A program for coupled-channel calculations with all order couplings for heavy-ion fusion reactions

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Abstract

A FORTRAN 77 program that calculates fusion cross sections and mean angular momenta of the compound nucleus under the influence of couplings between the relative motion and several nuclear collective motions is presented. The no-Coriolis approximation is employed to reduce the dimension of coupled-channel equations. The program takes into account the effects of nonlinear couplings to all orders, which have been shown to play an important role in heavy-ion fusion reactions at subbarrier energies. © 1999 Elsevier Science B.V. All rights reserved.

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Keywords: Heavy-ion subbarrier fusion reactions; Coupled-channel equations; Higher order coupling; No-Coriolis approximation; Incoming wave boundary condition; Fusion cross section; Mean angular momentum; Spin distribution; Fusion barrier distribution; Multi-dimensional quantum tunneling

PROGRAM SUMMARY

Title of program: CCFULL
Catalogue identifier: ADKM
Program Summary URL: http://www.cpc.cs.qub.ac.uk/cpc/summaries/ADKM
Program obtainable from: CPC Program Library, Queen’s University of Belfast, N. Ireland
Computer for which the program is designed and others on which it is operable: Any UNIX work-station or PC; the program has been tested on DEC and DEC-Alpha
Operating systems or monitor under which the program has been tested: UNIX

No. of bytes in distributed program, including test data, etc.: 80131
Distribution format: ASCII

Keywords: Heavy-ion subbarrier fusion reactions, coupled-channel equations, higher order coupling, no-Coriolis approximation, incoming wave boundary condition, fusion cross section, mean angular momentum, spin distribution, fusion barrier distribution, multi-dimensional quantum tunneling

Nature of physical problem
It has by now been well established that fusion reactions at energies near and below the Coulomb barrier are strongly influenced...
by couplings of the relative motion of the colliding nuclei to several nuclear intrinsic motions. Recently, precisely measured fusion cross sections have become available for several systems, and a distribution of the Coulomb barrier, which is originated from the channel couplings, have been extracted. It has been pointed out that the linear coupling approximation, which has often been used in coupled-channels calculations, is inadequate in order to analyze such high precision experimental data. The program CCFULL solves the coupled-channels equations to compute fusion cross sections and mean angular momenta of compound nucleus, taking into account the couplings to all orders.

Method of solution
CCFULL directly integrates coupled second order differential equations using the modified Numerov method. The incoming wave boundary condition is employed and a barrier penetrability is calculated for each partial wave. Nuclear coupling matrix elements are evaluated by using the matrix diagonalisation method once the physical space has been defined.

Restrictions on the complexity of the problem
The program is best suited for systems where the number of channels which strongly couple to the ground state is relatively small and where multi-nucleon transfer reactions play less important role compared with inelastic channels. It also relies on an assumption that the fusion process is predominantly governed by quantum tunneling over the Coulomb barrier. This assumption restricts a system which the program can handle to that where the sum of the charge of the projectile and the target nuclei $Z_p + Z_T$ is larger than around 12 and the charge product $Z_p Z_T$ less than around 1800. For most experimental data which were measured to aim at extracting fusion barrier distributions, this condition is well satisfied. The program also treats a vibrational coupling in the harmonic limit and a rotational coupling with a pure rotor. The program can be modified for general couplings by directly providing coupling strengths and excitation energies.

Typical running time
A few seconds for input provided. The computer time depends strongly upon the number of channels to be included. It will considerably increase if one wishes to include a large number of channels, as for instance 20.

1. Introduction

Fusion is defined as a reaction where two separate nuclei combine together to form a composite system. When the incident energy is not so large and the system is not so light, the reaction process is predominantly governed by quantum tunneling over the Coulomb barrier created by the strong cancellation between the repulsive Coulomb force and the attractive nuclear interaction. Extensive experimental as well as theoretical studies have revealed that fusion reactions at energies near and below the Coulomb barrier are strongly influenced by couplings of the relative motion of the colliding nuclei to several nuclear intrinsic motions [1]. Heavy-ion subbarrier fusion reactions thus provide a good opportunity to address the general problem on quantum tunneling in the presence of couplings, which has been a popular subject in the past decade in many branches of physics and chemistry.

