Heavy ion collisions における quark number scaling

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PHYSICAL REVIEW C 68, 044902 (2003)

京都大学　原子核理論
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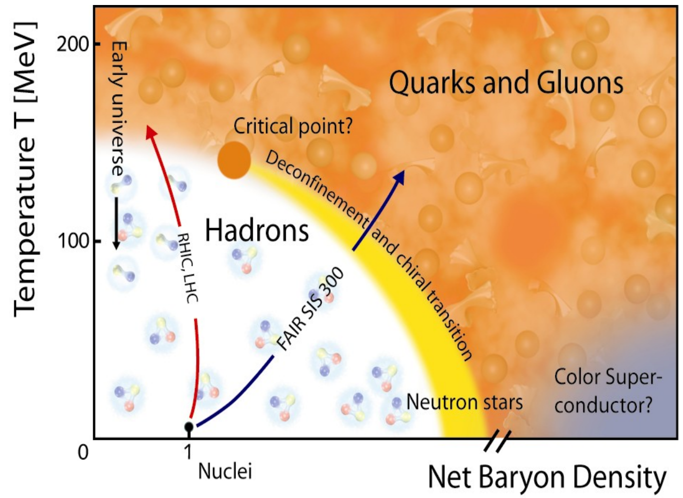
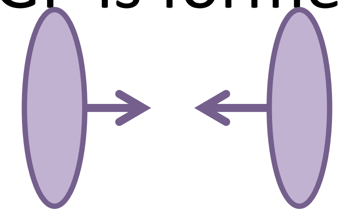
QGP

- QGP: quark gluon plasma at RHIC.

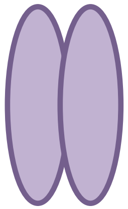


The Physics of High Baryon Densities International Workshop at ECT* in Trento May 29 - June 2, 2006

- QGP is formed by heavy ion collisions.



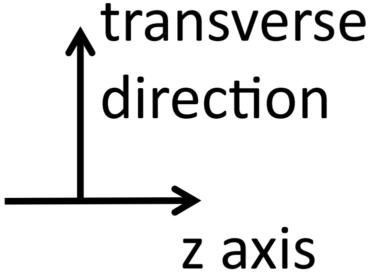
collisions

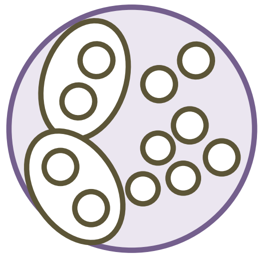


pre-equilibrium

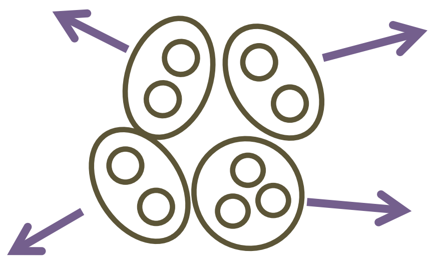
thermalization

Hydrodynamical expansion





hadronization

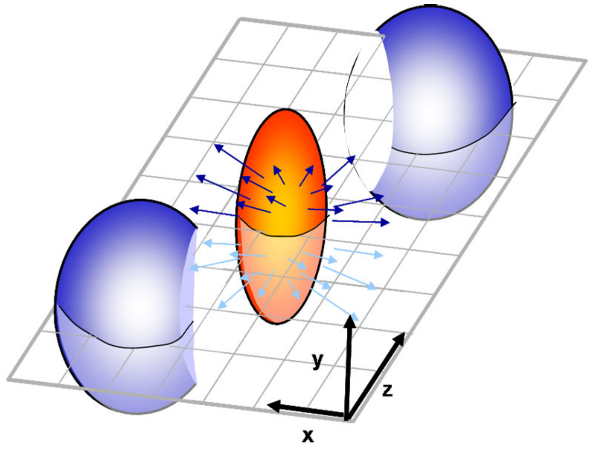


freeze-out

- One of the important measurement is elliptic flow.

Elliptic flow v_2

- To know particle emission dependence on azimuthal angle, expand in a Fourier cosine series.



Science grid this week, October 25, 2006

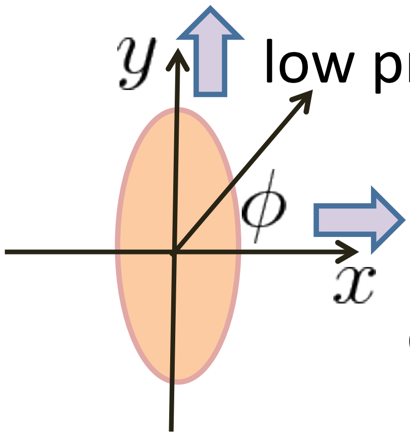
$$E\frac{d^3N}{d^3P} = \frac{d^2N}{2\pi P_T dP_T dy}\left(1 + \sum_n 2v_n \cos n\phi\right)$$

  – elliptic flow v_2

y : rapidity P_T: transverse momentum

- Elliptic flow reflects initial geometrical anisotropy.

QGP: hydrodynamics ⇨ difference of pressure gradient

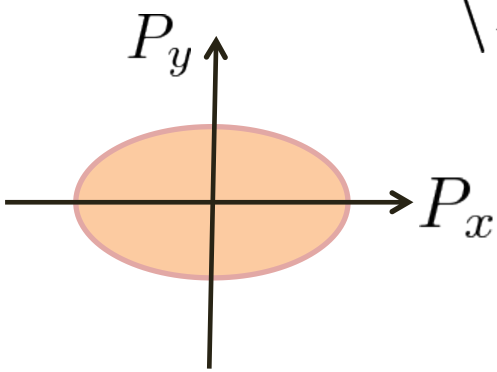


low pressure gradient

high pressure gradient

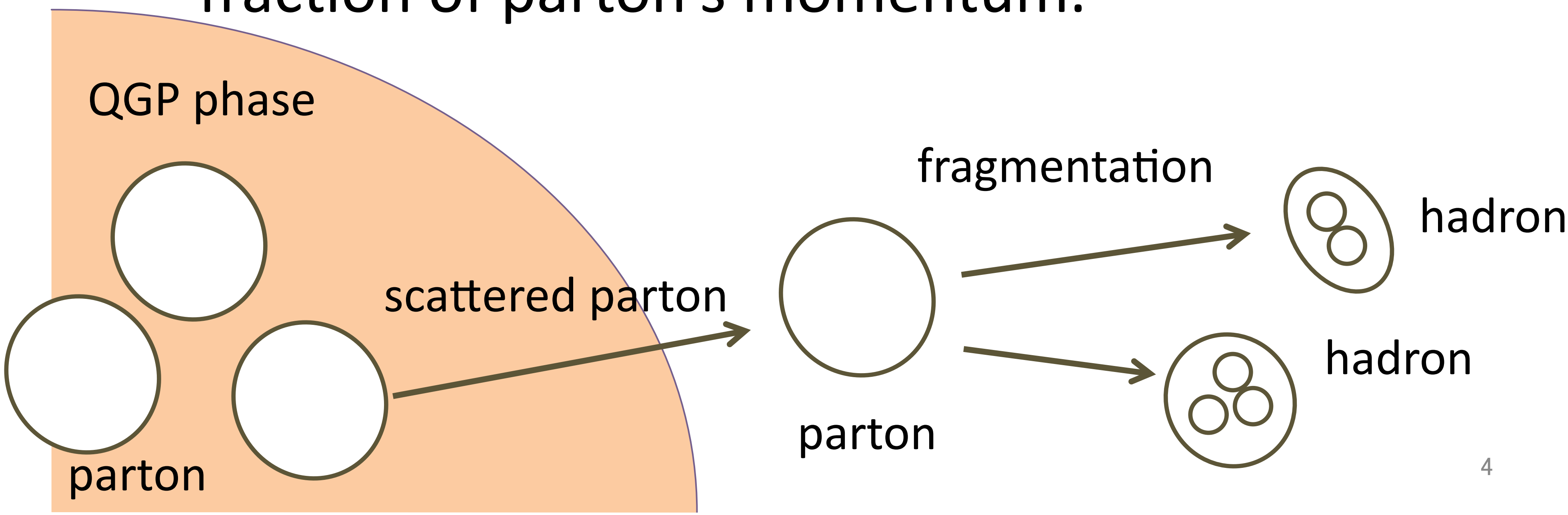
eccentlicity $\varepsilon = \frac{x^2 - y^2}{x^2 + y^2}$

