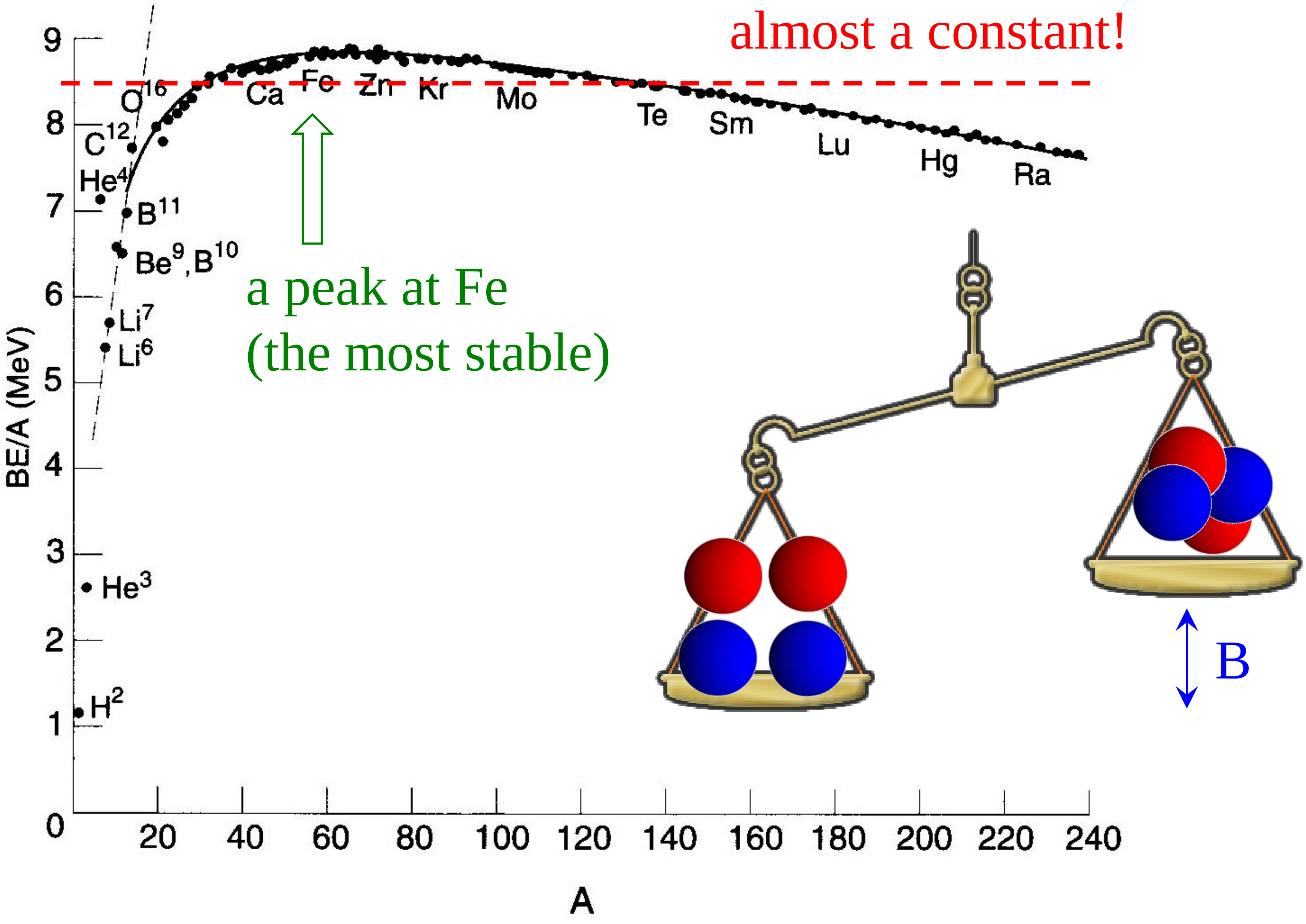
the origin of the peak



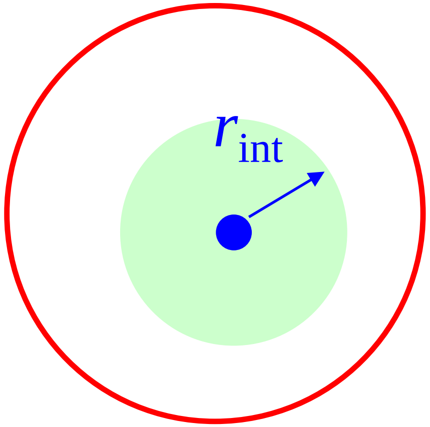
almost a constant!

a peak at Fe
(the most stable)

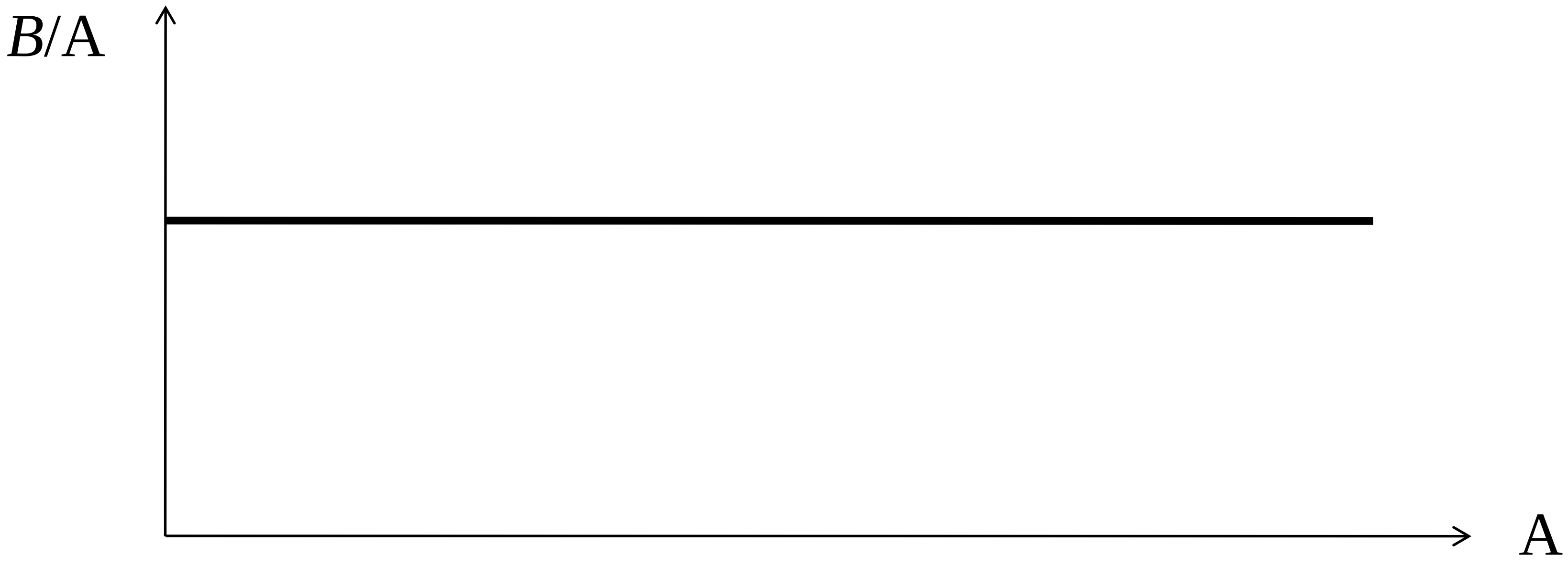
B

if each nucleon can interact only α-nucleons close by:

$$B \sim \alpha\, A/2 \longrightarrow B/A \sim \alpha/2 \text{ (const.)}$$



$$\alpha = \frac{4\pi}{3} r_{\text{int}}^{3} \cdot \rho$$



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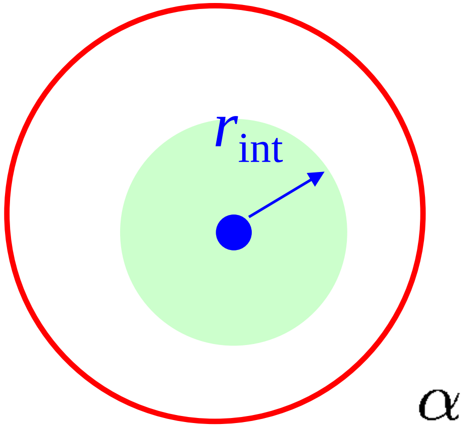
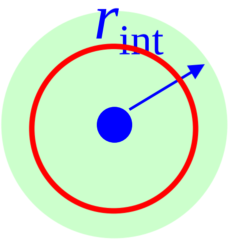
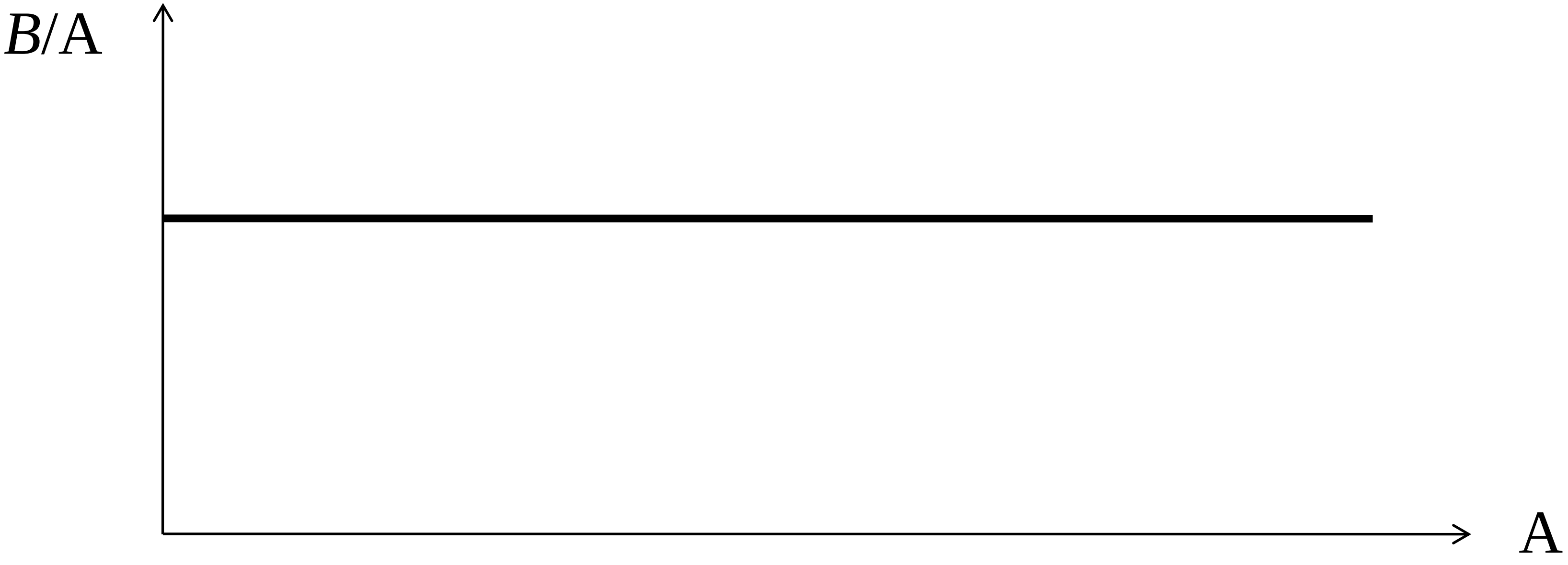


a small nucleus



$$\alpha = \frac{4\pi}{3} r_{\text{int}}^{3} \cdot \rho$$

$$\rightarrow B/A \propto A - 1$$



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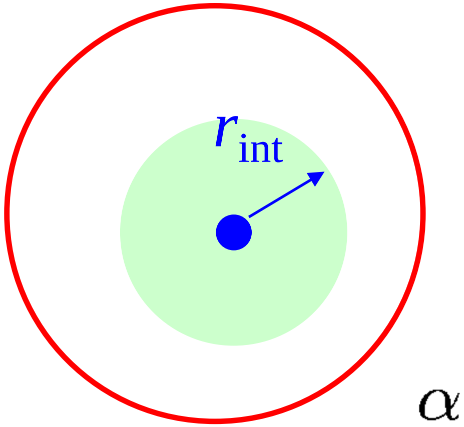

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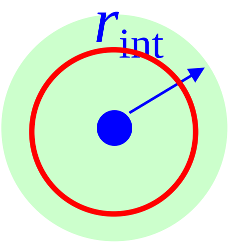

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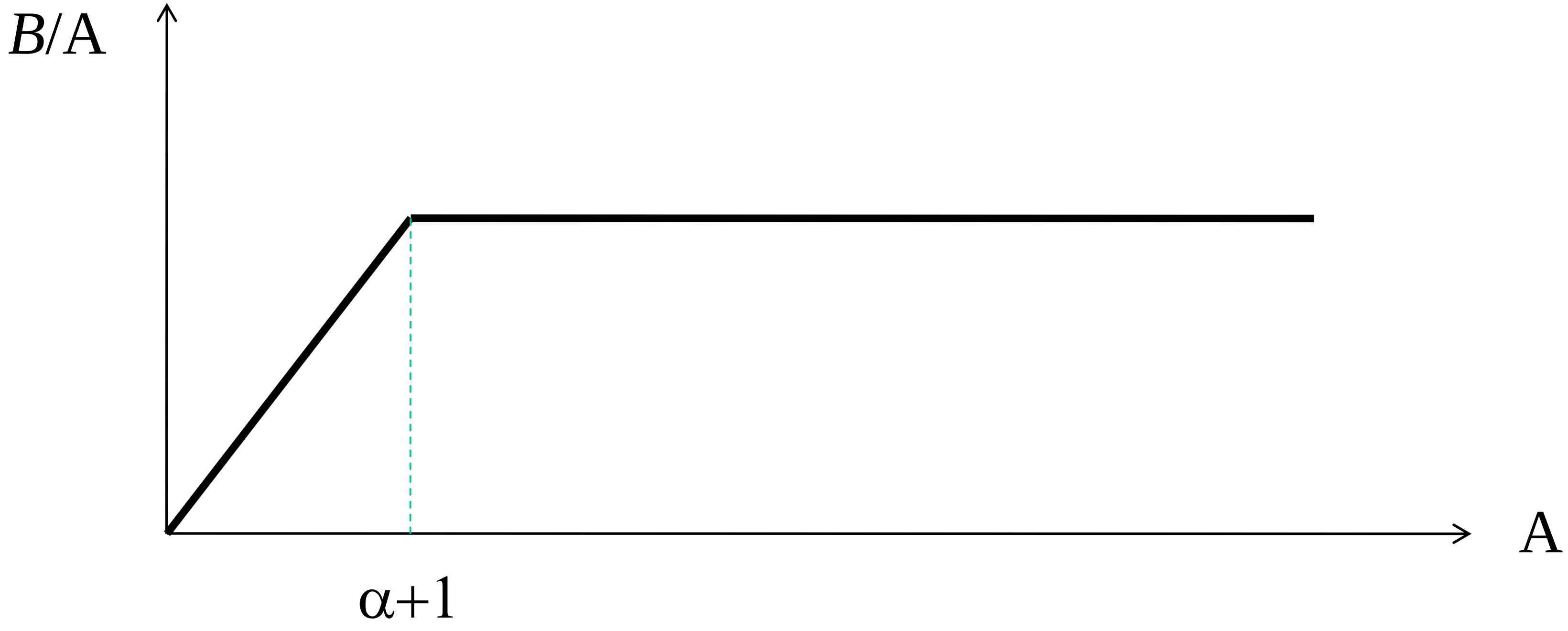
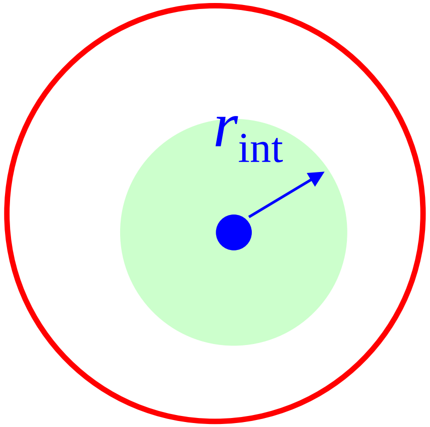
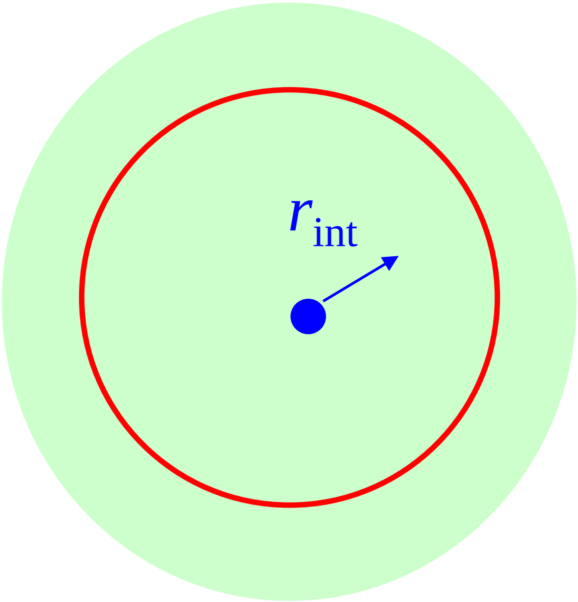


