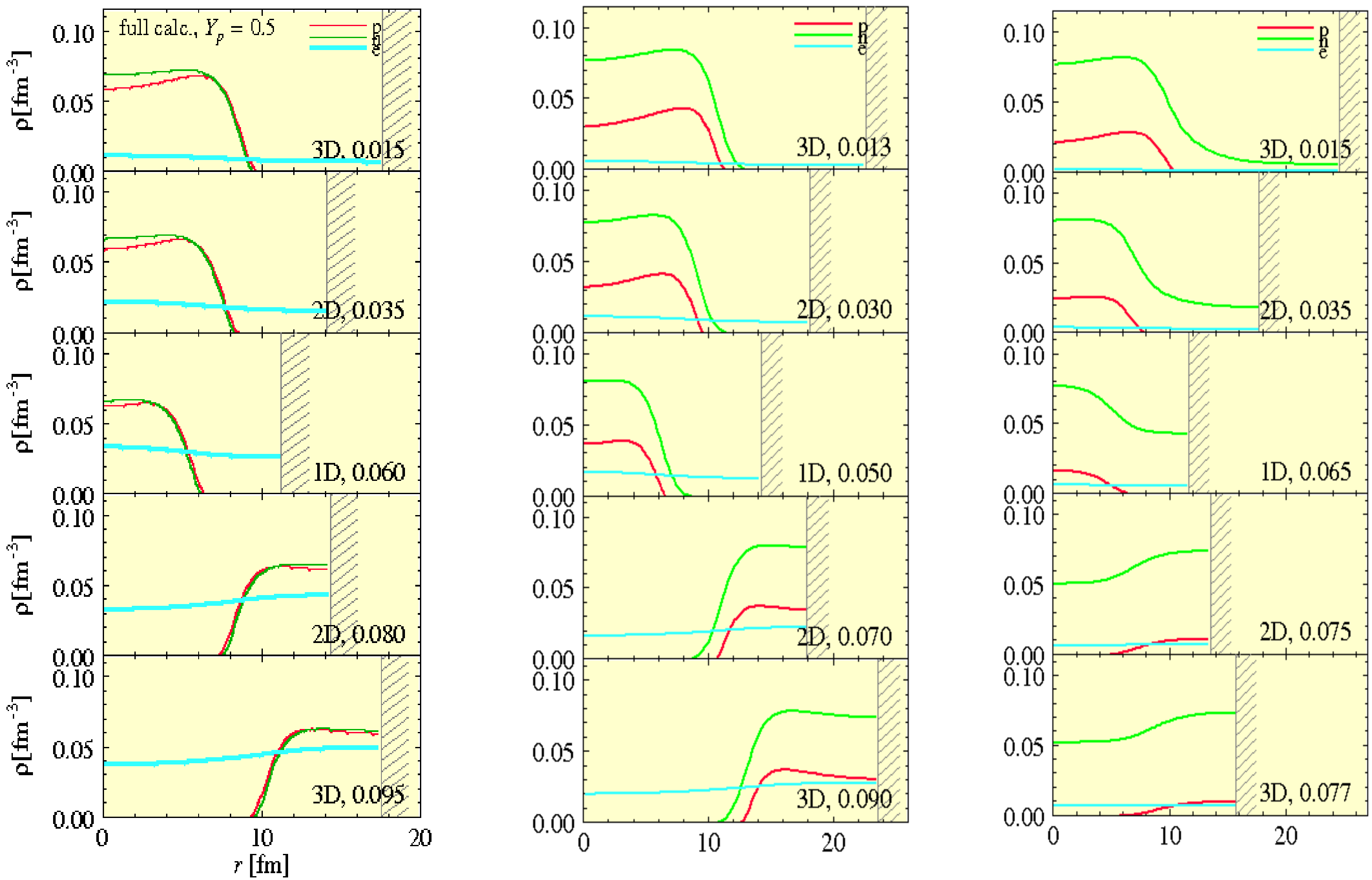


Oyamatsu (1993)



Newton (2009)

対称核物質 Y_p=0.5　　非対称核物質 Y_p=0.3　　非対称核物質 Y_p=0.1



T. Maruyama, T. Tatsumi Recent Res. Devel. in Phys. 7 (2006) 1.