$$v_2 = \langle \cos 2\phi \rangle = \left\langle \frac{p_x^2 - p_y^2}{p_x^2 + p_y^2} \right\rangle$$



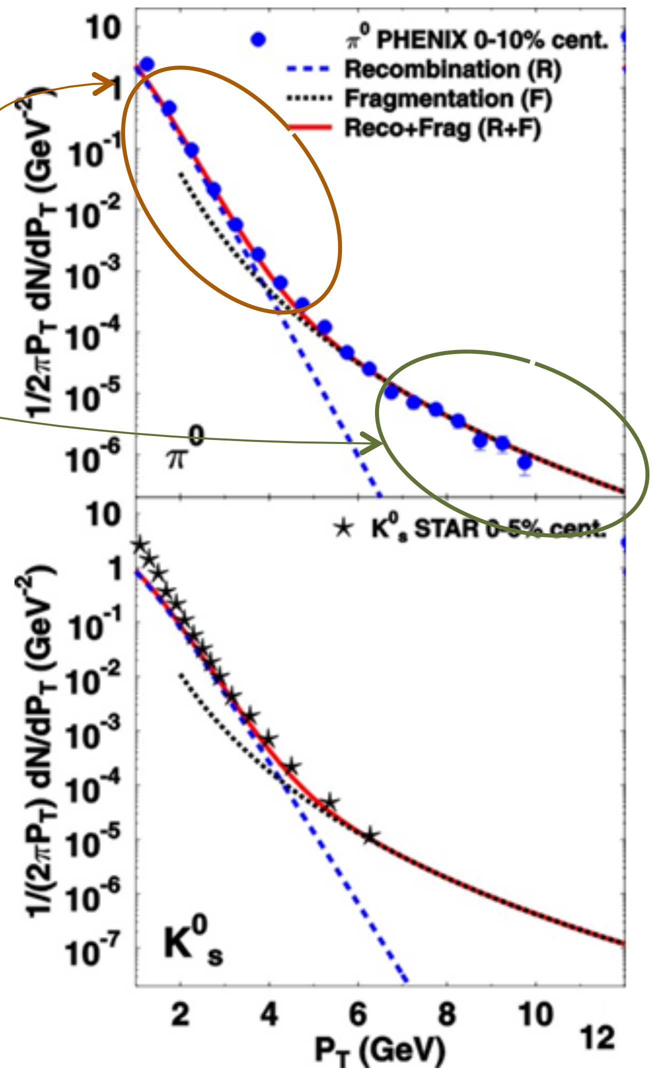
Existing mechanism ~ fragmentation

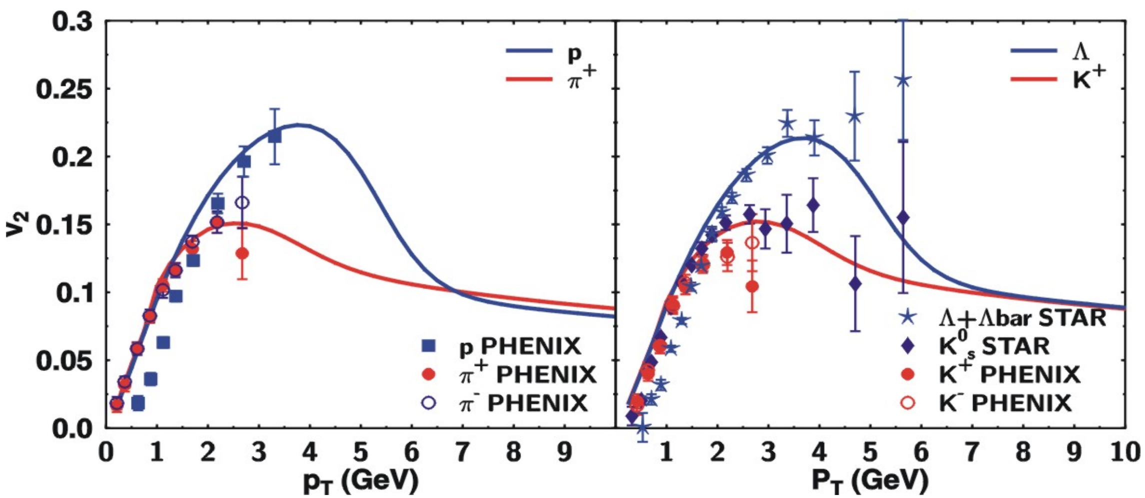
- Hadron production at sufficient large momentum can be described by pQCD.
- Fragmentation mechanism is a parton fragments into hadron carrying with a certain fraction of parton's momentum.



Fragmentation to Recombination

- The two component form of hadron transverse spectra, including an exponential part (thermalized phase)and a power law tail (pQCD).
- The particle dependence of the elliptic flow v_2 .





