Production of pions and clusters in heavy-ion collisions by the AMD+JAM approach

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Motivation: We like to understand how $\Delta$ resonances and pions are affected by the dynamics of neutrons and protons in HIC.

Pion and Symmetry energy

- Symmetry energy: soft / stiff
- Nucleon dynamics
- N/Z
- $\Delta$ resonance, Pion: $\Delta^-/\Delta^{++}$, $\pi^-/\pi^+$

Interest:
- High density $\rho \sim 2\rho_0$

Clear difference of $\text{N/Z}$ in high density due to different $S(\rho)$

Model predictions do not agree
Relation $\pi^-/\pi^+ \simeq (\text{N/Z})^2$ does not hold

J. Hong and P. Danielewicz, PRC90 (2014) 024605
Our study

Pion production in $^{132}$Sn + $^{124}$Sn Collision @E/A=300 MeV

Some effects
✓ Symmetry energy (soft/stiff)
✓ Cluster correlation
✓ Pauli blocking (NEW)

$^{132}$Sn + $^{124}$Sn, $^{108}$Sn + $^{112}$Sn Collision @270 MeV
- Experiment at RIKEN/RIBF
  SπRIT project
✓ Energy dependence
✓ Impact parameter dependence

➢ Theoretical Model:
  AMD + JAM
  - Nucleon dynamics
  - Treatment of cluster correlation
  - $\pi$, $\Delta$ production in the reaction process
  - hadronic cascade model

Symmetry energy
  soft / stiff

Nucleon
  N/Z

$\Delta$ resonance, Pion
  $\Delta^-/\Delta^{++}$, $\pi^-/\pi^+$
Transport model (AMD + JAM)

- Coupled equations for $f_\alpha(r, p, t)$ ($\alpha = N, \Delta, \pi$)

$$\frac{\partial f_N}{\partial t} + \frac{\partial h_N}{\partial p} \cdot \frac{\partial f_N}{\partial r} - \frac{\partial h_N[f_N, f_{\Delta, \pi}]}{\partial r} \cdot \frac{\partial f_N}{\partial p} = I_N[f_N, f_{\Delta, \pi}]$$

$$\frac{\partial f_{\Delta, \pi}}{\partial t} + \frac{\partial h_{\Delta, \pi}}{\partial p} \cdot \frac{\partial f_{\Delta, \pi}}{\partial r} - \frac{\partial h_{\Delta, \pi}[f_N, f_{\Delta, \pi}]}{\partial r} \cdot \frac{\partial f_{\Delta, \pi}}{\partial p} = I_{\Delta, \pi}[f_N, f_{\Delta, \pi}]$$

- Our model: JAM coupled with AMD

Perturbative treatment of pion and $\Delta$ particle production

$$I_N = I_N^{\text{el}}[f_N, 0] + \lambda I_N'[f_N, f_{\Delta, \pi}]$$

$$f_{\Delta, \pi} = O(\lambda) : \Delta \text{ and pion productions are rare}$$

$$f_N = f_N^{(0)} + \lambda f_N^{(1)} + ...$$

- Nucleon $f_N$ : Zeroth order equation

$$\frac{\partial f_N^{(0)}}{\partial t} + \frac{\partial h_N}{\partial p} \cdot \frac{\partial f_N^{(0)}}{\partial r} - \frac{\partial h_N[f_N^{(0)}, 0]}{\partial r} \cdot \frac{\partial f_N^{(0)}}{\partial p} = I_N^{\text{el}}[f_N^{(0)}, 0]$$

- $\Delta$ particle $f_\Delta$ and pion $f_\pi$ : First order equation

$$\frac{\partial f_{\Delta, \pi}}{\partial t} + \frac{\partial h_{\Delta, \pi}}{\partial p} \cdot \frac{\partial f_{\Delta, \pi}}{\partial r} - \frac{\partial h_{\Delta, \pi}[f_N^{(0)}, f_{\Delta, \pi}]}{\partial r} \cdot \frac{\partial f_{\Delta, \pi}}{\partial p} = I_{\Delta, \pi}[f_N^{(0)}, f_{\Delta, \pi}]$$
Transport model (AMD + JAM)

- **AMD (Antisymmetrized Molecular Dynamics)**
  A. Ono, H. Horiuchi, T. Maruyama, and A. Ohnishi, PTP87 (1992) 1185

