Heavy-ion fusion reactions: quantum tunneling with many degrees of freedom and superheavy elements

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မင်္ဂလာနံနက်ခင်းပါ
good morning

Heavy-ion fusion reactions: quantum tunneling with many degrees of freedom and superheavy elements

Kouichi Hagino
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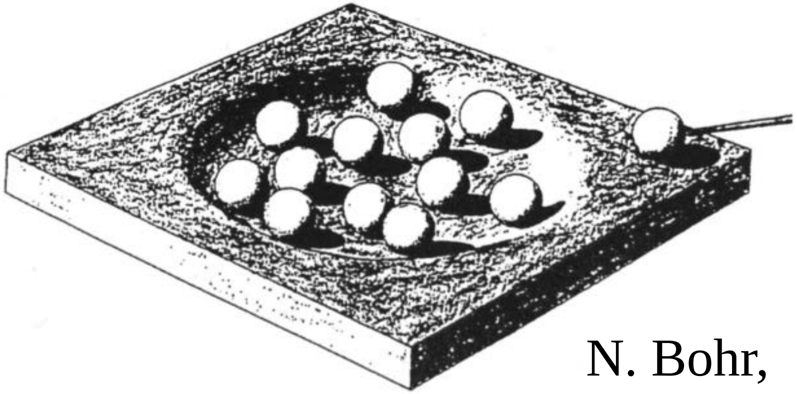
1. H.I. fusion reactions: why are they interesting?
2. Coupled-channels approach
3. Future perspectives: superheavy elements

Recent review article:
K. Hagino and N. Takigawa, Prog. Theo. Phys.128 ('12)1061.

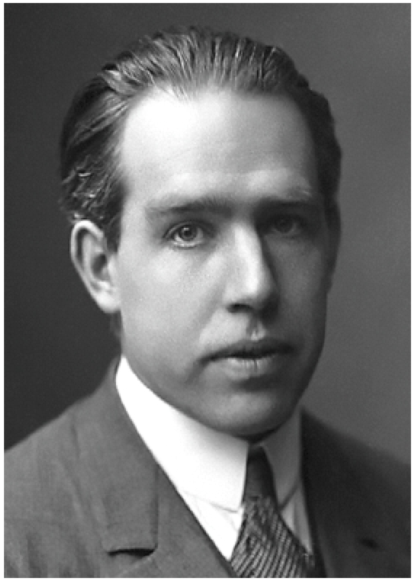
Fusion reactions: compound nucleus formation

Niels Bohr (1936)

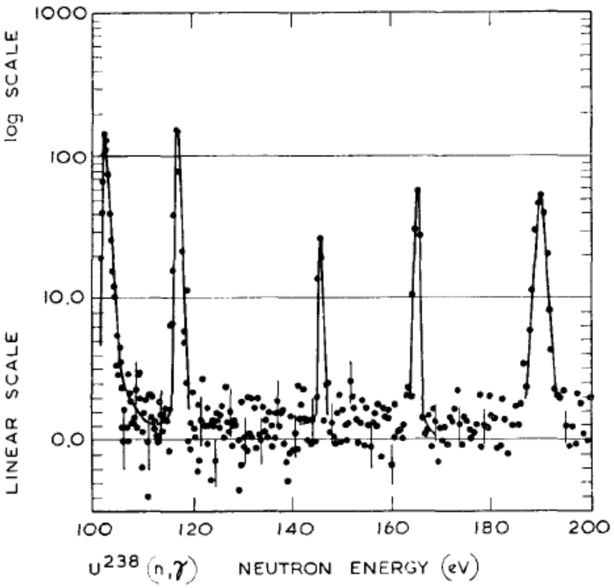
Neutron capture of nuclei → compound nucleus



N. Bohr,
Nature 137 (‘36) 351



Wikipedia



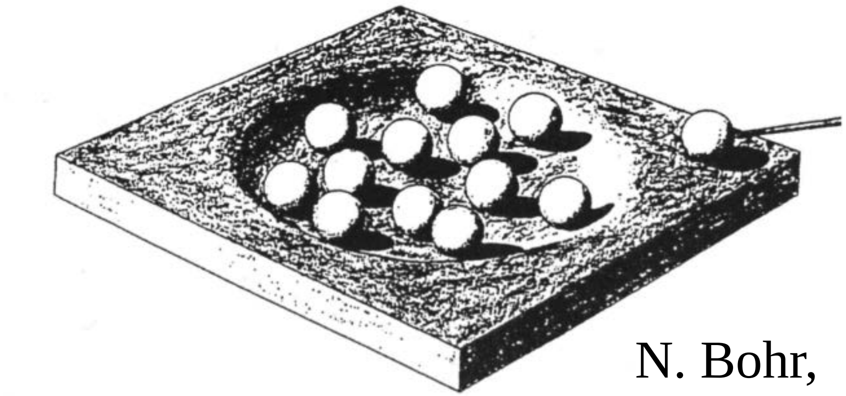
cf. Experiment of Enrico Fermi (1935)
many very narrow (=long life-time)
resonances (width ~ eV)

M. Asghar et al., Nucl. Phys. 85 (‘66) 305

Fusion reactions: compound nucleus formation

Niels Bohr (1936)

Neutron capture of nuclei → compound nucleus



N. Bohr,
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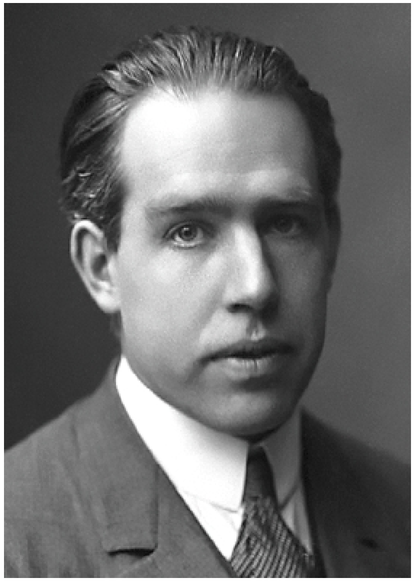
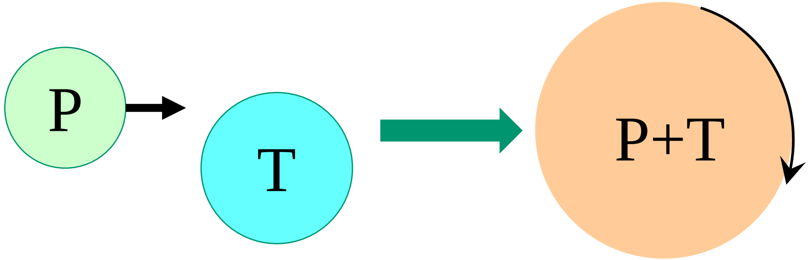


Wikipedia

forming a compound nucleus with heavy-ion reactions = H.I. fusion



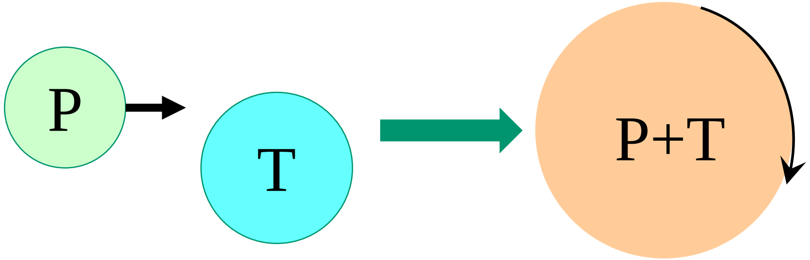
P

T

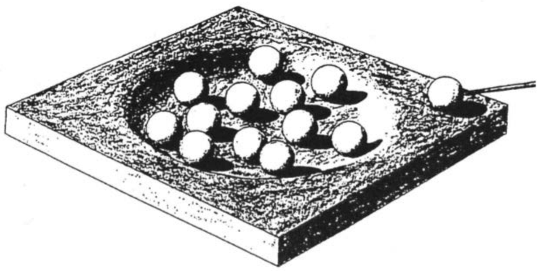
P+T

compound nucleus

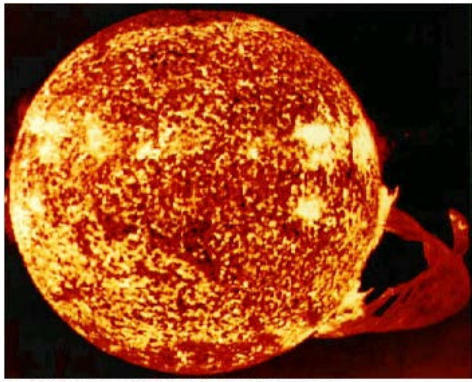
Fusion reactions: compound nucleus formation



compound
nucleus

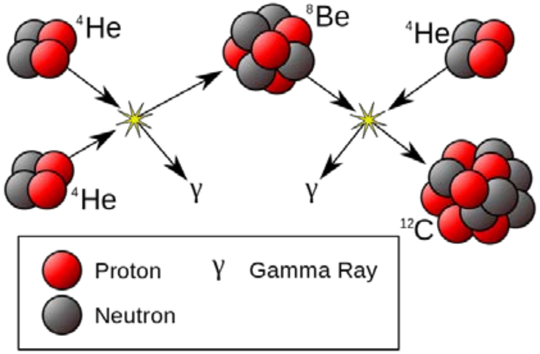


cf. Bohr '36

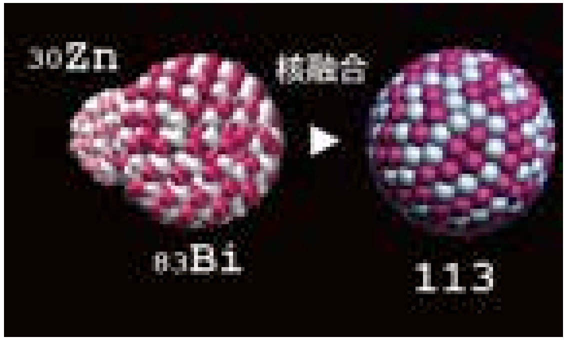


NASA, Skylab space station December 19, 1973, solar flare reaching 588 000 km off solar surface

energy production
in stars (Bethe '39)



nucleosynthesis

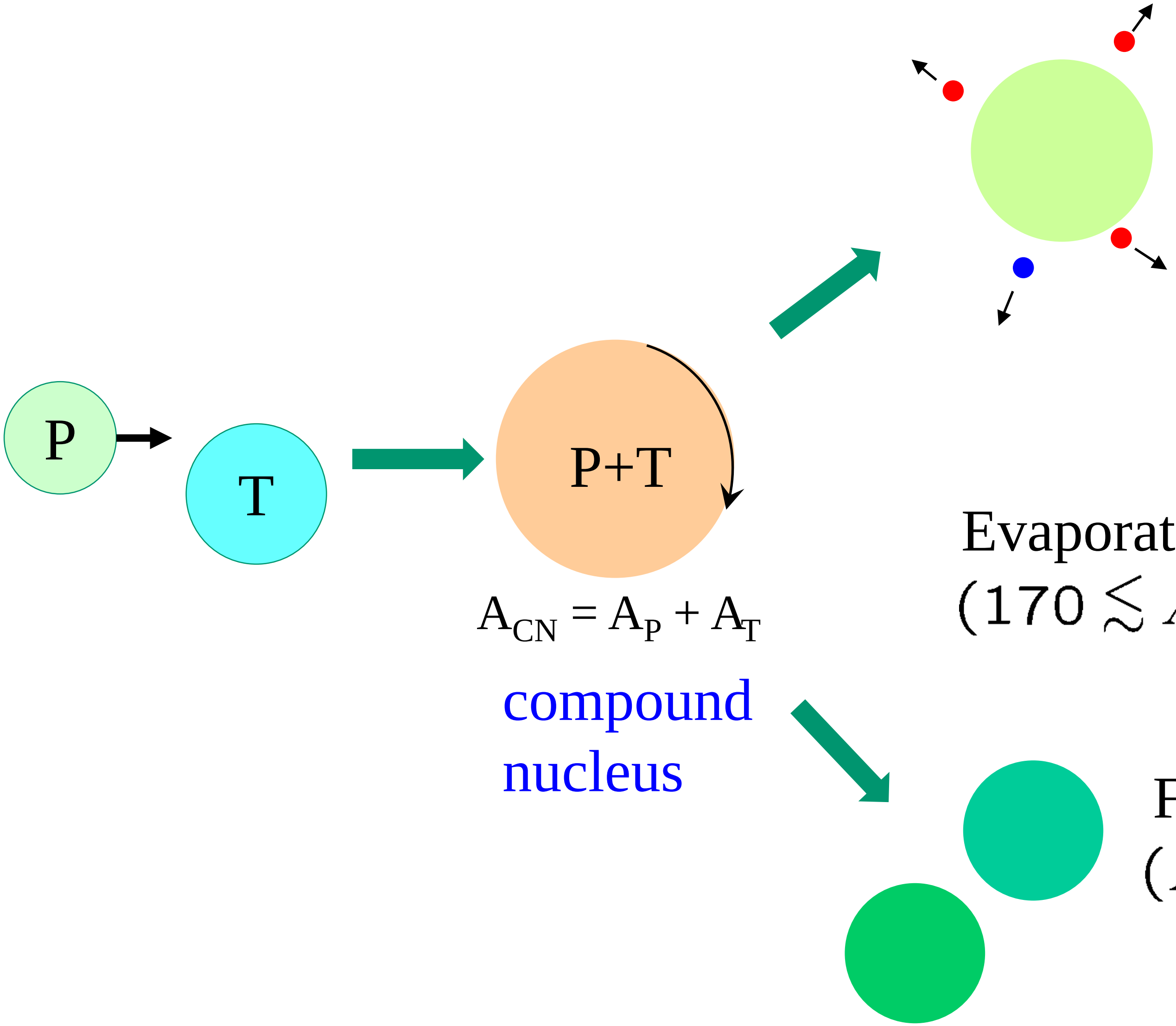


superheavy elements

Fusion and fission: large amplitude motions of quantum many-body systems with strong interaction

← microscopic understanding: an ultimate goal of nuclear physics

Fusion reactions: compound nucleus formation



$A_{CN} = A_P + A_T$

compound nucleus

Evaporation
$(A_{CN} \lesssim 170)$

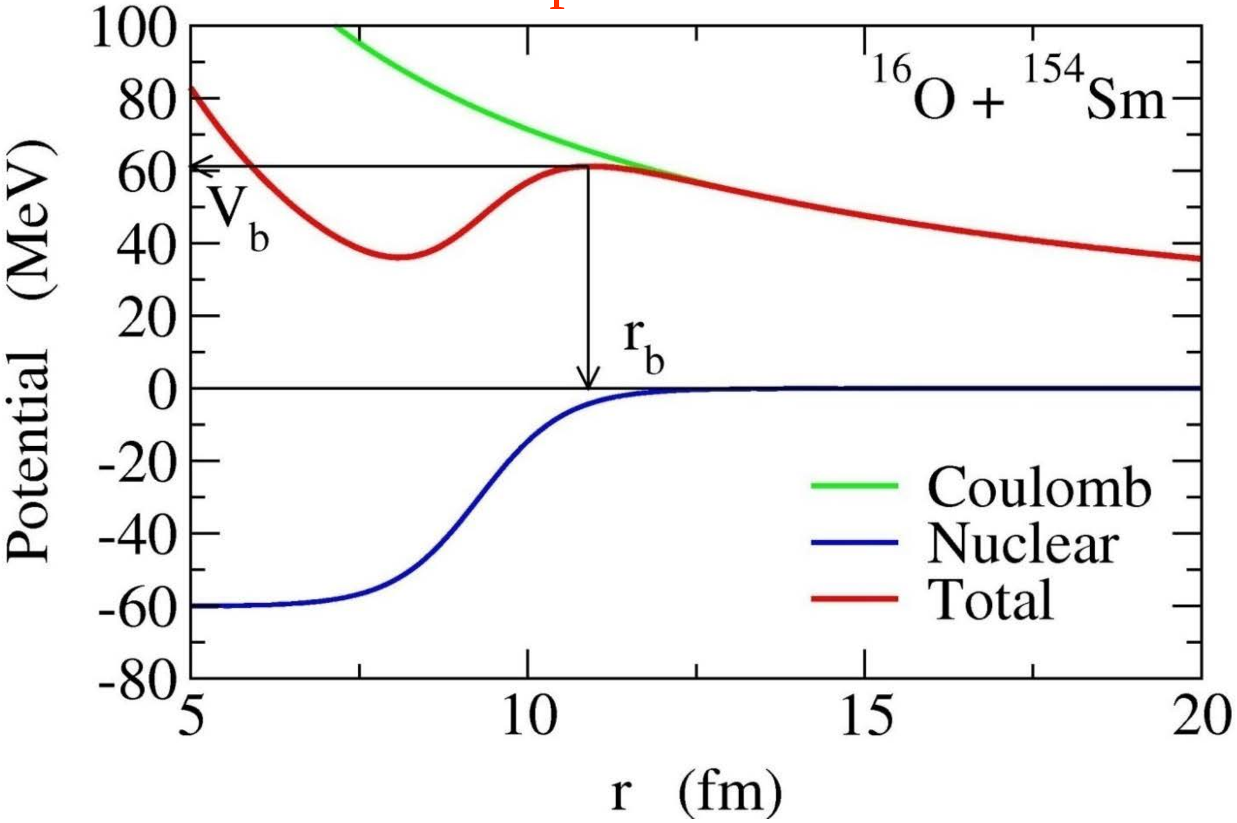
n,p,α emissions
γ decay

Evaporation + Fission
$(170 \lesssim A_{CN} \lesssim 220)$

Fission
$(A_{CN} \gtrsim 220)$

cf. superheavy elements

Inter-nucleus potential



Two interactions:

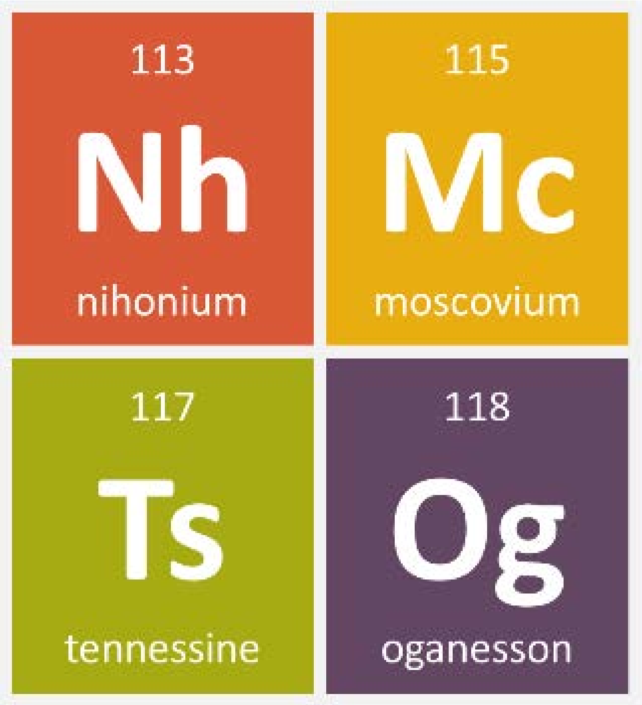
1. Coulomb force
   long range repulsion
2. Nuclear force
   short range attraction

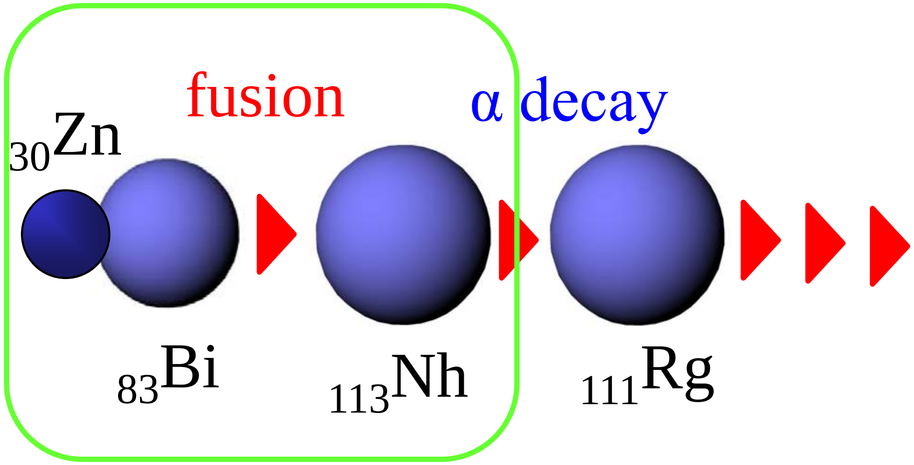
<ins>potential barrier</ins>
due to a cancellation
between the two
(Coulomb barrier)

- Above-barrier energies
- Sub-barrier energies
  (energies around the Coulomb barrier)
- Deep sub-barrier energies

Why sub-barrier fusion?

two obvious reasons:





superheavy elements

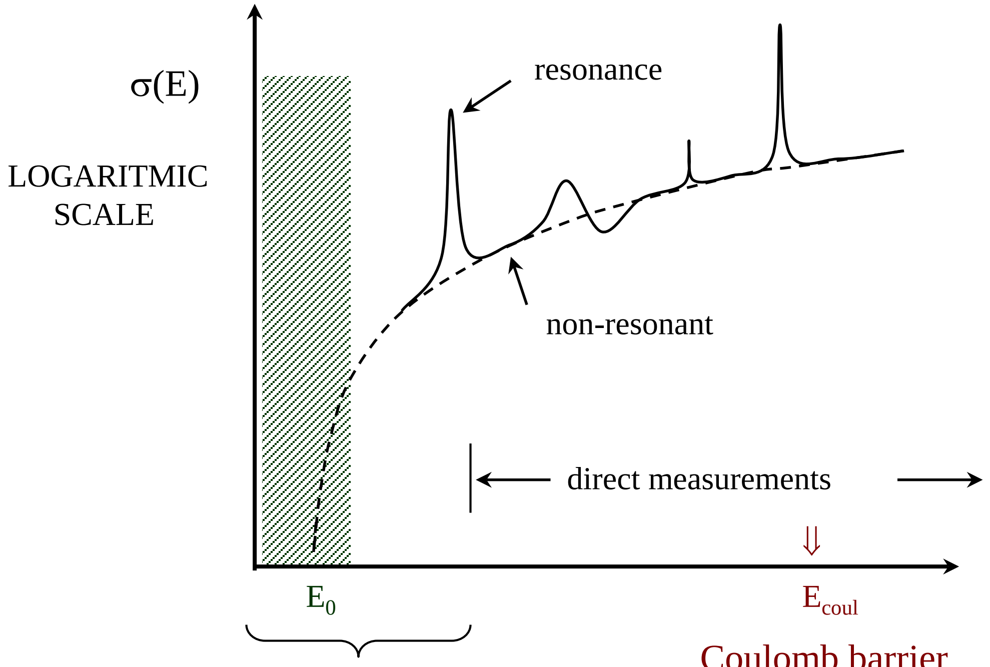
cf. ^{209}Bi (^{70}Zn,n) ^{278}Nh

$V_B \sim 260$ MeV

$E_{cm}^{(exp)} \sim 262$ MeV

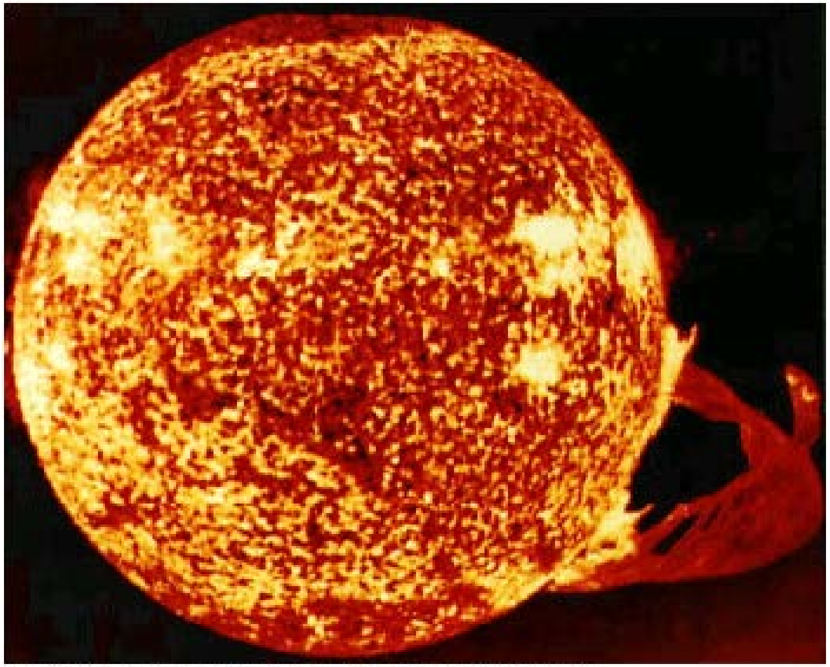
Why sub-barrier fusion?

two obvious reasons:



extrapolation
needed !