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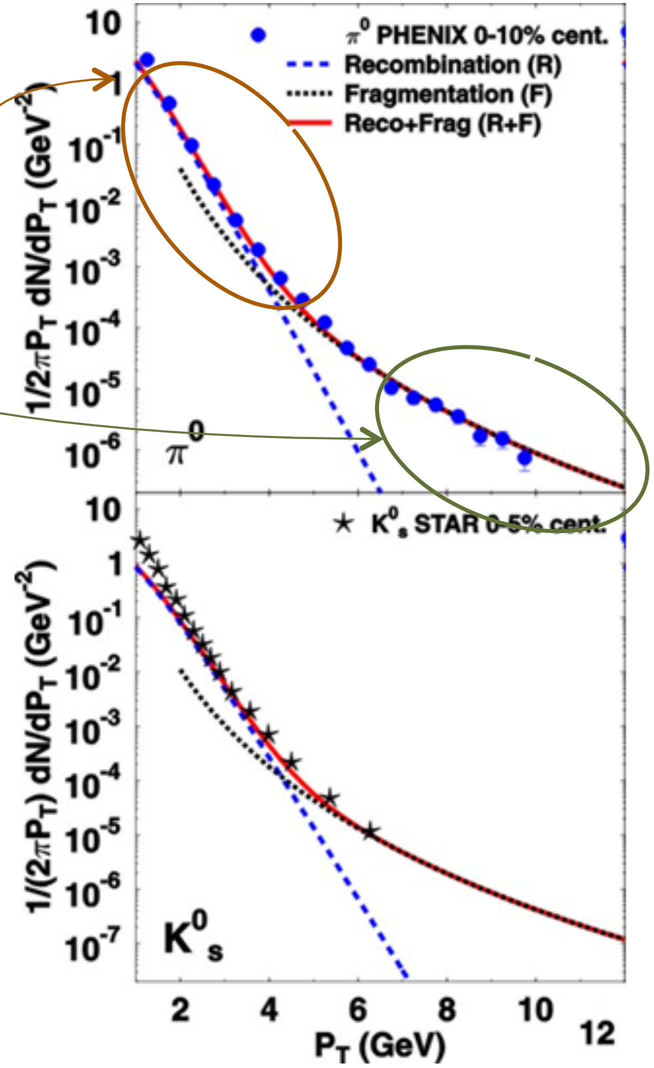
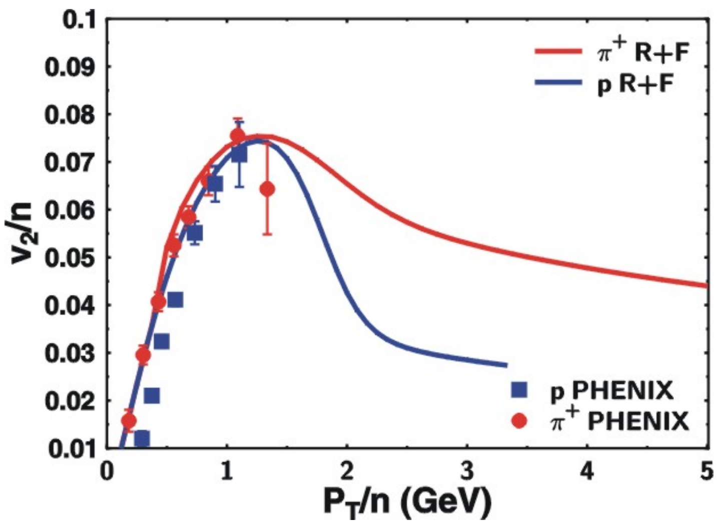
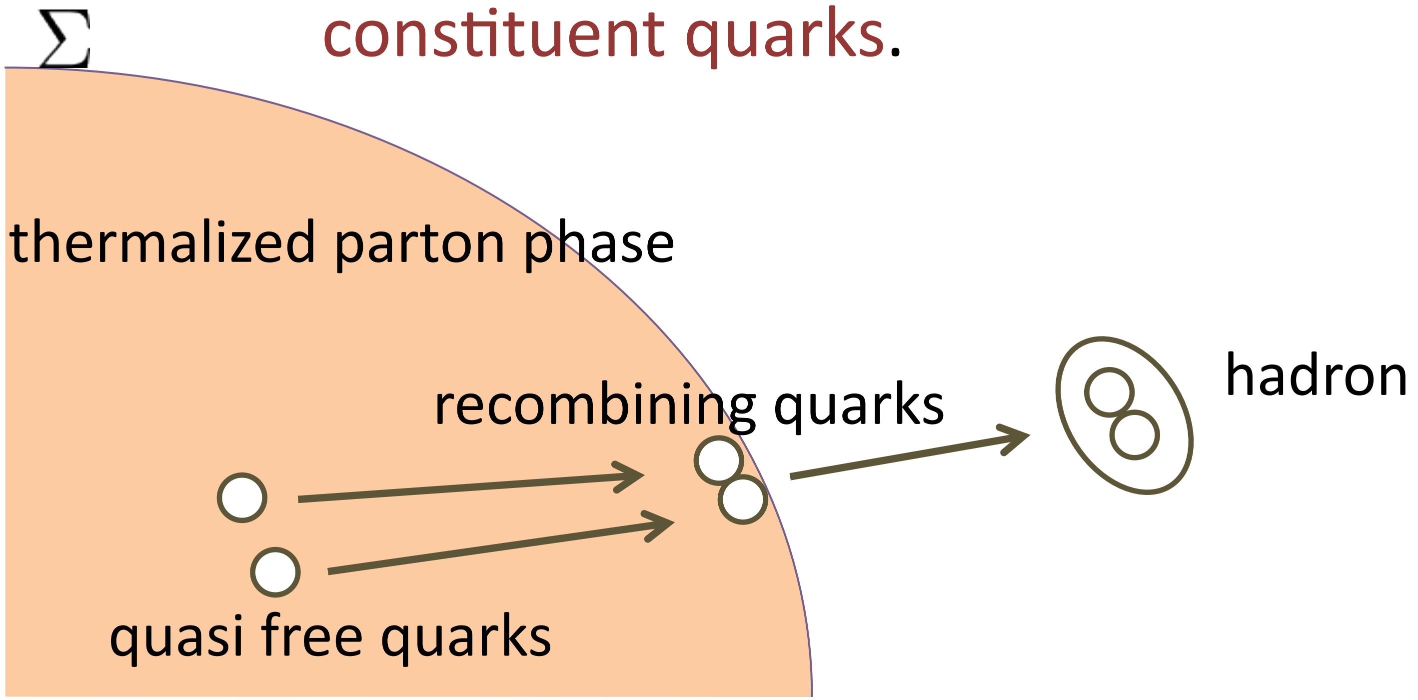
⇨ Quark number scaling





Recombination model

- Assumption
  - Recombination takes place in thermal phase.
  - An instantaneous recombination, corresponding to an infinitely thin hypersurface Σ.
  - No dynamical gluons. Taking into account only constituent quarks.