  - **AMD wave function**
    \[
    \langle \Phi(Z) \rangle = \det \{ \exp \left( -\nu \left( r_j - \frac{Z_i}{\sqrt{\nu}} \right)^2 \right) \} \chi_{\alpha_i(j)}
    \]
    \[
    Z_i = \sqrt{\nu} D_i + \frac{i}{2\hbar \sqrt{\nu}} K_i
    \]
    \[
    \nu : \text{Width parameter} = (2.5 \text{ fm})^{-2}
    \]
    \[
    \chi_{\alpha_i} : \text{Spin-isospin states} = p \uparrow, p \downarrow, n \uparrow, n \downarrow
    \]
    Solve the time evolution of the wave packet centroids \( Z \)

  - **Turn on/off Cluster correlation**
    - Without Cluster
      \[
      N_1 + N_2 \rightarrow N_1 + N_2
      \]
      \[
      N_1, N_2: \text{Colliding nucleons}
      \]
    - With Cluster
      \[
      N_1 + B_1 + N_2 + B_2 \rightarrow C_1 + C_2
      \]
      \[
      N_1, N_2: \text{Colliding nucleons}
      \]
      \[
      B_1, B_2: \text{Spectator nucleons/clusters}
      \]
      \[
      C_1, C_2: N, (2N), (3N), (4N) \text{ (up to } \alpha \text{ cluster)}
      \]

  - **Effective interaction**
    Skyrme force
Transport model (AMD + JAM)

- **Nucleon test Particles**

Test particles \((\mathbf{r}_1, \mathbf{p}_1), (\mathbf{r}_2, \mathbf{p}_2), \ldots, (\mathbf{r}_A, \mathbf{p}_A)\) are generated following the Wigner function \(f_{\text{AMD}}^\tau(\mathbf{r}, \mathbf{p})\) for \(\tau = \text{neutron or proton}\)

\[
f_{\text{AMD}}^\tau(\mathbf{r}, \mathbf{p}) = \frac{1}{2} \times 2^3 \sum_j \sum_k e^{-2\nu(r-R_{jk})^2-(p-P_{jk})^2/2h^2\nu} B_{jk} B_{k,j}^{-1}
\]

We send nucleon test particles \((\mathbf{r}_1, \mathbf{p}_1), (\mathbf{r}_2, \mathbf{p}_2), \ldots, (\mathbf{r}_A, \mathbf{p}_A)\) from AMD to JAM at every 2 fm/c with corrections for the conservation of baryon number and charge.

- **JAM** (Jet AA Microscopic transport model)


- Applied to high-energy collisions (1 ~ 158 A GeV)
- Hadron-Hadron reactions are based on experimental data and the detailed balance.
- No mean field (default)
- \(s\)-wave pion production (NN\(\rightarrow\)NN\(\pi\)) is turned off. ... etc.
Pion Calculations in central Au+Au collisions

- Pion multiplicity

Our calculation almost reproduces the experimental data reasonably well

Pion ratios are also larger than \((N/Z)^2_{\text{system}}\)

- Pion ratio

Exp. Data: Reisdorf et al., NPA 848 (2010) 366

J. Hong and P. Danielewicz, PRC90 (2014) 024605
Dynamics of neutrons and protons

$^{132}$Sn + $^{124}$Sn Collision @E/A=300 MeV

• with cluster  • without cluster  • JAM

Calculation set:
AMD + JAM
1. with cluster (asy-soft)
2. with cluster (asy-stiff)
3. without cluster (asy-soft)
4. without cluster (asy-stiff)
5. JAM (no mean field)

Effective interaction:
Skyrme force

Density maximum is different for cases with or without cluster
Clear difference of N/Z ratio due to different symmetry energy
Especially symmetry energy effect is weaker if there is cluster correlation
Final $\pi^-/\pi^+$ ratio

$^{132}\text{Sn} + ^{124}\text{Sn}$ Collision $@E/A=300$ MeV

1. **Symmetry energy dependence $S(\rho)$**

   $\pi^-/\pi^+$ ratio with soft $S(\rho)$ is larger
   
   $\rightarrow$ Similar result to IBUU

2. **Model dependence of nucleon dynamics**

   $S(\rho)$ effect is weaker with cluster correlations

3. $\pi^-/\pi^+$ ratio $> (N/Z)^2_{\text{system}}$

$\Rightarrow$ What is the origin of these behaviors?