Coulomb barrier



NASA, Skylab space station December 19. 1973, solar flare reaching 588 000 km off solar surface

figure: M. Aliotta

nuclear astrophysics
(nuclear fusion in stars)

cf. extrapolation of data

Why sub-barrier fusion?

two obvious reasons:

- ✓superheavy elements
- ✓ nuclear astrophysics

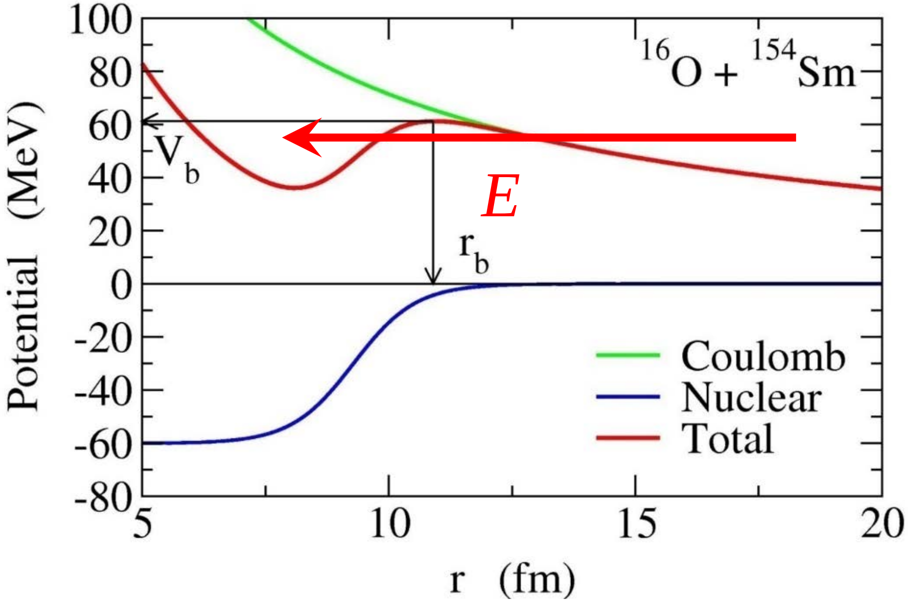
other reasons:

- ✓reaction dynamics

strong interplay between reaction and structure

cf. high *E* reactions: much simpler reaction mechanisms

- ✓many-particle tunneling



Why sub-barrier fusion?

two obvious reasons:

✓superheavy elements ✓ nuclear astrophysics

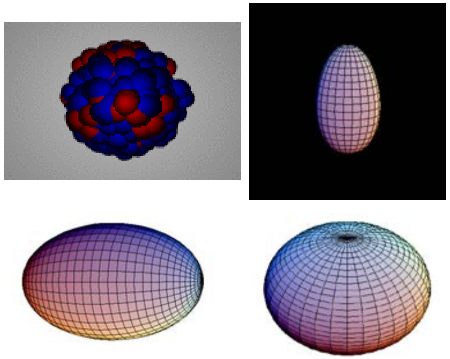
other reasons:

✓reaction dynamics

strong interplay between reaction and structure

cf. high *E* reactions: much simpler reaction mechanisms

✓many-particle tunneling



- many types of intrinsic degrees of freedom
  (several types of collective vibrations,
  deformation with several multipolarities)
- energy dependence of tunneling probability
  cf. alpha decay: fixed energy

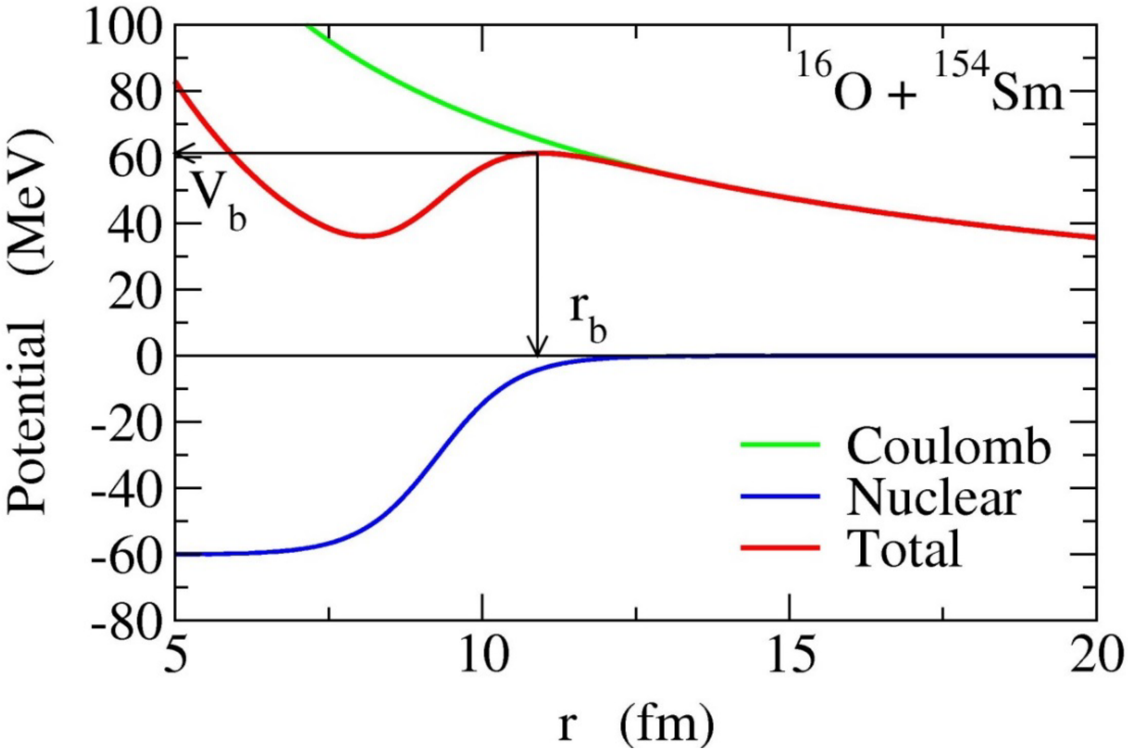
H.I. fusion reaction = an ideal playground to study quantum tunneling with many degrees of freedom

The simplest approach to fusion: potential model

Potential model: *V*(*r*) + absorption

$$\sigma_{\mathrm{fus}}(E) = \frac{\pi}{k^2} \sum_l (2l+1) P_l(E)$$

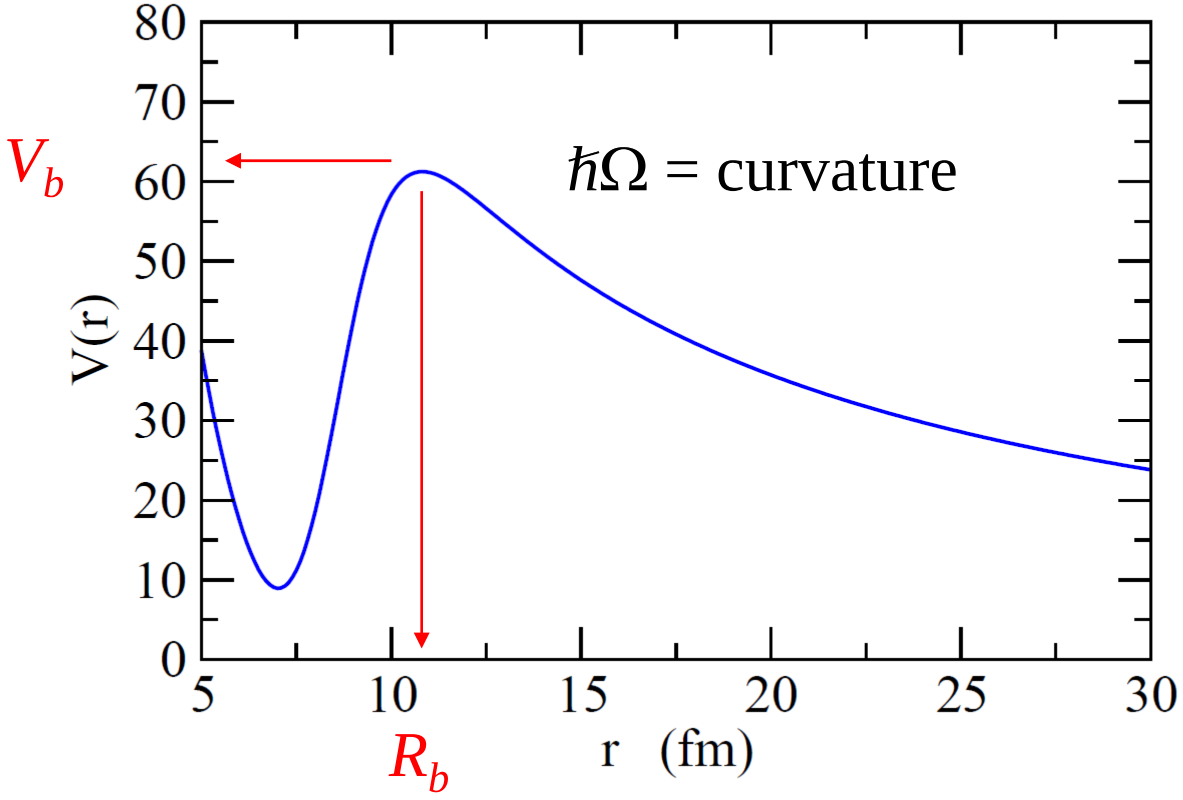
$P_l(E)$: barrier penetrability



Wong’s formula

C.Y. Wong, Phys. Rev. Lett. 31 (’73)766

$$\boxed{\sigma_{\mathrm{fus}}(E) = \frac{\hbar\Omega}{2E} R_b^2 \ln\left[1 + \exp\left(\frac{2\pi}{\hbar\Omega}(E - V_b)\right)\right]}$$



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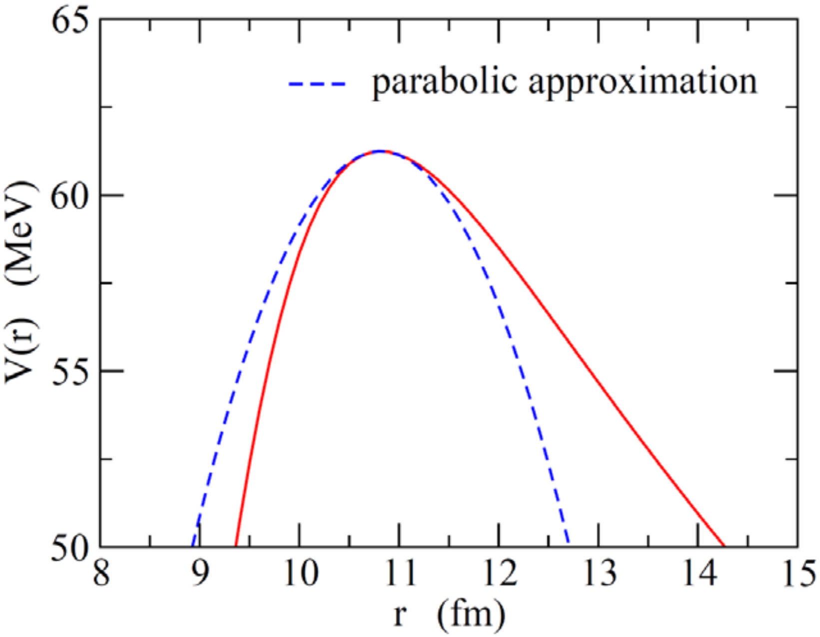
C.Y. Wong, Phys. Rev. Lett. 31 ('73)766

$$\boxed{\sigma_{\mathrm{fus}}(E) = \frac{\hbar\Omega}{2E} R_b^2 \ln\left[1 + \exp\left(\frac{2\pi}{\hbar\Omega}(E - V_b)\right)\right]}$$

i) Approximate the Coul. barrier by a parabola:

$$V(r) \sim V_b - \frac{1}{2}\mu\Omega^2 r^2$$

$$\longrightarrow \quad P_0(E) = \frac{1}{1 + \exp\left[\frac{2\pi}{\hbar\Omega}(V_b - E)\right]}$$

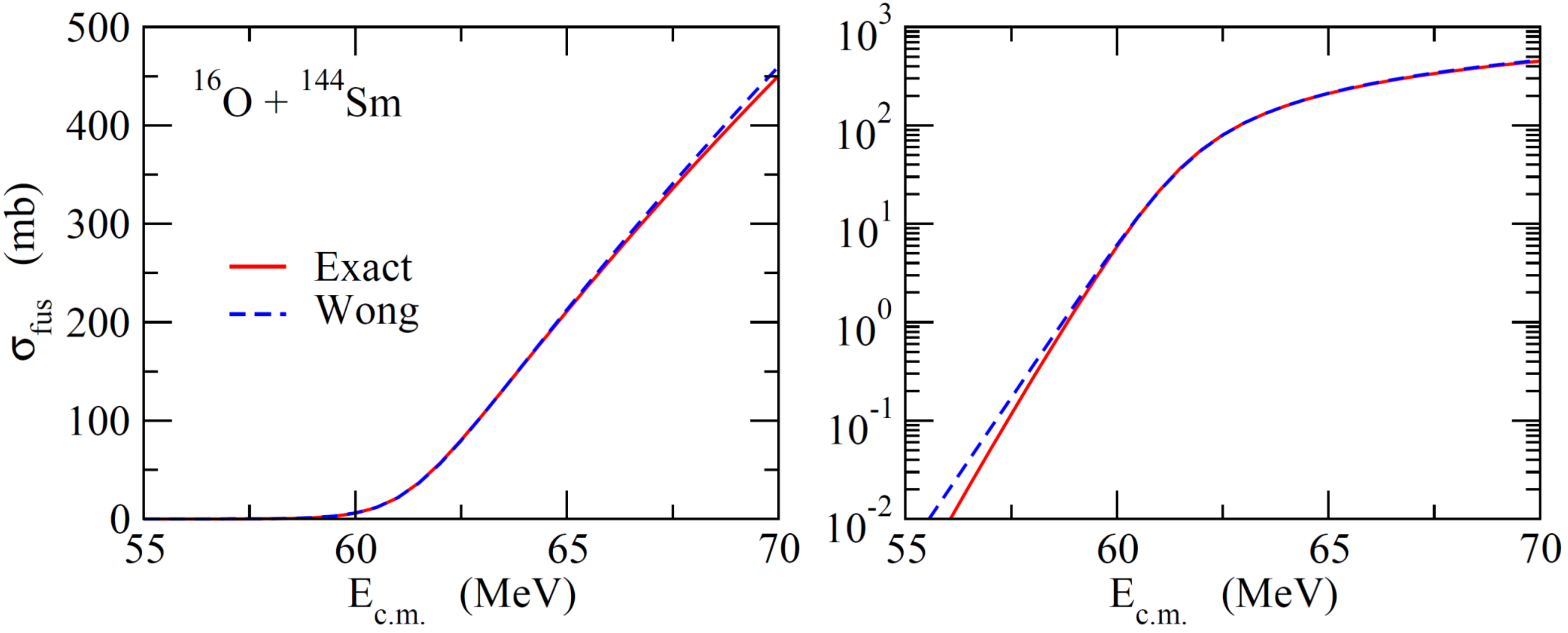


ii) *l*-independent barrier position and curvature:

$$\longrightarrow \quad P_l(E) \sim P_0\left(E - \frac{l(l+1)\hbar^2}{2\mu R_b^2}\right)$$

iii) Replace the sum of *l* with an integral

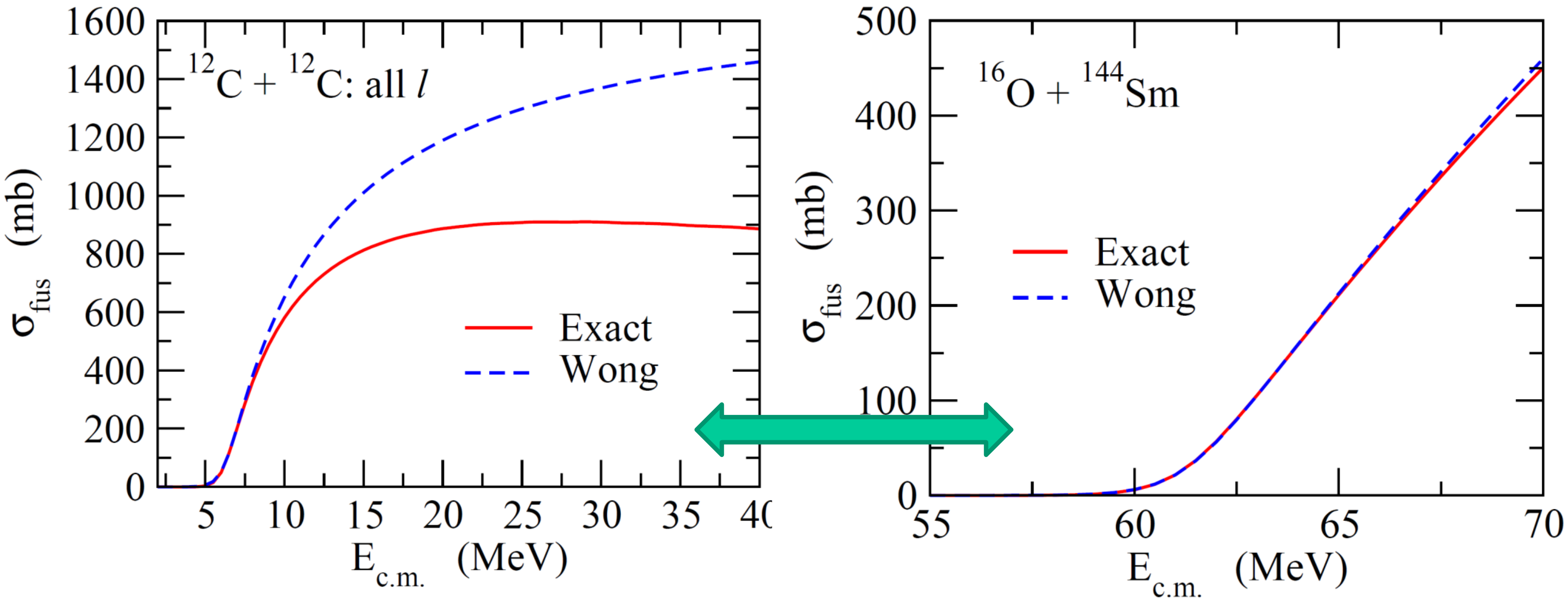
$$\sigma_{\mathrm{fus}}(E) = \frac{\pi}{k^2}\sum_l (2l+1)P_l(E) \to \frac{\pi}{k^2}\int dl\,(2l+1)P(l,E)$$