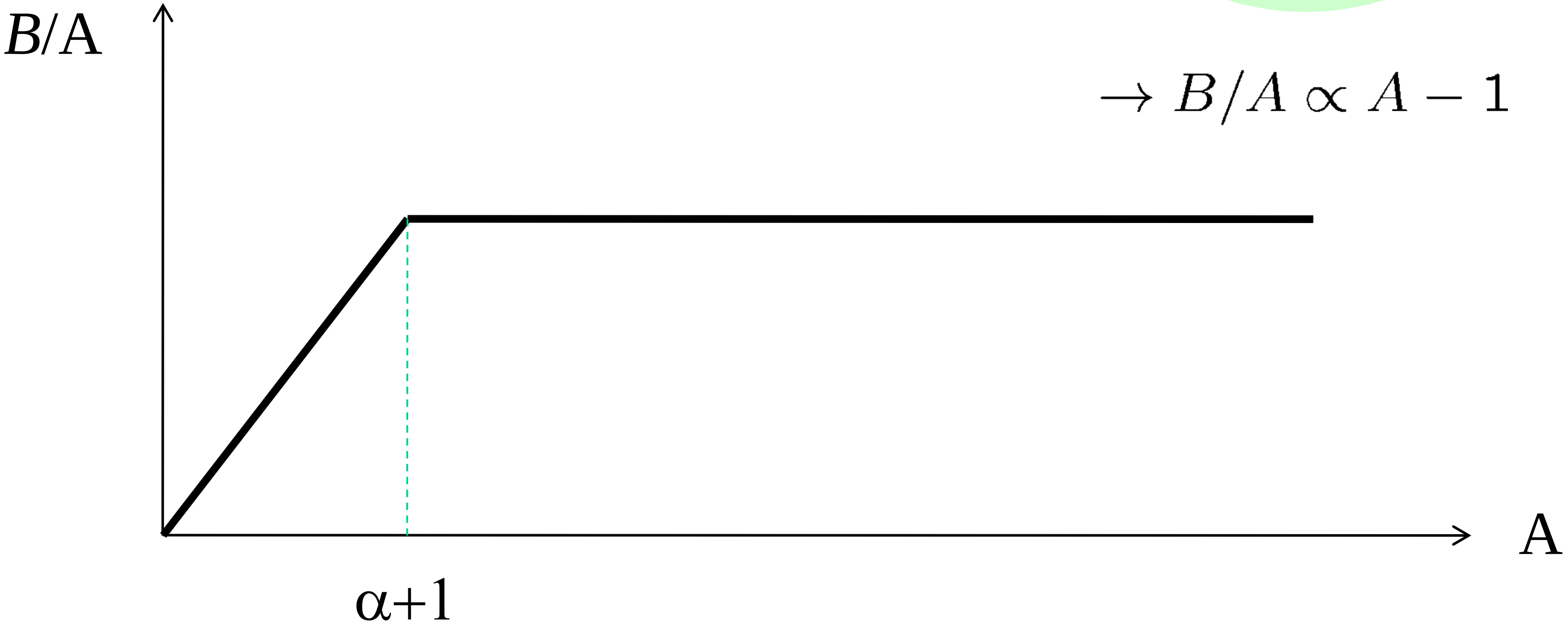
nuclear interaction



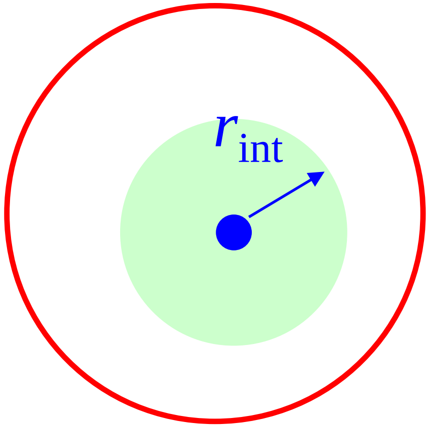
Coulomb interaction



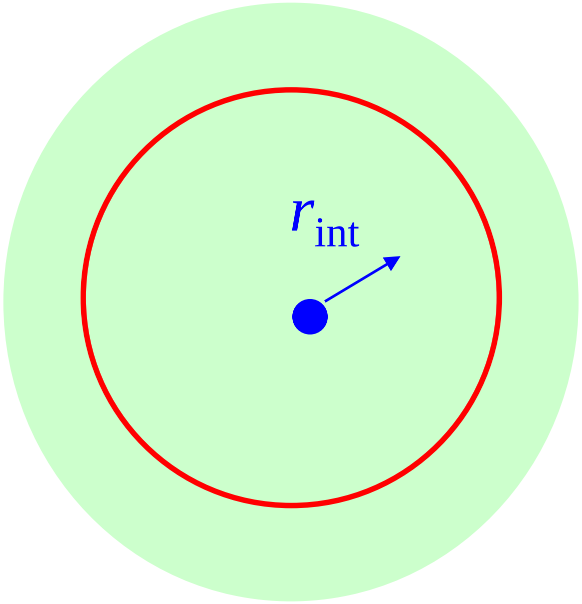
$\rightarrow B/A \propto A-1$



nuclear interaction



Coulomb interaction



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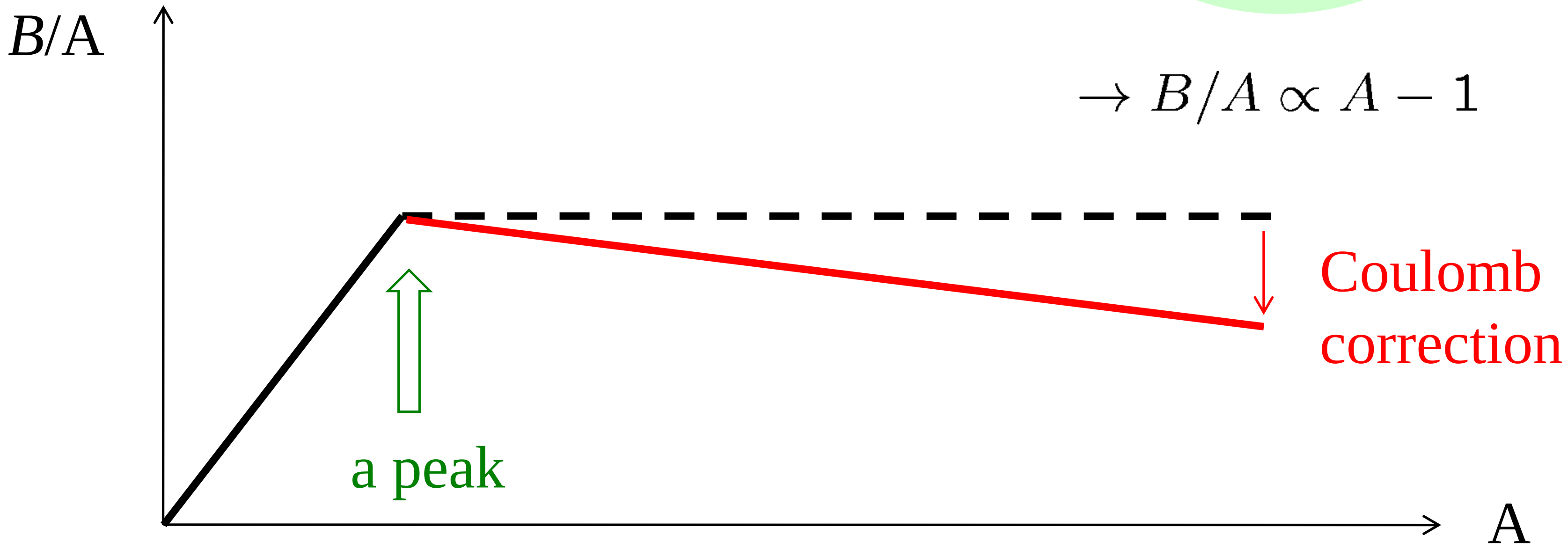
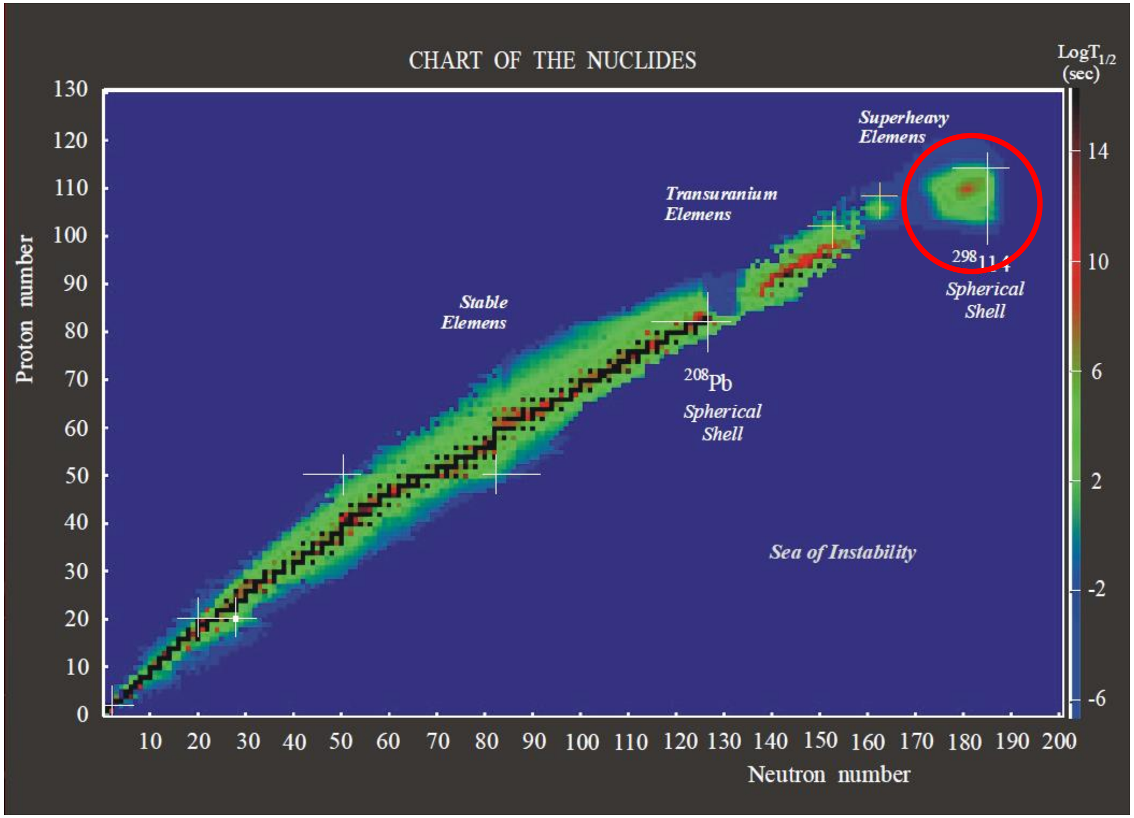
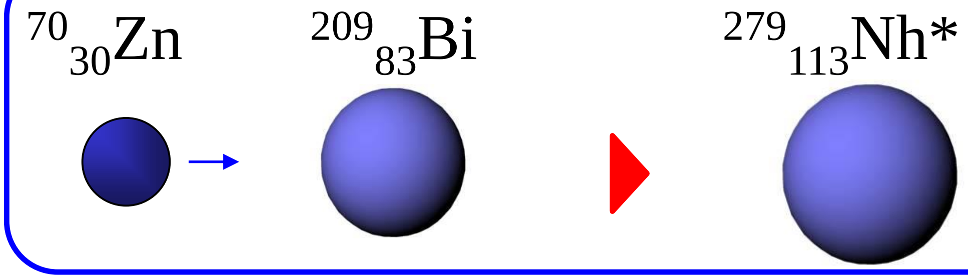


Superheavy elements

November, 2016

the island of stability（安定的島）



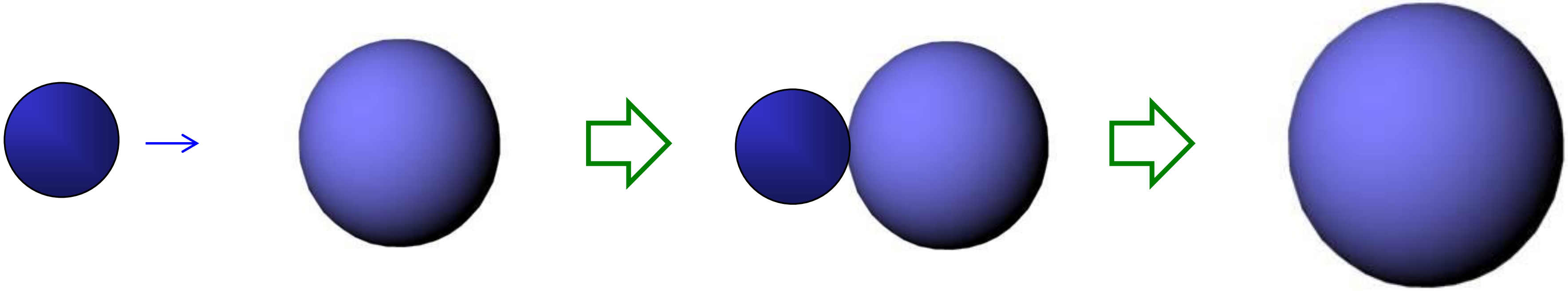


$^{70}_{30}\mathrm{Zn}$ $^{209}_{83}\mathrm{Bi}$ $^{279}_{113}\mathrm{Nh}^{*}$

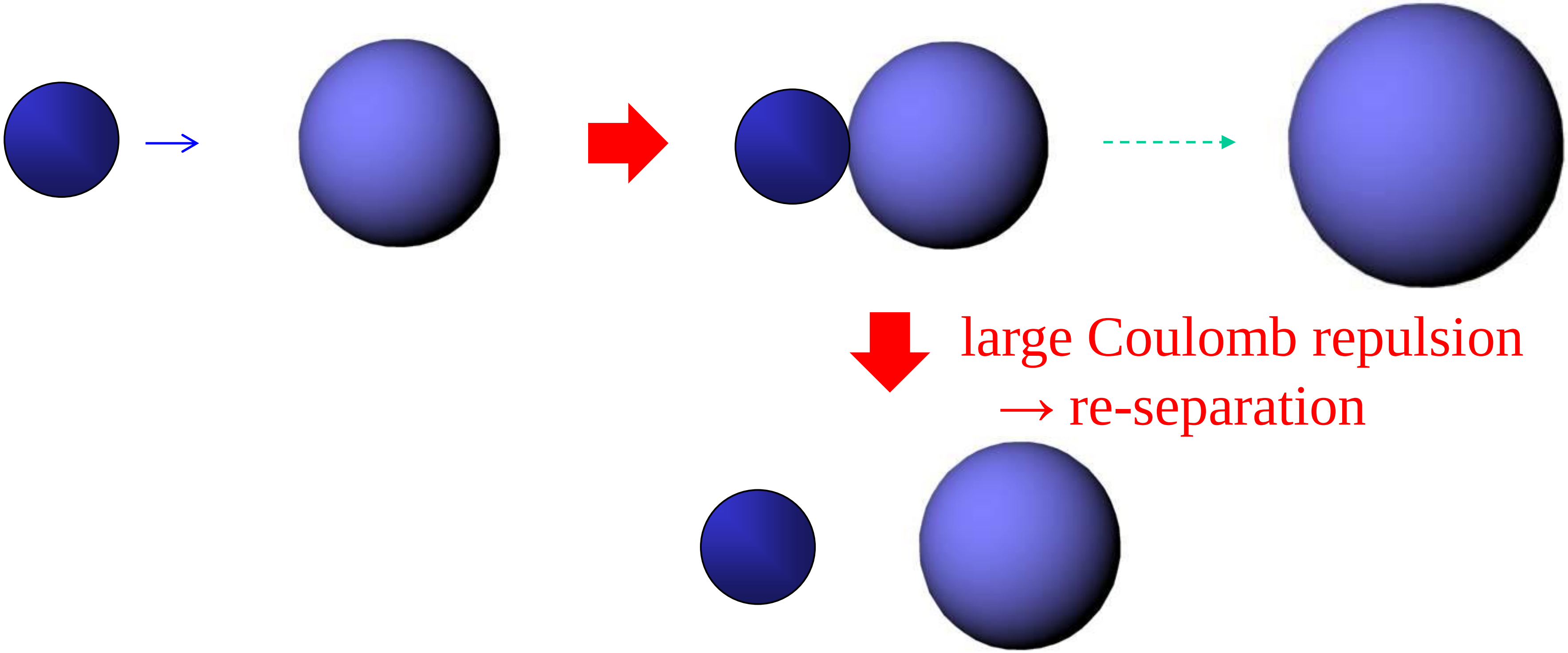
Heavy-ion fusion reaction

A complication

- Fusion of medium-heavy systems:



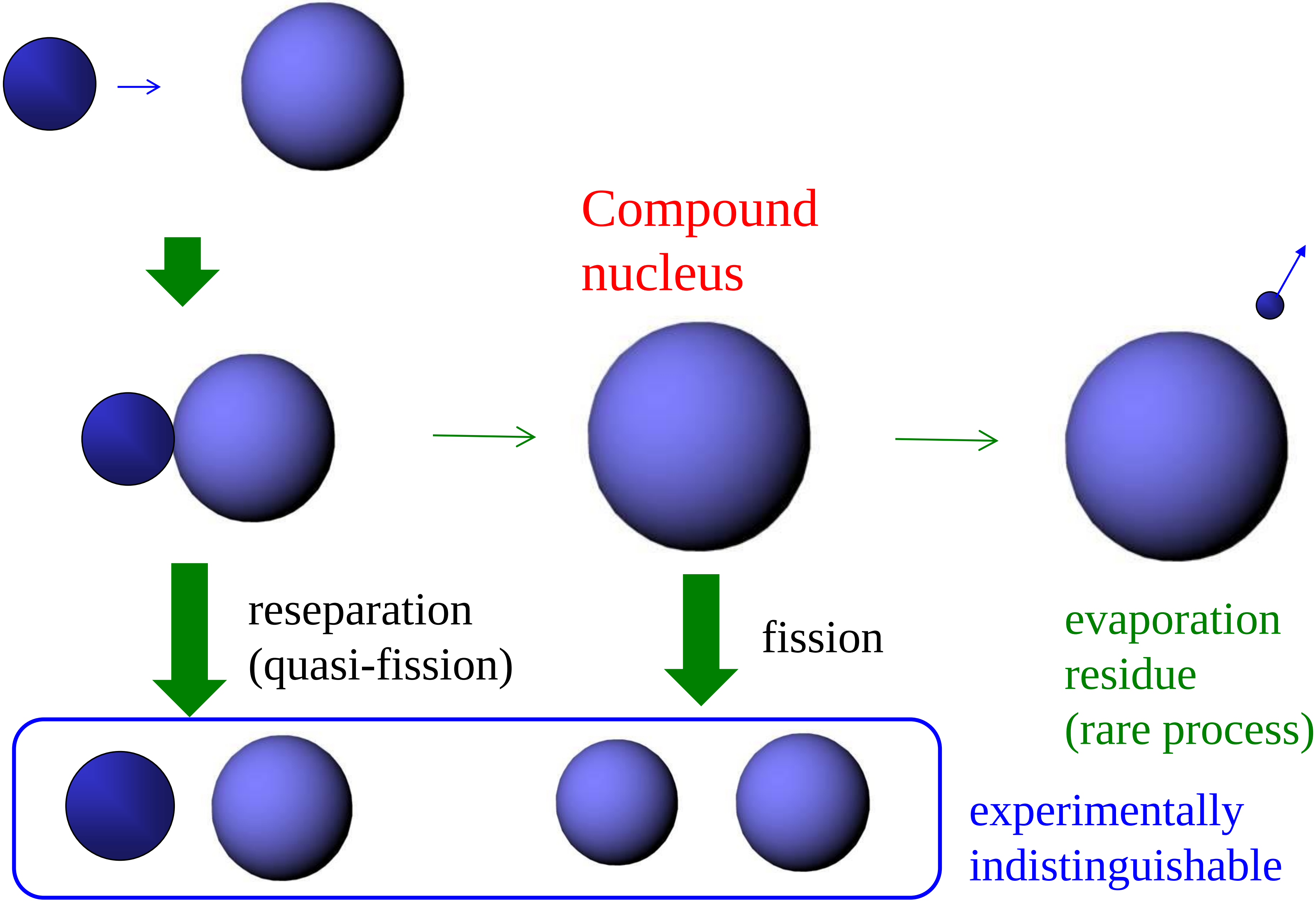
- Fusion of heavy and super-heavy systems:



large Coulomb repulsion

→ re-separation

Heavy-ion fusion reactions for superheavy elements



Compound nucleus

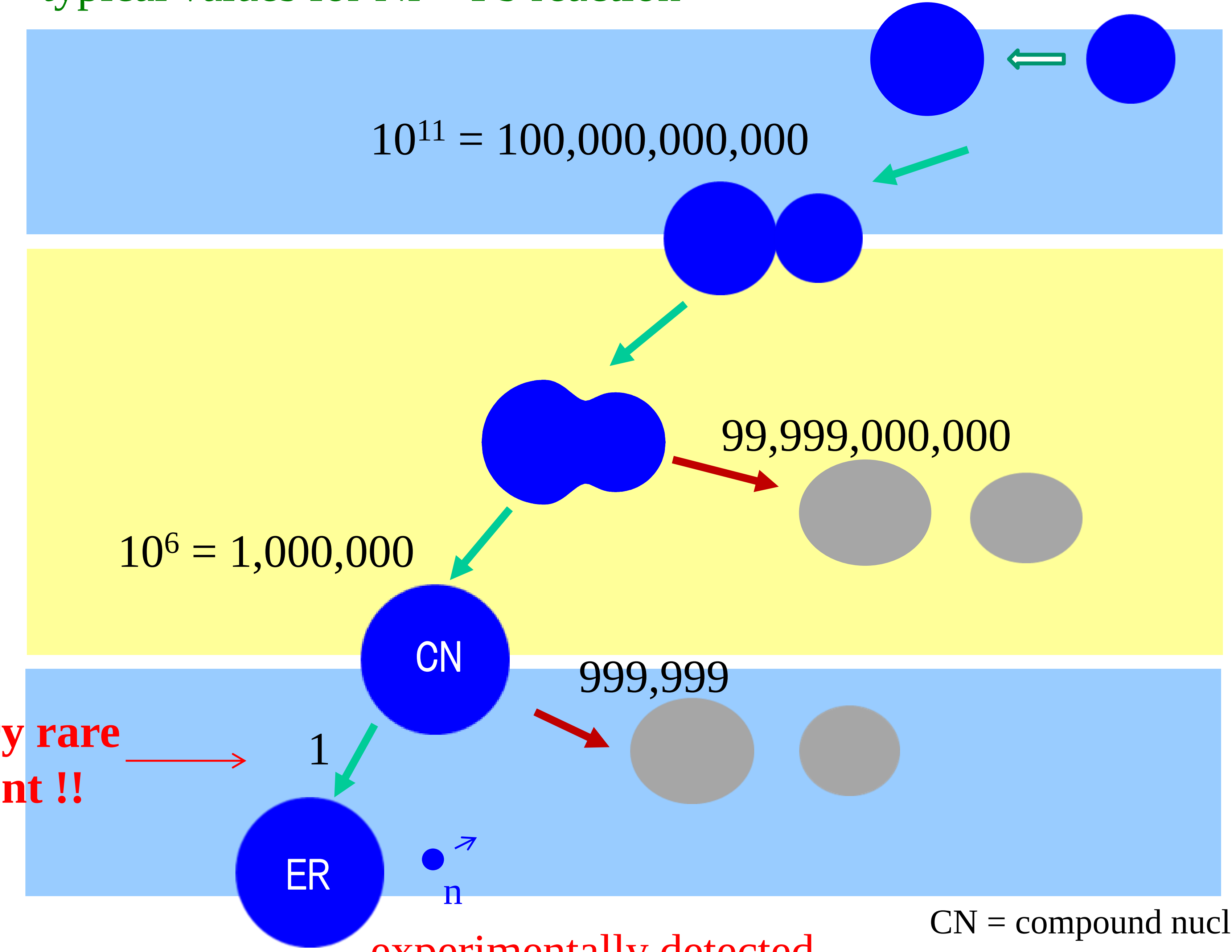
reseparation (quasi-fission)

fission

evaporation residue (rare process)

experimentally indistinguishable

typical values for Ni + Pb reaction



10^{11} = 100,000,000,000

99,999,000,000

10^{6} = 1,000,000

CN

999,999

very rare event !!

1

ER

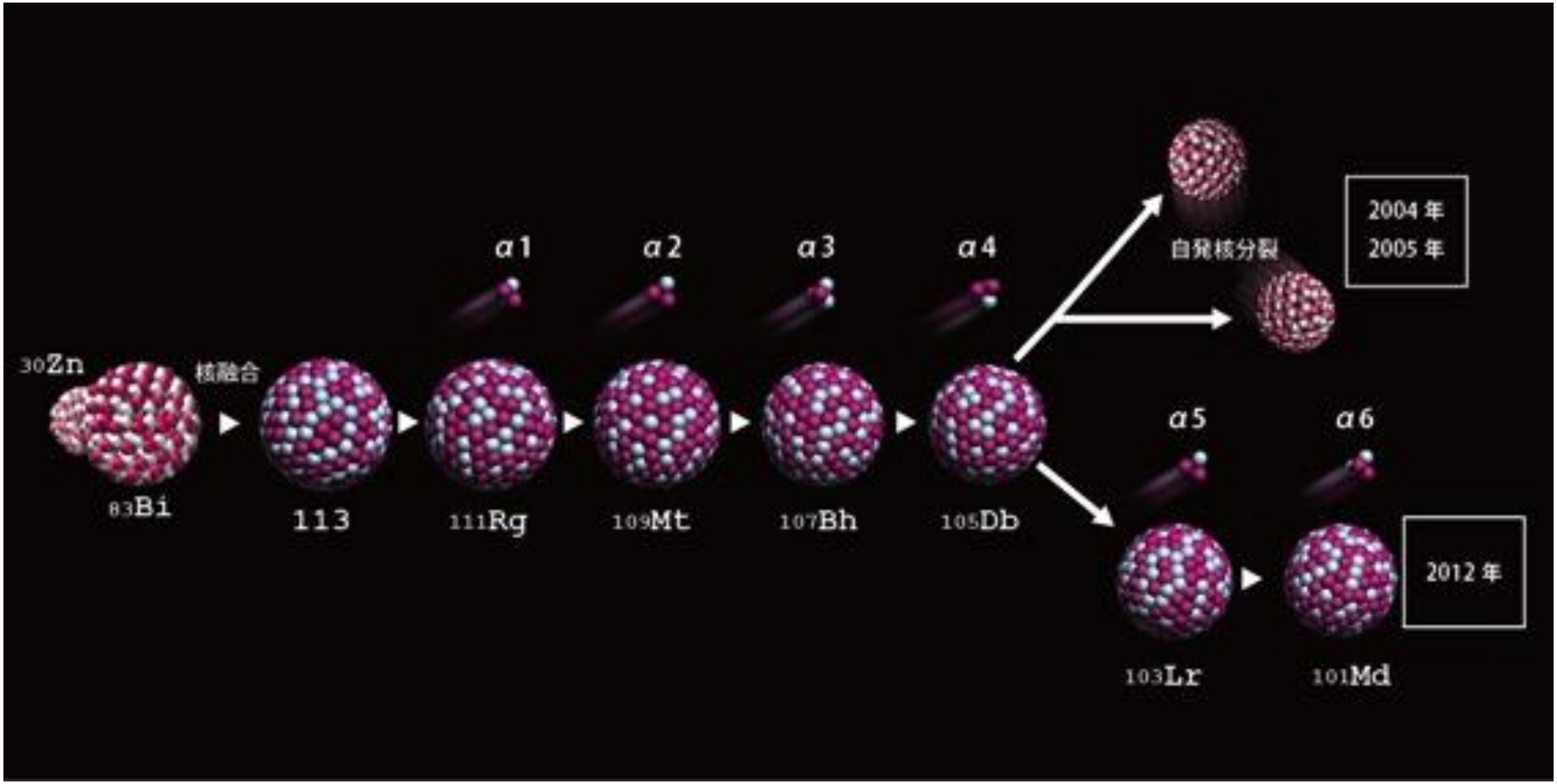
n

experimentally detected

CN = compound nucleus
ER = evaporation residue

Element 113 (RIKEN, K. Morita et al.)

$$^{70}\mathrm{Zn}\ (Z=30)\ +\ ^{209}\mathrm{Bi}\ (Z=83)\ \longrightarrow\ ^{278}\mathrm{Nh}\ (Z=113) + \mathrm{n}$$



K. Morita et al., J. Phys. Soc. Jpn. 81(‘12)103201

only 3 events for 553 days experiment

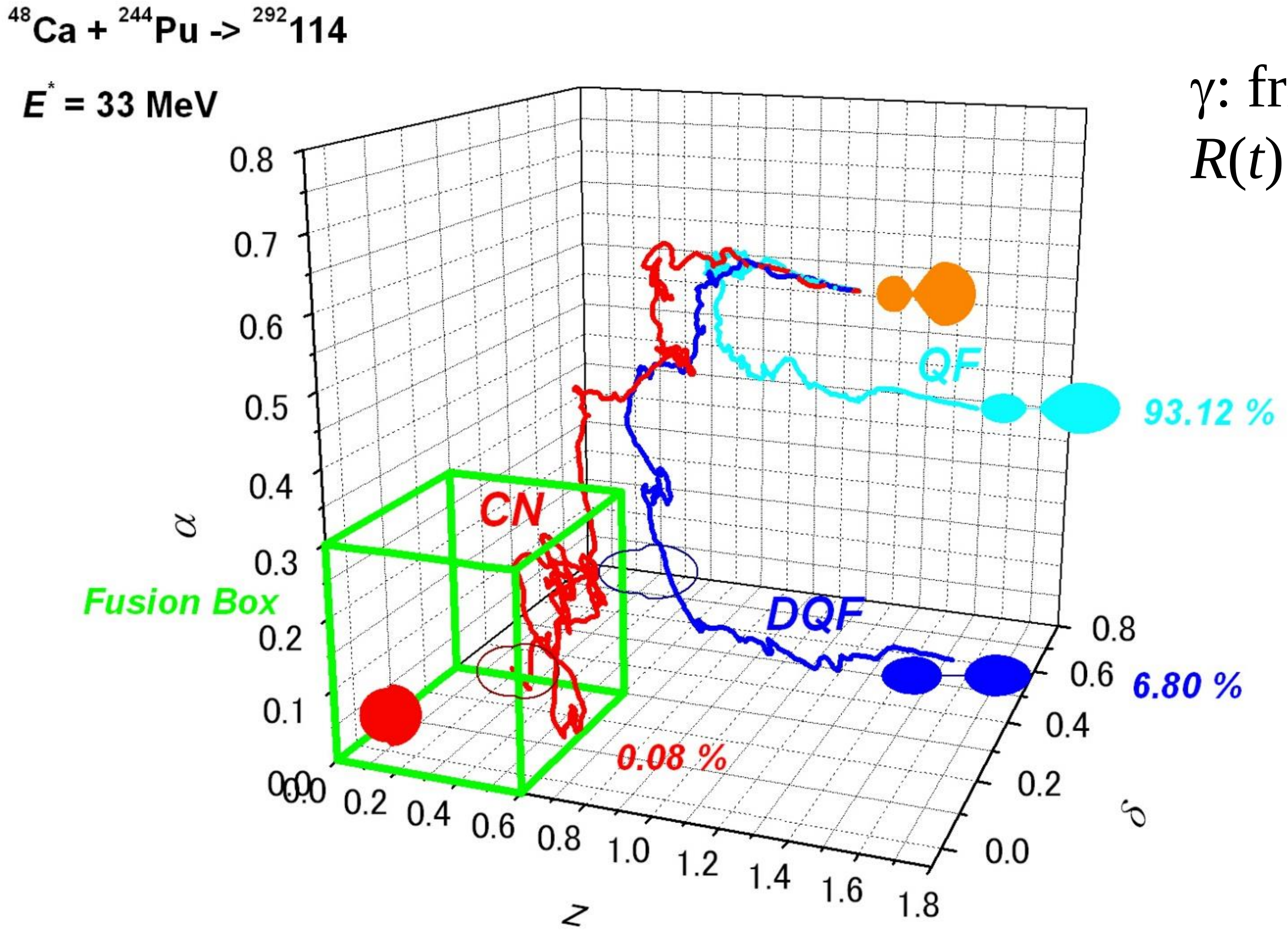
Theory: Lagenvin approach

multi-dimensional extension of:

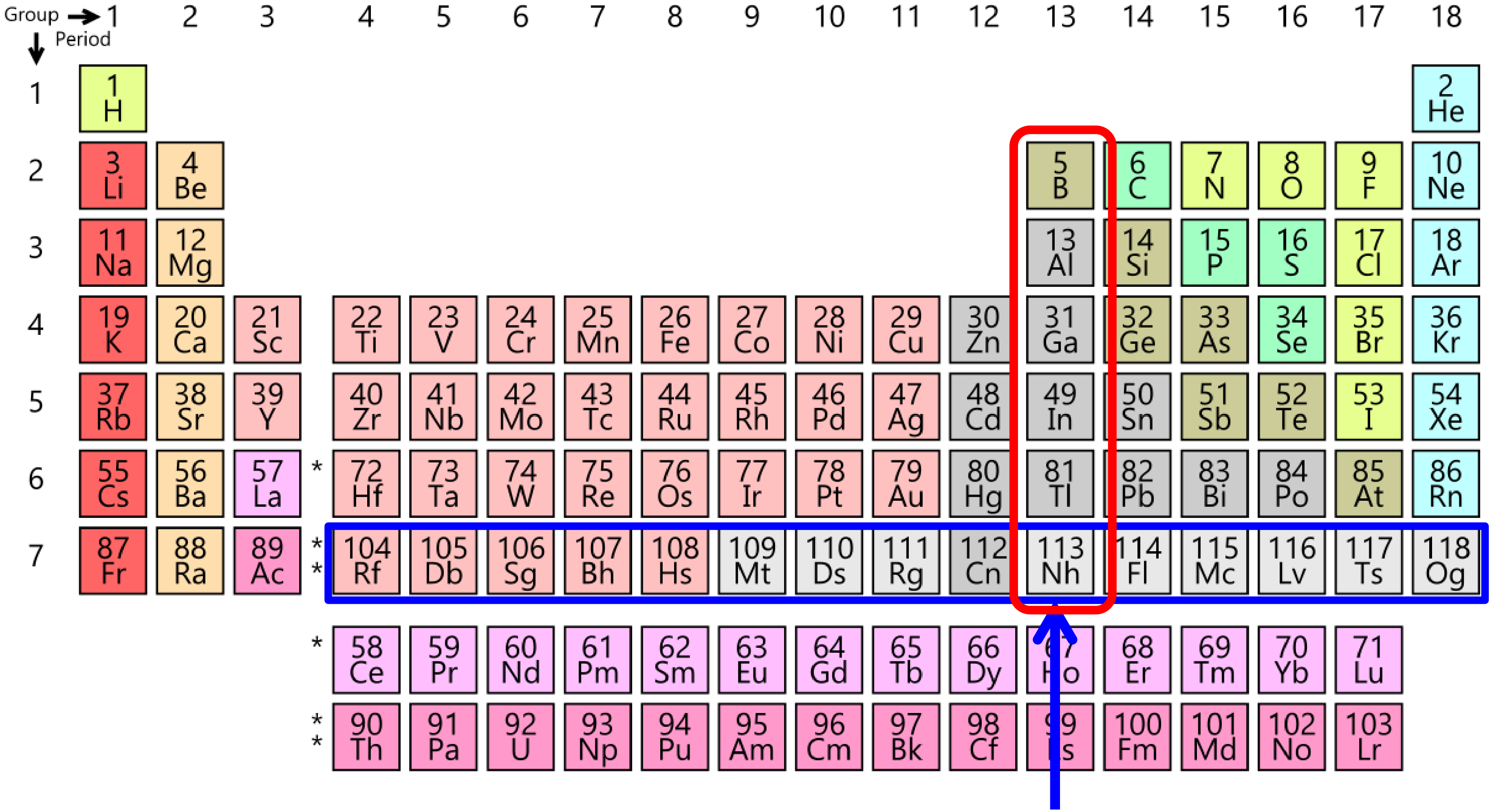
$$m\frac{d^2q}{dt^2} = -\frac{dV(q)}{dq} - \gamma\frac{dq}{dt} + R(t)$$

γ: friction coefficient
$R(t)$: random force

摩擦
乱雑力



Chemistry of superheavy elements



Group → Period ↓	1	2	3		4	5	6	7	8	9	10	11	12	13	14	15	16	17	18
1	1 H																		2 He
2	3 Li	4 Be												5 B	6 C	7 N	8 O	9 F	10 Ne
3	11 Na	12 Mg												13 Al	14 Si	15 P	16 S	17 Cl	18 Ar
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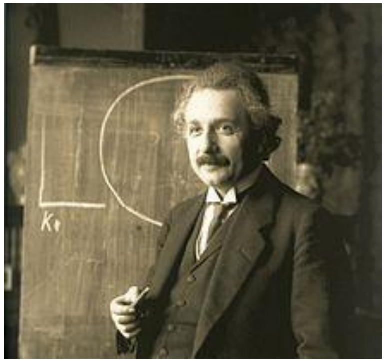
5	37 Rb	38 Sr	39 Y		40 Zr	41 Nb	42 Mo	43 Tc	44 Ru	45 Rh	46 Pd	47 Ag	48 Cd	49 In	50 Sn	51 Sb	52 Te	53 I	54 Xe
6	55 Cs	56 Ba	57 La	*	72 Hf	73 Ta	74 W	75 Re	76 Os	77 Ir	78 Pt	79 Au	80 Hg	81 Tl	82 Pb	83 Bi	84 Po	85 At	86 Rn
7	87 Fr	88 Ra	89 Ac	**	104 Rf	105 Db	106 Sg	107 Bh	108 Hs	109 Mt	110 Ds	111 Rg	112 Cn	113 Nh	114 Fl	115 Mc	116 Lv	117 Ts	118 Og
				*	58 Ce	59 Pr	60 Nd	61 Pm	62 Sm	63 Eu	64 Gd	65 Tb	66 Dy	67 Ho	68 Er	69 Tm	70 Yb	71 Lu	
				**	90 Th	91 Pa	92 U	93 Np	94 Pu	95 Am	96 Cm	97 Bk	98 Cf	99 Es	100 Fm	101 Md	102 No	103 Lr	

- ➢Are they here in the periodic table?
- ➢Does Nh show the same chemical properties as B, Al, Ga, In, and Tl?