NN $\leftrightarrow$ N$\Delta$  $\Delta$ $\leftrightarrow$ N$\pi$

We study what kind of information of nucleon is carried by $\Delta$ resonances.
Relation between $N/Z$ and $\Delta^-/\Delta^{++}$

**Simple expectation:**

$$\frac{\Delta^-}{\Delta^{++}} \sim (N/Z)^2$$

$$\frac{\Delta^-}{\Delta^{++}} = \frac{\text{Rate}(nn \rightarrow n\Delta^-)}{\text{Rate}(pp \rightarrow p\Delta^{++})}$$

- Nucleons in the sphere $\rho(r) \geq \rho_0$ centered at CM.
- Nucleons in the sphere $\rho(r) > \rho_0$ with high momentum.

(The collective radial momentum $p_{\text{rad}}$ is subtracted.)
From nucleons to pion ratios

- $\Delta^-/\Delta^{++} \sim (\pi^-/\pi^+)^{t=20}$
- Final stage:
  - $\pi^-/\pi^+$ is modified from $(\pi^-/\pi^+)^{t=20}$
  - ✓ $S(\rho)$ effect: 30% weaker
  - ✓ Cluster correlation $\rightarrow$ $\pi^-/\pi^+$ up

$\Rightarrow$ Cluster correlation played important roles for the pions.

Representative ratios:

$$\left(\frac{N}{Z}\right)^2 = \frac{\int_0^\infty N(t)^2 dt}{\int_0^\infty Z(t)^2 dt}$$

$$\frac{\Delta^-}{\Delta^{++}} = \frac{\int_0^\infty (nn \rightarrow p\Delta^-) dt}{\int_0^\infty (pp \rightarrow n\Delta^{++}) dt}$$

$N(t), Z(t)$: Numbers of nucleon which satisfy the conditions
Recently, we found that Pauli-blocking effect is important.

\[ f^\tau (r, p) = 4 \sum_{j \in \tau} e^{- (r - r_j)^2 / 2L - 2L (p - p_j)^2 / \hbar^2} \]

\( (\tau = \text{neutron or proton}) \)

\( \pi^-/\pi^+ \) and \( \Delta^-/\Delta^{++} \) ratios change due to Pauli-blocking effect.

PRC93, 044612 (2016)
N. Ikeno, A. Ono, Y. Nara, A. Ohnishi
Pauli-blocking effect

Pauli-blocking for the final nucleon(s) in two-body collisions

\[ \pi^- \text{ production} \]
\[ nn \rightarrow p\Delta^- \]
\[ \Delta^- \rightarrow n\pi^- \]

\[ \pi^+ \text{ production} \]
\[ pp \rightarrow n\Delta^{++} \]
\[ \Delta^{++} \rightarrow p\pi^+ \]

When Effect of Pauli-blocking is stronger
-> nucleons are blocked stronger
-> \( \Delta \) and \( \pi \) multiplicities are smaller

n-rich system
-> neutrons are blocked stronger
-> \( \Delta^{++} \) and \( \pi^+ \) multiplicities are smaller
-> \( \pi^-/\pi^+ \) ratio is larger

Pauli-blocking effect played an important roles for the pions.
Methods for Pauli blocking factor

➢ Use \( f \) of AMD for Pauli blocking

The Wigner function calculated for the AMD wave function, for \( \tau = \) neutron or proton, is

\[
\tilde{f}_{\text{AMD}}(r, p) = \frac{1}{2} \times 2^3 \sum_{j \in \tau} \sum_{k \in \tau} e^{-2\nu(r - R_{jk})^2 - (p - P_{jk})^2 / 2\hbar^2 \nu} B_{jk} B_{kj}^{-1}
\]

\( R_{jk} = (Z_j^* + Z_k) / \sqrt{\nu} \)
\( P_{jk} = 2i\hbar \sqrt{\nu} (Z_j^* - Z_k) \)
\( B_{jk} = \langle \varphi_j | \varphi_k \rangle \)

\( P_{\text{block}} = f_{\text{AMD}}^{\tau}(r_i, p'_i) \) for the final phase-space point \((r_i, p'_i)\).

Test particles \{(r_i, p'_i); i=1,2, ..., A\} are generated with the probability distribution \( f_{\text{AMD}}^{\tau}(r, p) \) and sent to JAM.