500
400
300
200
100
0
σ_{fus} (mb)
$^{16}O + ^{144}Sm$
Exact
Wong
55
60
65
70
$E_{c.m.}$ (MeV)
10^3
10^2
10^1
10^0
10^{-1}
10^{-2}
55
60
65
70
$E_{c.m.}$ (MeV)

not so bad

Wong formula for light heavy-ion fusion



Wong formula:

i) Approximate the Coul. barrier by a parabola

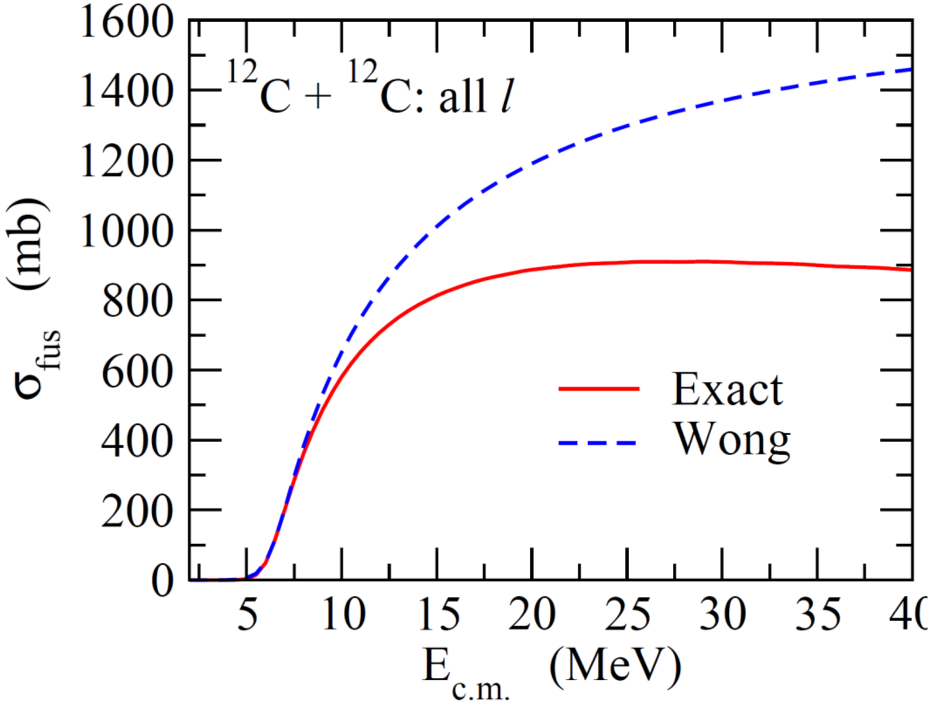
ii) *l*-independent barrier position and curvature ←

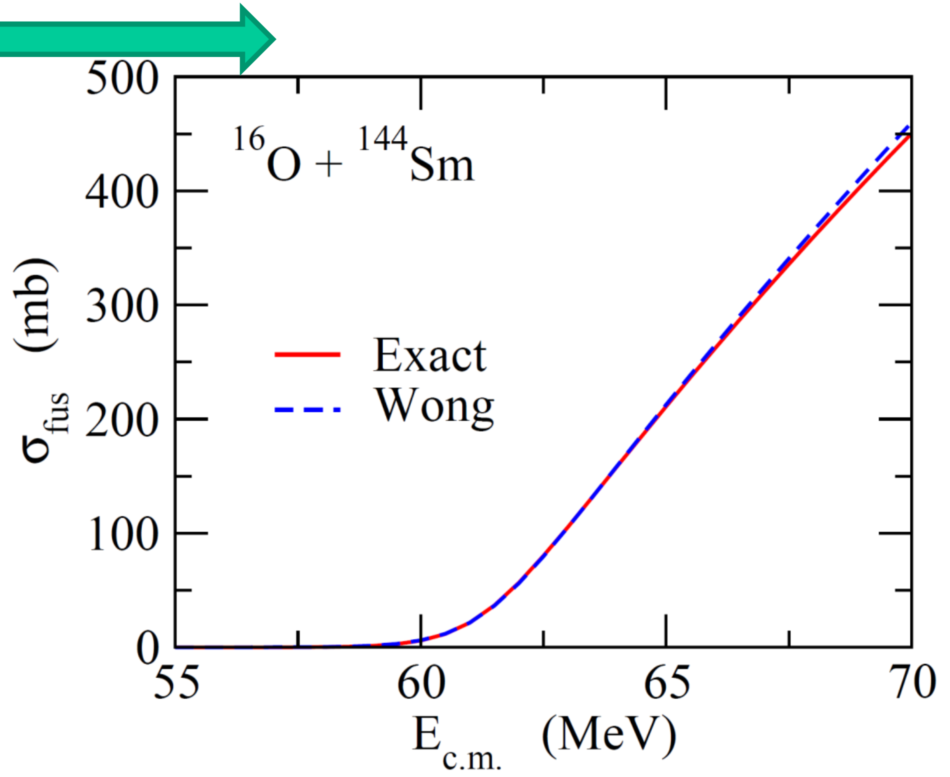
iii) Replace the sum of *l* with an integral

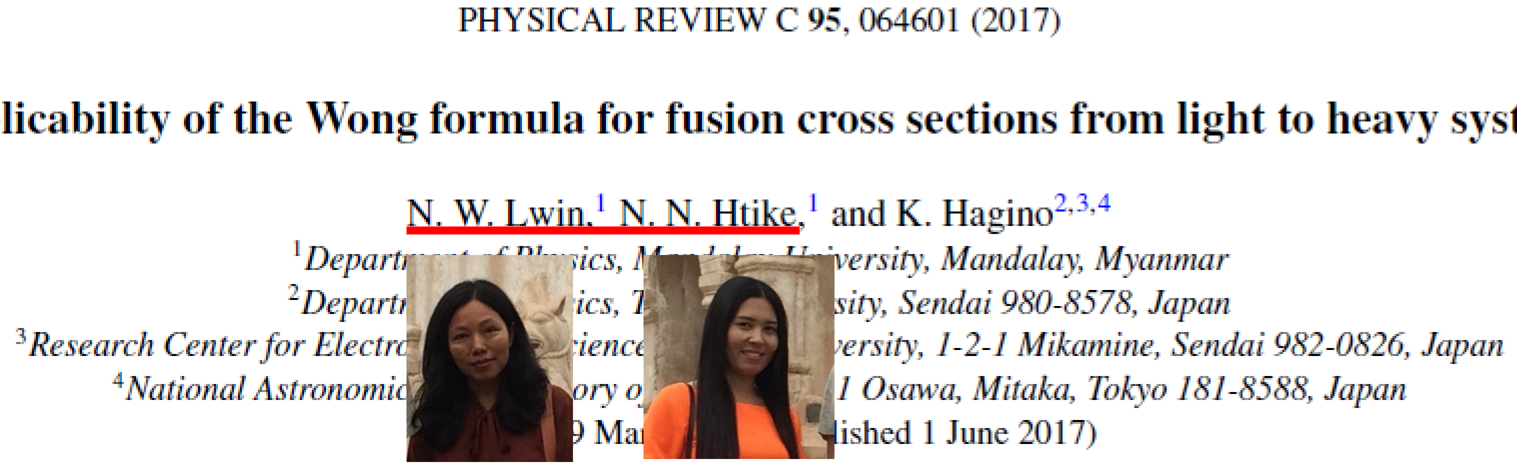
$$V_{\mathrm{cent}}(r) = \frac{l(l+1)\hbar^2}{2\mu r^2}$$

small

how is it evolved?







PHYSICAL REVIEW C **95**, 064601 (2017)

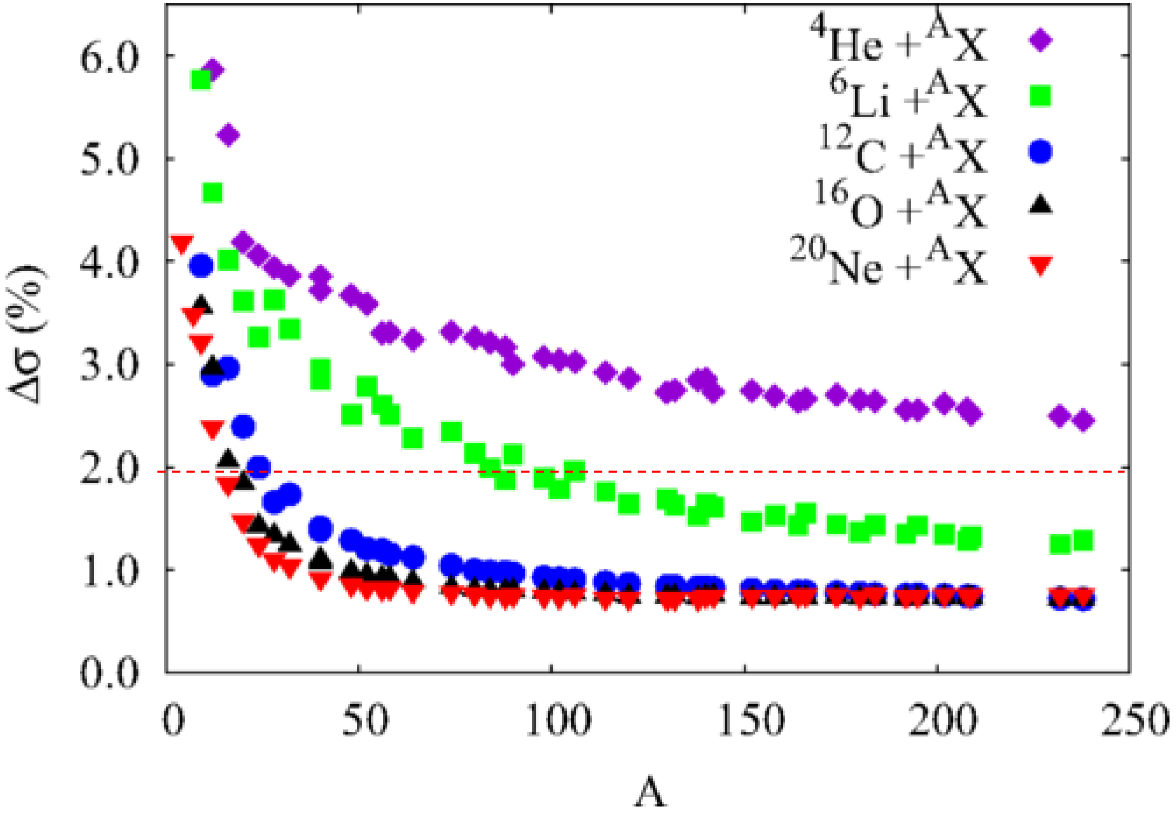
Applicability of the Wong formula for fusion cross sections from light to heavy systems

N. W. Lwin,[1] N. N. Htike,[1] and K. Hagino[2,3,4]

[1]*Depart f Physics, M niversity, Mandalay, Myanmar*
[2]*Depart ics, T sity, Sendai 980-8578, Japan*
[3]*Research Center for Electr ience versity, 1-2-1 Mikamine, Sendai 982-0826, Japan*
[4]*National Astronomic ory o 1 Osawa, Mitaka, Tokyo 181-8588, Japan*
9 Ma lished 1 June 2017)

$$\boxed{\Delta\sigma \equiv \frac{\int_{E_{\rm min}}^{E_{\rm max}} |\sigma_{\rm exact}(E) - \sigma_{\rm Wong}(E)|dE}{\int_{E_{\rm min}}^{E_{\rm max}} \sigma_{\rm exact}(E)dE}}$$