relativistic effect : important for large Z

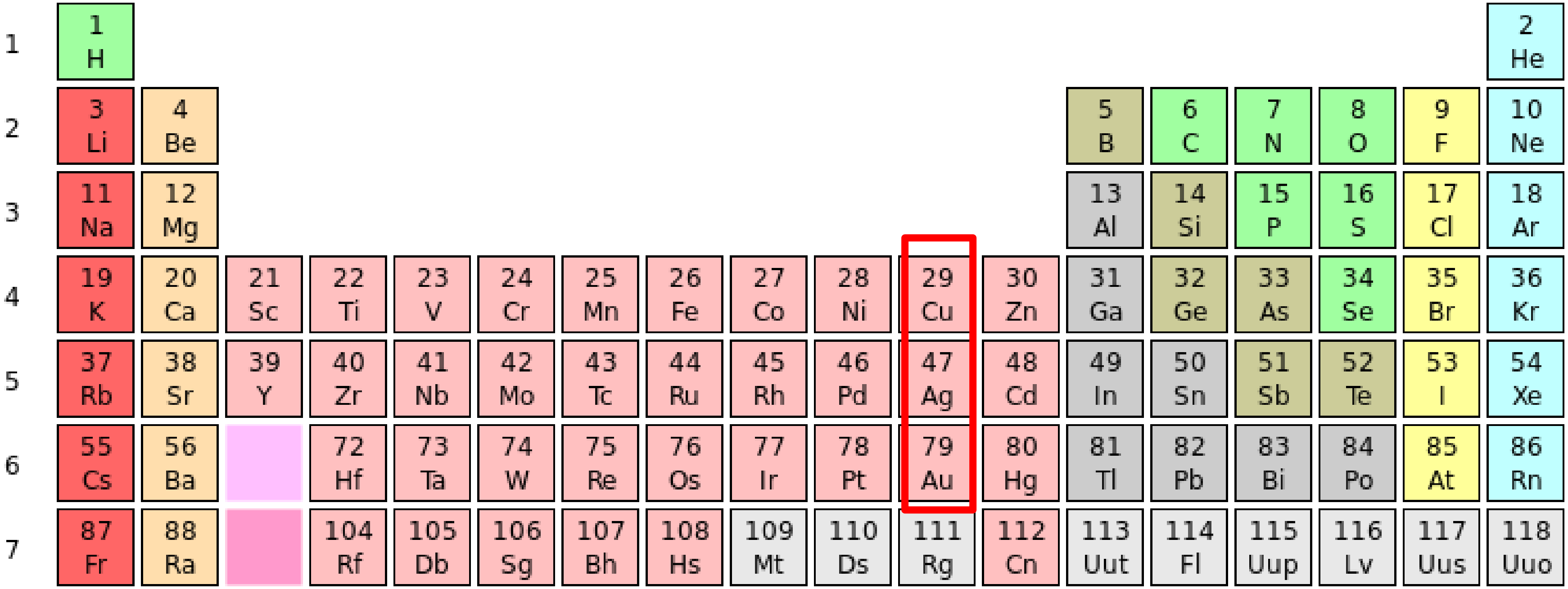
$$E = mc^2$$



Solution of the Dirac equation (relativistic quantum mechanics) for a hydrogen-like atom:

$$E_{1S} = mc^2\sqrt{1-(Z\alpha)^2} \sim mc^2\left(1 - \frac{(Z\alpha)^2}{2} \underbrace{- \frac{(Z\alpha)^4}{8} + \cdots}_{\text{relativistic effect}}\right)$$

Famous example of relativistic effects: the color of gold



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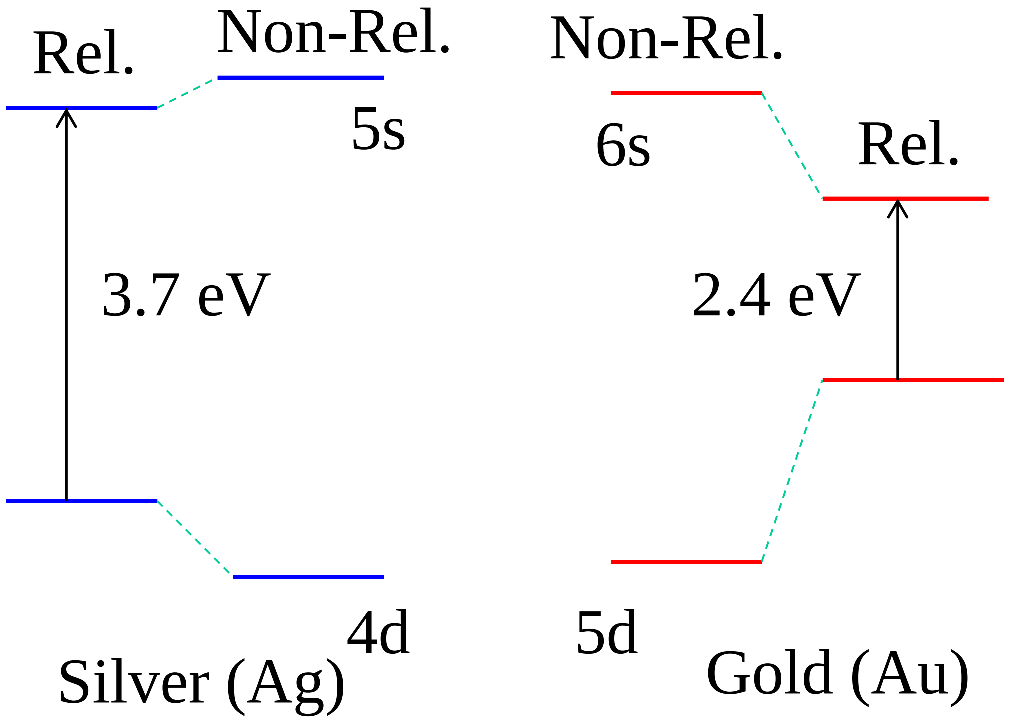
5	37 Rb	38 Sr	39 Y	40 Zr	41 Nb	42 Mo	43 Tc	44 Ru	45 Rh	46 Pd	47 Ag	48 Cd	49 In	50 Sn	51 Sb	52 Te	53 I	54 Xe
6	55 Cs	56 Ba		72 Hf	73 Ta	74 W	75 Re	76 Os	77 Ir	78 Pt	79 Au	80 Hg	81 Tl	82 Pb	83 Bi	84 Po	85 At	86 Rn
7	87 Fr	88 Ra		104 Rf	105 Db	106 Sg	107 Bh	108 Hs	109 Mt	110 Ds	111 Rg	112 Cn	113 Uut	114 Fl	115 Uup	116 Lv	117 Uus	118 Uuo

Gold looked like silver if there was no relativistic effects!



Rel.

Non-Rel.

5s

3.7 eV

4d

Silver (Ag)

Non-Rel.

6s

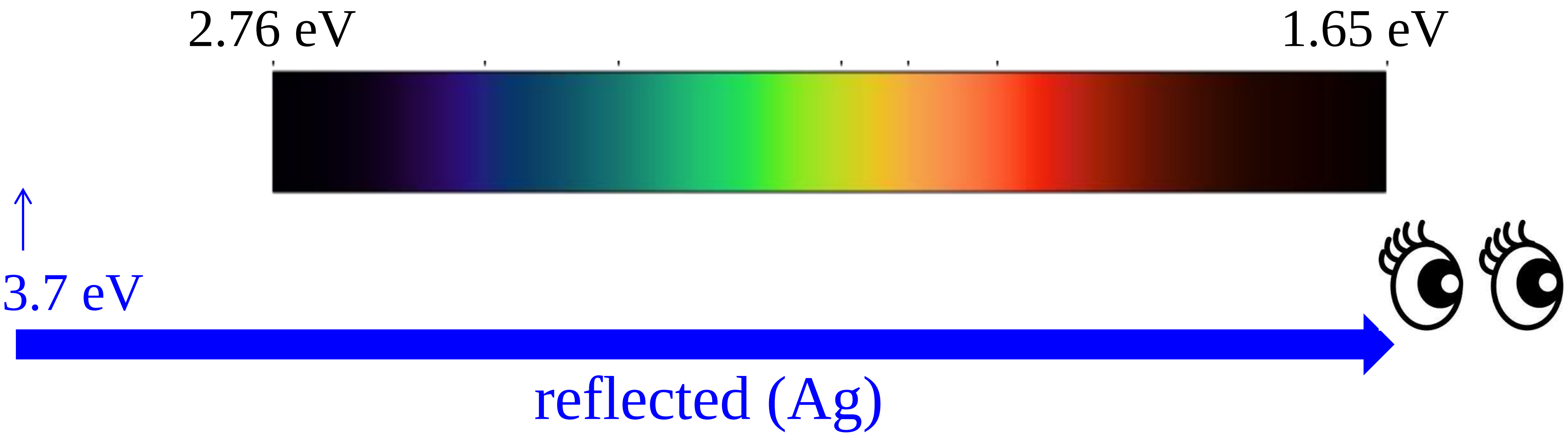
Rel.

2.4 eV

5d

Gold (Au)

cf. visible spectrum

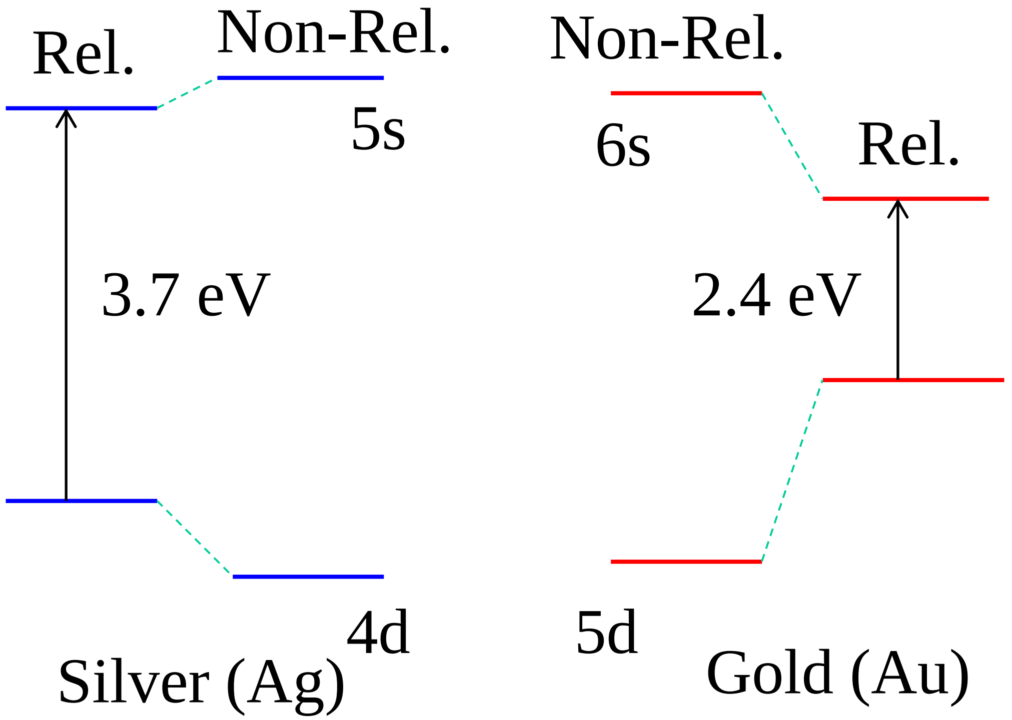


2.76 eV

1.65 eV

3.7 eV

reflected (Ag)



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Non-Rel.

5s

3.7 eV

4d

Silver (Ag)

Non-Rel.

6s

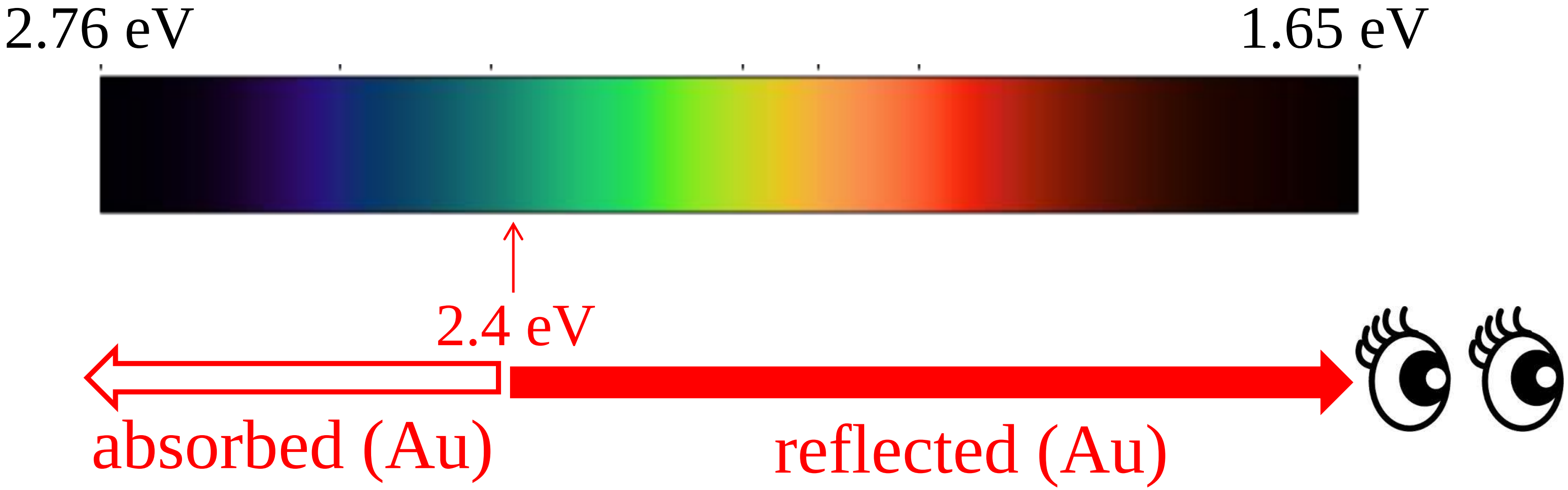
Rel.

2.4 eV

5d

Gold (Au)

cf. visible spectrum



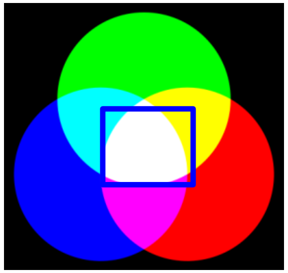
2.76 eV

1.65 eV

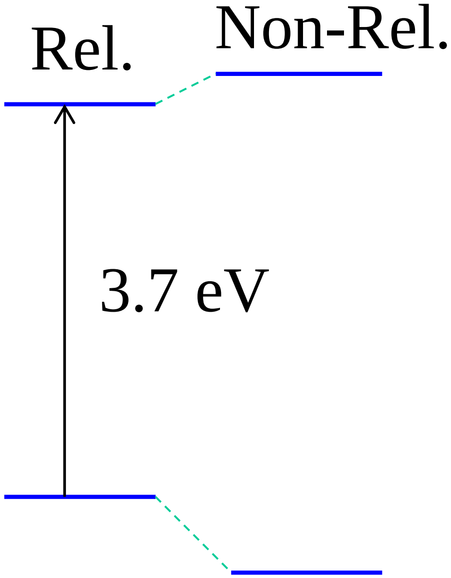
2.4 eV

absorbed (Au)

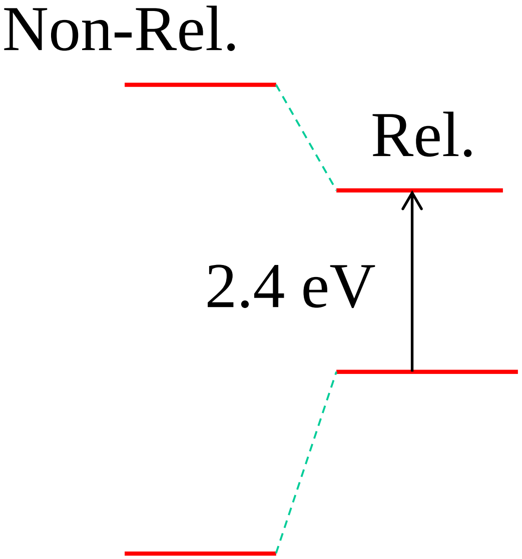
reflected (Au)



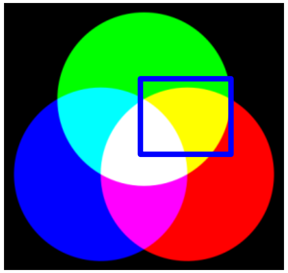
no color
absorbed



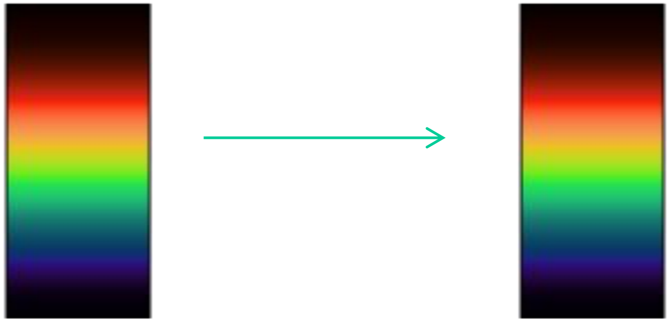
Silver (Ag)



Gold (Au)

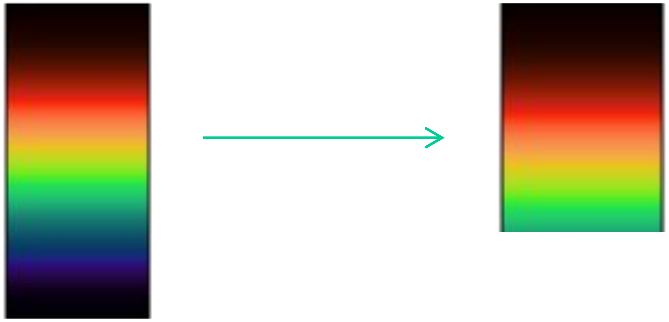


blue: absorbed



Ag

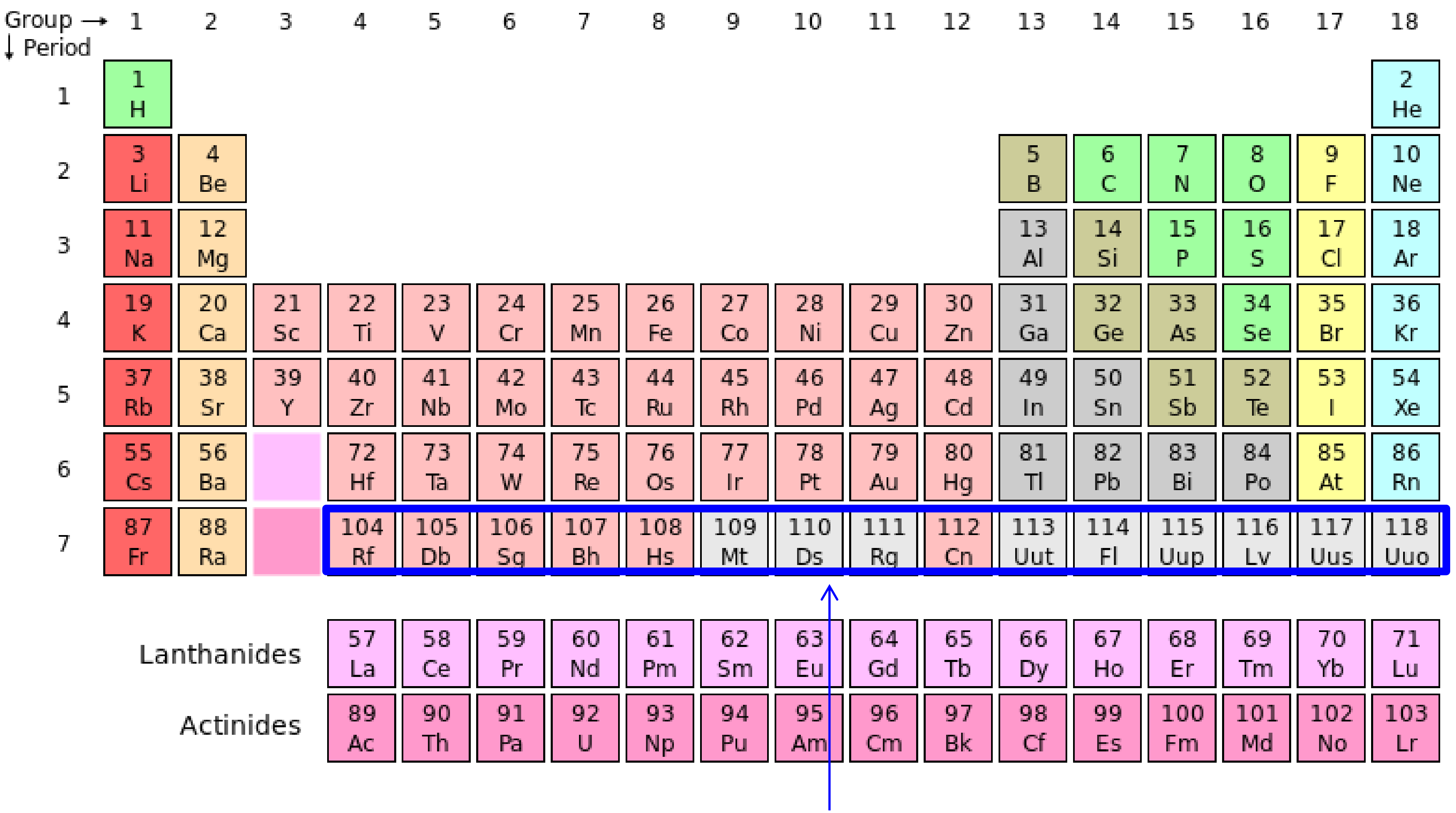
47th element



Au

79th element

Chemistry of superheavy elements



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Lanthanides	57 La	58 Ce	59 Pr	60 Nd	61 Pm	62 Sm	63 Eu	64 Gd	65 Tb	66 Dy	67 Ho	68 Er	69 Tm	70 Yb	71 Lu
Actinides	89 Ac	90 Th	91 Pa	92 U	93 Np	94 Pu	95 Am	96 Cm	97 Bk	98 Cf	99 Es	100 Fm	101 Md	102 No	103 Lr

How do the relativistic effects alter the periodic table for SHE?
What is the color of superheavy elements?