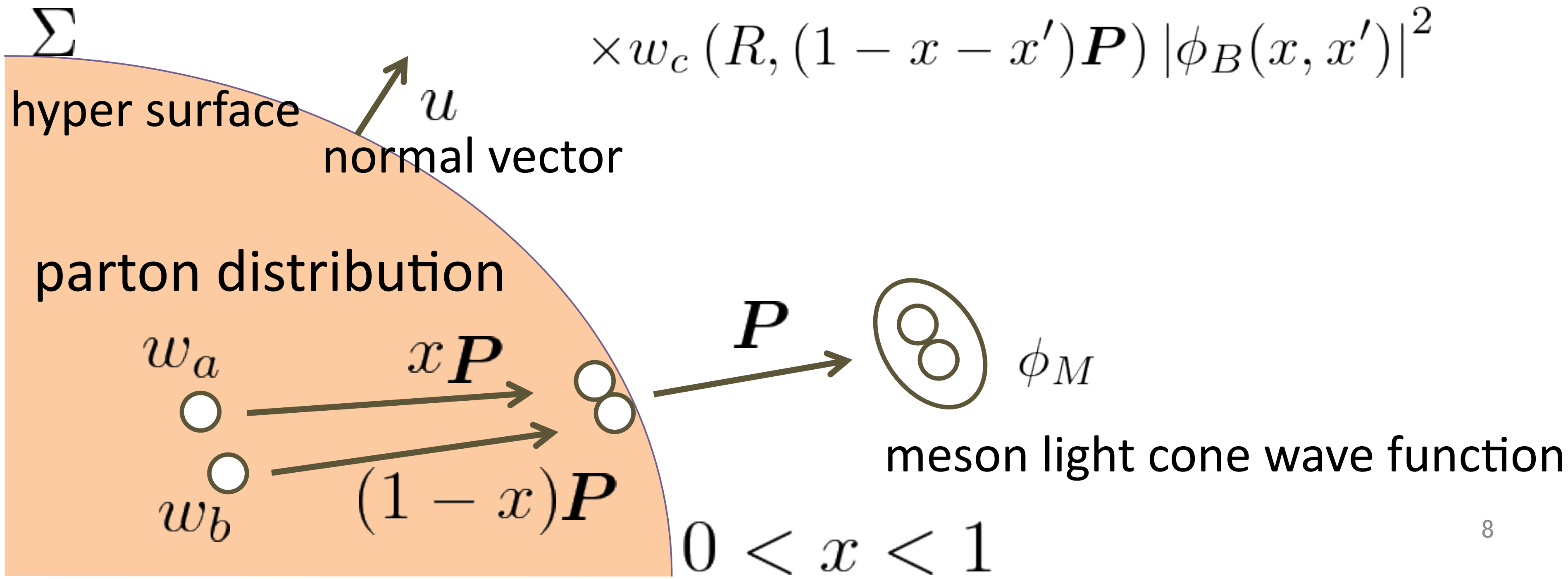
Recombination formalism

- Meson and Baryon spectrum is given by

$$E\frac{dN_M}{d^3P} = \sum_{a,b} \int_\Sigma \frac{d\sigma\, P\cdot u}{(2\pi)^3} \int_0^1 dx\, w_a\left(R, x\boldsymbol{P}\right) w_b\left(R, (1-x)\boldsymbol{P}\right) \left|\phi_M(x)\right|^2$$

$$E\frac{dN_B}{d^3P} = \sum_{a,b,c} \int_\Sigma \frac{d\sigma\, P\cdot u}{(2\pi)^3} \int_0^1 dx dx'\, w_a\left(R, x\boldsymbol{P}\right) w_b\left(R, x'\boldsymbol{P}\right)$$

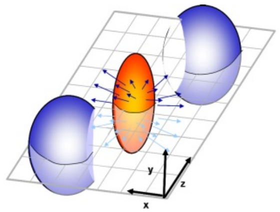
$$\times w_c\left(R, (1-x-x')\boldsymbol{P}\right) \left|\phi_B(x, x')\right|^2$$



scaling law of elliptic flow $v_2 = \langle \cos 2\phi \rangle$

- Assuming quarks and anti-quarks have a pure elliptic flow.

$$w_a(R,P) \rightarrow w_a(R,P)\left[1 + 2v_{2,q}(P_T)\cos 2\phi\right]$$



- In the thermalized region, elliptic flow of mesons and baryons are

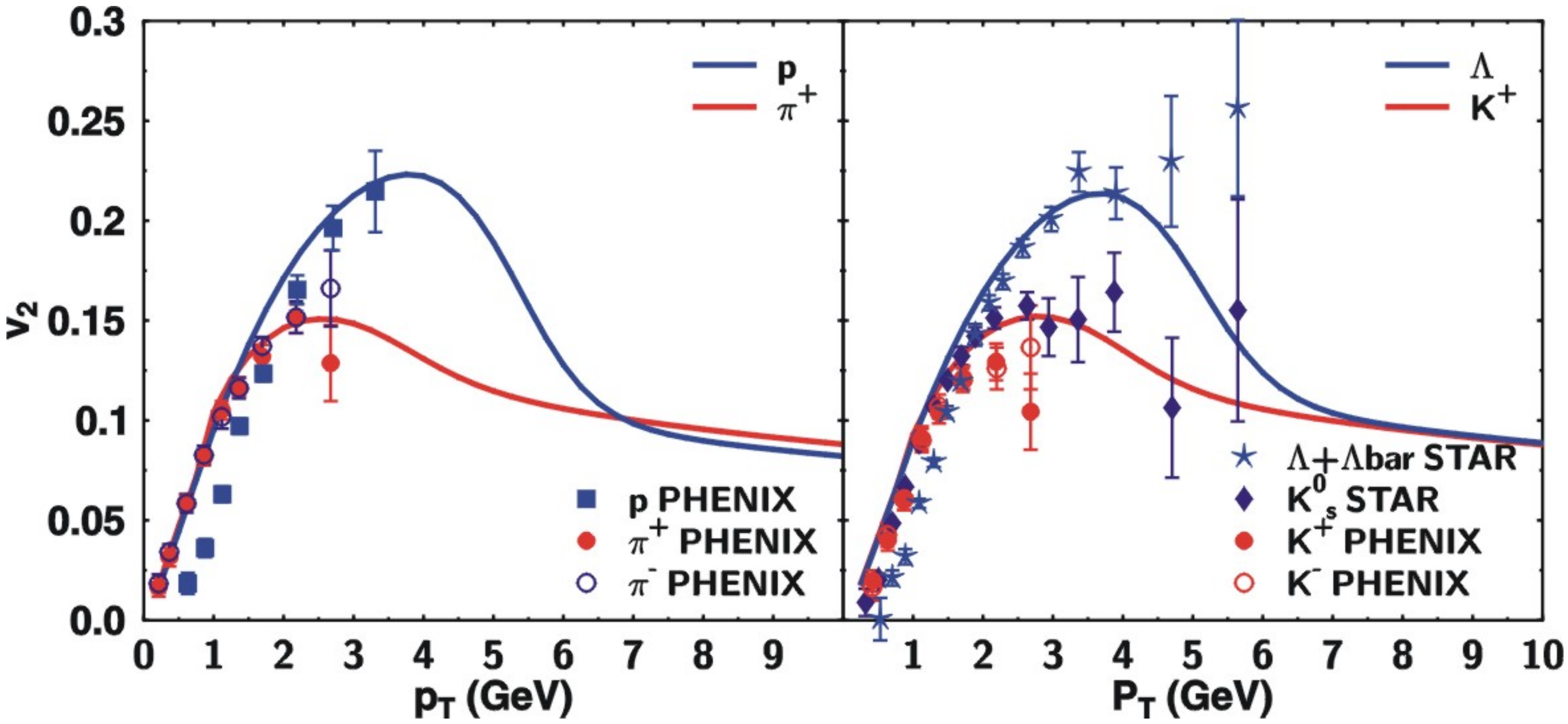
$$v_2^M(P_T) = \frac{2v_{2,q}\left(\frac{P_T}{2}\right)}{1 + 2v_{2,q}\left(\frac{P_T}{2}\right)^2},\ v_2^B(P_T) = \frac{3v_{2,q}\left(\frac{P_T}{3}\right) + 3v_{2,q}\left(\frac{P_T}{3}\right)^3}{1 + 6v_{2,q}\left(\frac{P_T}{3}\right)^2}.$$

- Ignoring quadratic and cubic terms, we can arrive at following simple scaling law:

$$v_2^H = n v_{2,q}\left(\frac{1}{n}P_T\right),$$ where *n* is # of costituent quarks.