$$E_{\rm min} = 0.9V_b,\;\; E_{\rm max} = 1.1V_b$$

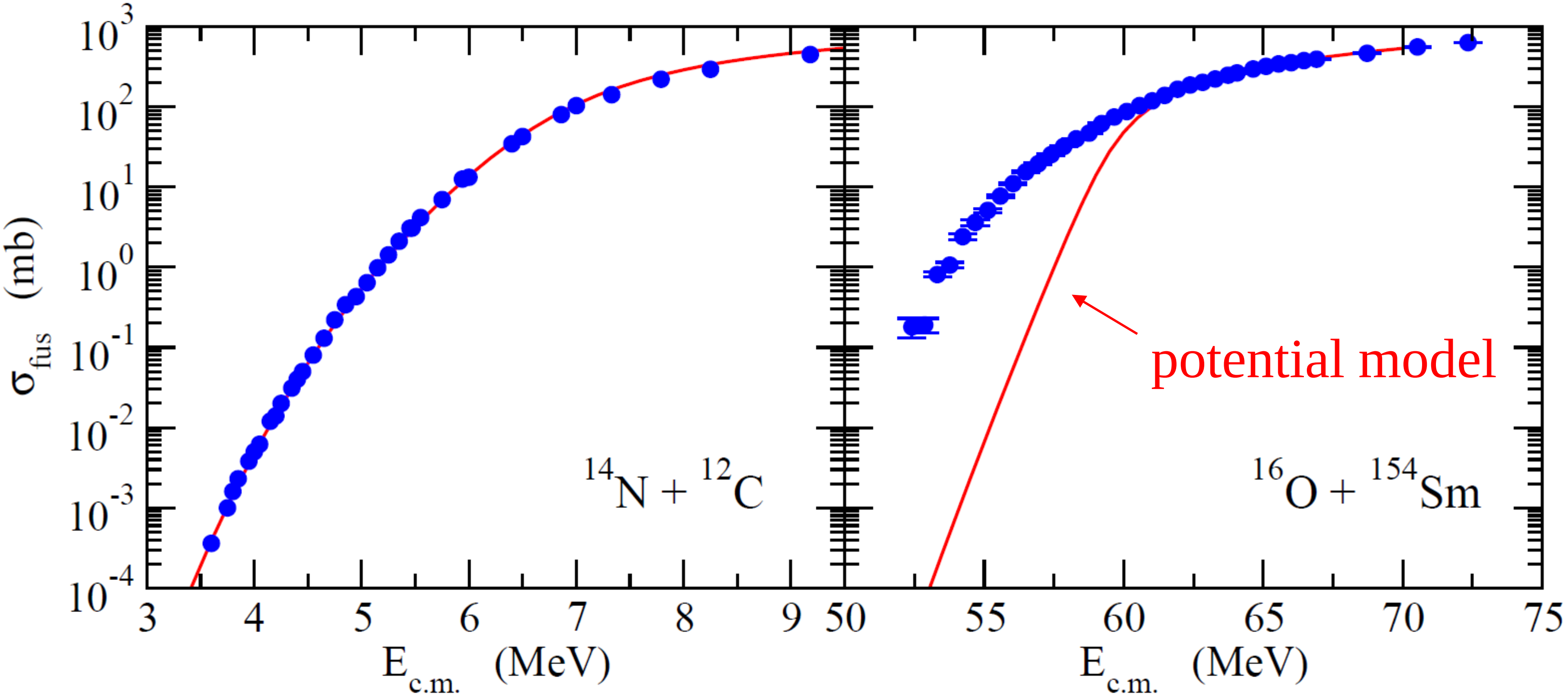


N.W. Lwin, N.N. Hitke, and K. Hagino, PRC95 (‘17) 064601

Comparison with experimental data: large enhancement of σ_{fus}

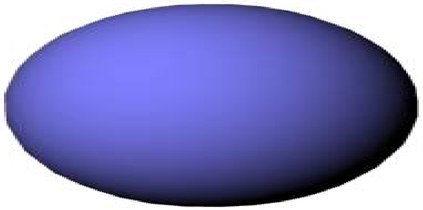
Potential model: $V(r)$ + absorption

$$\boxed{\sigma_{\rm fus} = \frac{\pi}{k^2}\sum_l (2l+1)(1-|S_l|^2)}$$

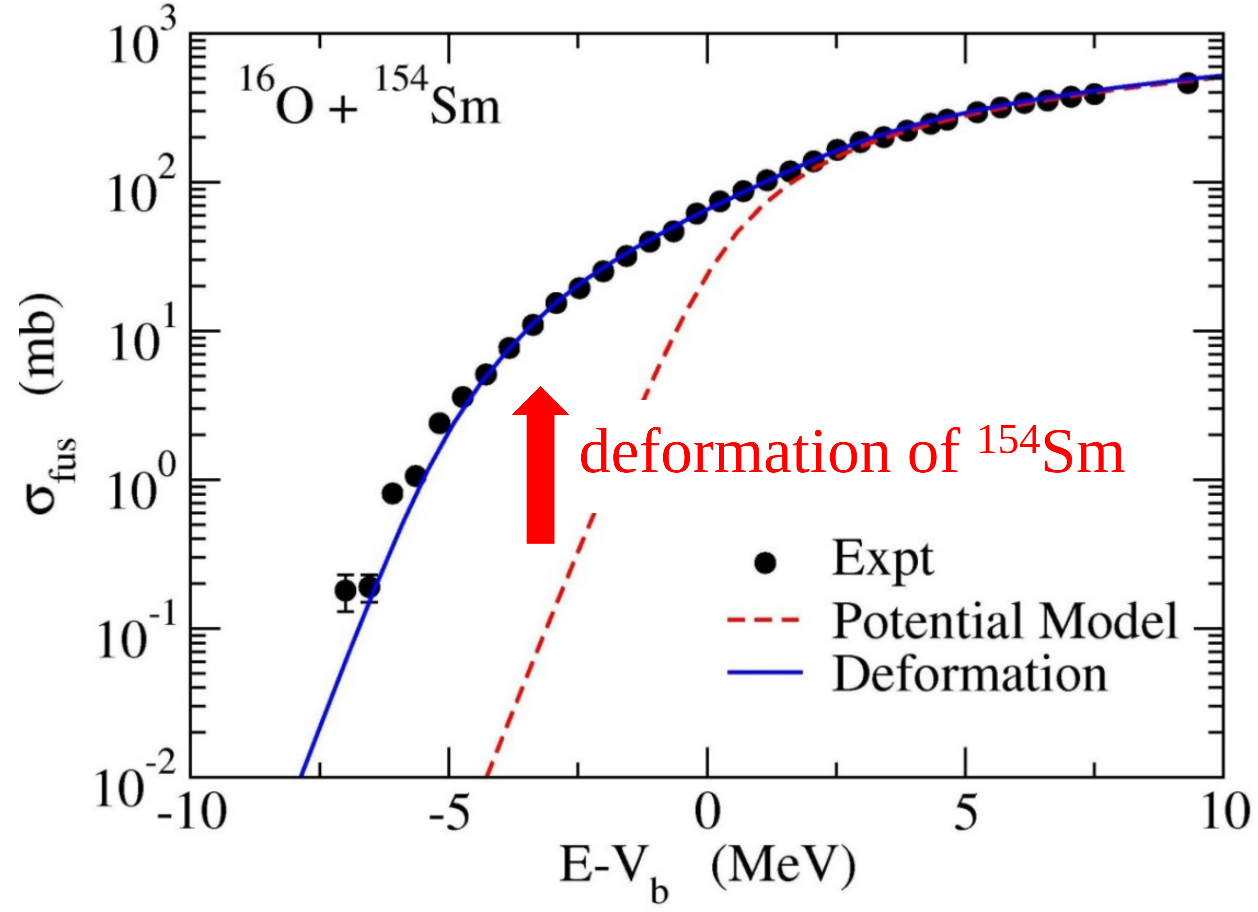


cf. seminal work:
R.G. Stokstad et al., PRL41('78) 465

^{154}Sm : a typical deformed nucleus



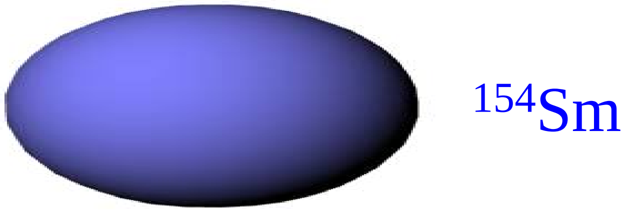
^{154}Sm

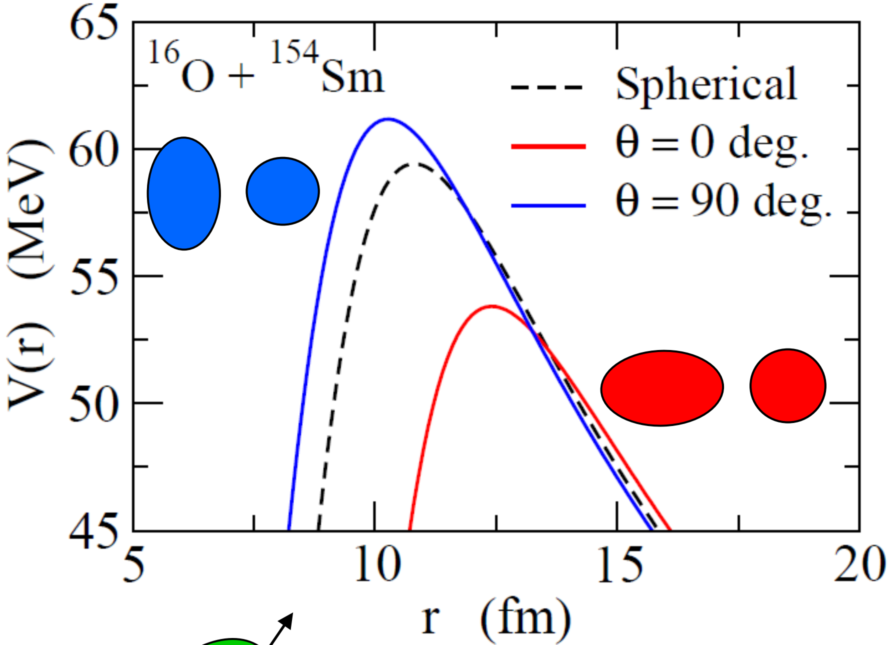


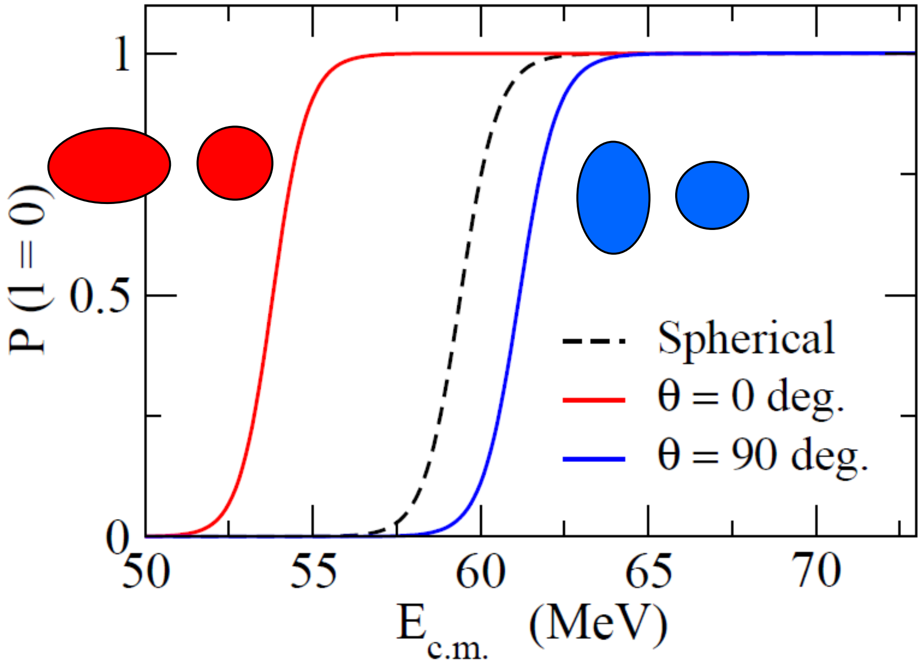
deformation of ^{154}Sm

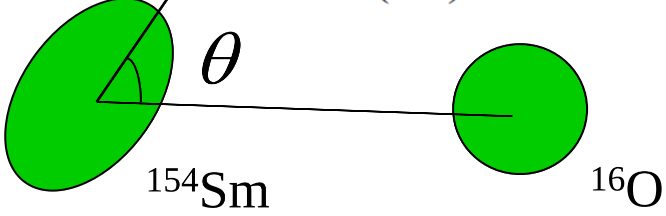
Effects of nuclear deformation

^{154}Sm : a typical deformed nucleus





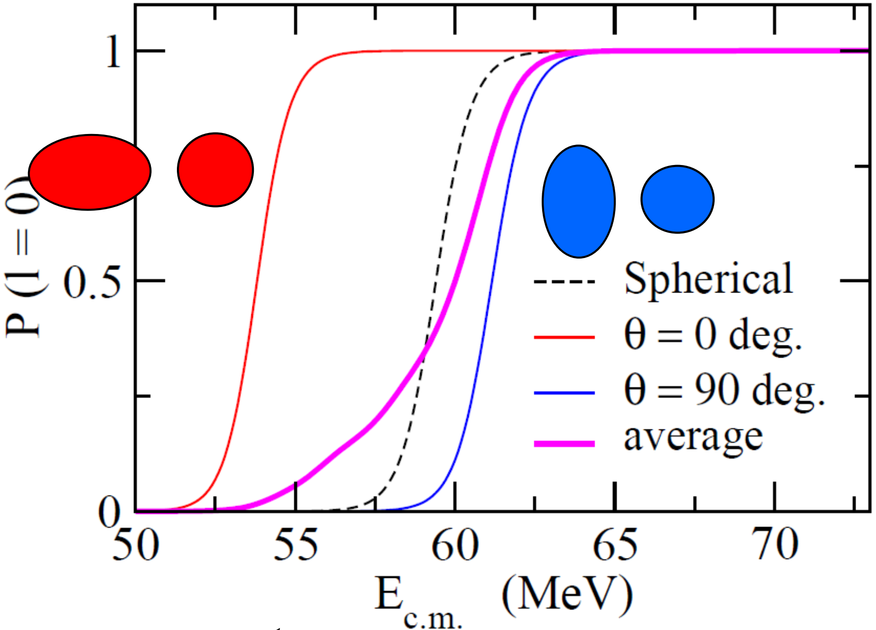


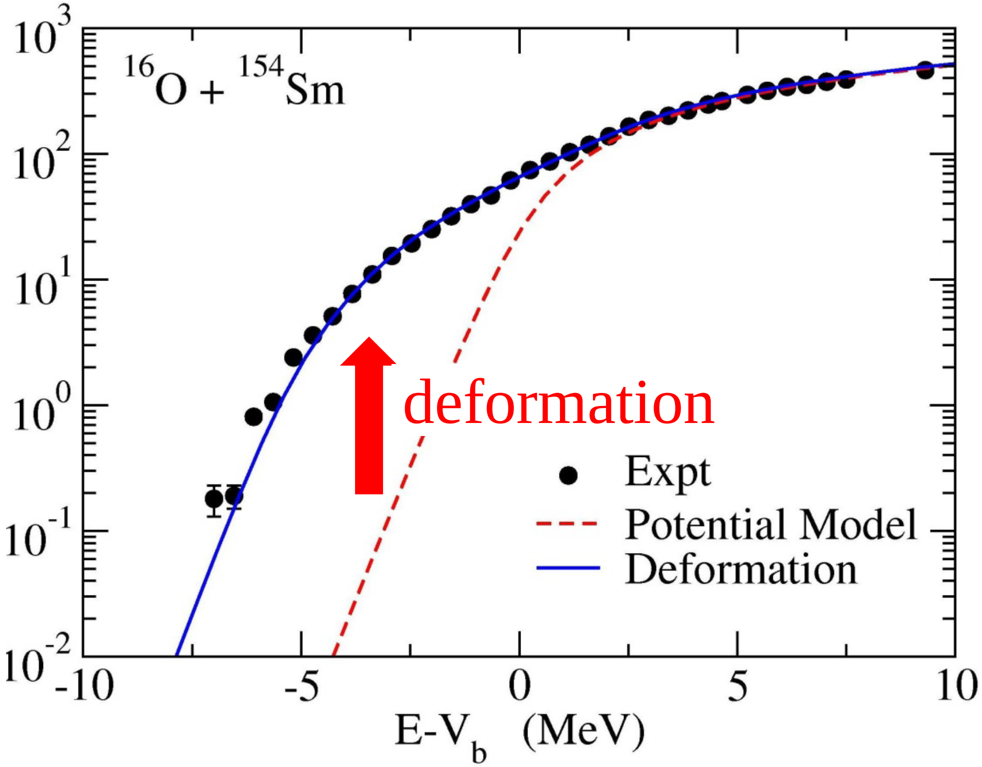


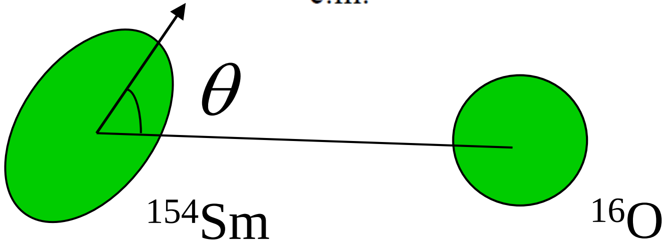
Effects of nuclear deformation

^{154}Sm : a typical deformed nucleus

$$\sigma_{\rm fus}(E)=\int_0^1 d(\cos\theta)\,\sigma_{\rm fus}(E;\theta)$$

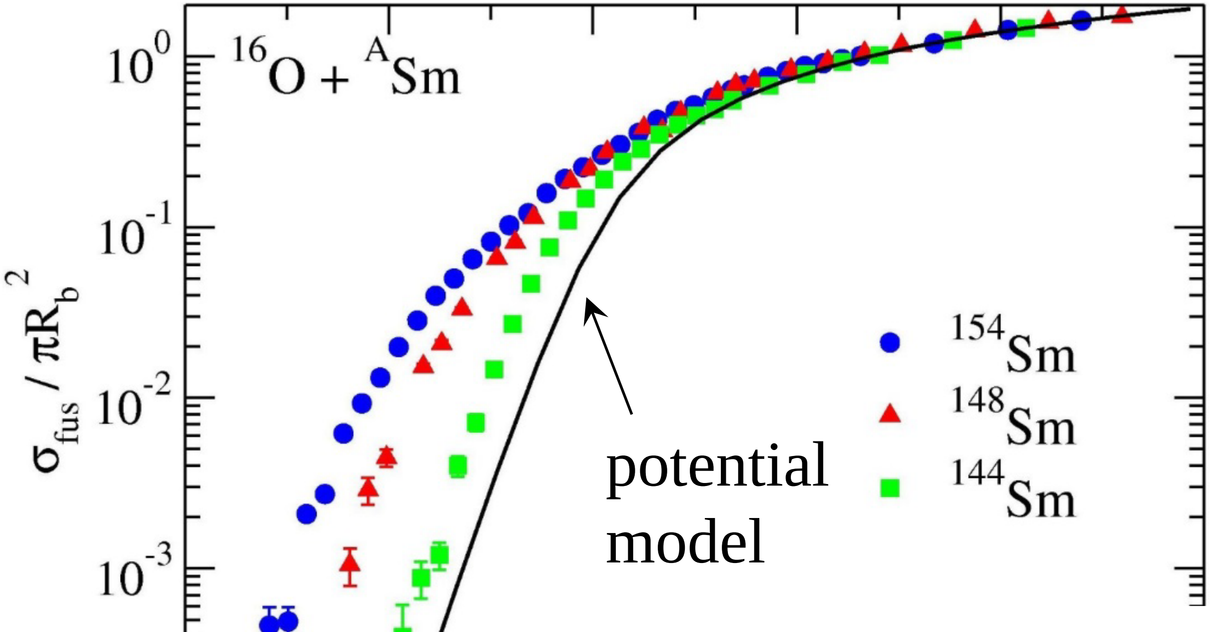




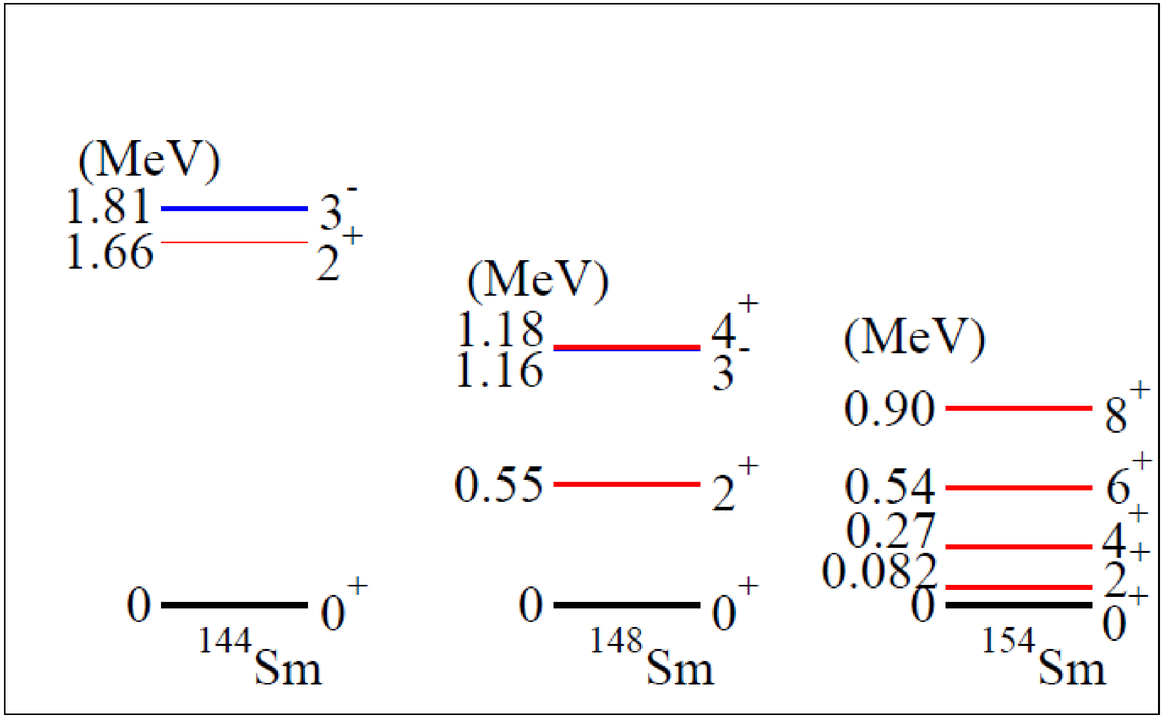


Fusion: strong interplay between nuclear structure and reaction

* Sub-barrier enhancement also for non-deformed targets:
couplings to low-lying collective excitations → coupling assisted tunneling



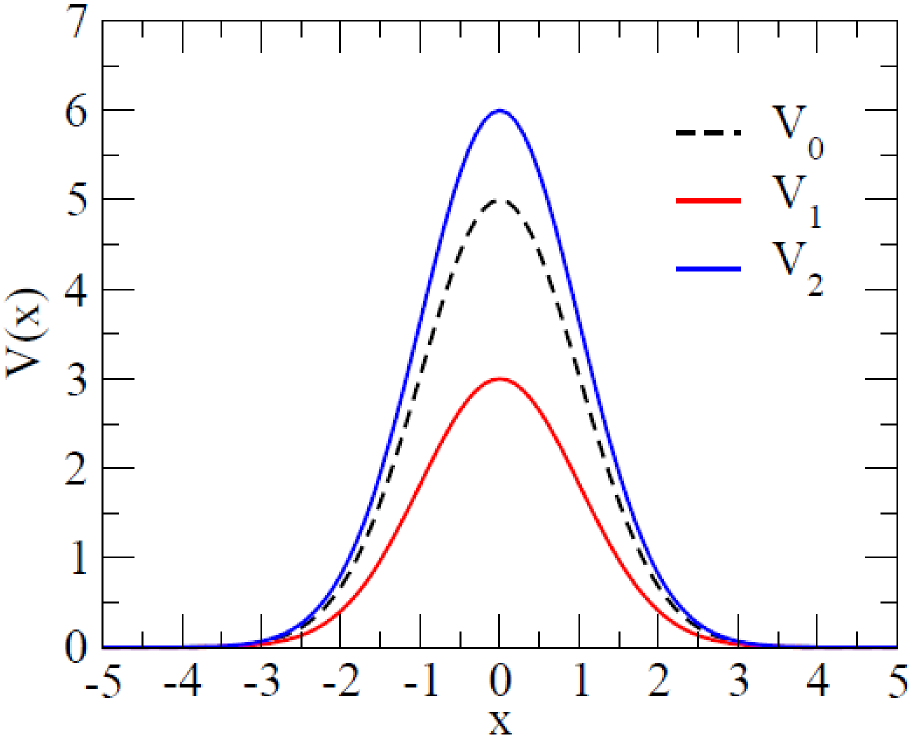
enhancement of fusion cross sections
: a general phenomenon

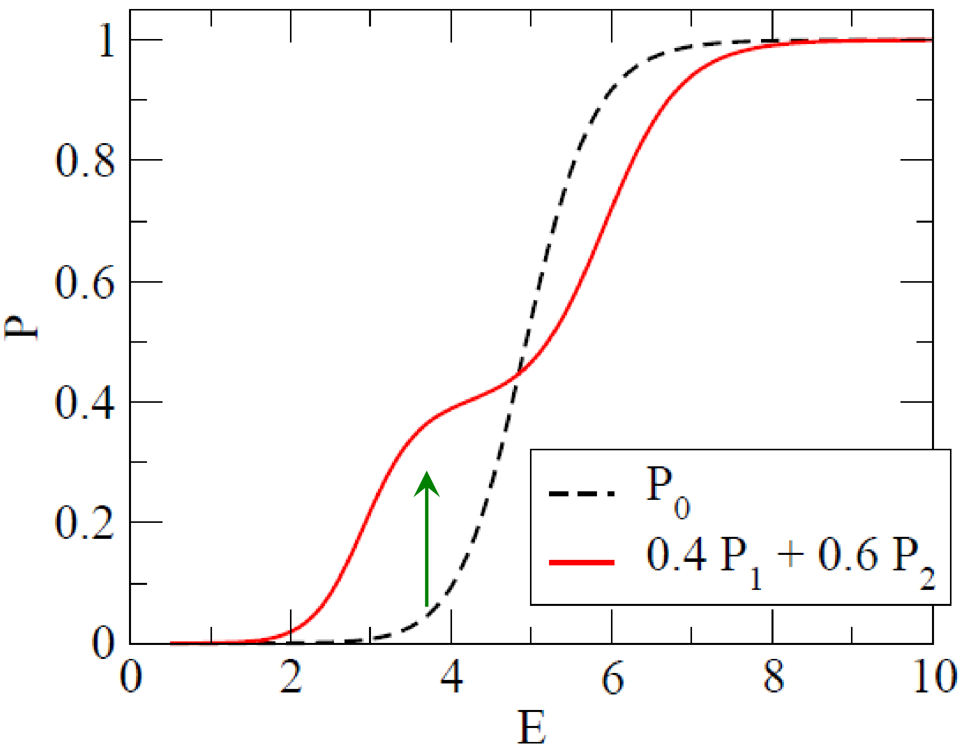


strong correlation with nuclear spectrum
→ coupling assisted tunneling

Enhancement of tunneling probability
: a problem of two potential barriers

$$P(E) = P(E;V_0) \rightarrow w_1 P(E;V_1) + w_2 P(E;V_2)$$

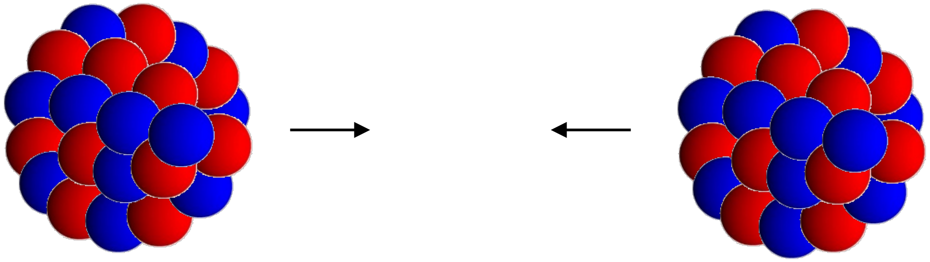




“barrier distribution” due to couplings to excited states
in projectile/target nuclei

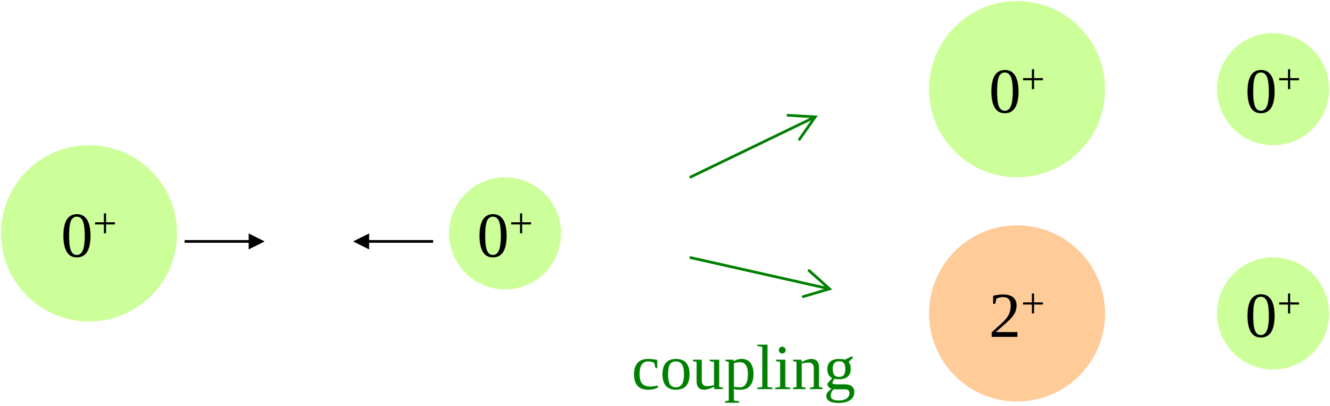
Coupled-channels method: a quantal scattering theory with excitations

many-body problem

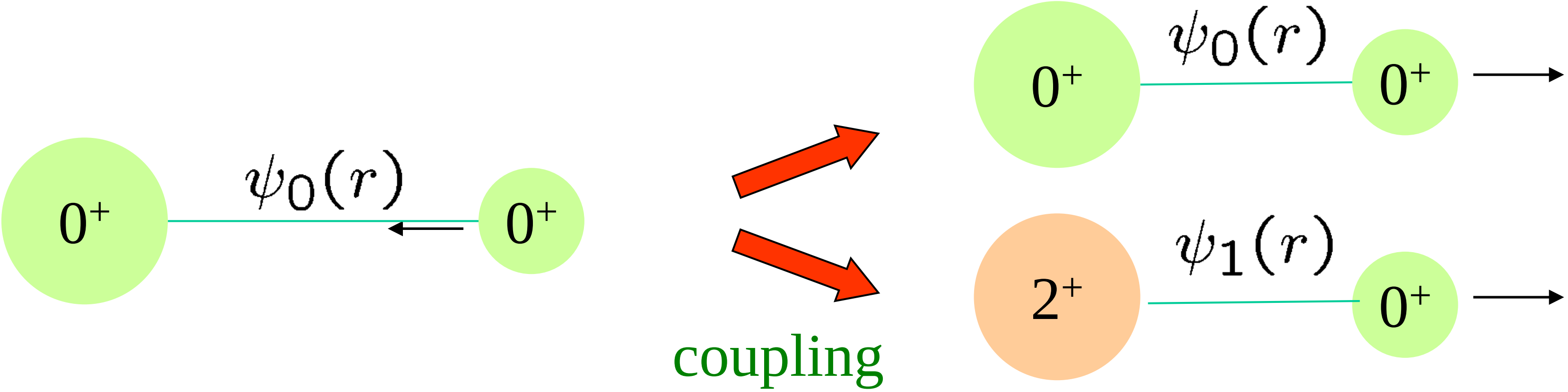


still very challenging

two-body problem, but with excitations
(coupled-channels approach)