⟶ big open questions

Magic Numbers of Atoms and Nuclei : quantum mechanics of many-Fermion systems

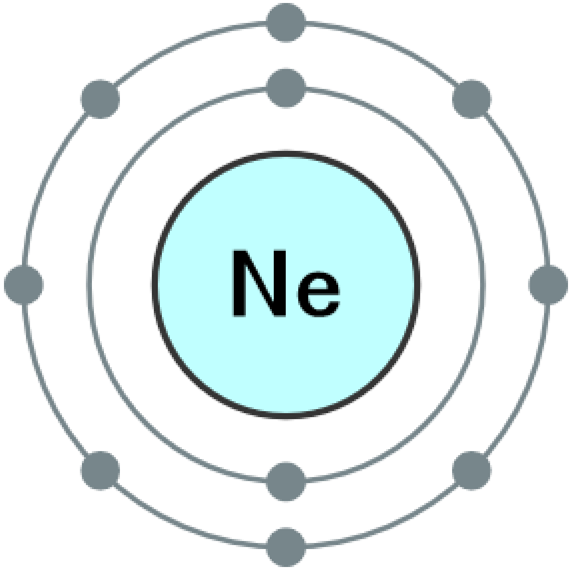
Kouichi Hagino

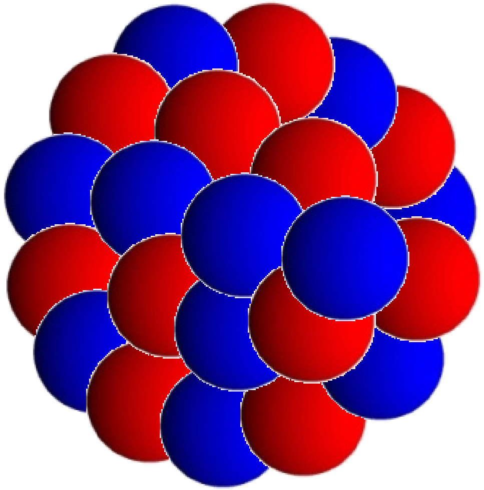
Tohoku University, Sendai, Japan

TOHOKU
UNIVERSITY

1. Identical particles: Fermions and Bosons
2. Simple examples: systems with two identical particles
3. Pauli principle
4. Magic numbers

Introduction





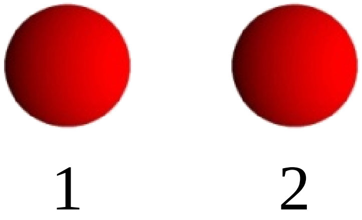
atom = nucleus
+ many electrons

nucleus = many protons
+ many neutrons

Quantum mechanics for those many Fermion systems?

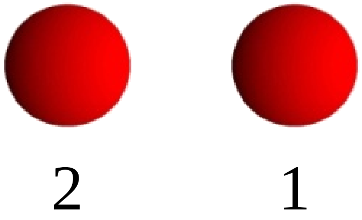
Exchange operator: Fermions and Bosons

a two-particle system



$$H(1,2) = \frac{\boldsymbol{p}_1^2}{2m} + \frac{\boldsymbol{p}_2^2}{2m} + V(\boldsymbol{r}_1, \boldsymbol{r}_2)$$

two particles are identical: particle 1 and 2 cannot be distinguished



$$H(2,1) \equiv \frac{\boldsymbol{p}_2^2}{2m} + \frac{\boldsymbol{p}_1^2}{2m} + V(\boldsymbol{r}_2, \boldsymbol{r}_1) = H(1,2)$$

Exchange operator: Fermions and Bosons

$$[H, P_{12}] = 0$$

where

$$P_{12}\Psi(1,2) = \Psi(2,1)$$

exchange operator

➡ wave functions have to be simultaneous eigen-states of H and P_{12}

Exchange operator: Fermions and Bosons

$$[H, P_{12}] = 0$$

where $$P_{12}\Psi(1,2) = \Psi(2,1)$$ exchange operator

→ wave functions have to be simultaneous eigen-states of H and P_{12}

Eigen-values of P_{12}

$$P_{12}\Psi(1,2) = \Psi(2,1)$$

→ $$(P_{12})^2\Psi(1,2) = P_{12}\Psi(2,1) = \Psi(1,2)$$

→ $$(P_{12})^2 = 1$$

→ $$\boxed{P_{12} = \pm 1}$$

Exchange operator: Fermions and Bosons

$$\boxed{P_{12} = \pm 1}$$

Natural Laws: each particle has a definite value of P_{12}
(independent of e.g., experimental setup and temperature)

- particles with a half-integer spin: P_{12} = -1 (“Fermion”)

electrons, protons, neutrons,…..

$$\Psi^{(-)}(1,2) = \frac{1}{\sqrt{2}}[\Psi(1,2) - \Psi(2,1)]$$

- particles with an integer spin: P_{12} = +1 (“Boson”)

photons, pi mesons,….

$$\Psi^{(+)}(1,2) = \frac{1}{\sqrt{2}}[\Psi(1,2) + \Psi(2,1)]$$

Simple examples: systems with two identical particles

Assume a spin-independent Hamiltonian for a two-particle system:

$$H = \frac{\boldsymbol{p}_1^2}{2m} + \frac{\boldsymbol{p}_2^2}{2m} + V(\boldsymbol{r}_1, \boldsymbol{r}_2)$$

→ separable between the space and the spin

$$\Psi(x_1, x_2) = \Psi_{\mathsf{space}}(\boldsymbol{r}_1, \boldsymbol{r}_2) \cdot \Psi_{\mathsf{spin}}$$

Simple examples: systems with two identical particles

Assume a spin-independent Hamiltonian for a two-particle system:

$$H = \frac{\boldsymbol{p}_1^2}{2m} + \frac{\boldsymbol{p}_2^2}{2m} + V(\boldsymbol{r}_1, \boldsymbol{r}_2)$$

→ separable between the space and the spin

$$\Psi(x_1, x_2) = \Psi_{\text{space}}(\boldsymbol{r}_1, \boldsymbol{r}_2) \cdot \Psi_{\text{spin}}$$

◆spin-zero bosons

no spin → symmetrize the spatial part

$$\begin{aligned} \Psi(\boldsymbol{r}_1, \boldsymbol{r}_2) &= \Psi_{\text{space}}(\boldsymbol{r}_1, \boldsymbol{r}_2) \\ &= \frac{1}{\sqrt{2}}[\phi(\boldsymbol{r}_1, \boldsymbol{r}_2) + \phi(\boldsymbol{r}_2, \boldsymbol{r}_1)] \end{aligned}$$

Simple examples: systems with two identical particles

$$\Psi(x_1,x_2)=\Psi_{\text{space}}(\boldsymbol{r}_1,\boldsymbol{r}_2)\cdot\Psi_{\text{spin}}$$

- spin-1/2 Fermions $\quad \Psi(x_1,x_2)=-\Psi(x_2,x_1)$

Spin part:

$$|S=1\rangle \;=\; |\uparrow\uparrow\rangle,\ \frac{1}{\sqrt{2}}(|\uparrow\downarrow\rangle+|\downarrow\uparrow\rangle),\ |\downarrow\downarrow\rangle$$

symmetric

$$|S=0\rangle \;=\; \frac{1}{\sqrt{2}}(|\uparrow\downarrow\rangle-|\downarrow\uparrow\rangle)$$

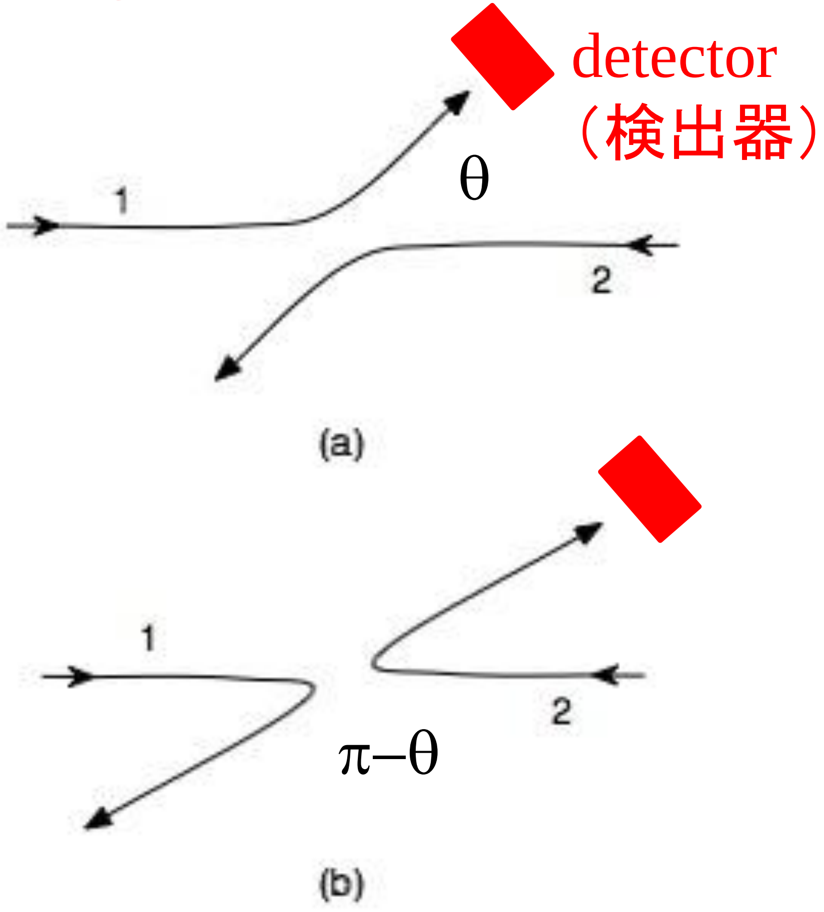
anti-symmetric

→ spatial part: anti-symmetric for *S* = 1
symmetric for *S* = 0

$$\Psi_{S=0}(x_1,x_2) \;=\; \frac{1}{\sqrt{2}}[\phi(\boldsymbol{r}_1,\boldsymbol{r}_2)+\phi(\boldsymbol{r}_2,\boldsymbol{r}_1)]|S=0\rangle$$

$$\Psi_{S=1}(x_1,x_2) \;=\; \frac{1}{\sqrt{2}}[\phi(\boldsymbol{r}_1,\boldsymbol{r}_2)-\phi(\boldsymbol{r}_2,\boldsymbol{r}_1)]|S=1\rangle$$

Scattering of identical particles



these two processes cannot be distinguished

↓

add two amplitudes and then take square

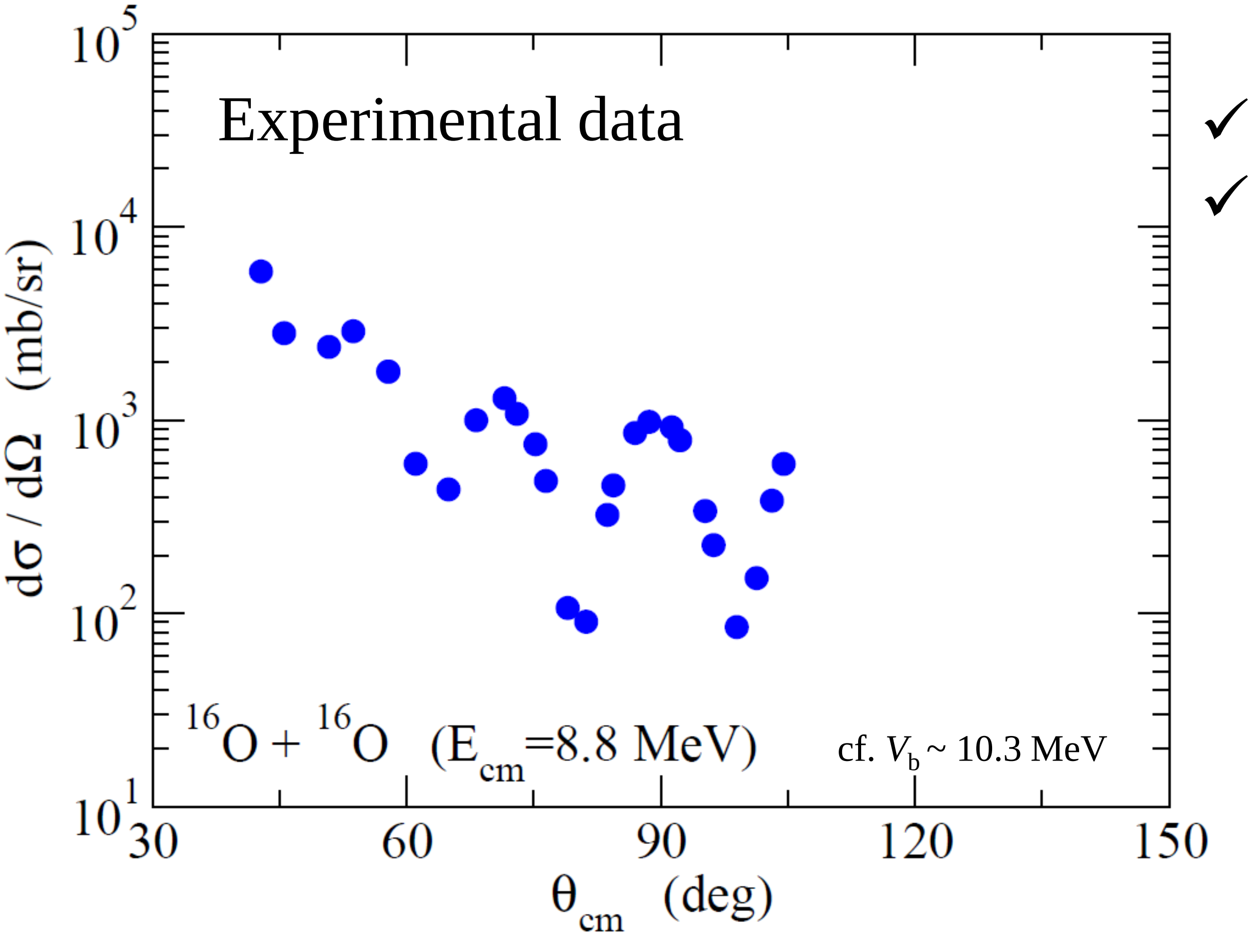
→ interference

干渉

$$\begin{aligned}\frac{d\sigma}{d\Omega} &= |f(\theta) \pm f(\pi-\theta)|^2 \\ &= |f(\theta)|^2 + |f(\pi-\theta)|^2 \pm f^*(\theta)f(\pi-\theta) \pm f(\theta)f^*(\pi-\theta)\end{aligned}$$