Thanks to the recent developments in experimental techniques, fusion cross sections can now be measured with high accuracy in small energy intervals. Such high precision experimental data have generated a renewed interest in heavy-ion subbarrier fusion reactions in recent years [2,3]. For instance, they have enabled a detailed study of the effects of couplings on fusion reactions through the so-called fusion barrier distribution [4,5] and have thus offered a good opportunity to test any theoretical framework for subbarrier fusion reactions.

Theoretically the standard way to address the effects of the coupling between the relative motion and the intrinsic degrees of freedom on fusion is to numerically solve the coupled-channel equations, including all the relevant channels. In the past, the coupled-channel calculations were often performed using the linear coupling approximation, where the coupling potential is expanded in powers of the deformation parameter, keeping only the linear term. It has been demonstrated that nonlinear couplings significantly affect the shape of fusion barrier distributions and thus the linear coupling approximation is inadequate in quantitative comparison with the recent high quality data of fusion cross sections [6,7]. The program CCFULL includes the couplings to full order and thus it does not introduce the expansion of the coupling potential. Since the dimension of the coupled-channel equations
with full space is in general too large for practical purposes, the program employs the no-Coriolis approximation, which is sometimes referred to as the isocentrifugal approximation too, to reduce the dimension \cite{8,9}. For heavy-ion fusion reactions, this approximation has been confirmed to work well \cite{10}. The program is otherwise exact and takes full account of the finite excitation energies of intrinsic motions. It includes Coulomb excitations and uses the incoming wave boundary condition inside the Coulomb barrier.

2. Coupled-channel equations

For heavy-ion fusion reactions, to a good approximation one can replace the angular momentum of the relative motion in each channel by the total angular momentum $J$ \cite{8,9}. This approximation, often referred to as no-Coriolis approximation or isocentrifugal approximation, is used in the program. The coupled-channel equations then read

$$
-\frac{\hbar^2}{2\mu} \frac{d^2}{dr^2} + \frac{J(J+1)\hbar^2}{2\mu r^2} + V_N^{(0)}(r) + \frac{Z_p Z_e e^2}{r} + \epsilon_n - E \psi_n(r) + \sum_m V_{nm}(r) \psi_m(r) = 0,
$$

where $r$ is the radial component of the coordinate of the relative motion and $\mu$ is the reduced mass, respectively. $E$ is the bombarding energy in the center of mass frame and $\epsilon_n$ is the excitation energy of the $n$th channel. $V_{nm}$ are the matrix elements of the coupling Hamiltonian, which in the collective model consist of Coulomb and nuclear components. These two components are detailed in the following section. $V_N^{(0)}$ is the nuclear potential in the entrance channel. In the program, the Woods–Saxon parametrisation

$$
V_N^{(0)}(r) = -\frac{V_0}{1 + \exp((r - R_0)/(a))}, \quad R_0 = r_0(A_p^{1/3} + A_T^{1/3}),
$$

is adopted for the nuclear potential $V_N^{(0)}$.

The coupled-channel equations are solved by imposing the boundary conditions that there are only incoming waves at $r = r_{\text{min}}$, and there are only outgoing waves at infinity for all channels except the entrance channel ($n = 0$), which has an incoming wave with amplitude one as well. This boundary condition is referred to as the incoming wave boundary condition (IWBC) \cite{11}, and is valid for heavy-ion reactions, where there is a strong absorption inside the Coulomb barrier. The program CCFULL adopts the minimum position of the Coulomb pocket inside the barrier for $r_{\text{min}}$. Practically the numerical solution is matched to a linear combination of incoming and outgoing and Coulomb wave functions at finite distance $r_{\text{max}}$ beyond which both the nuclear potential and the Coulomb coupling are sufficiently small. The boundary conditions are thus expressed as

$$
\psi_n(r) \rightarrow T_n \exp\left(-i \int_{r_{\text{min}}}^r k_n(r') \, dr'\right), \quad r \leq r_{\text{min}},
$$

$$
\rightarrow H_J^{(-)}(k_n r) \delta_{n,0} + R_n H_J^{(+)}(k_n r), \quad r > r_{\text{max}},
$$

where

$$
k_n(r) = \sqrt{\frac{2\mu}{\hbar^2}} \left( E - \epsilon_n - \frac{J(J+1)\hbar^2}{2\mu r^2} - V_N(r) - \frac{Z_p Z_e e^2}{r} - V_{nm}(r) \right)
$$

is the local wave number for the $n$th channel and $k_n = k_n(r = \infty)$. $H_J^{(-)}$ and $H_J^{(+)}$ in Eq. (4) are the incoming and the outgoing Coulomb functions, respectively.