➢ Do Pauli blocking within JAM

\( P_{\text{block}} = f_{\text{JAM}}^{\tau}(r_i, p'_i) \) with

\[
f_{\text{JAM}}^{\tau}(r, p) = \frac{1}{2} \times 2^3 \sum_{j \in \tau} e^{-(r - r_j)^2 / 2L - 2L(p - p_j)^2 / \hbar^2}
\]

\( L = 2.0 \text{ fm}^2 \)

=> We compare \( \frac{1}{4} f_{\text{JAM}}, f_{\text{JAM}} \) and \( f_{\text{AMD}} \), to see the effect and importance of Pauli blocking treatment
Different treatments for Pauli-blocking

- Production ratio $\Delta^-/\Delta^{++}$

$\Delta^-$ production

$\Delta^{++}$ production

- Clear difference of ratio due to different treatments for Pauli-blocking
Different treatments for Pauli-blocking

➢ Final $\pi^-/\pi^+$ ratio

zęp

Final $\pi^-/\pi^+$ ratio

1. Pauli blocking effect is stronger for $\pi^+$ in particular when cluster correlation is switched on.

2. Symmetry energy effect $S(\rho)$ is stronger for $\pi^-$ than for $\pi^+$.

3. Cluster correlation effect is stronger for $\pi^+$. 

 NN ↔ NΔ  Δ ↔ Nπ
Energy dependence of $^{132}$Sn + $^{124}$Sn system with cluster w/o cluster

Final $\pi^-/\pi^+$ ratio

E/A=270MeV : Experiment at SπRIT project

E/A=300MeV

$\pi^-/\pi^+$ vs $\pi^-/\pi^+$

$\langle N/Z \rangle^2_{\text{system}}$
Impact parameter dependence

$^{132}\text{Sn} + ^{124}\text{Sn} @ E/A = 270\text{MeV}$

Larger impact parameter $b$
- $\pi^-/\pi^+$ ratio is larger because of n-rich probably
- Multiplicities of $\pi^-, \pi^+$ are smaller

Final $\pi^-/\pi^+$ ratio

$(N/Z)^2_{\text{system}}$

$\pi^-/\pi^+$ ratio

impact parameter $b$ [fm]

Multiplicities

$\pi^-$

$\pi^+$

impact parameter $b$ [fm]
Different system @ E/A=270MeV : Experiment at SπRIT project

\[ ^{132}\text{Sn} + ^{124}\text{Sn} \]

\[ ^{108}\text{Sn} + ^{112}\text{Sn} \]

\[ \frac{\pi^-}{\pi^+} \]

\[ \frac{\text{(N/Z)}^2_{\text{system}}}{\text{ratio}} \]

\[ \text{with cluster} \]

\[ \text{with cl., soft} \]

\[ \text{with cl., stiff} \]
Summary: Pion production in Sn+Sn collisions by AMD+JAM

Motivation: To understand the mechanism how pions are produced reflecting the dynamics of neutrons and protons

- Pion ratio certainly carries the information on neutrons and protons
  - $\pi^-/\pi^+$ and $\Delta^-/\Delta^{++}$ ratios are related to the $(N/Z)^2$ in high-$p$ and high-$p$ region
  - In the final stage, $\pi^-/\pi^+$ ratio is modified from $(\pi^-/\pi^+)^{t=20}$ like

Important effects for pions

- Symmetry energy (soft/stiff)
  - $\pi^-/\pi^+$ ratio with soft $S(\rho)$ is larger
- Cluster correlation
  - $S(\rho)$ effect is weaker with cluster correlations
- Pauli-blocking effect
  - Multiplicity and ratio change

Future work:

We calculate to compare with experimental data.
We need to investigate pions but also other observables (cluster correlation)
- $\Delta$ resonance production threshold
Different treatments for Pauli-blocking

Final $\pi^-/\pi^+$ ratio

![Graph showing different treatments for Pauli-blocking with various ratios and labels for $f_{\text{JAM}}$, $f_{\text{AMD}}$, and $f_{\text{JAM}}$ (Wigner), $f_{\text{JAM}}$ (Husimi).]
Phase space distribution function $f$

Wigner function

$$f^{\tau}_{\text{AMD}}(r,p) = \frac{1}{2} \times 2^3 \sum_{j \in \tau} \sum_{k \in \tau} e^{-2\nu(r-R_{jk})^2 - (p-P_{jk})^2 / 2\hbar^2 \nu} B_{jk} B_{kj}^{-1}$$