+: for spatially symmetric, and – : for spatially anti-symmetric

$^{16}O + ^{16}O$ elastic scattering

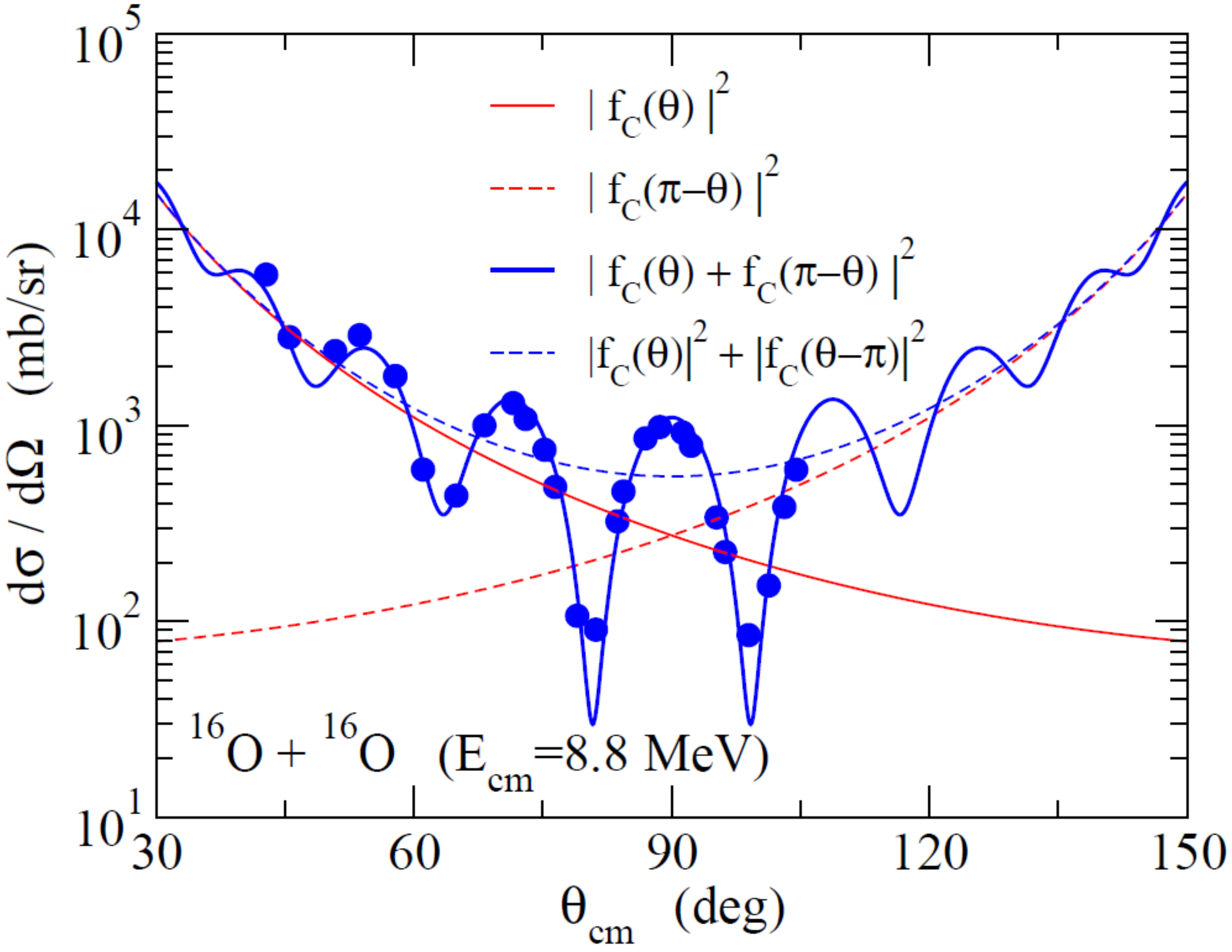


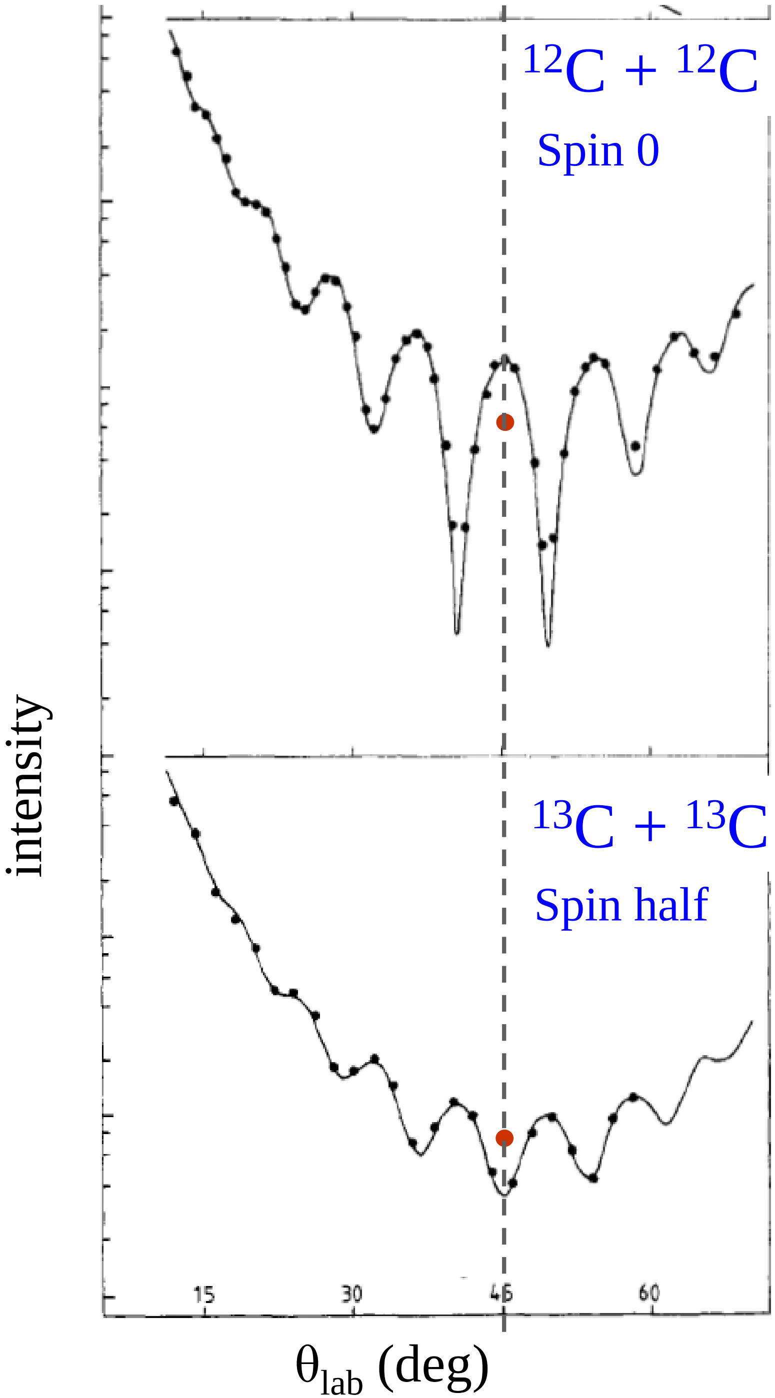
✓Symmetric at 90 deg.
✓clear oscillations

D.A. Bromley et al.,
Phys. Rev. 123('61)878

^{16}O + ^{16}O elastic scattering

^{16}O: spin-zero Boson → $$\frac{d\sigma}{d\Omega} = |f(\theta) + f(\pi - \theta)|^2$$





identical Bosons

→ constructive interference

$$\frac{d\sigma}{d\Omega} = |f(\theta) + f(\pi - \theta)|^2$$

identical Fermions

→ destructive interference

S=1

$$\frac{d\sigma}{d\Omega} = \frac{3}{4}|f(\theta) - f(\pi - \theta)|^2 + \frac{1}{4}|f(\theta) + f(\pi - \theta)|^2$$

S=0

Pauli exclusion principle and Slater determinants

Pauli exclusion principle: two identical Fermion cannot take the same state

Let us assume:

$$H(1,2) = \underbrace{\frac{\boldsymbol{p}_1^2}{2m} + V(\boldsymbol{r}_1)}_{\equiv h_1} + \underbrace{\frac{\boldsymbol{p}_2^2}{2m} + V(\boldsymbol{r}_2)}_{\equiv h_2}$$

(no interaction between 1 and 2)

separation of variables → a product form of wave function

$$\left(\frac{\boldsymbol{p}^2}{2m} + V(\boldsymbol{r})\right)\phi_n(x) = \epsilon_n\phi_n(x); \qquad x \equiv (\boldsymbol{r}, \sigma)$$

$$\longrightarrow \quad \Psi^{(-)}(x_1, x_2) = \frac{1}{\sqrt{2}}[\phi_n(x_1)\phi_{n'}(x_2) - \phi_n(x_2)\phi_{n'}(x_1)]$$

Pauli exclusion principle and Slater determinants

Pauli exclusion principle: two identical Fermion cannot take the same state

$$H(1,2) = \frac{\boldsymbol{p}_1^2}{2m} + V(\boldsymbol{r}_1) + \frac{\boldsymbol{p}_2^2}{2m} + V(\boldsymbol{r}_2)$$

separation of variables → a product form of wave function

$$\left(\frac{\boldsymbol{p}^2}{2m} + V(\boldsymbol{r})\right)\phi_n(x) = \epsilon_n\phi_n(x); \qquad x \equiv (\boldsymbol{r},\sigma)$$

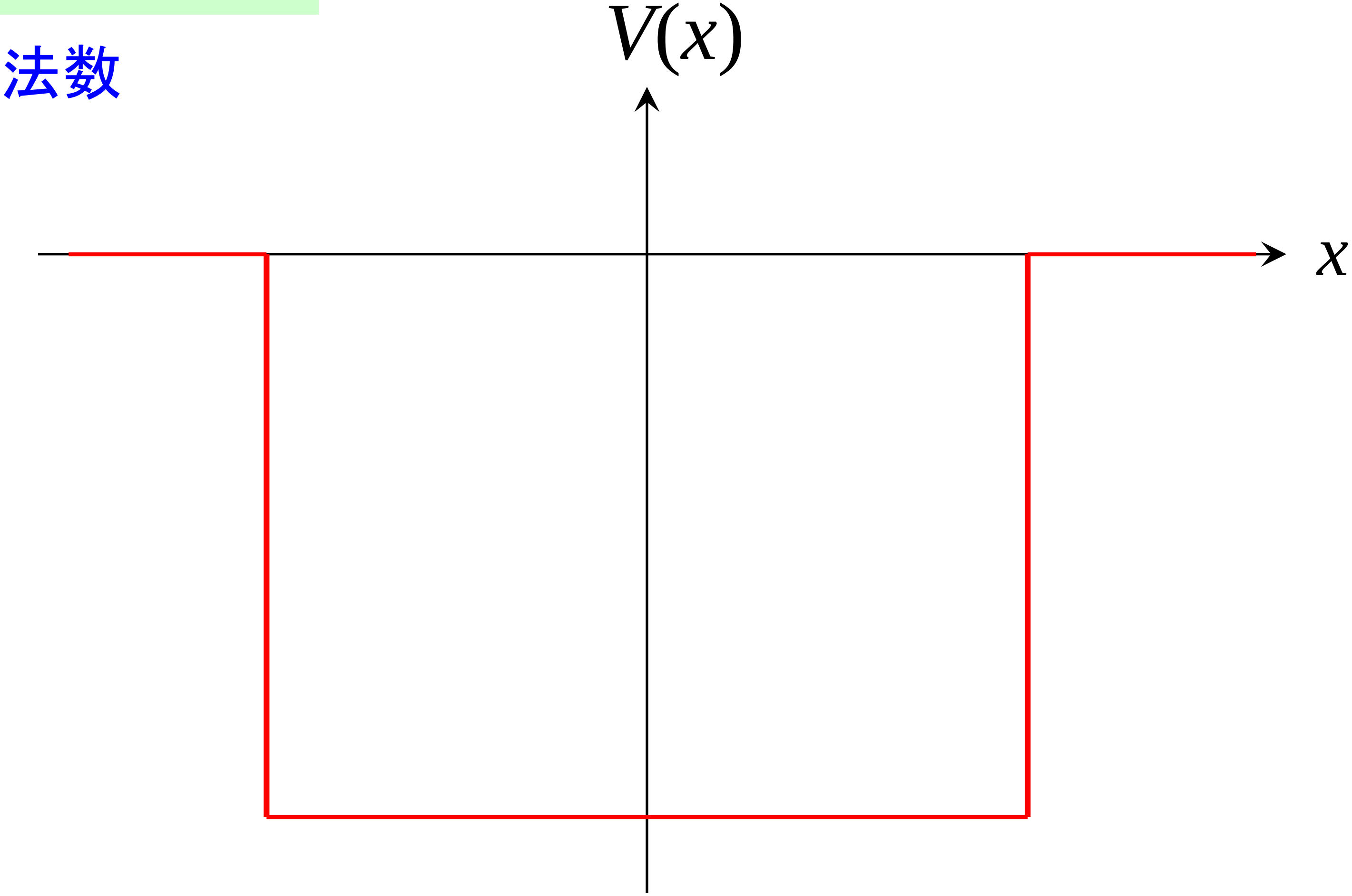
$$\longrightarrow \quad \Psi^{(-)}(x_1,x_2) = \frac{1}{\sqrt{2}}[\phi_n(x_1)\phi_{n'}(x_2) - \phi_n(x_2)\phi_{n'}(x_1)]$$

$$\boxed{\Psi^{(-)}(x_1,x_2) = 0 \quad \text{if} \ \ n = n'}$$

(Pauli principle)

Magic numbers

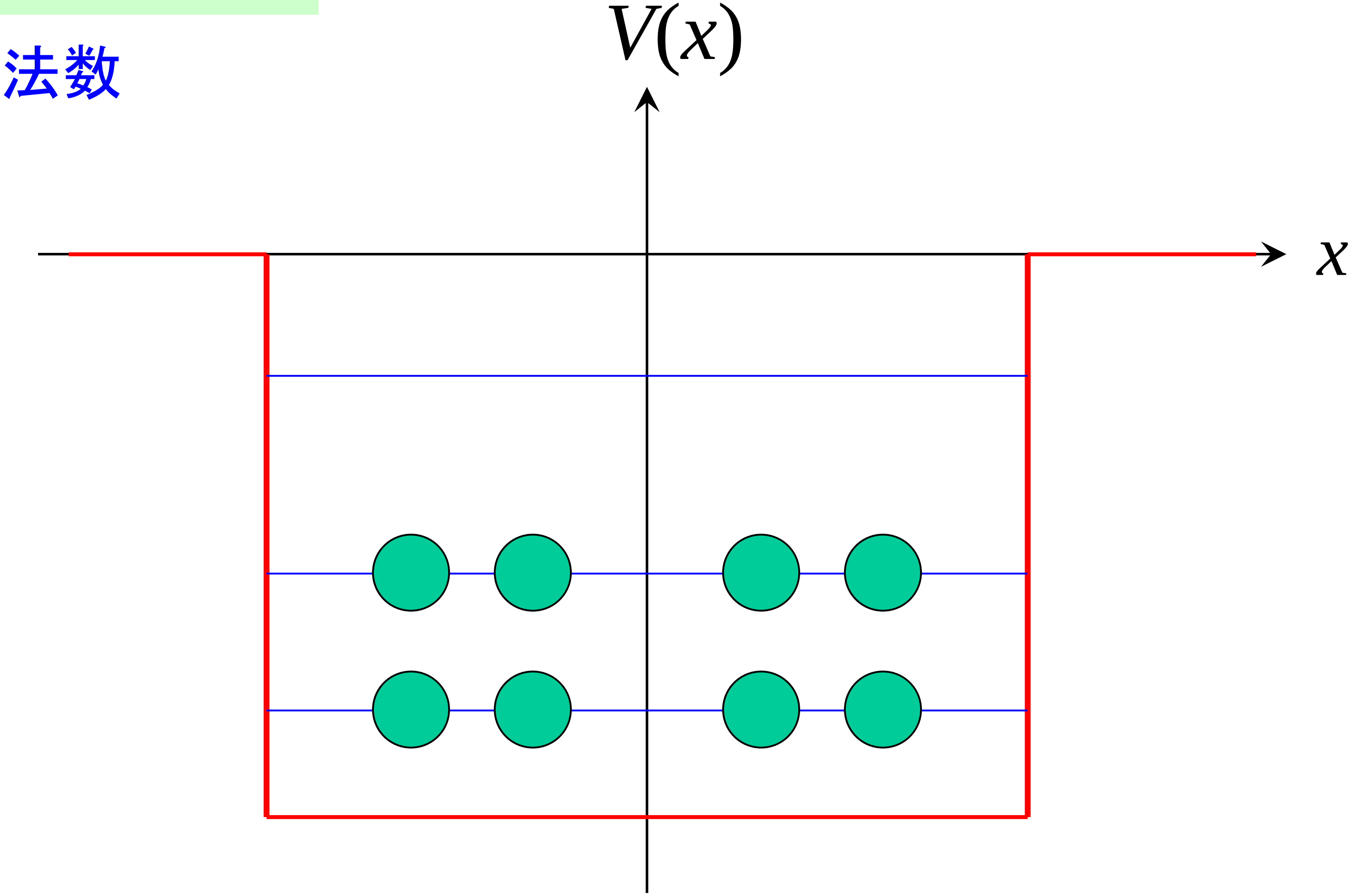
魔法数



$$\left(-\frac{\hbar^2}{2m}\frac{d^2}{dx^2}+V(x)\right)\psi(x)=E\psi(x)$$

Magic numbers

魔法数

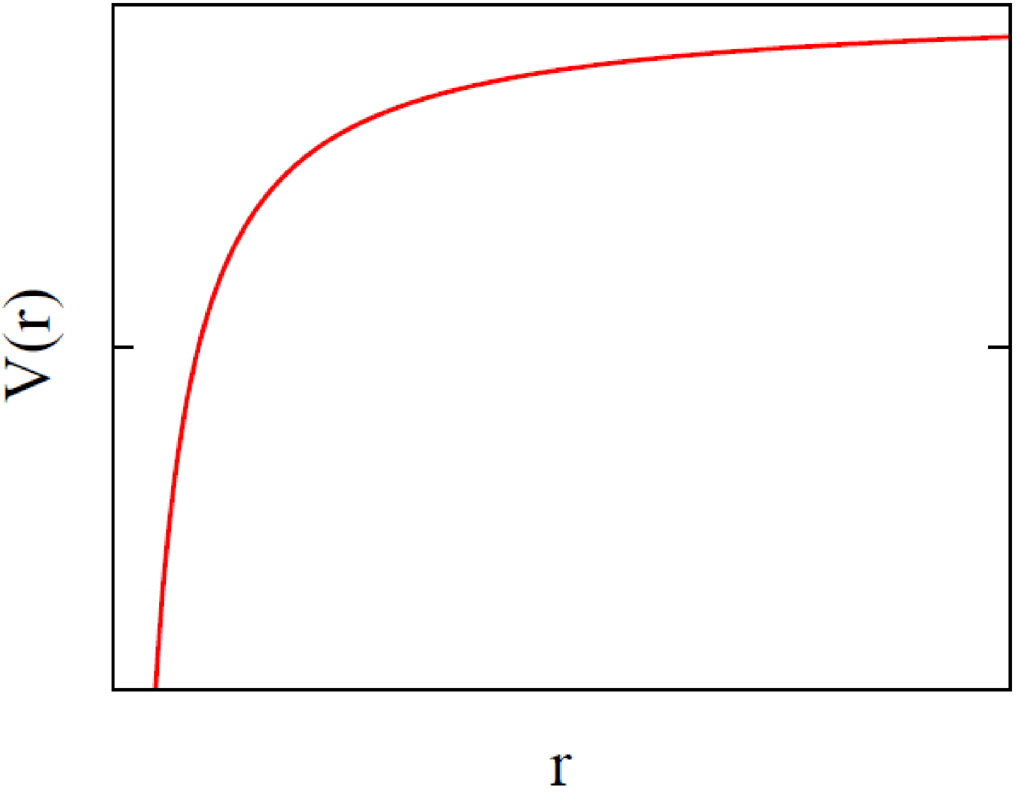


discrete bound states

The lowest state of many-Fermion systems
= put particles from the bottom of the potential well (Pauli principle)