In order to ensure that there are only incoming waves at $r \rightarrow r_{\text{min}}$, the program CCFULL solves the coupled-channel equations outwards from $r_{\text{min}}$, first by setting \cite{12}
\[ \psi_n(r_{\text{min}}) = 1, \quad \psi_m(r_{\text{min}}) = 0 \quad (m \neq n), \]  
\[ \frac{d}{dr} \psi_n(r_{\text{min}}) = -i k_n(r_{\text{min}}), \quad \frac{d}{dr} \psi_m(r_{\text{min}}) = 0 \quad (m \neq n). \]  

Since the first derivative of the wave functions at \( r_{\text{min}} \) has been explicitly written down from Eq. (3), the wave functions at \( r = r_{\text{min}} + h \), \( h \) being the radial mesh to integrate the equations, can be determined by the Runge–Kutta method. After the wave functions at \( r = r_{\text{min}} + h \) have been thus obtained, CCFULL solves the coupled-channel equations from \( r = r_{\text{min}} + h \) to \( r = r_{\text{max}} \) by the modified Numerov methods [13], since the Runge–Kutta method may not be so efficient to solve the second order differential equations. The modified Numerov method relates the wave functions at \( r_{i+1} \equiv r_{\text{min}} + (i + 1)h \) to those at \( r_i \) and \( r_{i-1} \) as

\[ \psi^{i+1} = \left(1 - \frac{h^2}{12} A^{i+1}\right)^{-1} \left[ \left(\frac{h^2}{12} A^i + \sqrt{3}\right)^2 - 1\right] \left(1 - \frac{h^2}{12} A^{i-1}\right) \psi^i - \frac{h^2}{12} A^i \psi^{i-1} \right], \]  

where \( A_{nm}(r) \) is defined by

\[ A_{nm}(r) = \frac{2 \mu}{h^2} \left( V^{(0)}(r) + J(J + 1) \frac{h^2}{2 \mu r^2} + \frac{Z_F Z_T e^2}{r} + \epsilon_n - E \right) k_{n,m} - V_{nm}(r) \].

and \( \psi^i \) are the wave functions at \( r_i \).

Let \( \chi_{nm}(r) \) be the wave function of the \( m \)th channel thus obtained, i.e. it is \( \psi_m(r) \) which satisfies the boundary conditions (3) at \( r = r_{\text{min}} \). At \( r = r_{\text{max}} \), \( \chi_{nm} \) can be expressed by a superposition of the incoming and outgoing Coulomb waves as

\[ \chi_{nm}(r) = C_{nm} H_{J}^{(-)}(k_m r) + D_{nm} H_{J}^{(+)}(k_m r), \quad r \rightarrow r_{\text{max}}. \]  

The coefficients \( C_{nm} \) and \( D_{nm} \) are determined either by matching the logarithmic derivatives at \( r_{\text{max}} \) or by matching the ratio of the wave functions at \( r_{\text{max}} = h \) to those at \( r_{\text{max}} + h \). Since the modified Numerov methods do not automatically generate the derivative of the wave functions, the latter procedure is more suitable here. The coefficients are then obtained as

\[ C_{nm} = \frac{H_{Jm}^{(+)(i+1)}(k_{n,m}) - H_{Jm}^{(+)(i-1)}(k_{n,m})}{H_{Jm}^{(+)(i+1)}(k_{n,m}) - H_{Jm}^{(+)(i-1)}(k_{n,m})} \]  

and

\[ D_{nm} = \frac{H_{Jm}^{(-)(i+1)}(k_{n,m}) - H_{Jm}^{(-)(i-1)}(k_{n,m})}{H_{Jm}^{(-)(i+1)}(k_{n,m}) - H_{Jm}^{(-)(i-1)}(k_{n,m})}, \]

respectively. We have defined \( H_{Jm}^{(+)(i+1)} \equiv H_{J}^{(+)}(k_m \cdot (r_{\text{max}} + h)) \), etc., and \( \chi_{nm}^{i+1} \equiv \chi_{nm}(r_{\text{max}} + h) \), etc. This procedure is repeated for all \( n \) and \( m \) to determine the matrices \( C \) and \( D \).