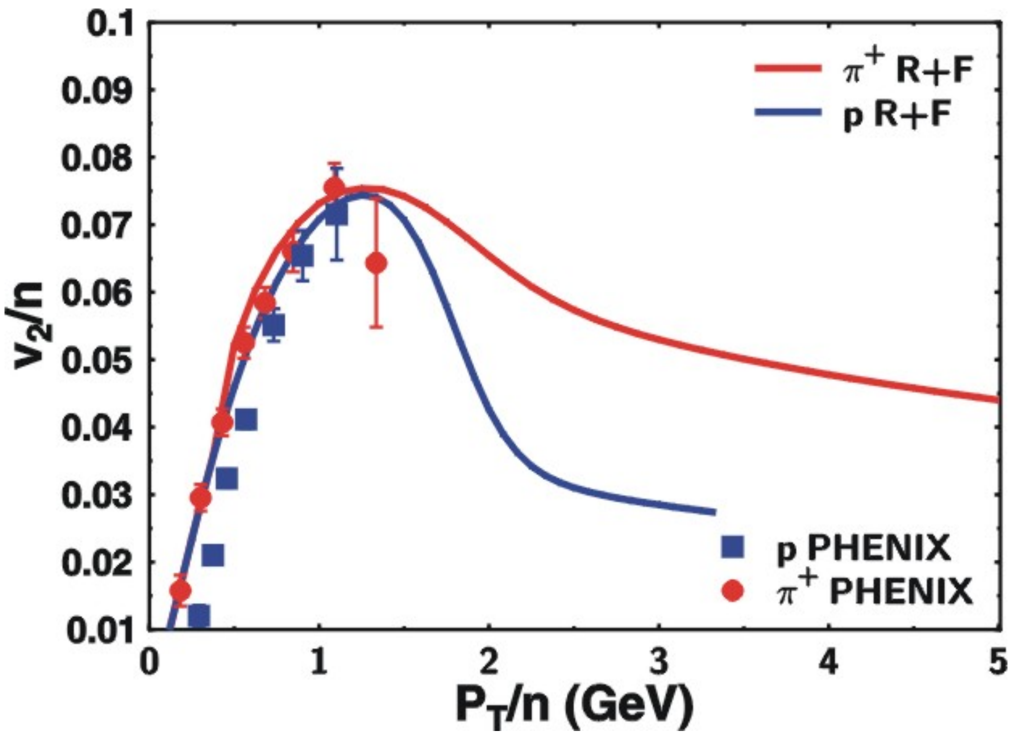
Results

- The particle dependence of elliptic flow.
- The different behavior of mesons.
  - protons and pions. kaons and Λ's.



Scaling law

- Follow one universal curve.
- Scaling law breaks down in the pQCD domain.

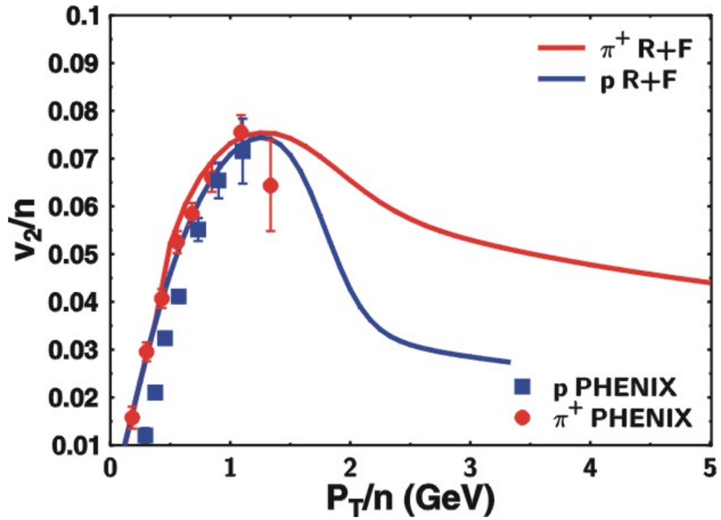


$$v_2^H = n v_{2,q}\left(\frac{1}{n}P_T\right)$$

Conclusions

- Recombination model explains scaling law.

- Recombination model implies …
  - the existence of thermalized parton phase (QGP).
  - hydrodynamical evolution on a quark level!



Back up

Modeling parton phase

- Low P_T domain
  - The thermal region is parameterized as :

$$w_a = \gamma_a e^{-p\cdot v/T} e^{-\eta^2/2\Delta^2} f(\rho, \phi)$$

γ_a : a flavor fugasicy factor

T : temperature

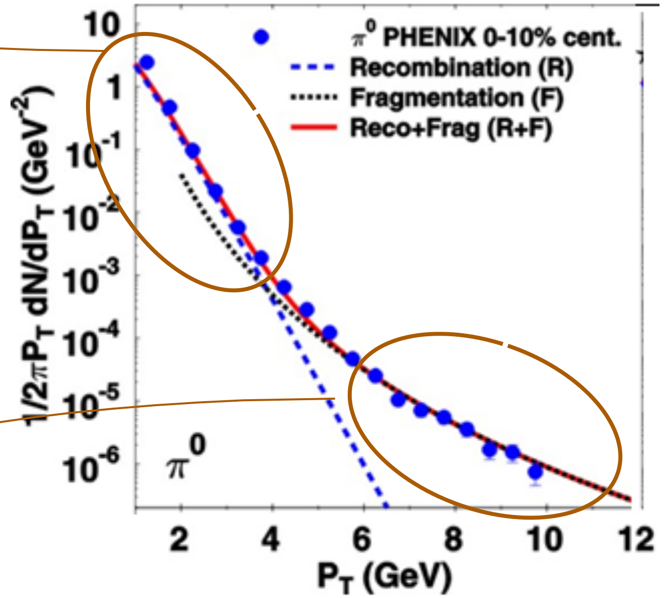
$f(\rho, \phi)$: transverse ditribution

Δ : rapidity distribution width

- High P_T domain
  - The pQCD component is parameterized as:

$$\left.\frac{dN_a}{P_T dP_T dy}\right|_{y=0} = K \frac{C}{(1 + P_T/B)^{\beta}}$$

C, *B* and *β* is taken from a pQCD.