Coupled-channels method: a quantal scattering theory with excitations

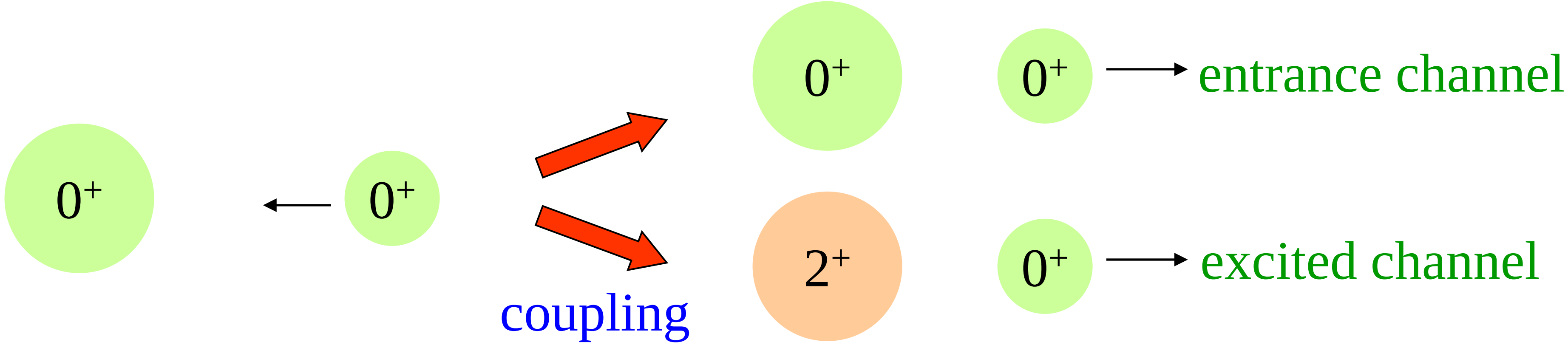


$$\boxed{\left[-\frac{\hbar^2}{2\mu}\nabla^2 + \overleftrightarrow{V}(r) - \overleftrightarrow{E}\right]\vec{\psi}(\boldsymbol{r}) = 0}$$

if written down more explicitly:

$$\boxed{\left[-\frac{\hbar^2}{2\mu}\nabla^2 + V_0(r) + \underset{\substack{\uparrow \\ \text{excitation energy}}}{\epsilon_k} - E\right]\psi_k(\boldsymbol{r}) + \sum_{k'}\langle\phi_k|\underset{\substack{\uparrow \\ \text{excitation operator}}}{V_{\mathrm{coup}}}|\phi_{k'}\rangle\psi_{k'}(\boldsymbol{r}) = 0}$$

Coupled-channels method: a quantal scattering theory with excitations



coupling

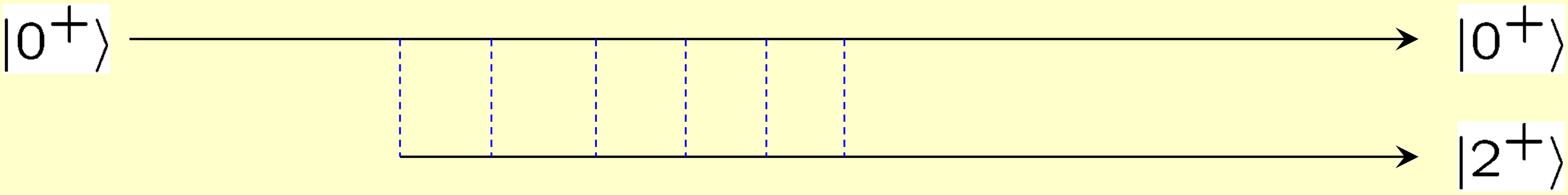
entrance channel

excited channel

$$\boxed{\left[-\frac{\hbar^2}{2\mu}\nabla^2+V_0(r)+\epsilon_k-E\right]\psi_k(\boldsymbol{r})+\sum_{k'}\langle\phi_k|V_{\rm coup}|\phi_{k'}\rangle\psi_{k'}(\boldsymbol{r})=0}$$

excitation energy

excitation operator



full order treatment of excitation/de-excitation dynamics during reaction

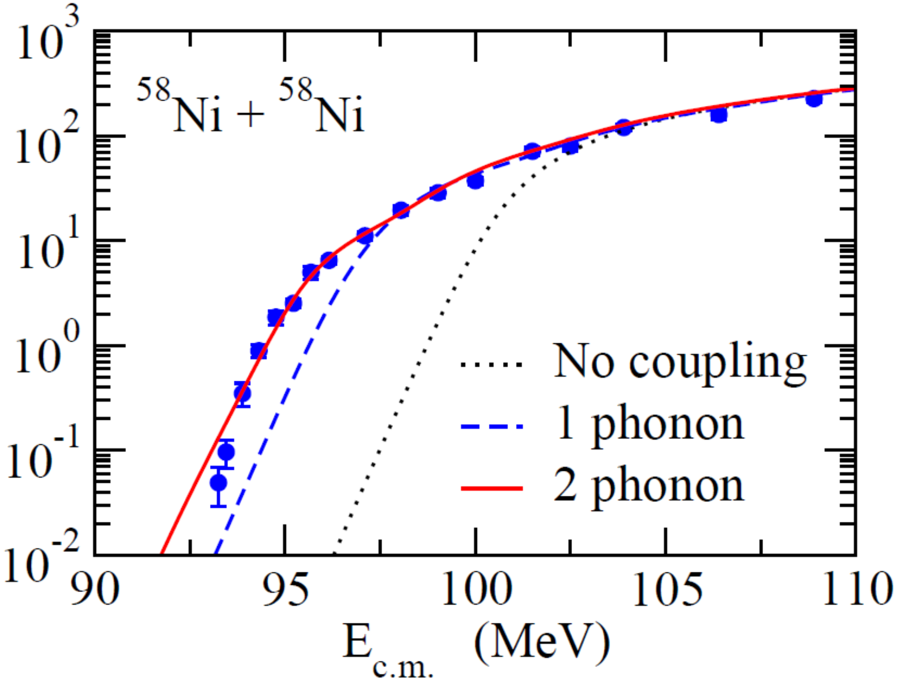
Inputs for C.C. calculations

i) Inter-nuclear potential

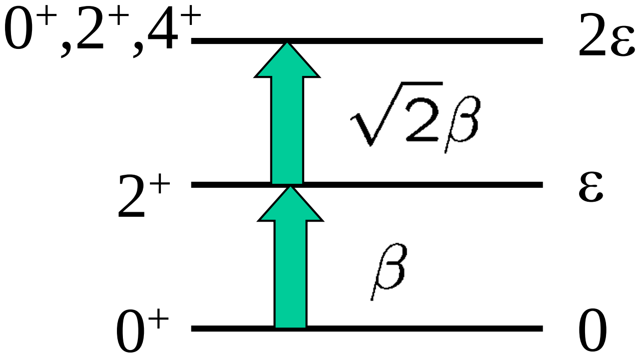
a fit to experimental data at above barrier energies

ii) Intrinsic degrees of freedom

in most of cases, (macroscopic) collective model
(rigid rotor / harmonic oscillator)



simple harmonic oscillator

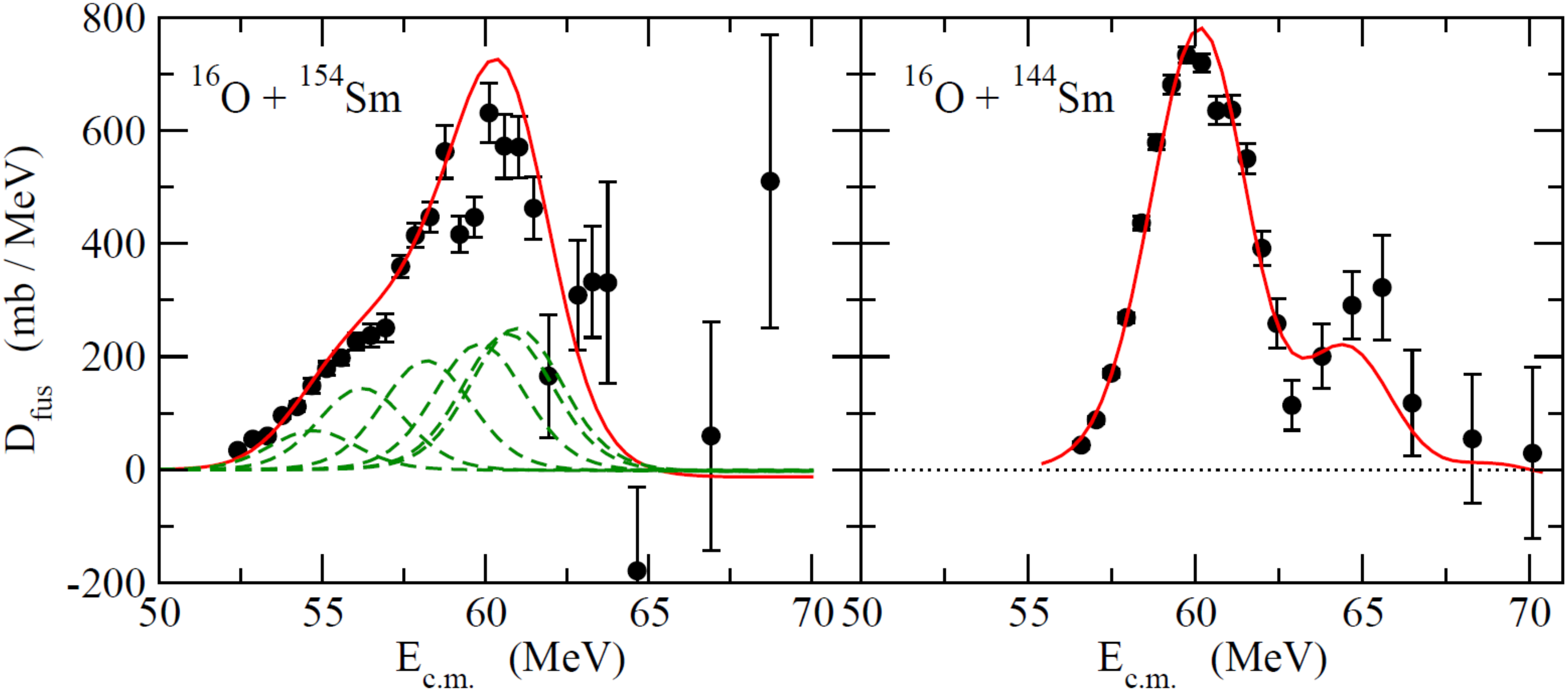


C.C. approach: a standard tool for sub-barrier fusion reactions

cf. CCFULL (K.H., N. Rowley, A.T. Kruppa, CPC123 (‘99) 143)

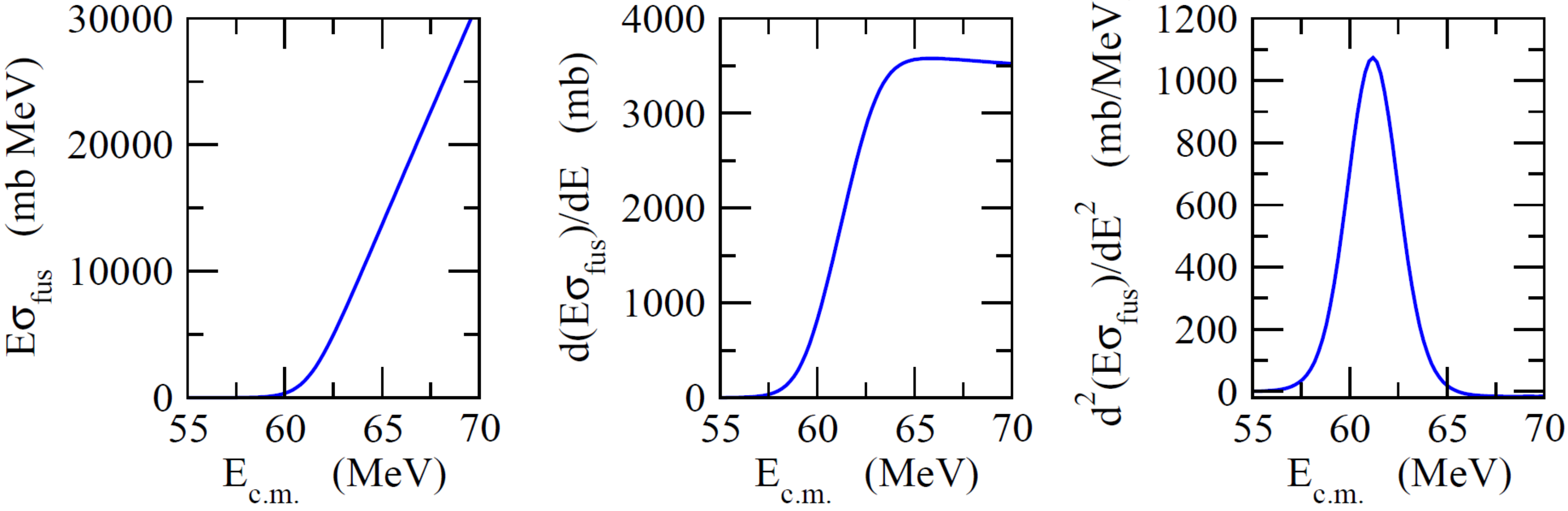
✓ Fusion barrier distribution (Rowley, Satchler, Stelson, PLB254(‘91))

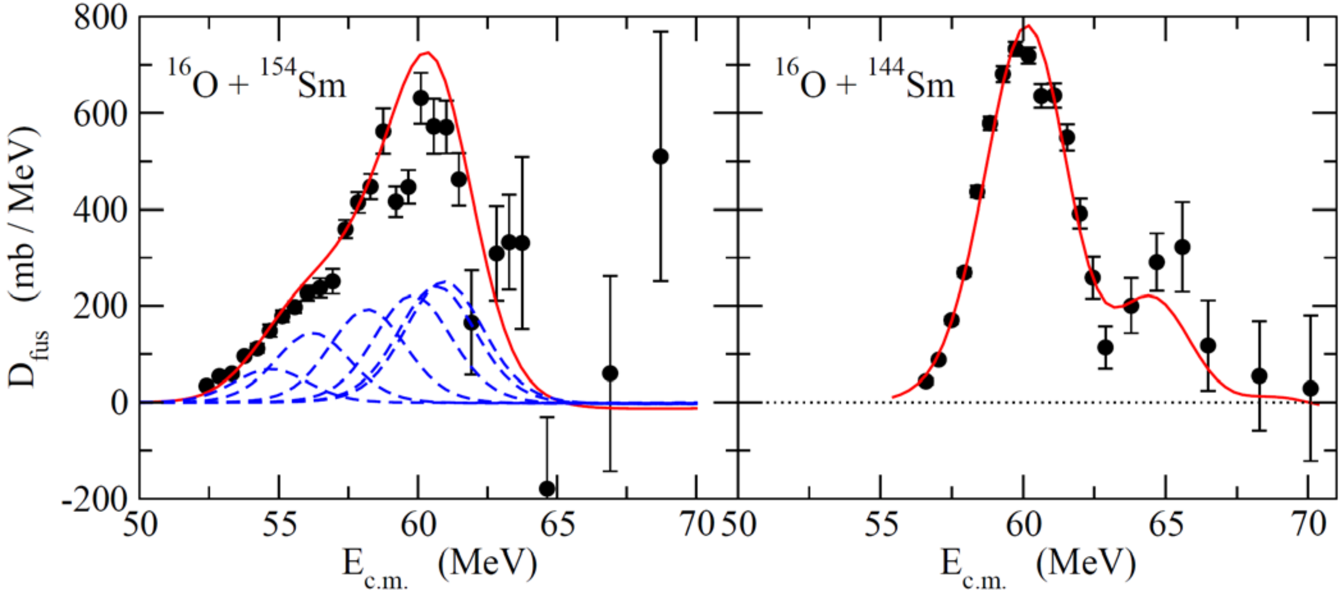
$$D_{\text{fus}}(E) = \frac{d^2(E\sigma_{\text{fus}})}{dE^2}$$



K.H., N. Takigawa, PTP128 (‘12) 1061

$\sim \pi R_b^2 \, P_{l=0}(E)$ $\sim \pi R_b^2 \cdot dP_{l=0}/dE$

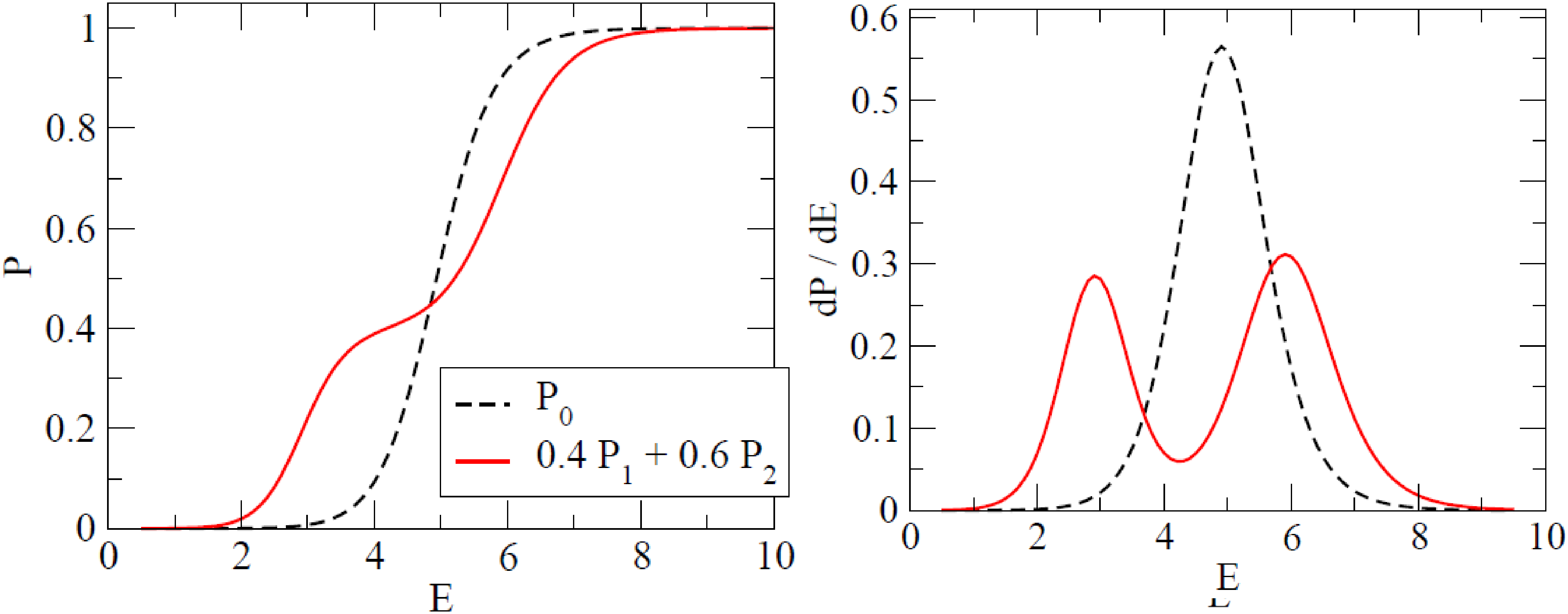




K.H. and N. Takigawa, PTP128 (‘12) 1061

barrier distribution: a problem of two potential barriers

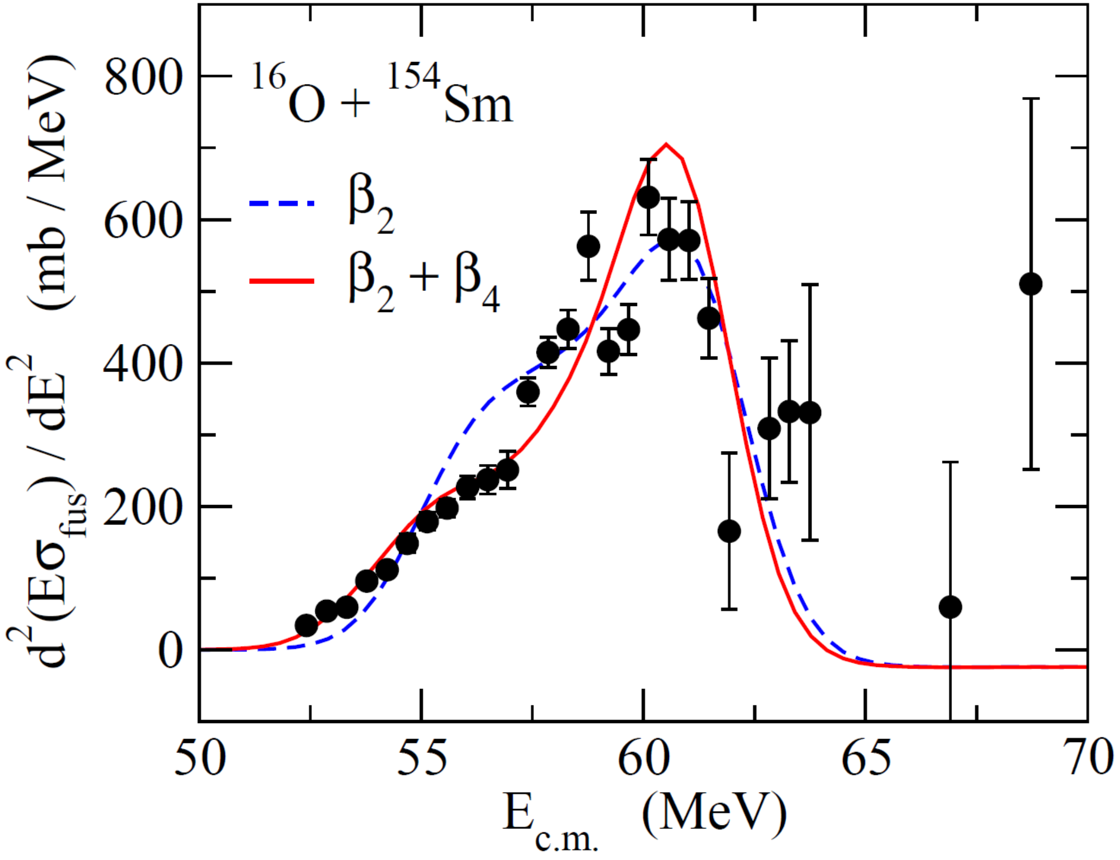
$$P(E) = P(E; V_0) \rightarrow w_1 P(E; V_1) + w_2 P(E; V_2)$$



Fusion barrier distribution

$$D_{\text{fus}}(E) = \frac{d^2(E\sigma_{\text{fus}})}{dE^2}$$

- ◆ N. Rowley, G.R. Satchler, and P.H. Stelson, PLB254 (‘91) 25
- ◆ J.X. Wei, J.R. Leigh et al., PRL67 (’91) 3368
- ◆ M. Dasgupta et al., Annu. Rev. Nucl. Part. Sci. 48 (’98) 401
- ◆ A.M. Stefanini et al., Phys. Rev. Lett. 74 (‘95) 864