The solution of the coupled-channel equations with the proper boundary conditions (3) and (4) is given by a linear combination of \( \chi_{nm} \) as

\[ \psi_m(r) = \sum_n T_n \chi_{nm}(r). \]

This equation satisfies the boundary condition (3) at \( r = r_{\text{min}} \). At \( r = r_{\text{max}} \), it leads to

\[ \psi_m(r_{\text{max}}) = \sum_n T_n \chi_{nm}(r_{\text{max}}) = \sum_n T_n (C_{nm} H_{J}^{(-)}(k_m r_{\text{max}}) + D_{nm} H_{J}^{(+)}(k_m r_{\text{max}})). \]
By comparing Eqs. (4) and (14), one finds
\[
\sum_{n} T_n C_{nm} = \delta_{m,0}. \tag{15}
\]
The transmission coefficients are then finally obtained by
\[
T_n = (C^{-1})_{n0}. \tag{16}
\]
For many examples, we are interested only in the inclusive process, where the intrinsic degree of freedom emerges in any final state. Taking a summation over all possible intrinsic states, the inclusive penetrability is given by
\[
P_J(E) = \sum_n \frac{k_n(r_{\text{min}})}{k_0} |T_n|^2. \tag{17}
\]
The fusion cross section and the mean angular momentum of compound nucleus are then calculated by
\[
\sigma_{\text{fus}}(E) = \sum_J \sigma_J(E) = \frac{\pi}{k_0^2} \sum_J (2J + 1) P_J(E), \tag{18}
\]
\[
\langle l \rangle = \frac{\sum_J J \sigma_J(E)}{\sum_J \sigma_J(E)} = \left( \frac{\pi}{k_0^2} \sum_J (2J + 1) P_J(E) \right) \left( \sum_J (2J + 1) P_J(E) \right), \tag{19}
\]
respectively. In the program CCFULL, the summation over the partial wave is truncated at the angular momentum whose contribution to the cross section is less than $10^{-4}$ times the total cross section.

3. Coupling matrix elements

3.1. Rotational coupling

In this section, we give explicit expressions for the coupling matrix elements $V_{nm}(r)$ in Eq. (1). Let us first consider a rotational coupling in the target nucleus. The nuclear coupling Hamiltonian can be generated by changing the target radius in the nuclear potential (2) to a dynamical operator
\[
R_0 \rightarrow R_0 + \hat{O} = R_0 + \beta_2 R_T Y_{20} + \beta_4 R_T Y_{40}, \tag{20}
\]
where $R_T$ is parametrised as $r_{\text{coup}} A_T^{1/3}$, and $\beta_2$ and $\beta_4$ are the quadrupole and hexadecapole deformation parameters of the deformed target nucleus, respectively. The nuclear coupling Hamiltonian is thus given by
\[
V_N(r, \hat{O}) = -\frac{V_0}{1 + \exp((r - R_0 - \hat{O})/\alpha)}. \tag{21}
\]
We need matrix elements of this coupling Hamiltonian between the $|m\rangle = |J0\rangle$ and $|n\rangle = |I0\rangle$ states of the ground rotational band of the target. These can be easily obtained using a matrix algebra [14]. In this algebra, one first looks for the eigenvalues and eigenvectors of the operator $\hat{O}$ which satisfies
\[
\hat{O}|\alpha\rangle = \lambda_{\alpha} |\alpha\rangle. \tag{22}
\]
In the program CCFULL, this is done by diagonalising the matrix $\hat{O}$, whose elements are given by
\[
\hat{O}_{ll'} = \sqrt{\frac{5(2J + 1)(2I' + 1)}{4\pi \beta_2 R_T}} \begin{pmatrix} I & 0 & 0 \end{pmatrix}^2 + \sqrt{\frac{9(2J + 1)(2I' + 1)}{4\pi \beta_4 R_T}} \begin{pmatrix} I & 0 \end{pmatrix}^2 \begin{pmatrix} 0 & I' & 0 \end{pmatrix}^2. \tag{23}
\]
The nuclear coupling matrix elements are then evaluated as