Recombination formalism 1

- By introducing the density matrix $\hat{\rho}$ the number of quark-antiquark states that we interpret as mesons is given by

$$N_M = \sum_{a,b} \int \frac{d^3 P}{(2\pi)^3} \langle M; \boldsymbol{P} \mid \hat{\rho}_{ab} \mid M; \boldsymbol{P} \rangle$$

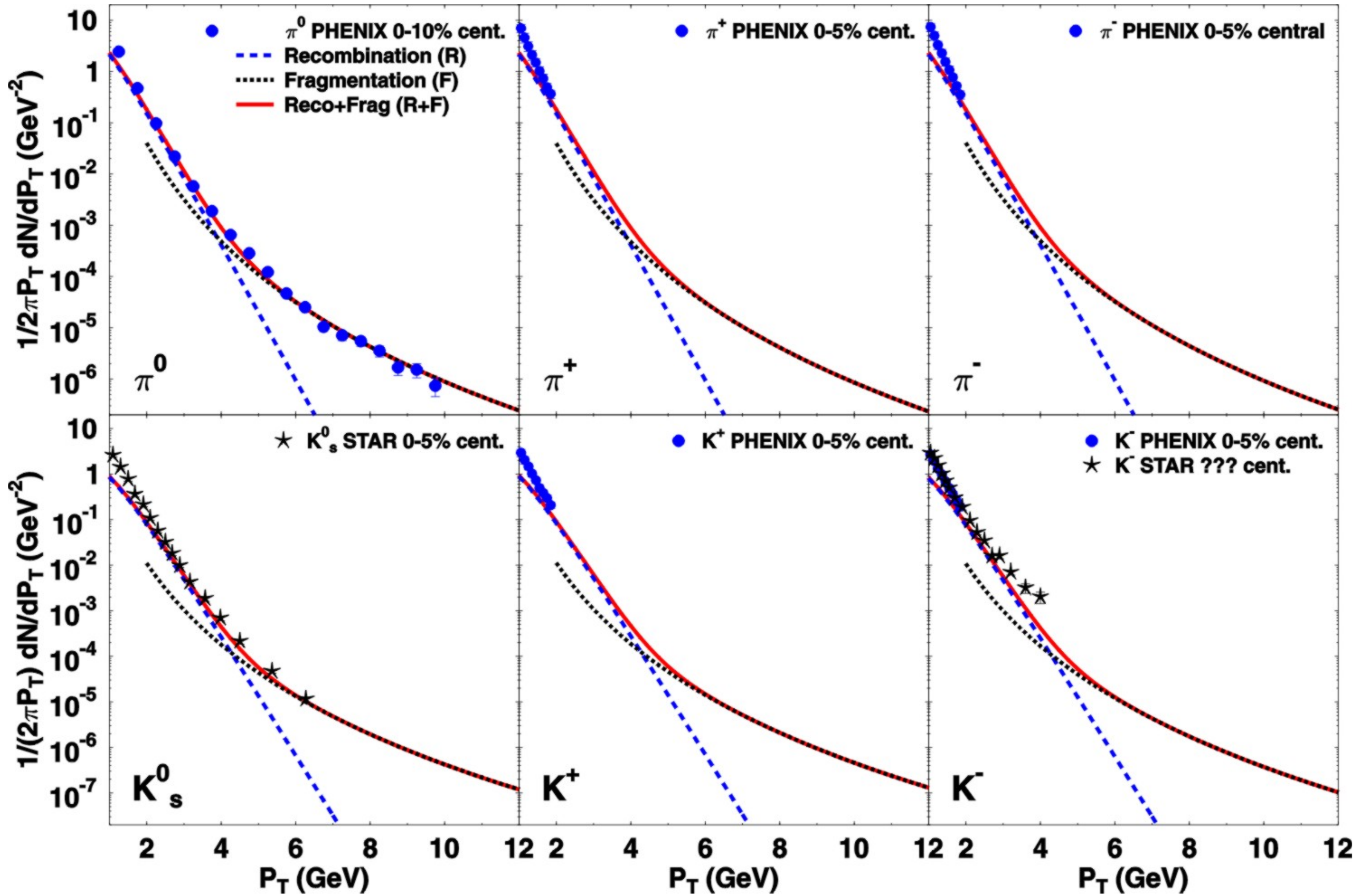
Sum is over all combination of quantum numbers

– flavor, helicity and color.

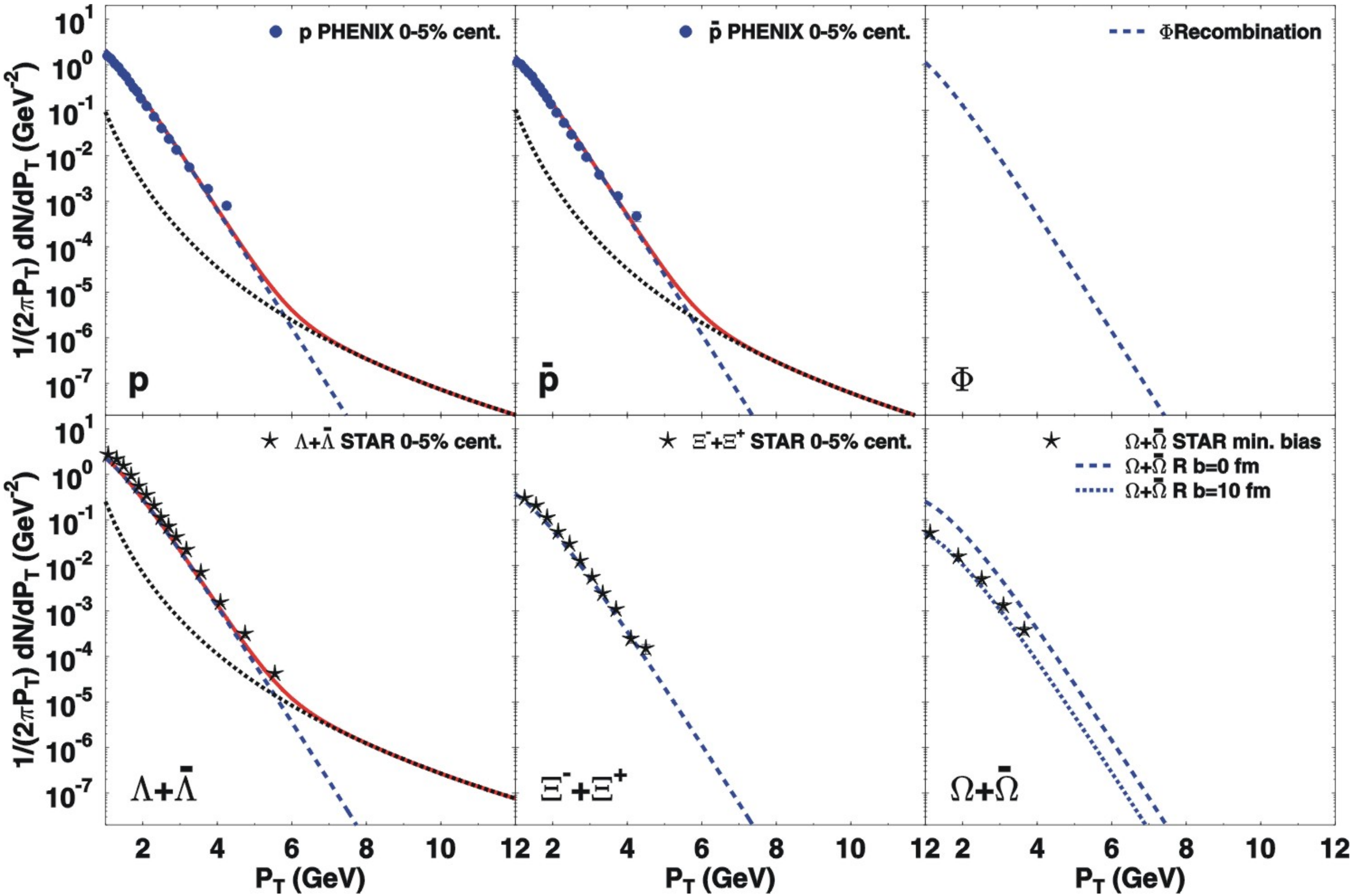
- Introducing Wigner functions *W*, *Φ* for two-quarks and mesons respectively.