Husimi function

$$f^{\tau}_{\text{AMD}}(r,p) = \frac{1}{2} \times \sum_{j \in \tau} \sum_{k \in \tau} e^{-\nu(r-R_{jk})^2 - (p-P_{jk})^2 / 4\hbar^2 \nu} B_{jk} B_{kj}^{-1}$$
Pion spectra

AMD + JAM with cluster (asy-soft)

- With Coulomb
  - Acceleration of $\pi^+$
  - Deceleration of $\pi^-$
  $\Rightarrow$ Changes of pion spectra

- Without Coulomb

<table>
<thead>
<tr>
<th></th>
<th>$\pi^-$</th>
<th>$\pi^+$</th>
<th>$\pi^-/\pi^+$</th>
</tr>
</thead>
<tbody>
<tr>
<td>with Coulomb</td>
<td>0.577</td>
<td>0.192</td>
<td>3.01(1)</td>
</tr>
<tr>
<td>w/o Coulomb</td>
<td>0.582</td>
<td>0.193</td>
<td>3.02(1)</td>
</tr>
</tbody>
</table>

$\Rightarrow$ Coulomb effect has almost no effect on the pion multiplicities and the pion ratio.
Clusters at high density?

In the calculation, cluster correlation played important roles for the pions. But, in the high density region, should cluster correlations really exist?

3 Options: Treatment of cluster correlations

1. **With cluster**
   Clusters are formed at any density.

2. **Without cluster**
   Clusters are **not** formed at all.

3. **With cluster** ($\rho < 0.16 \text{ fm}^{-3}$)
   Clusters are formed in the **low** density region ($\rho < 0.16 \text{ fm}^{-3}$)
   Clusters are **not** formed in the **high** density region ($\rho > 0.16 \text{ fm}^{-3}$)
Preliminary result with cluster \((\rho < 0.16 \, \text{fm}^{-3})\)

- **Dynamics of neutrons and protons**

1. with cluster \(t=22\,\text{fm}/c\)
2. without cluster
3. With cluster \((\rho<0.16\,\text{fm}^{-3})\)

- ✓ Density maximum is not as high as the case with cluster
Preliminary result with cluster ($\rho < 0.16$ fm$^{-3}$)

- Final $\pi^-/\pi^+$ ratio

- With cluster ($\rho < 0.16$ fm$^{-3}$)
  Closer to the case without cluster

![Graph showing $\pi^-/\pi^+$ ratio with and without cluster at different densities.](image_url)
Potential for $\Delta$ and pion

In JAM, reaction thresholds are the same as in free space. (The production and absorption reactions for $\Delta$ and pions occur in the JAM calculation as in the free space)

Nucleons feel potential in the AMD calculation.

Therefore AMD+JAM assumes

\[
\begin{align*}
\text{NN} & \leftrightarrow \text{N}\Delta \\
U^{(N)}_{\tau_1} + U^{(N)}_{\tau_2} &= U^{(N)}_{\tau_3} + U^{(\Delta)}_{\tau_4}, \\
\text{\Delta} & \leftrightarrow \text{N}\pi \\
U^{(\Delta)}_{\tau_1} &= U^{(N)}_{\tau_3} + U^{(\pi)}_{\tau_4}
\end{align*}
\]

for $\tau_1(\tau_2) = \tau_3 + \tau_4$

This is equivalent to the choice in the pBUU calculation
c.f. Hong and Danielewicz, PRC 90 (2014) 024605

\[
\begin{align*}
v_{asy}(\Delta^-) &= 2v_{asy}(n) - v_{asy}(p) = 3v_{asy}(n), \\
v_{asy}(\Delta^0) &= v_{asy}(n), \\
v_{asy}(\Delta^+) &= v_{asy}(p) = -v_{asy}(n), \\
v_{asy}(\Delta^{++}) &= 2v_{asy}(p) - v_{asy}(n) = -3v_{asy}(n).
\end{align*}
\]

* Different choice, cf. Bao-An Li

\[
\begin{align*}
v_{asy}(\Delta^-) &= v_{asy}(n), \\
v_{asy}(\Delta^0) &= \frac{2}{3}v_{asy}(n) + \frac{1}{3}v_{asy}(p) = \frac{1}{3}v_{asy}(n), \\
v_{asy}(\Delta^+) &= \frac{1}{3}v_{asy}(n) + \frac{2}{3}v_{asy}(p) = -\frac{1}{3}v_{asy}(n), \\
v_{asy}(\Delta^{++}) &= v_{asy}(p) = -v_{asy}(n).
\end{align*}
\]