Magic numbers

Hydrogen-like potential: $$V(r) = -\frac{Ze^2}{r}$$



$$E_n = -\frac{(Z\alpha)^2}{2n^2} mc^2$$

$$\alpha = \frac{e^2}{\hbar c} \sim \frac{1}{137}$$

$$n = n_r + l + 1$$

Magic numbers

Hydrogen-like potential: $$V(r) = -\frac{Ze^2}{r}$$

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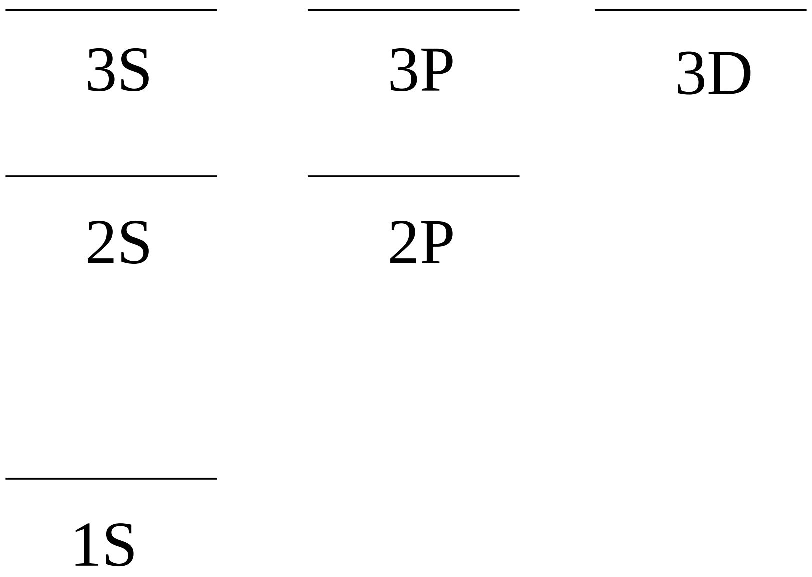

Magic numbers

Hydrogen-like potential:

$$V(r) = -\frac{Ze^2}{r}$$

degeneracy = 2 * (2 *l* +1)

(spin x l_z)

$$E_n = -\frac{(Z\alpha)^2}{2n^2} mc^2$$

3S [2] 3P [6] 3D [10]

2S [2] 2P [6]

1S [2]

$$\alpha = \frac{e^2}{\hbar c} \sim \frac{1}{137}$$

$$n = n_r + l + 1$$

Magic numbers

Hydrogen-like potential: $V(r) = -\frac{Ze^2}{r}$

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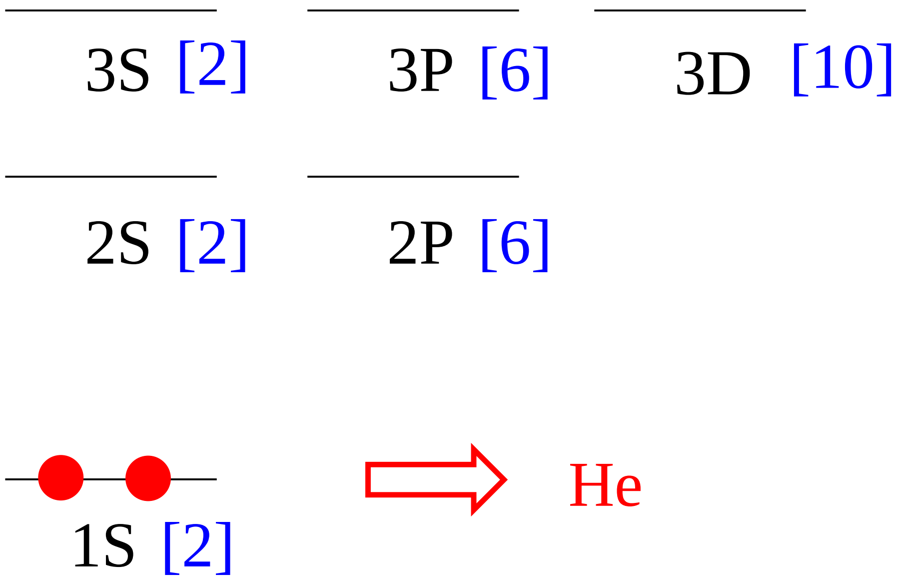


3S [2]

3P [6]

3D [10]

2S [2]

2P [6]

1S [2]

He

Magic numbers

Hydrogen-like potential:

$$V(r) = -\frac{Ze^2}{r}$$

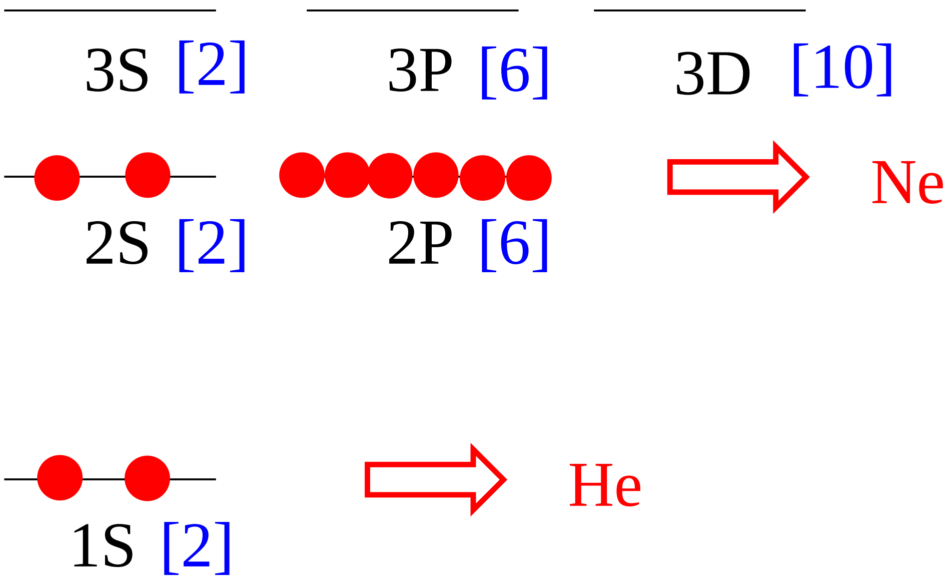
degeneracy = 2 * (2 *l* +1)

(spin x l_z)

$$E_n = -\frac{(Z\alpha)^2}{2n^2}mc^2$$

$$\alpha = \frac{e^2}{\hbar c} \sim \frac{1}{137}$$

$$n = n_r + l + 1$$



3S [2] 3P [6] 3D [10]

2S [2] 2P [6] ⇒ Ne

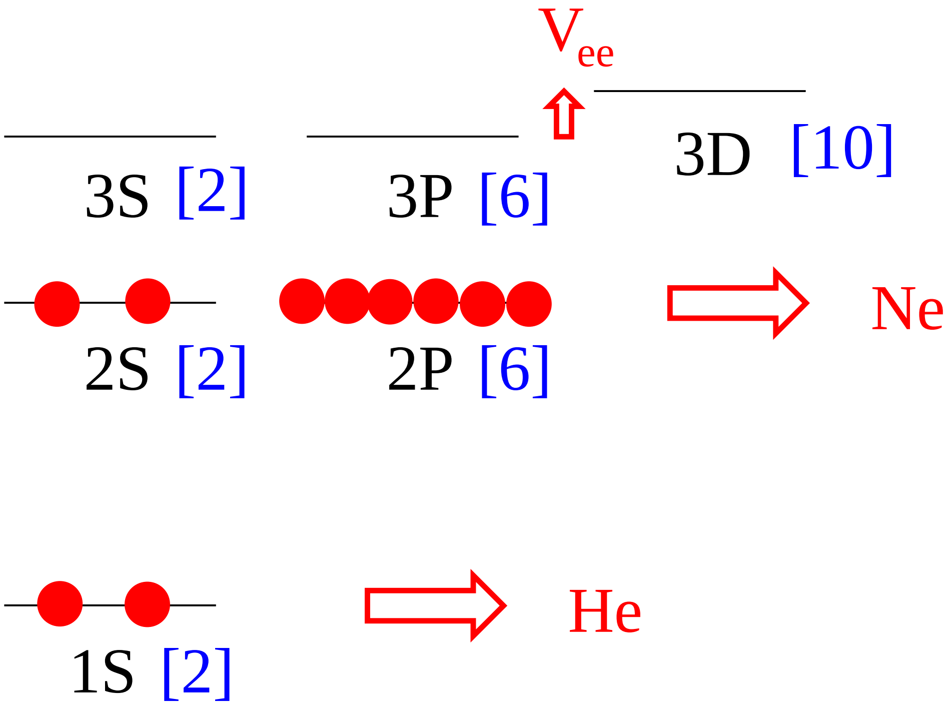
1S [2] ⇒ He

Magic numbers

Hydrogen-like potential:

$$V(r) = -\frac{Ze^2}{r}$$

degeneracy = 2 * (2 *l* +1)



$$E_n = -\frac{(Z\alpha)^2}{2n^2} mc^2$$

$$\alpha = \frac{e^2}{\hbar c} \sim \frac{1}{137}$$

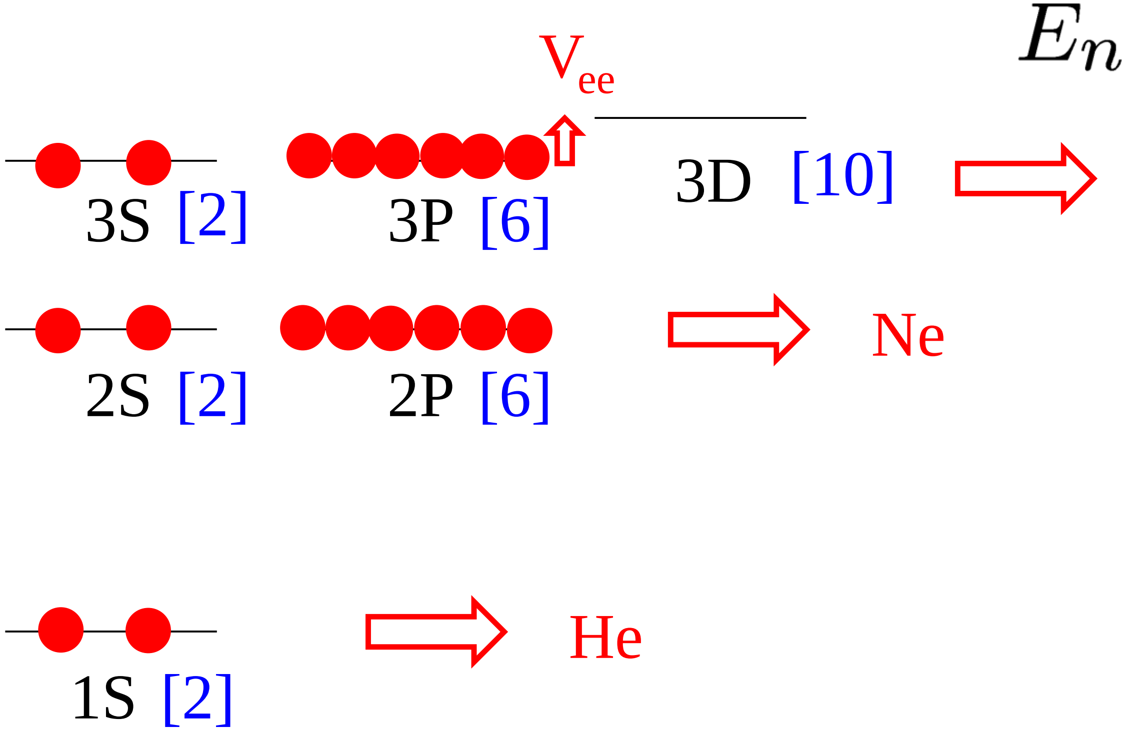
$$n = n_r + l + 1$$

Magic numbers

Hydrogen-like potential: $$V(r) = -\frac{Ze^2}{r}$$

degeneracy = 2 * (2 *l* +1)

$$E_n = -\frac{(Z\alpha)^2}{2n^2} mc^2$$



V_{ee}

3S [2] 3P [6] 3D [10] ⇨ Ar

2S [2] 2P [6] ⇨ Ne

1S [2] ⇨ He

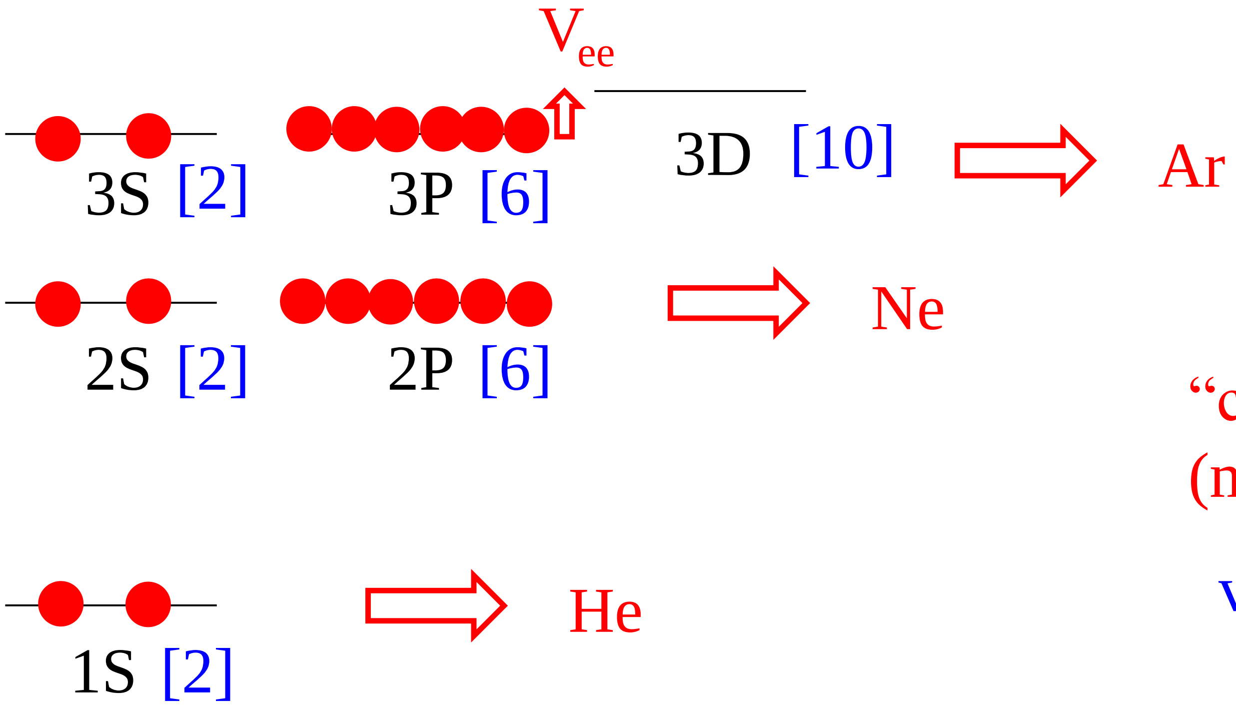
$$\alpha = \frac{e^2}{\hbar c} \sim \frac{1}{137}$$

$$n = n_r + l + 1$$

Magic numbers

Hydrogen-like potential: $$V(r) = -\frac{Ze^2}{r}$$

degeneracy = 2 * (2 *l* +1)

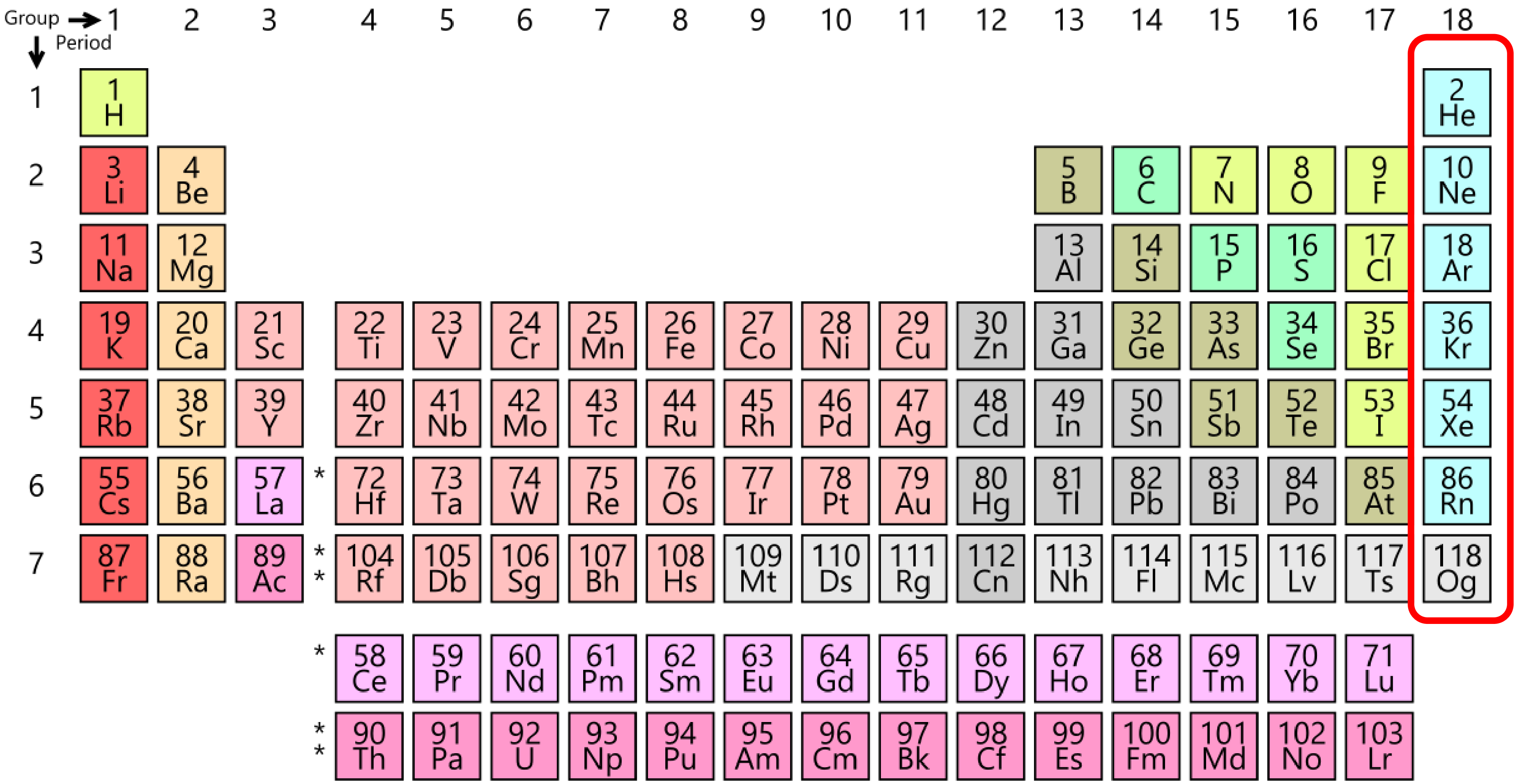


“closed shell”
(magic numbers)