$$\frac{dN_M}{d^3 P} = \sum_{a,b} \int \frac{d^3 R}{(2\pi)^3} \int \frac{d^3 q d^3 r}{(2\pi)^3} W_{ab} \left(\boldsymbol{R} + \frac{\boldsymbol{r}}{2}, \boldsymbol{R} - \frac{\boldsymbol{r}}{2}, \frac{\boldsymbol{P}}{2} + \boldsymbol{q}, \frac{\boldsymbol{P}}{2} - \boldsymbol{q} \right) \Phi_M^W (\boldsymbol{r}, \boldsymbol{q})$$

Hadron spectra 1



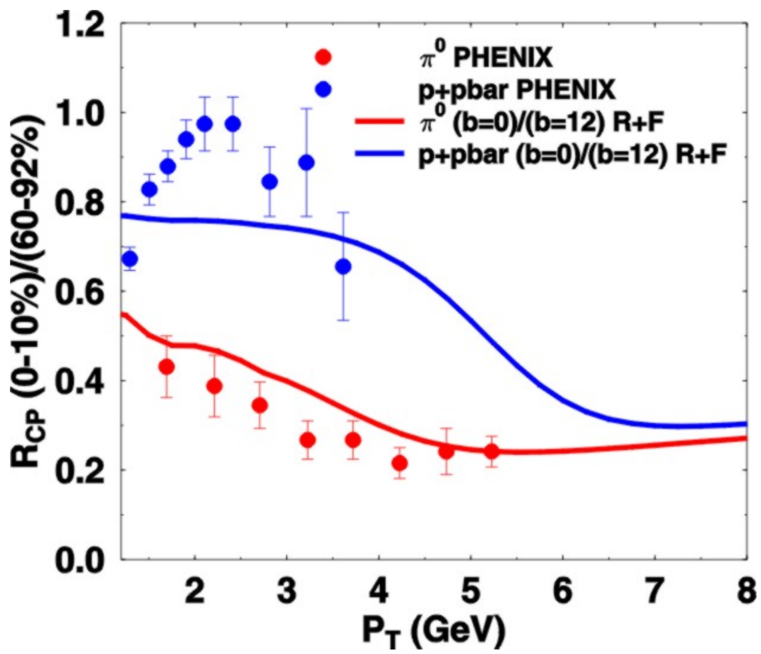
Hadron spectra 2

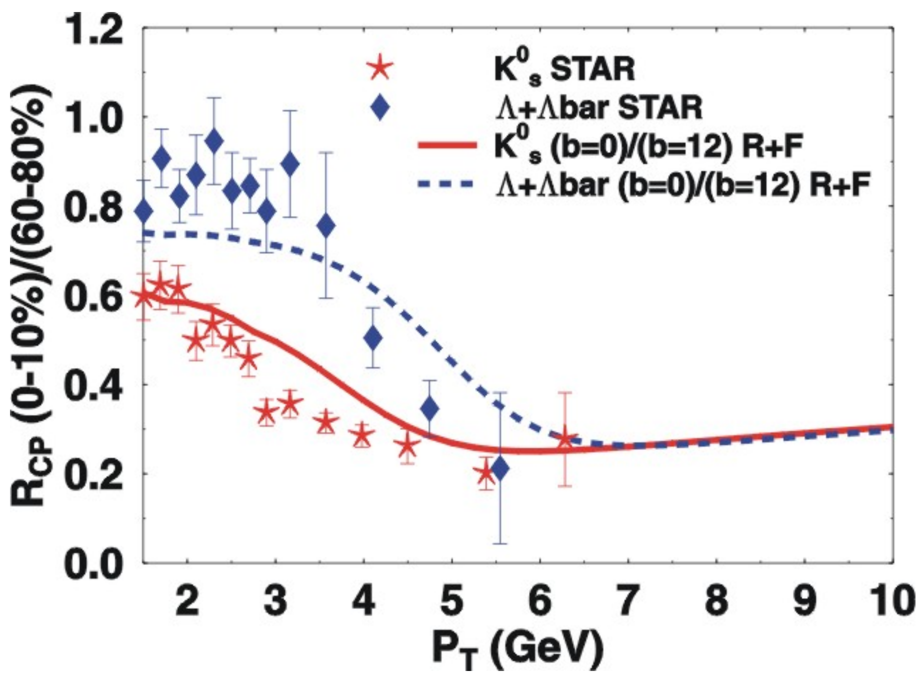


RCP

- The ratio of particle production in central and peripheral collisions normalized by the number of binary collisions.

$$R_{CP} = \frac{\frac{1}{N_{\text{binary}}^{\text{central}}} \frac{dN^{\text{central}}}{dP_T}}{\frac{1}{N_{\text{binary}}^{\text{peripheral}}} \frac{dN^{\text{peripheral}}}{dP_T}}$$





Rapidity

- Boost along the z axis,

$$\beta = \frac{\beta_1 + \beta_2}{1 + \beta_1 \beta_2}$$

- Rapidity : $y = \text{arctan}\,\beta$

$$\beta = \frac{\beta_1 + \beta_2}{1 + \beta_1 \beta_2} \Rightarrow y = y_1 + y_2$$

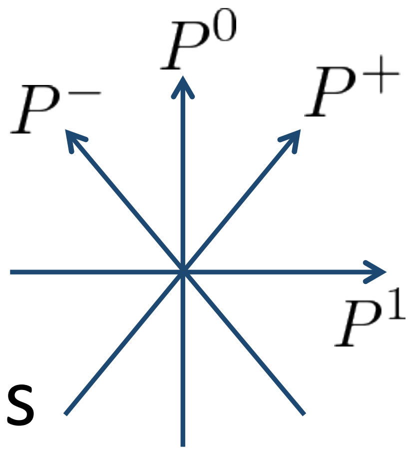
- other expressions

$$y = \frac{1}{2} \ln \frac{1 + \beta}{1 - \beta} = \frac{1}{2} \ln \frac{E + P_z}{E - Pz}$$

- For small β ,

$$y \approx \beta$$

Light cone coordinates



- Light cone coordinates

$$x^+ = \frac{1}{\sqrt{2}}\left(x^0 + x^1\right) \quad x^- = \frac{1}{\sqrt{2}}\left(x^0 - x^1\right)$$