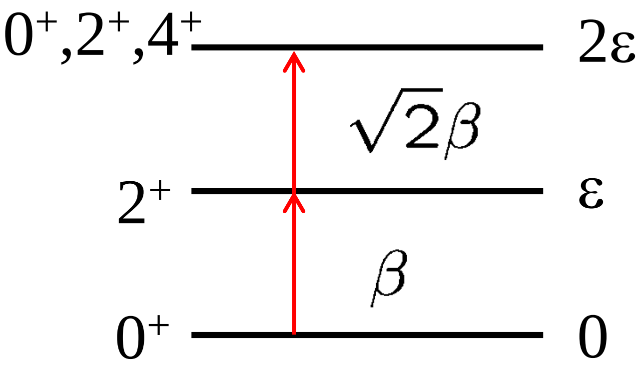
sensitive to nuclear structure

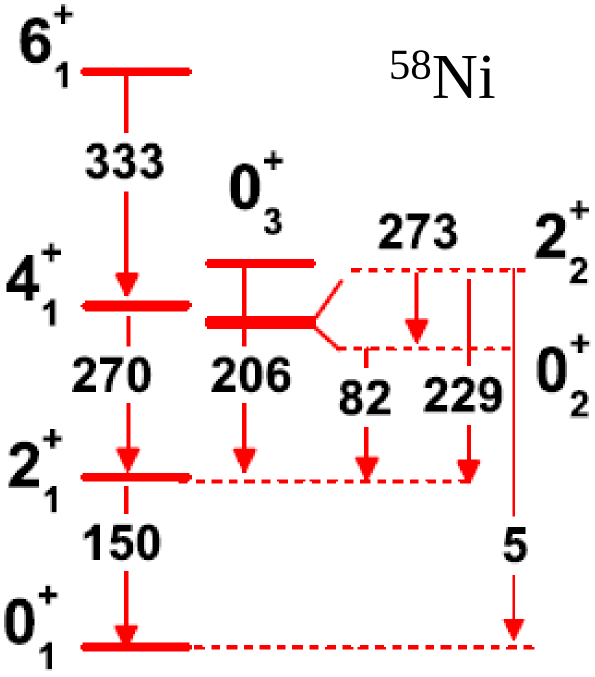
Further development: semi-microscopic modelling

K.H. and J.M. Yao, PRC91('15) 064606

CCFULL
\+ microscopic nuclear structure
calculations
(GCM, Shell Model, IBM…..)

simple harmonic
oscillator

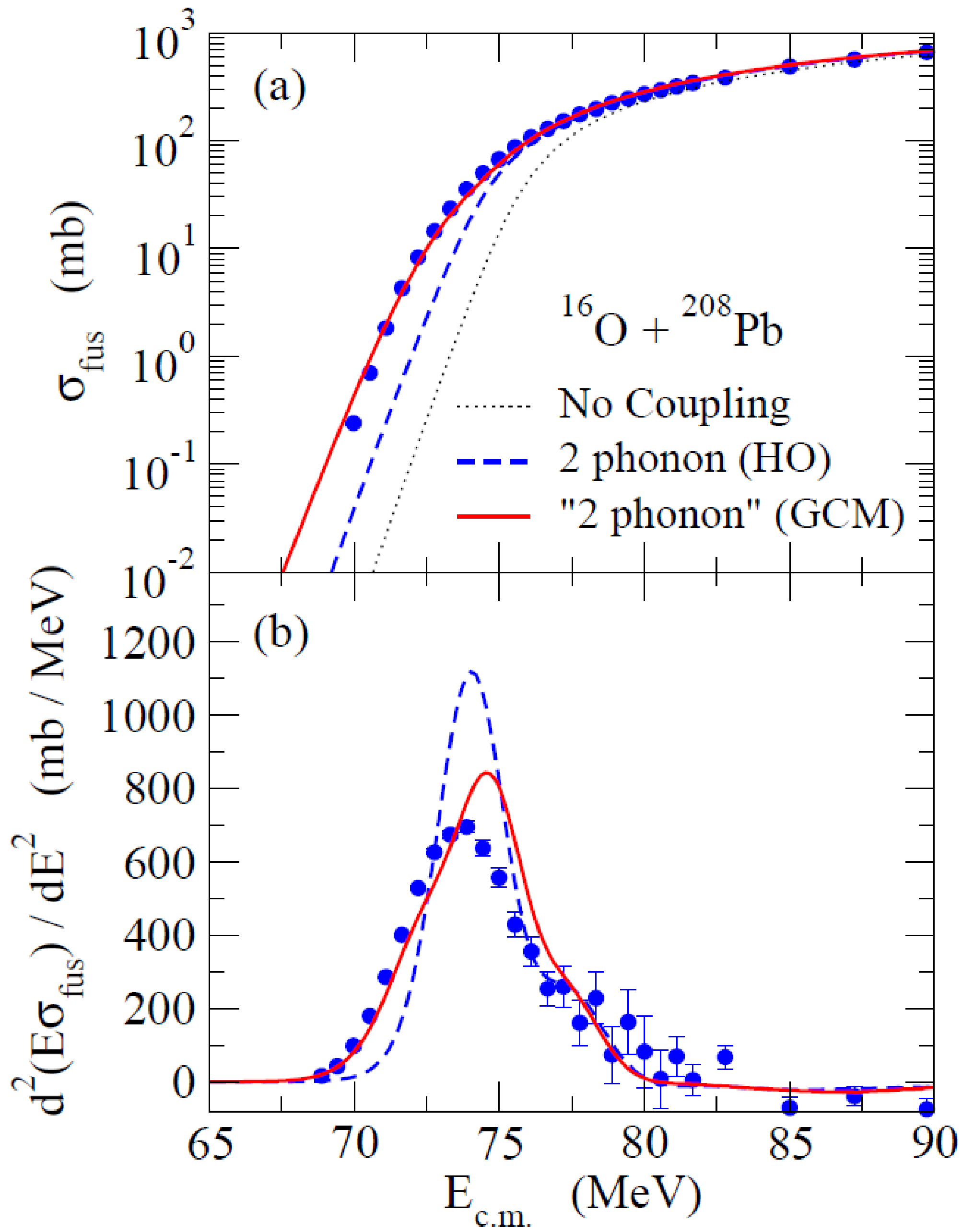


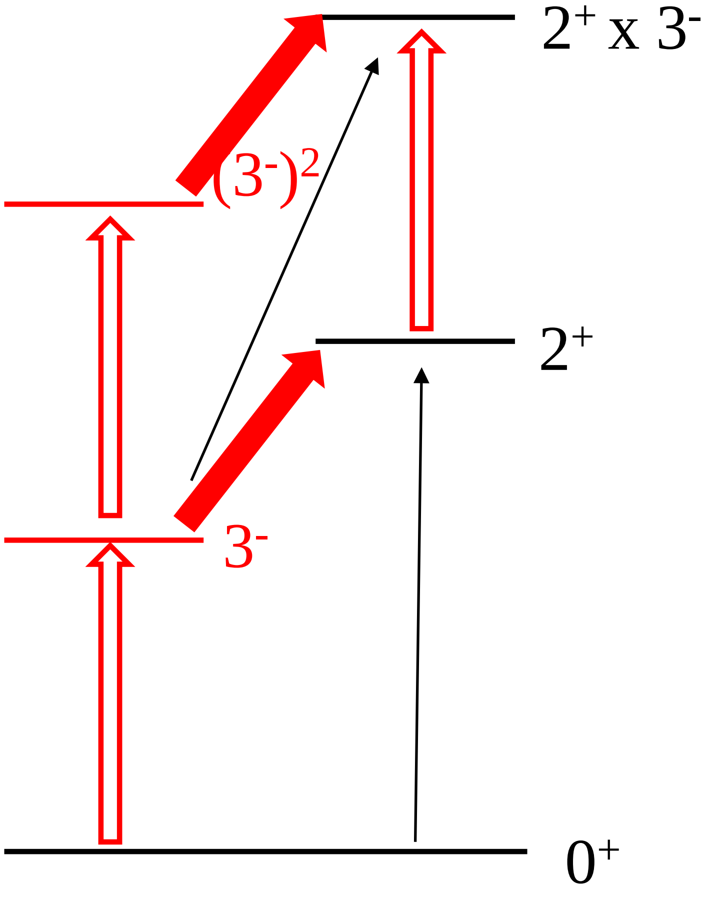


relativistic MF + GCM

anharmonicity of phonon spectra

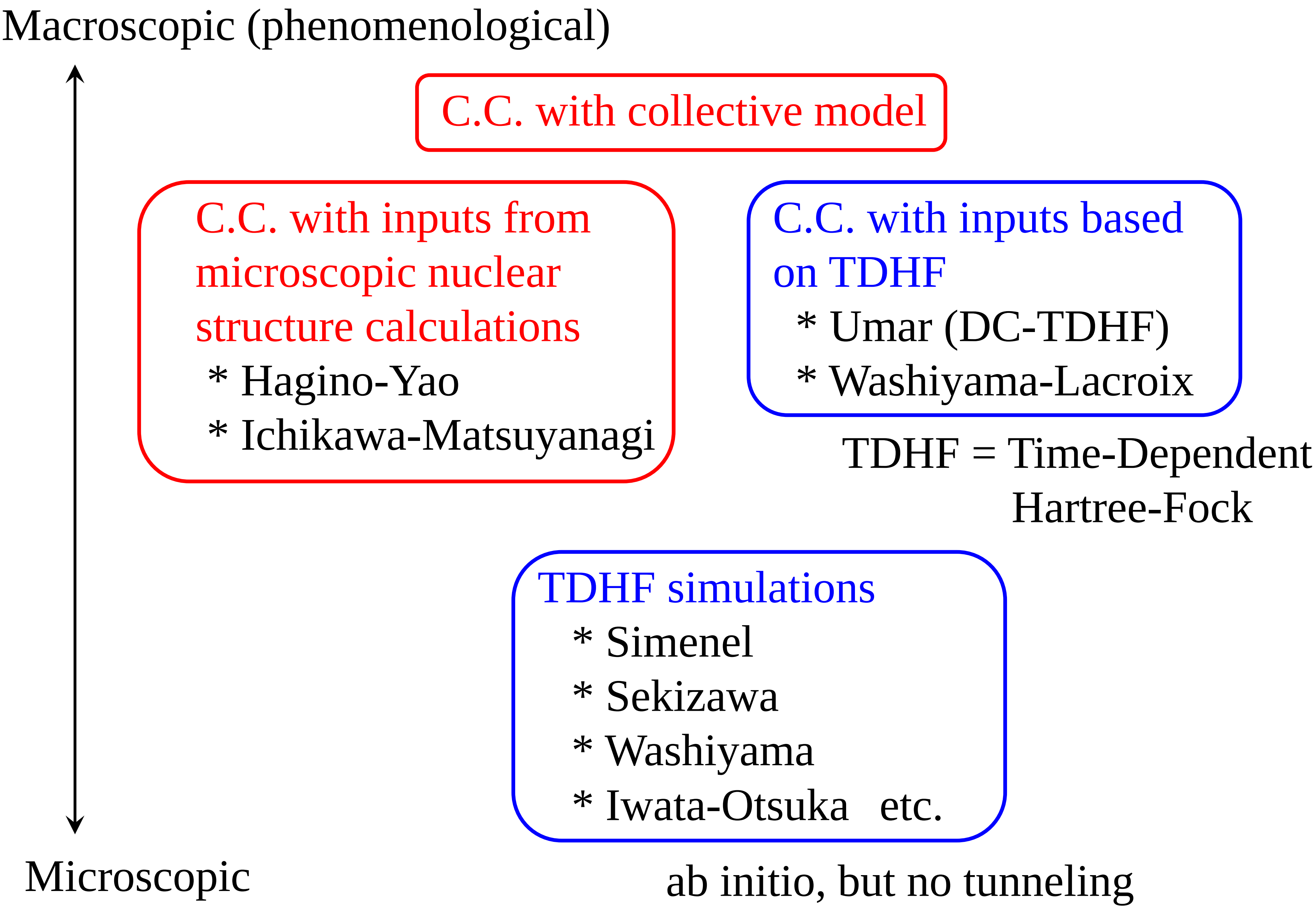
CCFULL with RMF+GCM





J.M. Yao and K.H.,
PRC94 (‘16) 11303(R)

From phenomenological approach to microscopic approach



Macroscopic (phenomenological)

C.C. with collective model

C.C. with inputs from microscopic nuclear structure calculations
- Hagino-Yao
- Ichikawa-Matsuyanagi

C.C. with inputs based on TDHF
- Umar (DC-TDHF)
- Washiyama-Lacroix

TDHF = Time-Dependent
Hartree-Fock

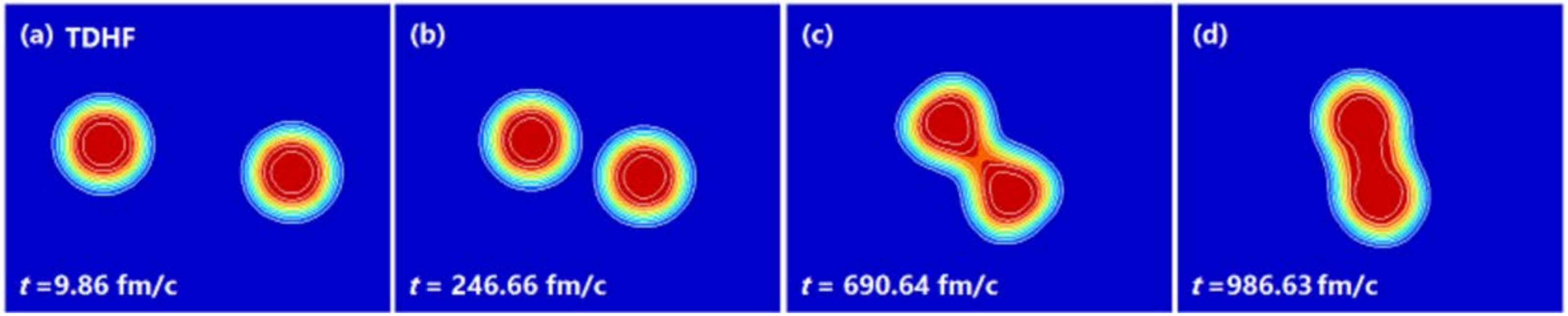
TDHF simulations
- Simenel
- Sekizawa
- Washiyama
- Iwata-Otsuka etc.

Microscopic

ab initio, but no tunneling

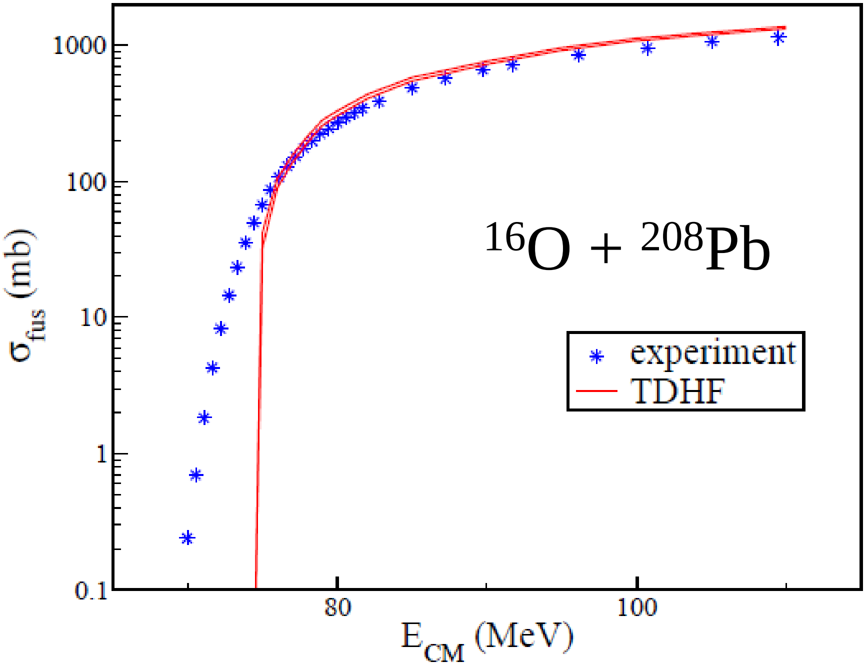
TDHF simulation

TDHF = Time Dependent Hartree-Fock



S. Ebata, T. Nakatsukasa, JPC Conf. Proc. 6 ('15) 020056

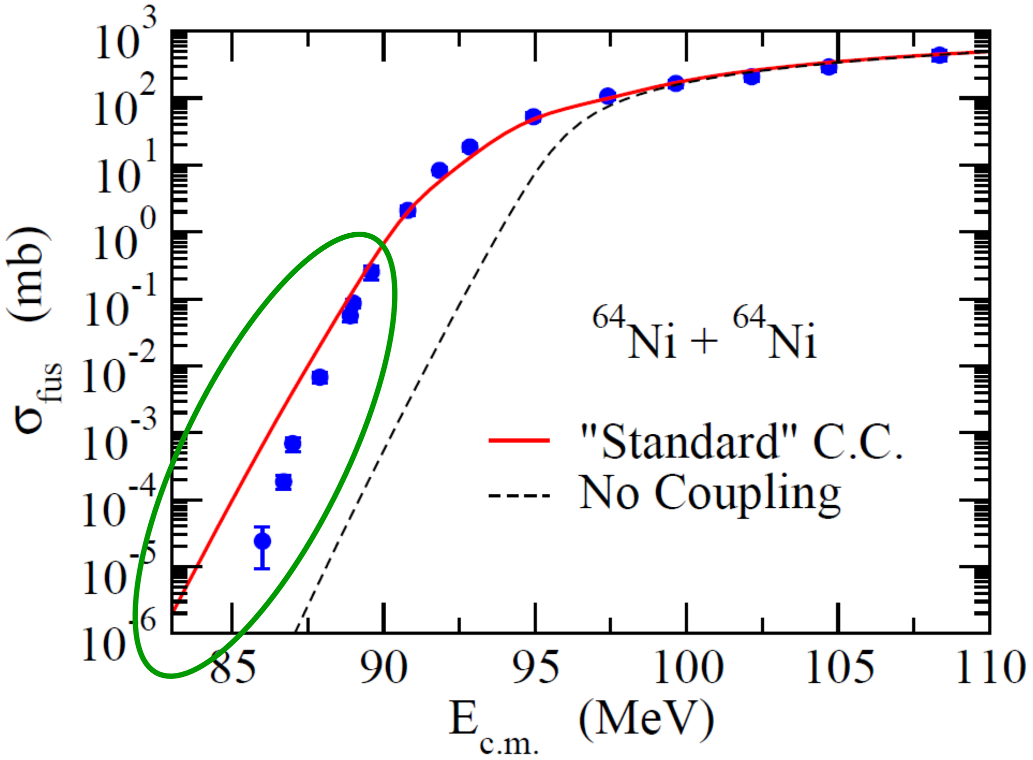
ab-initio, but no tunneling



C. Simenel,
EPJA48 ('12) 152

One of the remaining theoretical challenges

✓Deep sub-barrier hindrance of fusion cross sections



C.L. Jiang et al., PRL89('02)052701;
PRL93('04)012701

Theoretical models:

- ➢ Sudden model

  S. Misicu and H. Esbensen,
  PRL96('06)112701

  ✓ frozen density
  ✓ repulsive inner core

  ⟶ shallow potential

- ➢ Adiabatic model

  <u>T. Ichikawa</u>, K.H., and
  A. Iwamoto,
  PRL103('09)202701

  ✓ density change after the touching
  ✓ neck formation

  ⟶ deep and thick potential

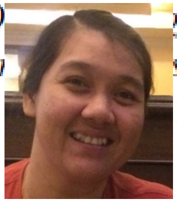
PHYSICAL REVIEW C 85, 057602 (2012)

Transition from subbarrier to deep-subbarrier regimes in heavy-ion fusion reactions

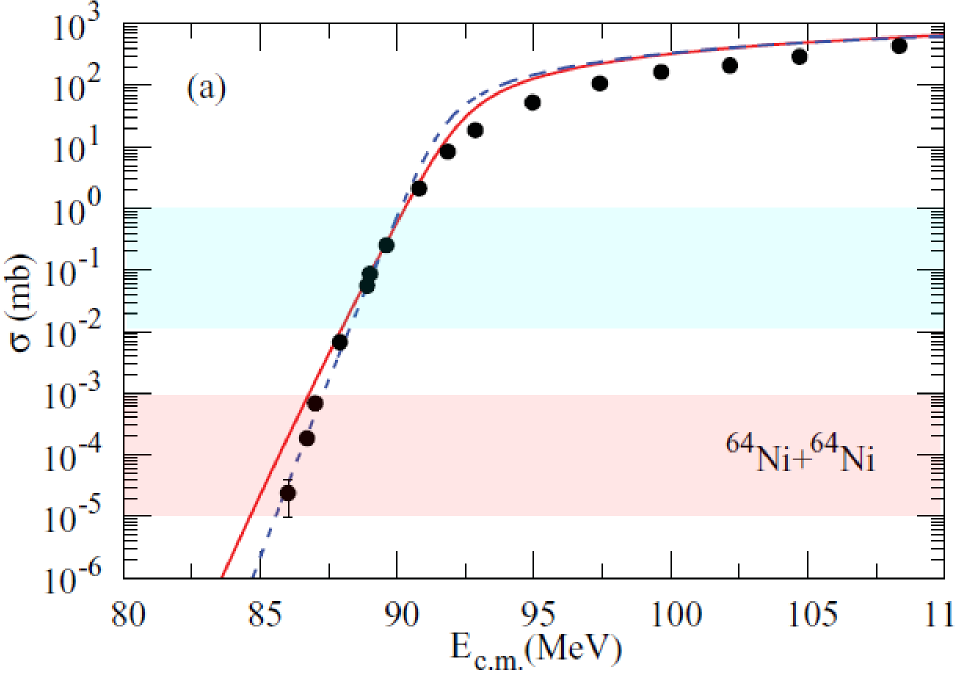
Ei Shwe Zin Thein,[1] N. W. Lwin,[1] and K. Hagino[2]