\[ V_{nm}^{(N)} = \langle I'0|V_N(r, \hat{O})|I'0 \rangle - V_{N}^{(0)}(r)\delta_{n,m} \]

\[ = \sum_{\alpha} \langle I'0|\alpha|I'0 \rangle V_N(r, \lambda_{\alpha}) - V_{N}^{(0)}(r)\delta_{n,m}. \quad (24) \]

The last term in this equation is included to avoid the double counting of the diagonal component.

For the Coulomb interaction of the deformed target, the program CCFULL includes up to the second order with respect to \( \lambda_2 \) and to the first order of \( \lambda_4 \). Contrary to the nuclear couplings, the higher order couplings of the Coulomb interaction have been shown to play a rather minor role [6]. The matrix elements are then given by

\[ V_{nm}^{(C)} = \frac{3Z_p Z_T}{5} \frac{R_T^2}{r^3} \sqrt{\frac{5(2I + 1)(2I' + 1)}{4\pi}} \left( \beta_2 + \frac{2}{7} \sqrt{\frac{5}{\pi}} \beta_2^2 \right)^2 \left( \begin{array}{ccc} I & 2 & I' \\ 0 & 0 & 0 \end{array} \right)^2 \]

\[ + \frac{3Z_p Z_T}{9} \frac{R_T^4}{r^5} \sqrt{\frac{9(2I + 1)(2I' + 1)}{4\pi}} \left( \beta_4 + \frac{9}{7} \beta_2^2 \right)^2 \left( \begin{array}{ccc} I & 4 & I' \\ 0 & 0 & 0 \end{array} \right)^2. \quad (25) \]

The total coupling matrix element is given by the sum of \( V_{nm}^{(N)} \) and \( V_{nm}^{(C)} \).

### 3.2. Vibrational coupling

We next consider a vibrational coupling. Ref. [6] discusses all order nuclear couplings for the case where the vibration can be approximated by the harmonic oscillator. In a realistic case, however, phonon spectra are often truncated at some level, and thus the intrinsic motion deviates from the harmonic limit even when the excitation energies are equally spaced and/or the electromagnetic transitions do not alter in the linear approximation. (See Ref. [15] for a discussion on differences between the harmonic oscillator and the truncated oscillator, i.e. spin systems.) In such a situation, the matrix formalism discussed in the previous section still provides a convenient and powerful technique to evaluate the coupling matrix elements [2]. For vibrational coupling, the operator \( \hat{O} \) in the nuclear coupling Hamiltonian is given by

\[ \hat{O} = \frac{\beta_{\lambda}}{\sqrt{4\pi}} R_T (a_{\lambda 0}^\dagger + a_{\lambda 0}), \quad (26) \]

where \( \lambda \) is the multipolarity of the vibrational mode and \( a_{\lambda 0}^\dagger(a_{\lambda 0}) \) is the creation (annihilation) operator of the phonon. The matrix element of this operator between the \( n \)-phonon state \( |n \rangle \) and the \( m \)-phonon state \( |m \rangle \) is given by

\[ \hat{O}_{nm} = \frac{\beta_{\lambda}}{\sqrt{4\pi}} R_T (\sqrt{m} \delta_{n,m-1} + \sqrt{n} \delta_{n,m+1}). \quad (27) \]

The rest of the procedure to evaluate the nuclear coupling matrix element is exactly the same as for the rotational case. The operator \( \hat{O} \) is diagonalised in a physical space and then the nuclear coupling matrix elements are calculated according to Eq. (24).