very stable

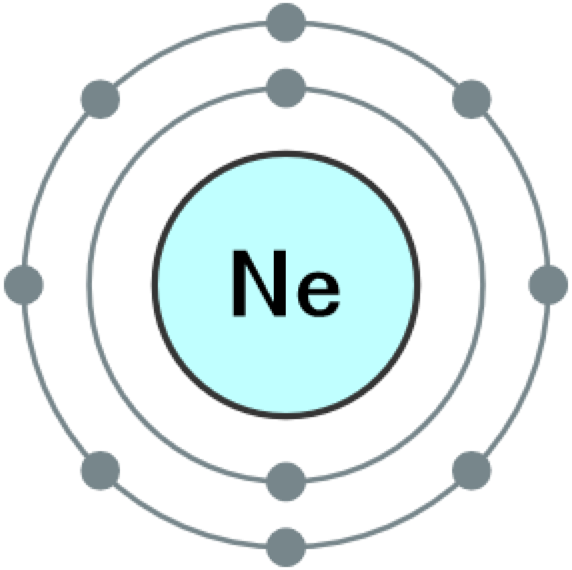
Periodic Table of elements

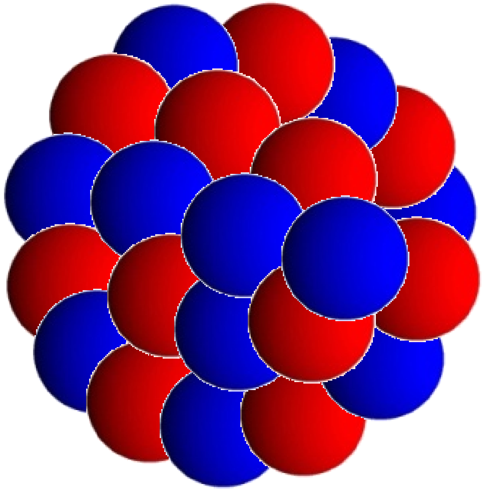
noble gas



Group → Period ↓	1	2	3		4	5	6	7	8	9	10	11	12	13	14	15	16	17	18
1	1 H																		2 He
2	3 Li	4 Be												5 B	6 C	7 N	8 O	9 F	10 Ne
3	11 Na	12 Mg												13 Al	14 Si	15 P	16 S	17 Cl	18 Ar
4	19 K	20 Ca	21 Sc		22 Ti	23 V	24 Cr	25 Mn	26 Fe	27 Co	28 Ni	29 Cu	30 Zn	31 Ga	32 Ge	33 As	34 Se	35 Br	36 Kr
5	37 Rb	38 Sr	39 Y		40 Zr	41 Nb	42 Mo	43 Tc	44 Ru	45 Rh	46 Pd	47 Ag	48 Cd	49 In	50 Sn	51 Sb	52 Te	53 I	54 Xe
6	55 Cs	56 Ba	57 La	*	72 Hf	73 Ta	74 W	75 Re	76 Os	77 Ir	78 Pt	79 Au	80 Hg	81 Tl	82 Pb	83 Bi	84 Po	85 At	86 Rn
7	87 Fr	88 Ra	89 Ac	**	104 Rf	105 Db	106 Sg	107 Bh	108 Hs	109 Mt	110 Ds	111 Rg	112 Cn	113 Nh	114 Fl	115 Mc	116 Lv	117 Ts	118 Og
				*	58 Ce	59 Pr	60 Nd	61 Pm	62 Sm	63 Eu	64 Gd	65 Tb	66 Dy	67 Ho	68 Er	69 Tm	70 Yb	71 Lu	
				**	90 Th	91 Pa	92 U	93 Np	94 Pu	95 Am	96 Cm	97 Bk	98 Cf	99 Es	100 Fm	101 Md	102 No	103 Lr	

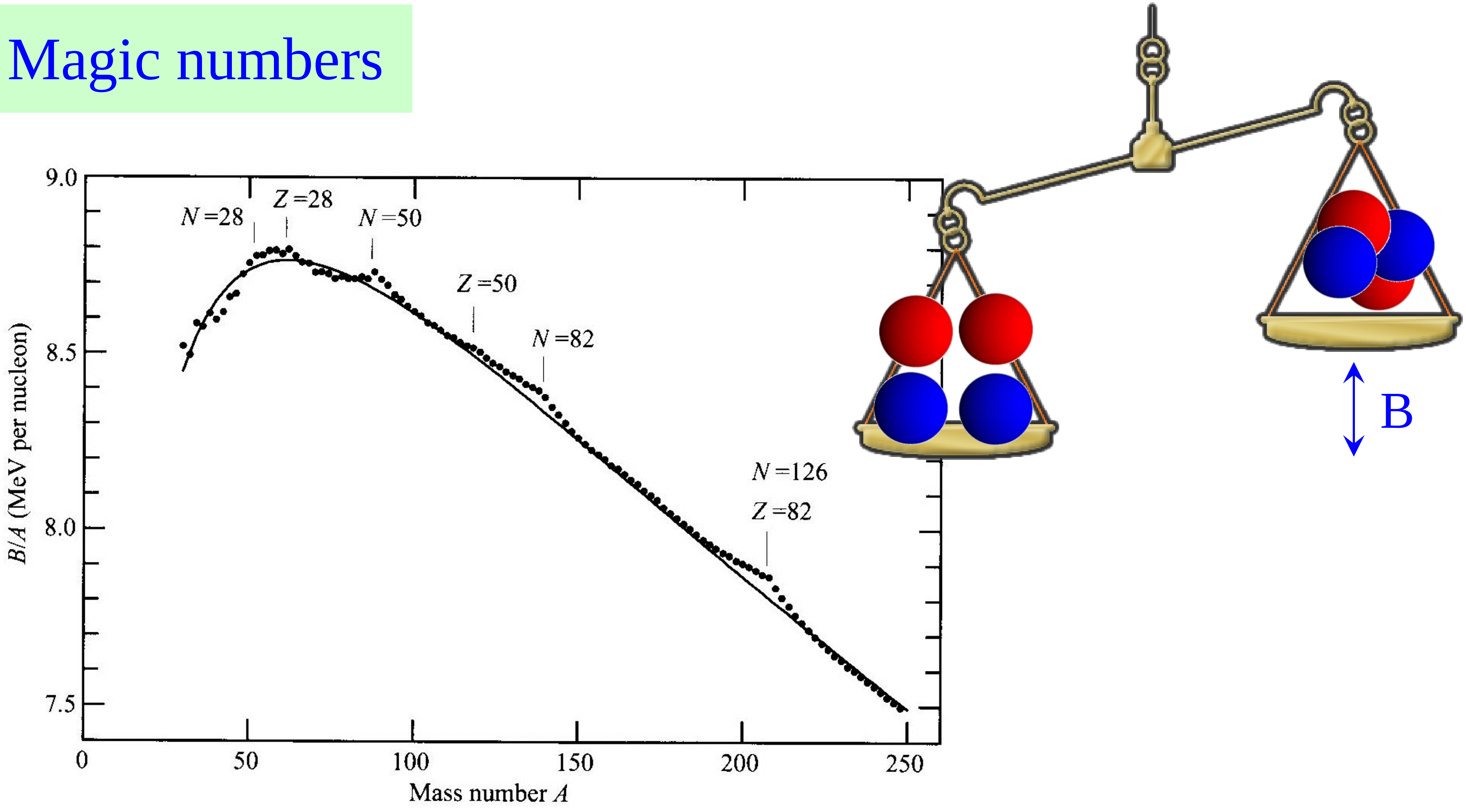
Magic numbers





similar magic numbers also in atomic nuclei

Magic numbers

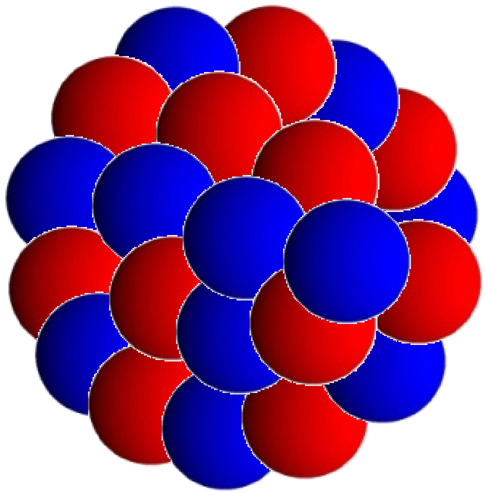


Extra binding for N or Z = 2, 8, 20, 28, 50, 82, 126 (magic numbers)

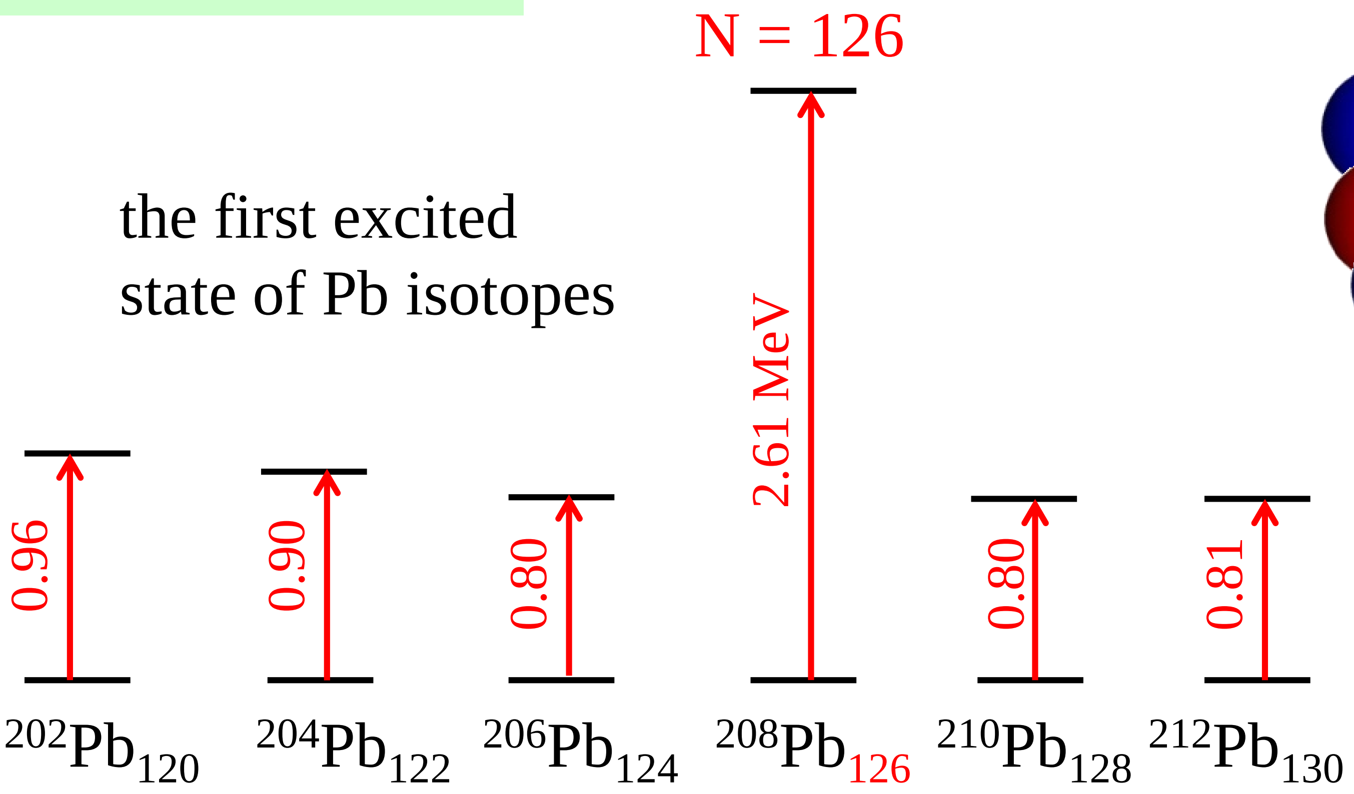
⇨ Very stable

$^{4}_{2}He_{2}$, $^{16}_{8}O_{8}$, $^{40}_{20}Ca_{20}$, $^{48}_{20}Ca_{28}$, $^{208}_{82}Pb_{126}$

Magic numbers



the first excited
state of Pb isotopes



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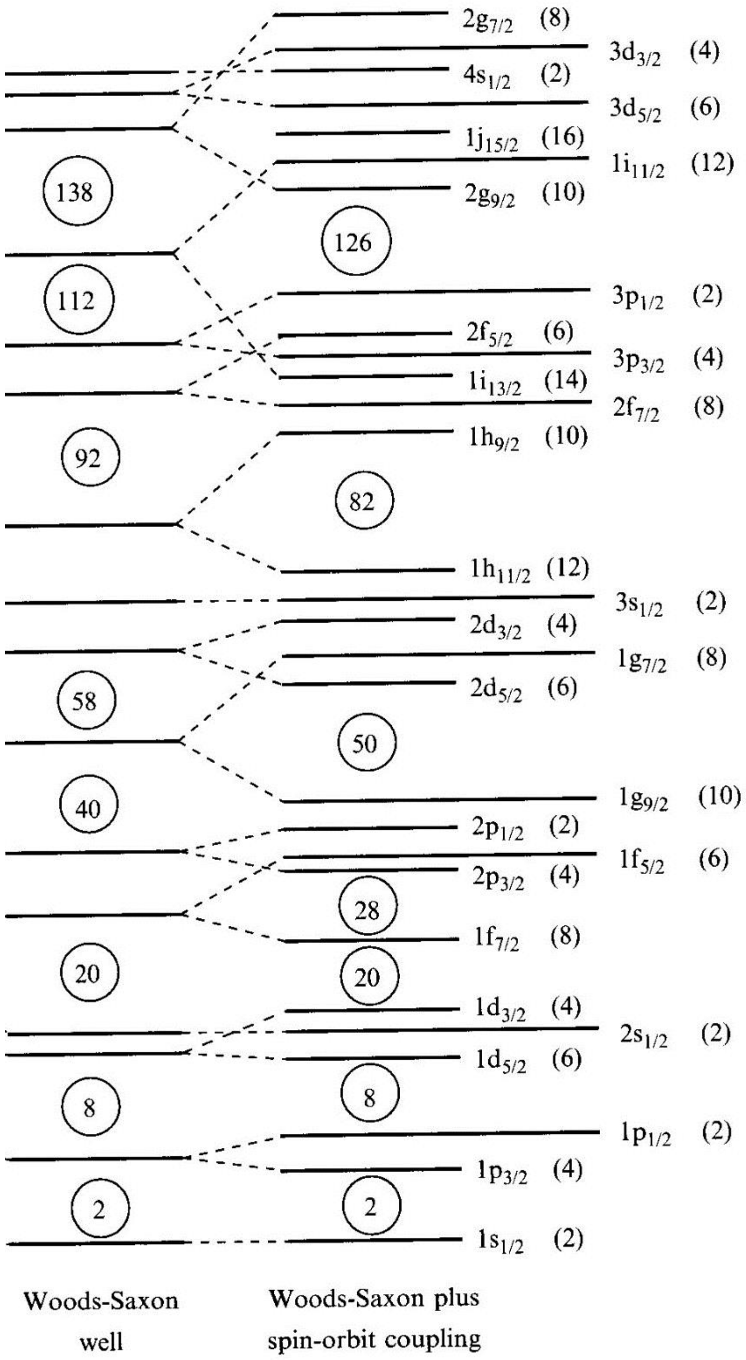
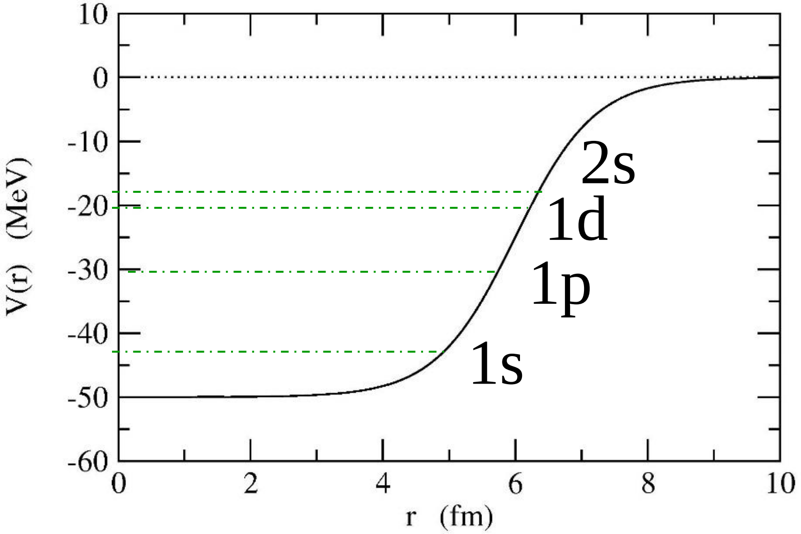


$$V(r) = \frac{-V_0}{1 + \exp((r - R_0)/a)}$$



+ spin-orbit potential

$$V_{ls}(r)\boldsymbol{l} \cdot \boldsymbol{s}$$

Lucky accident for the origin of lie

Atomic magic numbers

electron #: 2, 10, 18, 36, 54, 86

元素の周期表

	1A	2A	3A	4A	5A	6A	7A	8			1B	2B	3B	4B	5B	6B	7B	0
1	H																	He
2	Li	Be											B	C	N	O	F	Ne
3	Na	Mg											Al	Si	P	S	Cl	Ar
4	K	Ca	Sc	Ti	V	Cr	Mn	Fe	Co	Ni	Cu	Zn	Ga	Ge	As	Se	Br	Kr
5	Rb	Sr	Y	Zr	Nb	Mo	Tc	Ru	Rh	Pd	Ag	Cd	In	Sn	Sb	Te	I	Xe
6	Cs	Ba	L	Hf	Ta	W	Re	Os	Ir	Pt	Au	Hg	Tl	Pb	Bi	Po	At	Rn
7	Fr	Ra	A															
		L	La	Ce	Pr	Nd	Pm	Sm	Eu	Gd	Tb	Dy	Ho	Er	Tm	Yb	Lu	
		A	Ac	Th	Pa	U	Np	Pu	Am	Cm	Bk	Cf	Es	Fm	Md	No	Lr	

🟧 典型金属元素
🟩 半金属元素
🟦 非金属元素
🟨 遷移金属元素
🟥 希ガス

inert gas：He, Ne, Ar, Kr, Xe, Rn

Nuclear magic numbers

proton # or neutron #
2, 8, 20, 28, 50, 82, 126

→ e.g., $^{16}_{8}O_{8}$ (double magic)

→ many oxygen nuclei: produced during nucleosynthesis

→ oxygen: chemically active

→ several complex chemical reactions, leading to the birth of life

参考：望月優子 ビデオ「元素誕生の謎にせまる」 http://rarfaxp.riken.go.jp/~motizuki/contents/genso.html

Summary

Everything is made from atoms.

Nuclear Physics is important for many things.

Nuclei have very rich nature.

Nuclear Physics is interesting!

一清、二白、三紅、四緑、五黄

clear lectures（清）?

謝謝 !