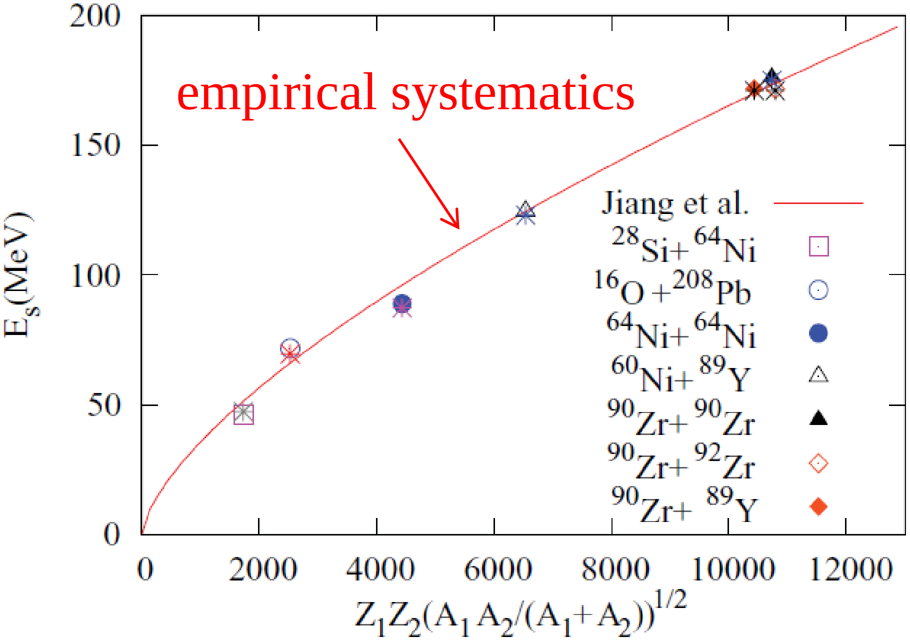
[1]D f Physics, University, Myanmar

[2]Departı sics, Toho ity, Sendai 980-8578, Japan



3 April 20 hed 16 May 2012)

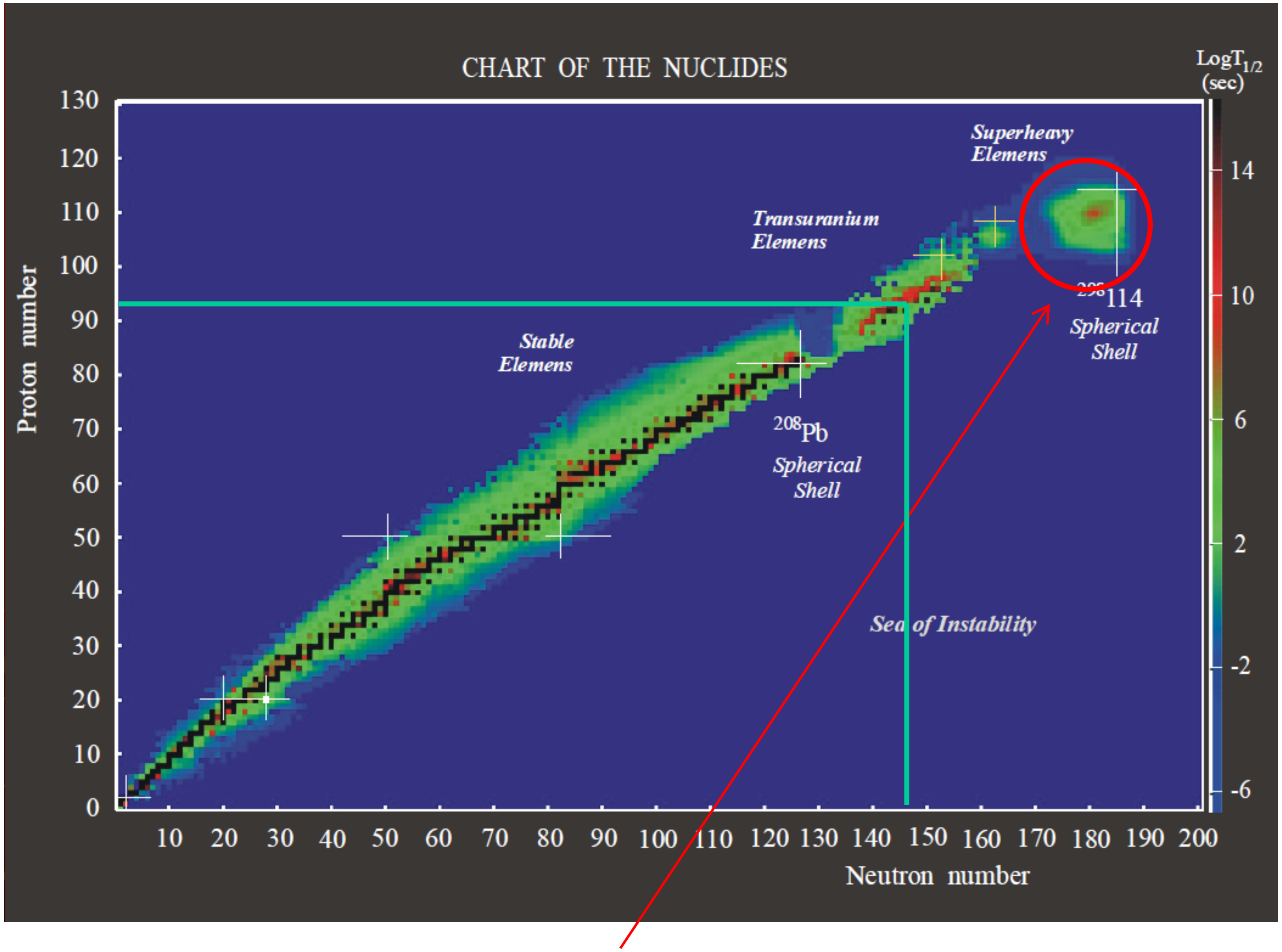




two region fits

→ crossing: threshold energy for hindrance

Future perspectives: Superheavy elements

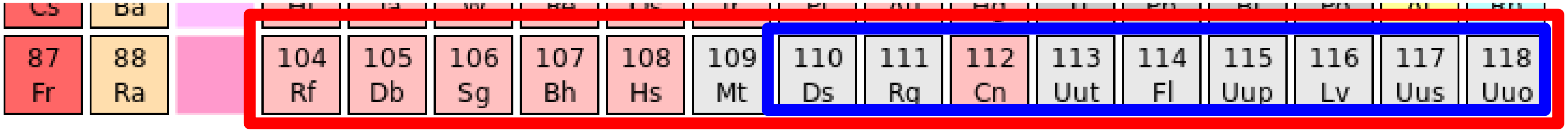


Yuri Oganessian

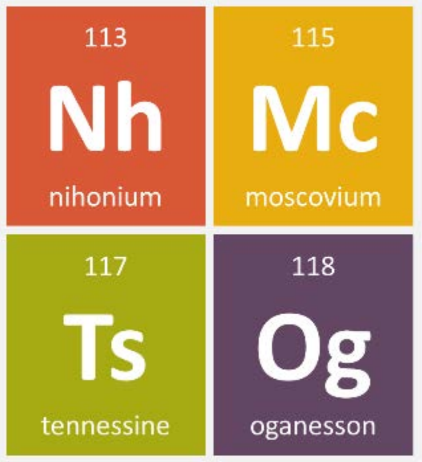
island of stability around Z=114, N=184

W.D. Myers and W.J. Swiatecki (1966), A. Sobiczewski et al. (1966)

who is she?



Z=110	Darmstadtium (Ds)	1994 Germany
Z=111	Roentgenium (Rg)	1994 Germany
Z=112	Copernicium (Cn)	1996 Germany
Z=113	**Nihonium (Nh)**	2003 Russia / 2004 Japan
Z=114	Flerovium (Fl)	1999 Russia
Z=115	**Moscovium (Mc)**	2003 Russia
Z=116	Livermoriun (Lv)	2000 Russia
Z=117	**Tennessine (Ts)**	2010 Russia
Z=118	**Oganesson (Og)**	2002 Russia

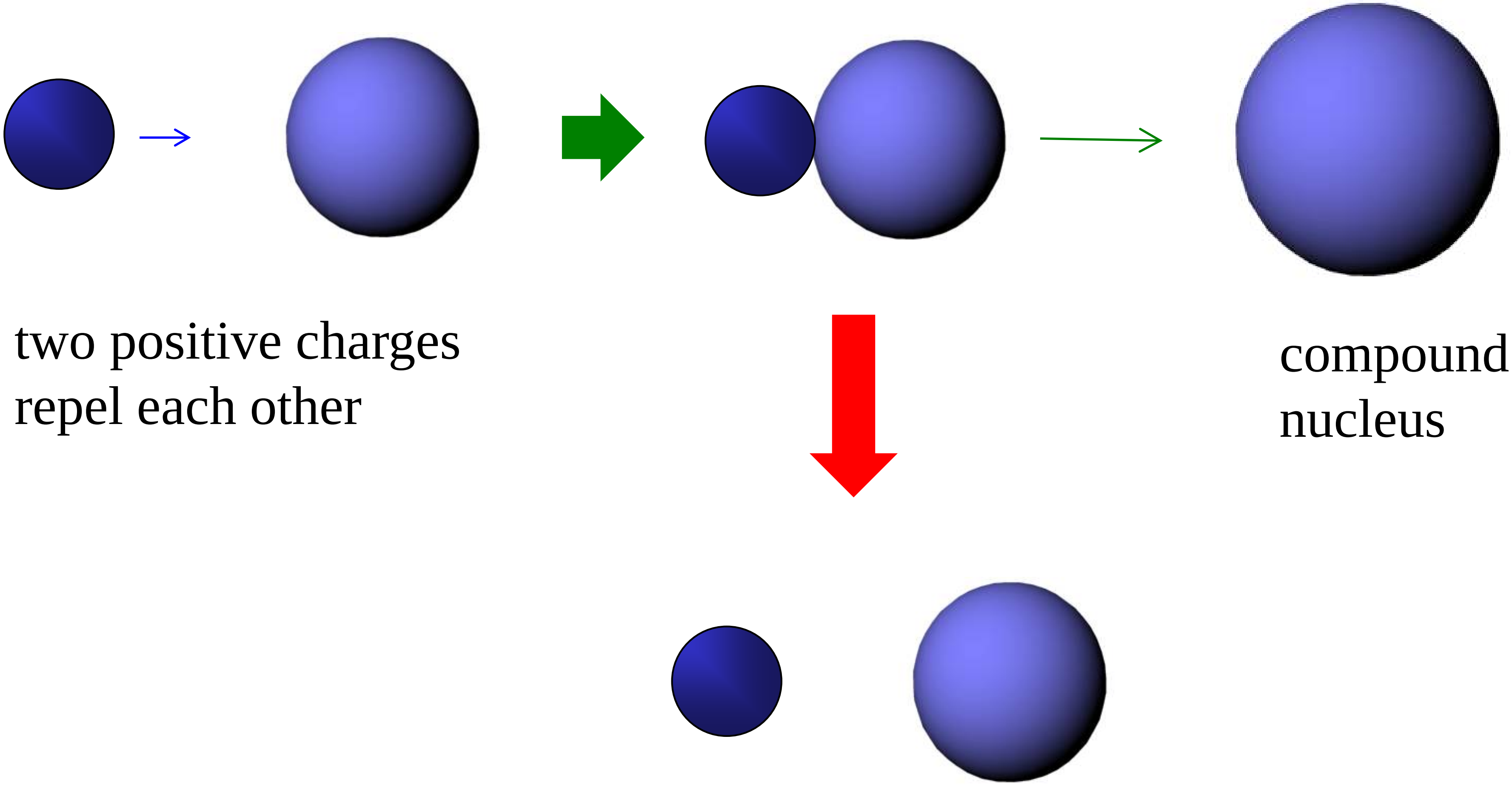


How to synthesize SHE?

Nuclear fusion reactions

e.g.,

$^{70}\mathrm{Zn} + {}^{209}\mathrm{Bi} \longrightarrow {}^{279}113$

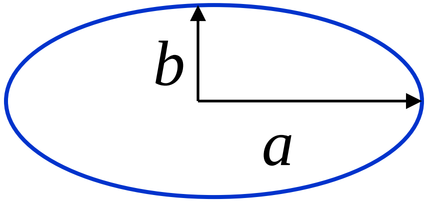


two positive charges
repel each other

compound
nucleus

re-separation

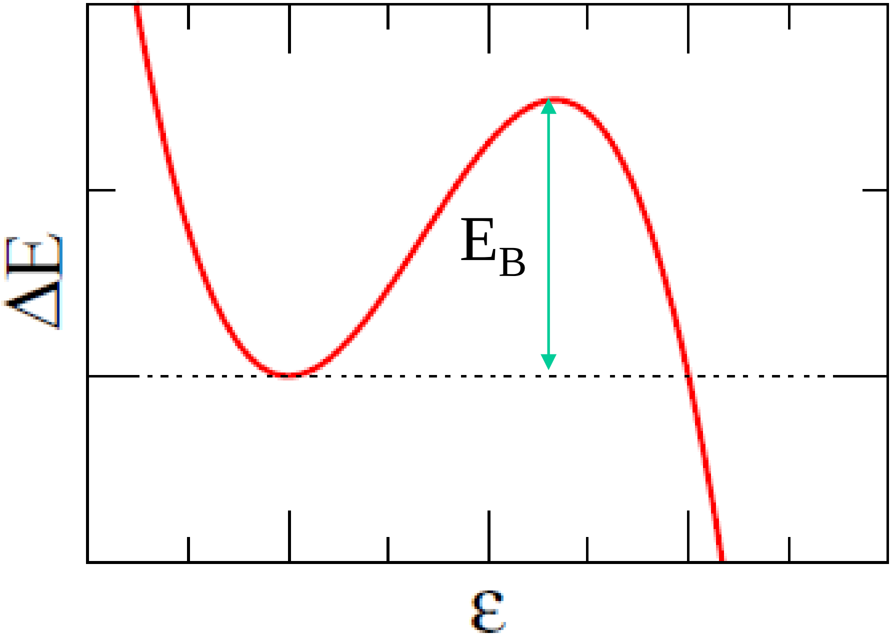
(note) fission barrier in the liquid drop model



$$\begin{aligned} a &= R\cdot(1+\epsilon) \\ b &= R\cdot(1+\epsilon)^{-1/2} \end{aligned}$$

$$ab^2 = R^3 = \text{constant}$$

$$\begin{aligned} \Delta E &= \Delta E_{\rm surf} + \Delta E_{\rm coul} \\ &= E_S^{(0)}\left\{\frac{2}{5}(1-x)\epsilon^2 - \frac{4}{105}(1+2x)\epsilon^3 + \cdots\right\} \end{aligned}$$

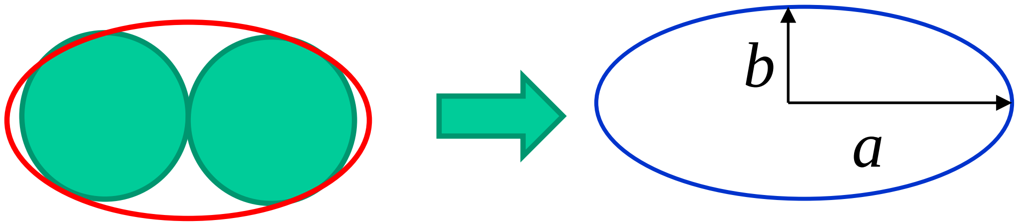


$$E_S^{(0)} = +a_S A^{2/3}$$

$$x \equiv \frac{E_C^{(0)}}{2E_S^{(0)}} = \frac{a_C}{2a_S}\cdot\frac{Z^2}{A} \sim \frac{1}{53.3}\cdot\frac{Z^2}{A}$$

$$E_C^{(0)} = a_C Z(Z-1)/A^{1/3}$$

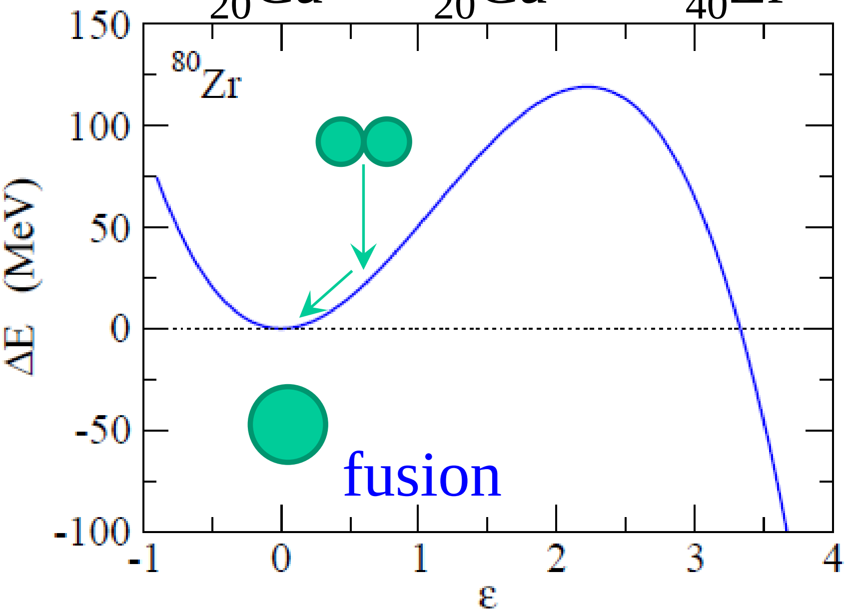
if two identical nuclei contact :



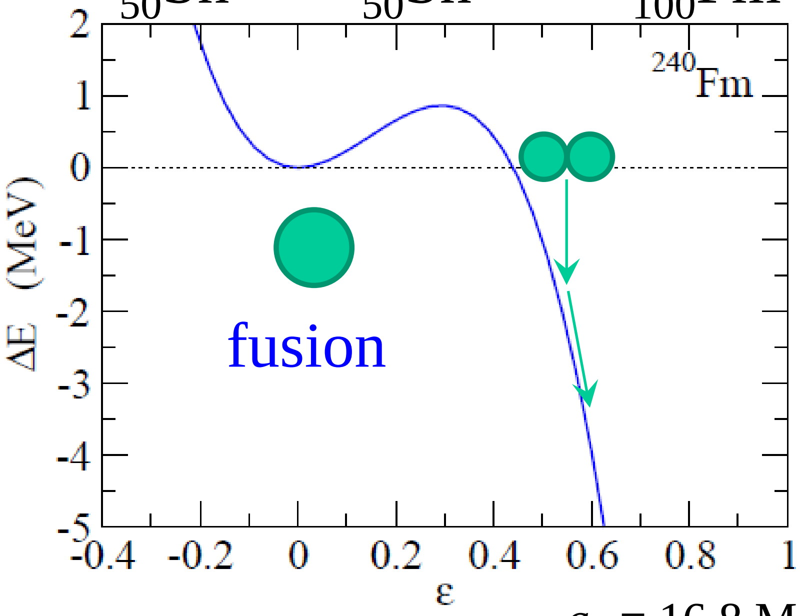
$$a = R_0 \cdot (1+\epsilon)$$
$$b = R_0 \cdot (1+\epsilon)^{-1/2}$$

$$\frac{a}{b} \sim \frac{2R}{R} = 2 \to \epsilon \sim 0.587$$

${}^{40}_{20}\mathrm{Ca} + {}^{40}_{20}\mathrm{Ca} \longrightarrow {}^{80}_{40}\mathrm{Zr}$