The program CCFULL uses the linear coupling approximation for the Coulomb coupling of the vibrational degree of freedom. The Coulomb coupling matrix elements thus read as

\[ V_{nm}^{(C)}(r) = \frac{\beta_{\lambda}}{\sqrt{4\pi}} \frac{3}{2\lambda + 1} Z_p Z_T e^2 \frac{R_T^4}{r^{\lambda+1}} \left( \sqrt{m} \delta_{n,m-1} + \sqrt{n} \delta_{n,m+1} \right). \quad (28) \]

Again the total coupling matrix element is given by the sum of \( V_{nm}^{(N)} \) and \( V_{nm}^{(C)} \).
Table 1
Input to the computer code CCFULL

<table>
<thead>
<tr>
<th>Line</th>
<th>Parameters</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>AP, ZP, AT, ZT</td>
</tr>
<tr>
<td>2</td>
<td>RP, IVIBROTP, RT, IVIBROTT</td>
</tr>
<tr>
<td>3</td>
<td>OMEGAT, BETAT, LAMBDA, NPHONONT (if IVIBROTT=0)</td>
</tr>
<tr>
<td></td>
<td>E2T, BETA2T, BETA4T, NROTT (if IVIBROTT=1)</td>
</tr>
<tr>
<td>4</td>
<td>OMEGAT2, BETAT2, LAMBDA2, NPHONONT2</td>
</tr>
<tr>
<td>5</td>
<td>OMEGAP, BETAP, LAMBDAP, NPHONONP (if IVIBROTP=0)</td>
</tr>
<tr>
<td></td>
<td>E2P, BETA2P, BETA4P, NROTP (if IVIBROTP=1)</td>
</tr>
<tr>
<td>6</td>
<td>NTRANS, QTRANS, FTR</td>
</tr>
<tr>
<td>7</td>
<td>V0, R0, A0</td>
</tr>
<tr>
<td>8</td>
<td>EMIN, EMAX, DE</td>
</tr>
<tr>
<td>9</td>
<td>RMAX, DR</td>
</tr>
</tbody>
</table>

3.3. Transfer coupling

The program CCFULL includes a pair-transfer coupling between the ground states. It uses the macroscopic coupling form factor given by [16]

\[
F_{\text{trans}}(r) = F_t \frac{dV^{(0)}_N}{dr},
\]

where \(F_t\) is the coupling strength.

4. Program input and test run

A description of the format for the input parameters is given in Table 1. All parameters are entered in free format. The first line contains the parameters specifying the system. AP (AT) is the projectile (target) mass and ZP (ZT) is the projectile (target) charge. The second line is for the coupling Hamiltonian. RP (RT) is the radius parameter \(r_{\text{coup}}\) of the projectile (target) used in the coupling Hamiltonian. Note that this is in general different from the radius parameter used in the nuclear potential (2), which is defined in the seventh line. IVIBROTP (IVIBROTT) is an option which specifies the property of the intrinsic motion of the projectile (target). If it is set to be \(1\), the projectile (target) is assumed to be inert and the fifth (the third and the fourth) line will be ignored. The fusion cross sections and the mean angular momentum in the absence of channel coupling can be therefore obtained by setting both the IVIBROTP and the IVIBROTT to \(1\). When IVIBROTP (IVIBROTT) is set to zero, the CCFULL assumes that the coupling in the projectile (target) is vibrational, while if it is set to one, the rotational coupling is assumed.

The third line is for detailed information on the target excitation. If IVIBROTR is zero (i.e., the vibrational coupling), the CCFULL reads OMEGAT, BETAT, LAMBDA, and NPHONONT. OMEGAT is the excitation energy of the single phonon state, BETAT is the deformation parameter, and LAMBDA is the multipolarity of the vibrational excitation. NPHONONT is the maximum phonon number to be included. For example, if it is two, up to two phonon states are included in the calculation. The CCFULL assumes the harmonic oscillator for a vibrational coupling. The excitation energy of the \(n\)-phonon state is thus given by \(n\) times OMEGAT. Sometimes a user may want to use a different value of deformation parameter for the nuclear coupling from that for the Coulomb coupling. The CCFULL therefore will ask the user interactively before a run whether he/she intends to use a
different value of the coupling strength for the nuclear coupling. If IVIBROTT is one (i.e., the rotational coupling),
the CCFULL reads E2T, BET2T, BET4T, and NROTT. E2T is the excitation energy of the first \(2^+\) state in
the ground rotational band of the target nucleus, BET2T and BET4T are the quadrupole and hexadecapole
deformation parameters, respectively. NROTT is the number of levels in the rotational band to be included. For
instance, if it is 3, the \(2^+, 4^+, \text{ and } 6^+\) states are included together with the ground state. The CCFULL assumes a
pure rotor for a deformed nucleus, and the excitation energy of the \(I^+\) state is given by \(I(I + 1) \cdot E2T/6\).