- The line elements in *D*=*d*+1 dimensions

$$ds^2 = -2dx^+dx^- + \left(dx^2\right)^2 + \cdots + \left(dx^d\right)^2$$

- Momentum on the light cone

$$P^+ = \frac{1}{\sqrt{2}}\left(P^0 + P^1\right) \quad P^- = \frac{1}{\sqrt{2}}\left(P^0 - P^1\right)$$

fragmentation

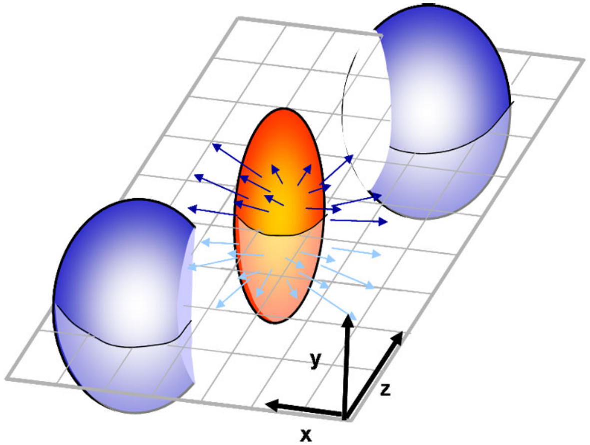
- The invariant cross section for a hadron *h* with momentum *P* can be given in factorized form,

$$E\frac{d\sigma_h}{d^3P} = \sum_{a,b}\int_0^1 \frac{dx}{x^2}D_{a\to h}(x)\frac{d\sigma_a}{d^3\left(P/x\right)}$$

- $D_{a\to h}$:fragmentation function
- x: momentum fraction carried by each quark

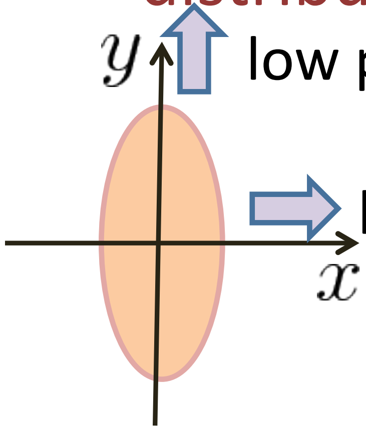
QGP

- QGP is formed by heavy ion collisions.
- QGP is described by hydrodynamics, which leads to the difference of pressure gradient.



Science grid this week, October 25, 2006

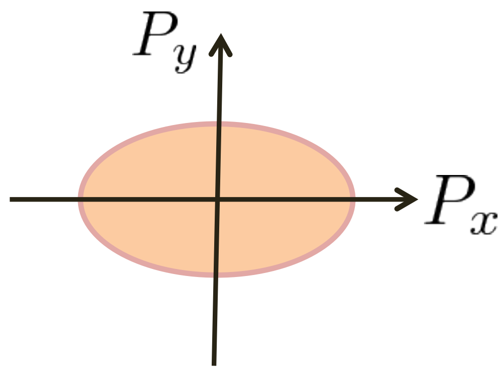
- The anisotropy of hadron momentum distributions.



low pressure gradient

high pressure gradient

Azimuthal distribution



Elliptic flow v_2

- To know particle emission dependence on azimuthal angle, expand in a Fourier cosine series. The 2nd coefficient is called elliptic flow.

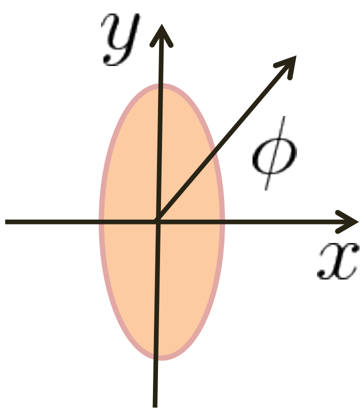
$$E\frac{d^3N}{d^3P} = \frac{d^2N}{2\pi P_T dP_T dy}\left(1 + \sum_n 2v_n \cos n\phi\right)$$

y : rapidity

P_T: transverse momentum

- Elliptic flow reflects initial geometrical anisotropy.

$$v_2 = \langle \cos 2\phi \rangle = \left\langle \frac{p_x^2 - p_y^2}{p_x^2 + p_y^2} \right\rangle$$



eccentlicity

$$\varepsilon = \frac{x^2 - y^2}{x^2 + y^2}$$

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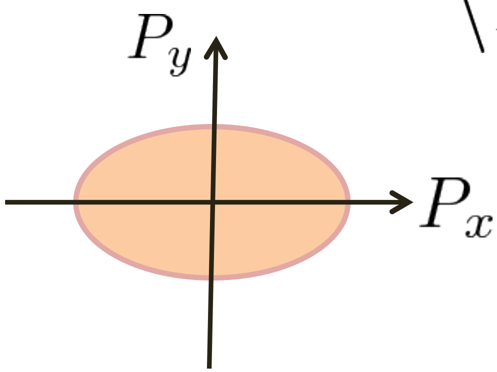

scaling law of elliptic flow v_2

$$v_2 = \langle \cos 2\phi \rangle$$

- In the thermalized region, elliptic flow of mesons and baryons

$$v_2^M(P_T) = \frac{2v_2\left(\frac{P_T}{2}\right)}{1 + 2v_2\left(\frac{P_T}{2}\right)^2} \qquad v_2^B(P_T) = \frac{3v_2\left(\frac{P_T}{3}\right) + 3v_2\left(\frac{P_T}{3}\right)^3}{1 + 6v_2\left(\frac{P_T}{3}\right)^2}$$

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$$\times w_c\left(R, (1-x-x')\boldsymbol{P}\right)\left|\phi_B(x,x')\right|^2$$

- Choose a hypersurface Σ for hadronization.
- w_a : single particle distribution function for quarks at hadornization.
- ϕ_M and ϕ_B : light cone wave function for the meson and baryon.
- x, x' : momentum fraction carried by each quark.
- u : the future oriented unit vector orthogonal to the hypersurface.

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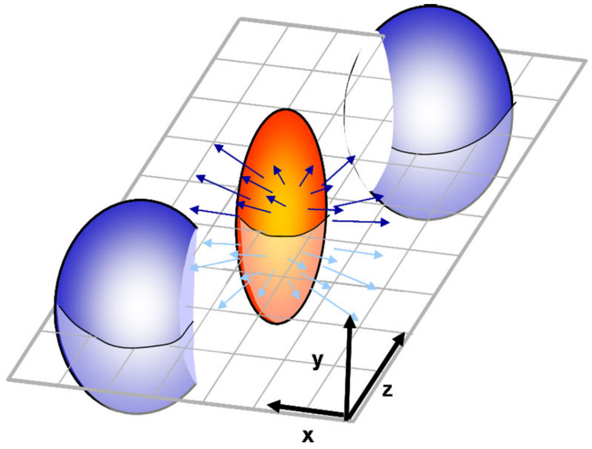

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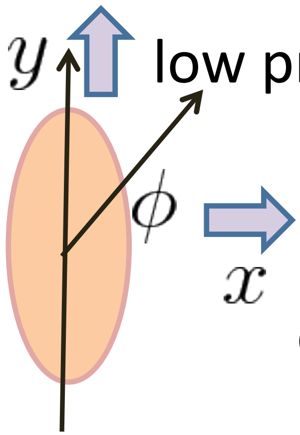

low pressure gradient

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