In many applications, there are two vibrational modes of excitations in the target nucleus. A typical example
is the octupole and quadrupole vibrational excitations in \(^{144}\text{Sm}\). The fourth line is for the second mode of
excitation in the target nucleus. The meaning of OMEGAT2, BETAT2, LAMBDAT2 and NPHONT2 is the
same as OMEGAT, BETAT, LAMBDAT and NPHONT, respectively. The second mode is not included when
NPHONT2 is set to zero. OMEGAT2, BETAT2, and LAMBDAT2 are then ignored. When NPHONT2 is
not zero, the user will be asked before a run which of the mutual excitation channels he/she intends to include in
the calculation.

The fifth line is the same as the third line, but for the projectile excitations. If there exist excitations both in the
projectile and the target, the CCFULL takes into account all the possible mutual excitation channels between the
projectile and the target excitations.

The sixth line is for the pair transfer coupling. QTRANS is the Q-value for the pair transfer channel, while FTR
is the coupling strength defined by Eq. (29). NTRANS is the number of the pair transfer channel. In the present
version of the CCFULL, NTRANS is restricted to be either one or zero. If it is zero, the pair transfer channel is not
included and QTRANS and FTR are ignored.

The seventh line is for the nuclear potential in the entrance channel (2). V0 is the depth parameter of the Woods–
Saxon potential, R0 is the radius parameter \(r_0\) in Eq. (2), and A0 is the surface diffuseness parameter \(a\).

EMIN, EMAX, and DE in the next line are the minimum and the maximum value of the colliding energy in the
center of mass frame and the interval in the energy scale, respectively. The CCFULL constructs the distribution
of partial cross sections \(\sigma_J\) as a function of \(J\) if a single value of the energy is entered, i.e. either when EMIN =
EMAX or DE = 0.

The accuracy of the calculation is controlled by the matching radius RMAX and the mesh for the integration DR
in the ninth line. For many applications, especially for asymmetric systems such as \(^{16}\text{O} + ^{144}\text{Sm}\), RMAX = 30 fm
and DR = 0.05 fm provides sufficiently accurate results. For heavier systems, such as \(^{64}\text{Ni} + ^{92}\text{Zr}\), RMAX may
have to be extended to a value as large as 50 fm.

The test case shows the fusion cross sections and the mean angular momentum of the compound nucleus for
the \(^{16}\text{O} + ^{144}\text{Sm}\) reaction. The projectile nucleus \(^{16}\text{O}\) is assumed to be inert, while the single octupole phonon excitation in \(^{144}\text{Sm}\) is taken into account. The transfer channel is not included in this calculation.

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References

TEST RUN INPUT

16., 8., 144., 62.
1.2, -1, 1.06, 0
1.81, 0.205, 3, 1
1.66, 0.11, 2, 0
6.13, 0.733, 3, 0
0, 0., 0.3
105.1, 1.1, 0.75
55., 72., 1.
36, 0.05

TEST RUN OUTPUT

\[ ^{160} \text{O} + ^{144}\text{Sm} \] Fusion reaction
------------------------------------------------- 
Phonon Excitation in the targ.: beta_N= 0.205, beta_C= 0.205, r0= 1.06(fm) 
\omega= 1.81(MeV), Lambda= 3, Nph= 1
-------------------------------------------------
Potential parameters: VO= 105.10(MeV), r0= 1.10(fm), a= 0.75(fm)
Uncoupled barrier: Rb=10.82(fm), Vb= 61.25(MeV), Curv= 4.25(MeV)
-------------------------------------------------

<table>
<thead>
<tr>
<th>Ecm (MeV)</th>
<th>sigma (mb)</th>
<th>&lt;l&gt;</th>
</tr>
</thead>
<tbody>
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