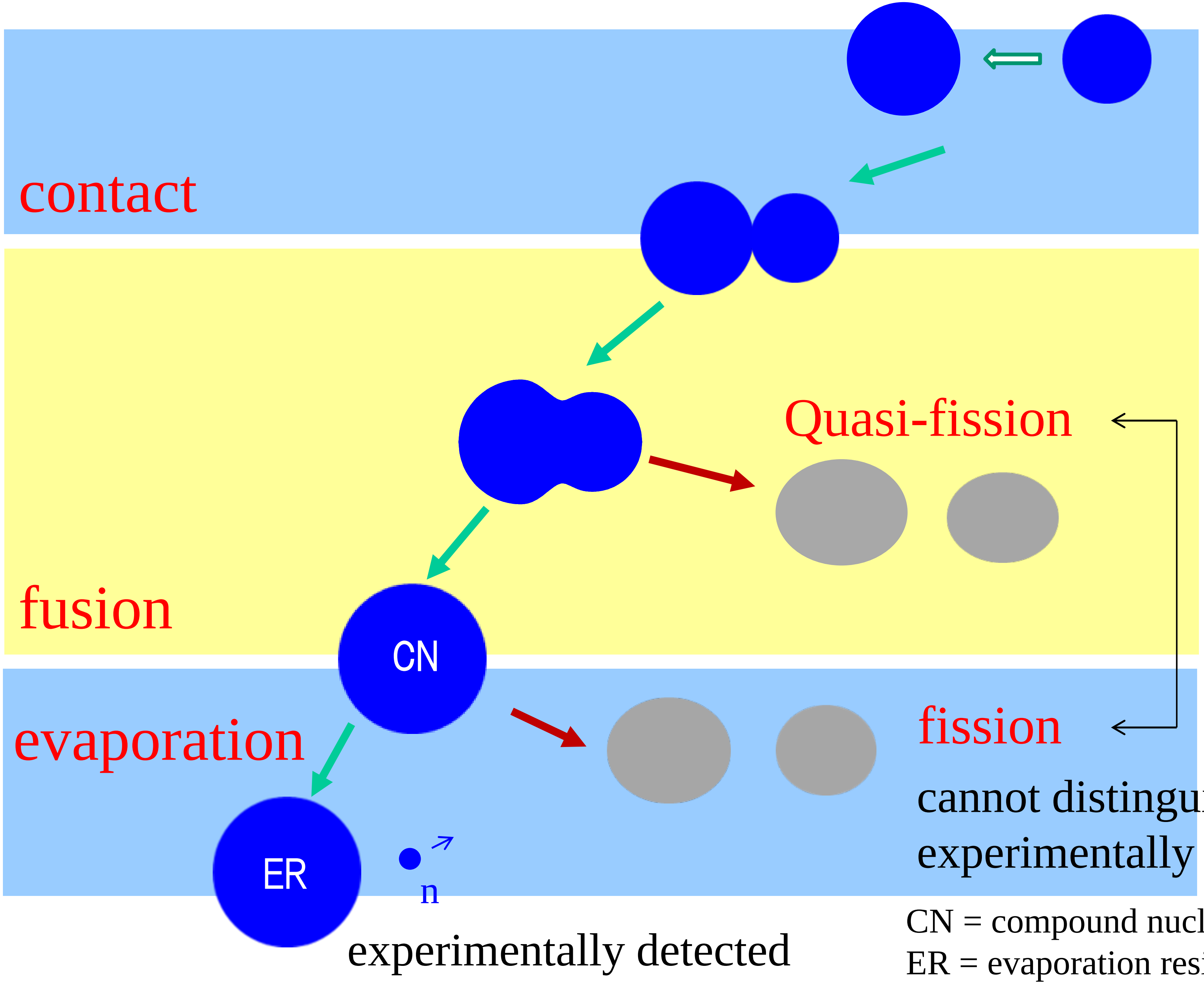


${}^{120}_{50}\mathrm{Sn} + {}^{120}_{50}\mathrm{Sn} \longrightarrow {}^{240}_{100}\mathrm{Fm}$



threshold: $Z_1 * Z_2 = 1600 \sim 1800$

$a_S = 16.8$ MeV
$a_C = 0.72$ MeV



contact

fusion

Quasi-fission

evaporation

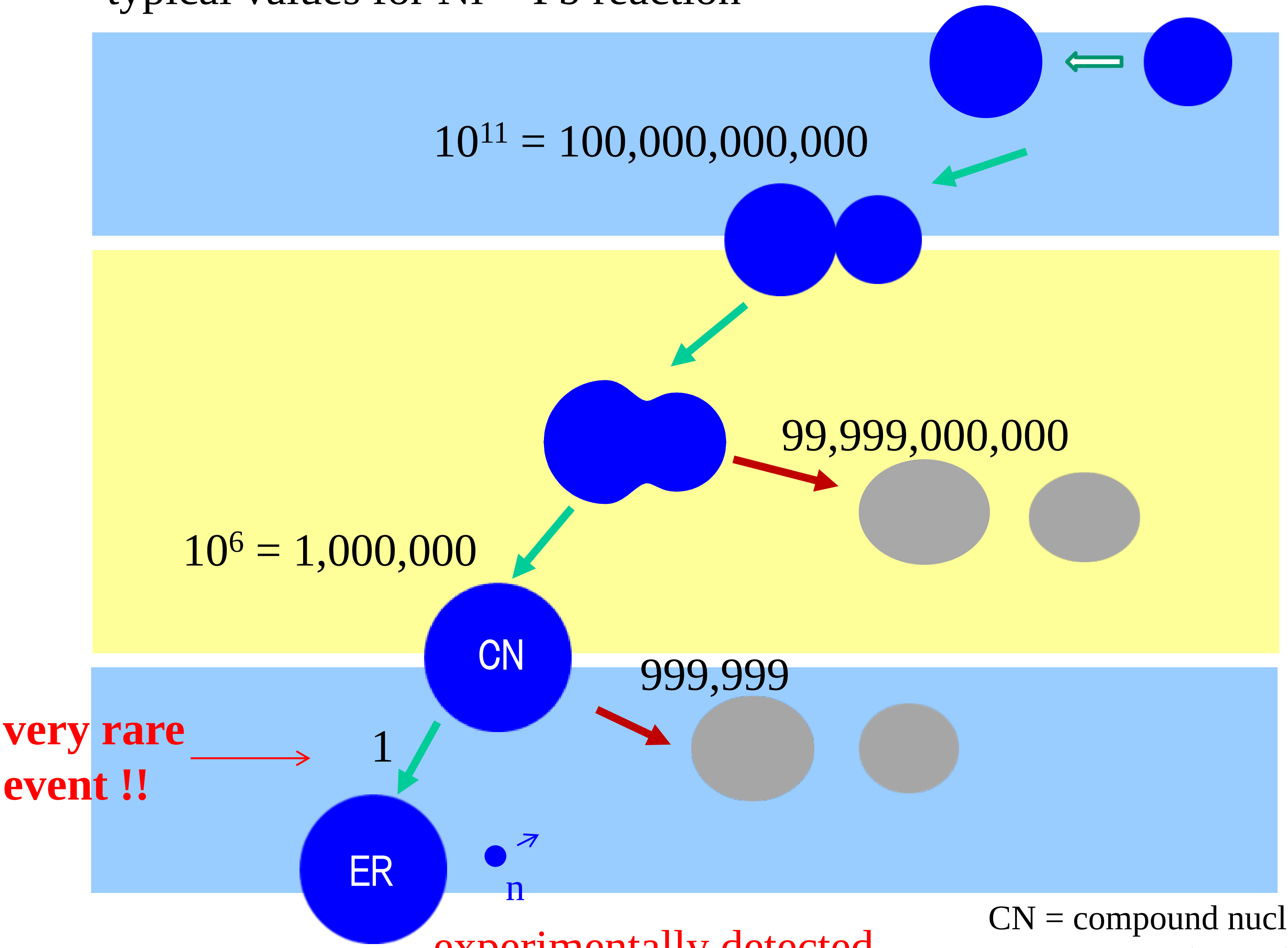
fission

cannot distinguish experimentally

experimentally detected

CN = compound nucleus
ER = evaporation residue

typical values for Ni + Pb reaction



10^{11} = 100,000,000,000

99,999,000,000

10^6 = 1,000,000

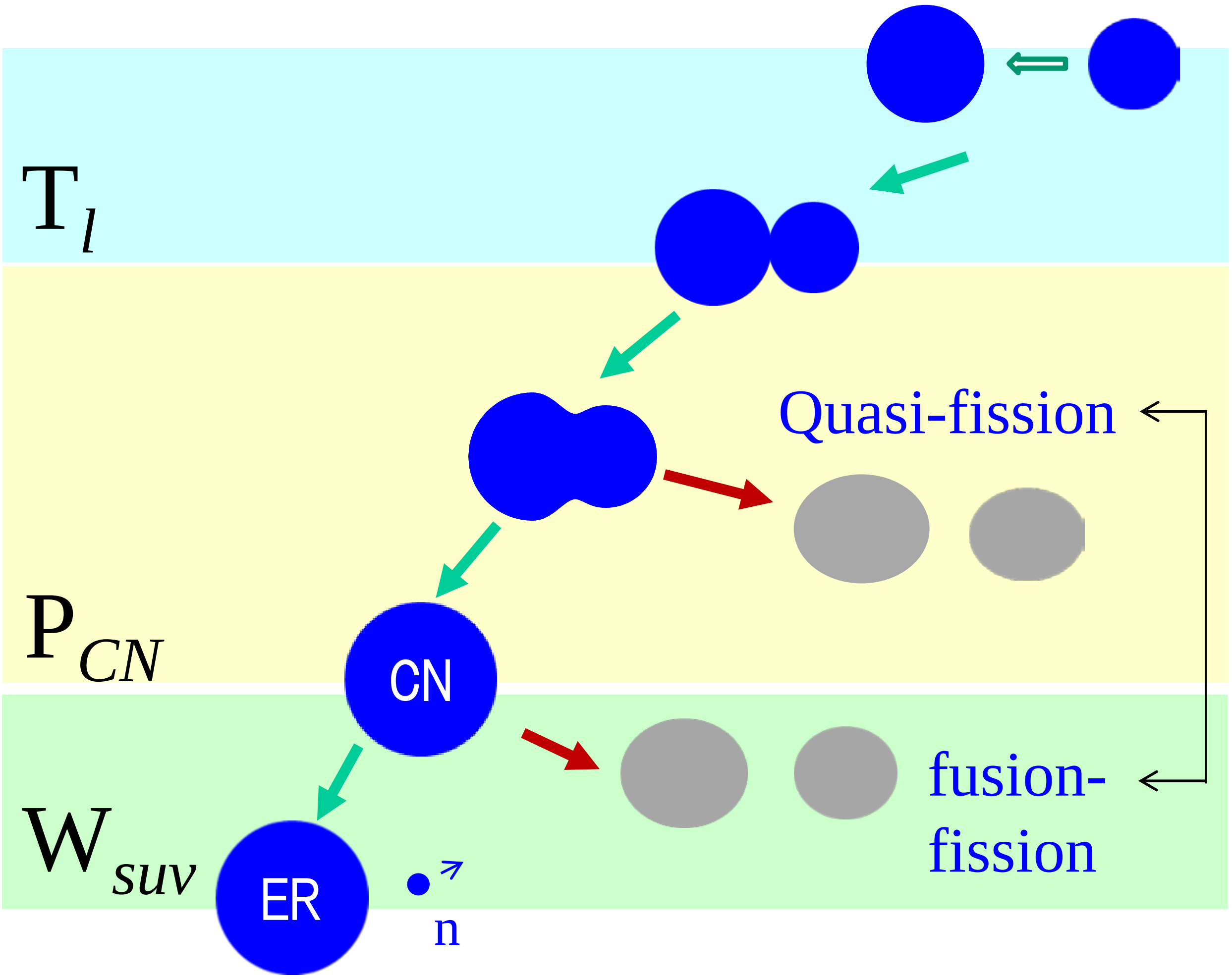
999,999

very rare event !!

1

experimentally detected

CN = compound nucleus
ER = evaporation residue



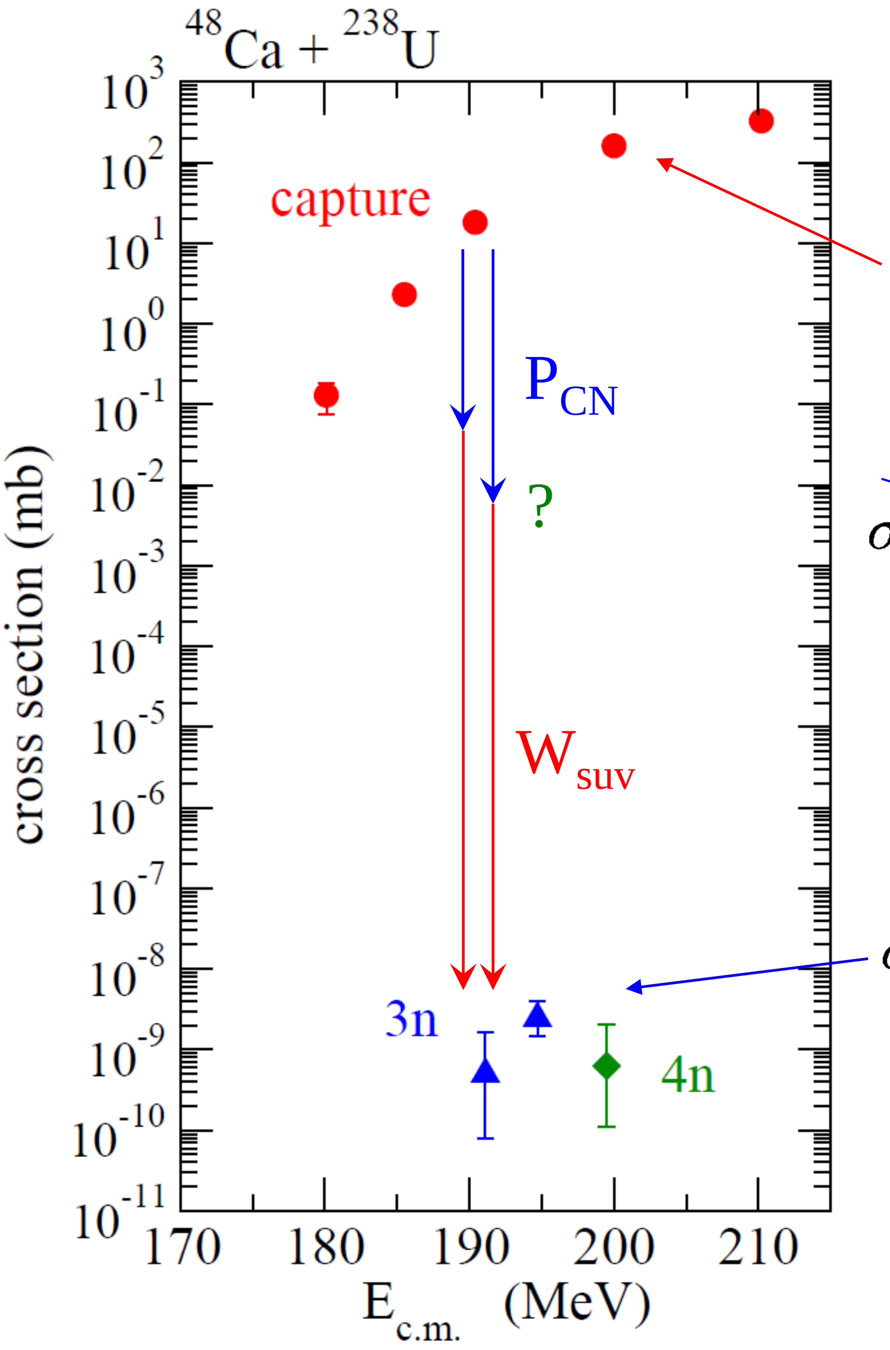
Coupled-channels

Langevin approach

statistical model

CN = compound nucleus
ER = evaporation residue

$$\boxed{\sigma_{\mathrm{ER}}(E) = \frac{\pi}{k^2}\sum_l (2l+1)T_l(E)\,P_{\mathrm{CN}}(E,l)\,W_{\mathrm{suv}}(E^*,l)}$$



no experimental data for P_{CN}

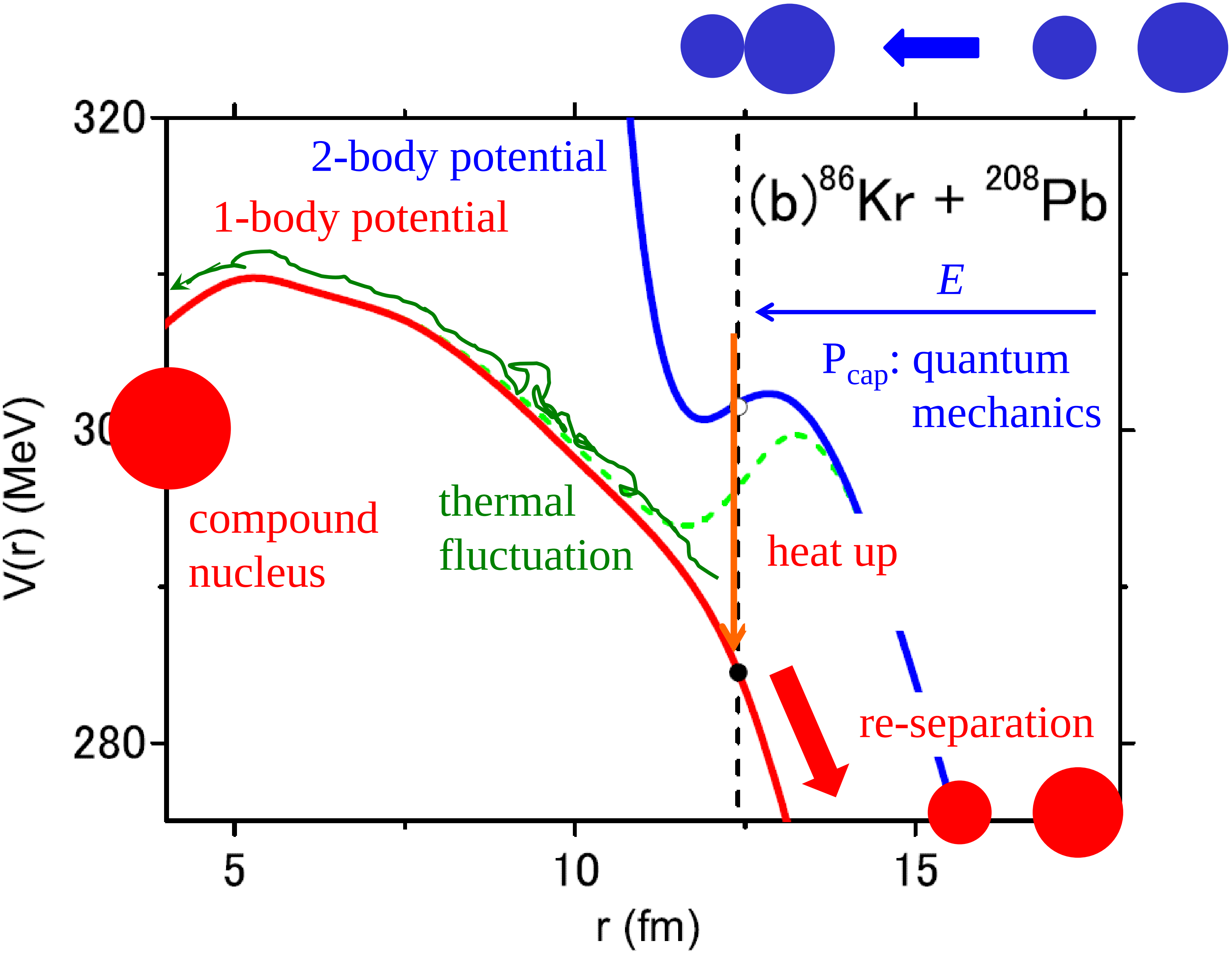
$$\sigma_{\rm cap}(E) = \frac{\pi}{k^2}\sum_l (2l+1)T_l(E)$$

$$\xcancel{\sigma_{\rm CN}(E) = \frac{\pi}{k^2}\sum_l (2l+1)T_l(E) \times P_{\rm CN}}$$

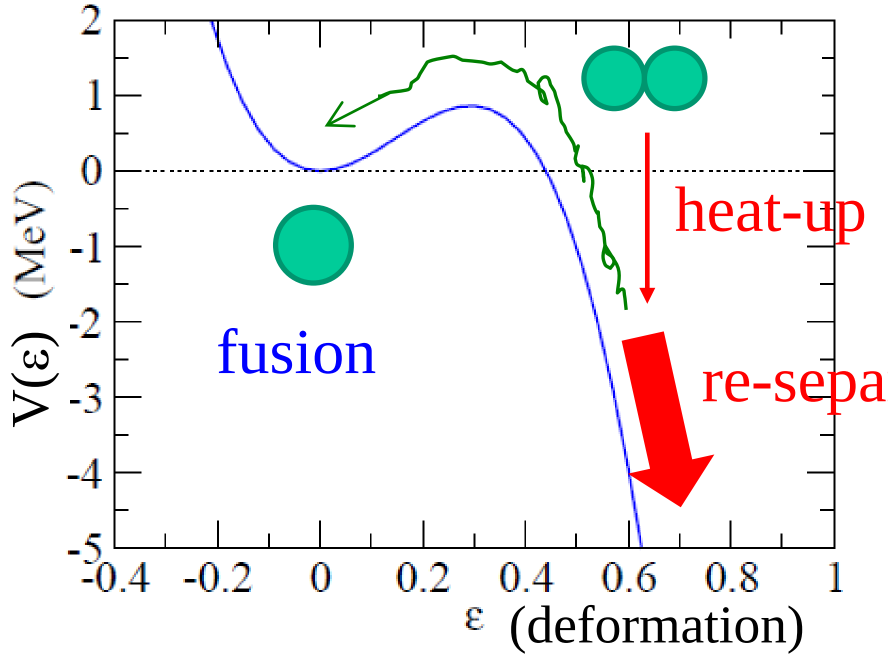
not available

$$\sigma_{\rm ER}(E) = \frac{\pi}{k^2}\sum_l (2l+1)T_l(E) \times \boxed{P_{\rm CN}\cdot W_{\rm suv}}$$

large uncertainties


2-body potential
1-body potential
(b)^{86}Kr + ^{208}Pb
E
P_{cap}: quantum mechanics
compound nucleus
thermal fluctuation
heat up
re-separation
320
30
280
5
10
15
r (fm)
V(r) (MeV)

Langevin approach



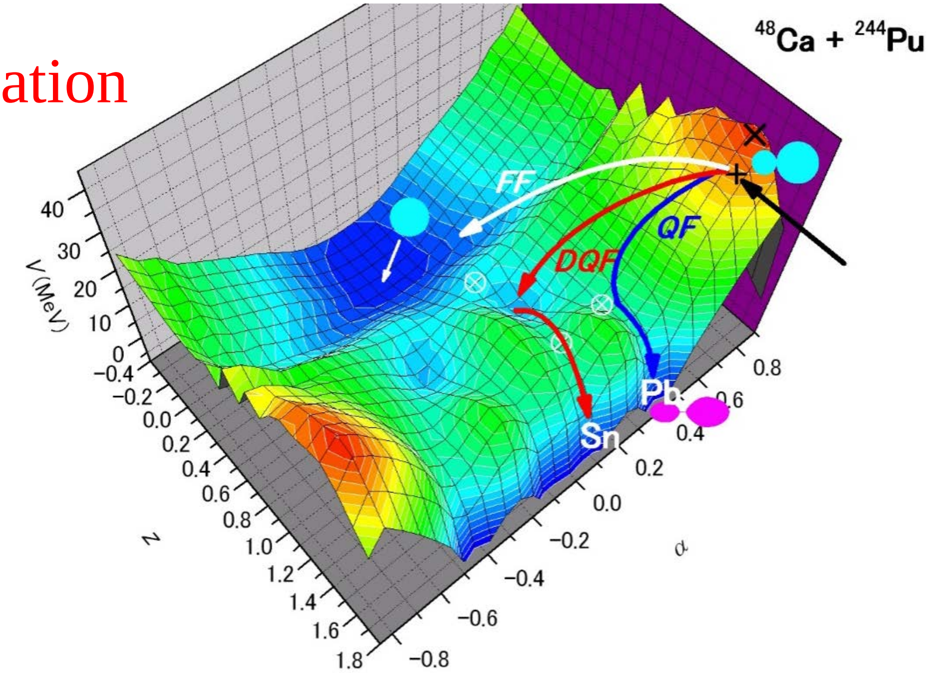
thermal fluctuation

⟶ Langevin method
(Brownian method)

$$m\frac{d^2q}{dt^2} = -\frac{dV(q)}{dq} - \gamma\frac{dq}{dt} + R(t)$$

multi-dimensional extention

q:
- internuclear separation,
- deformation,
- asymmetry of the two fragments



γ: friction coefficient

R(*t*): random force

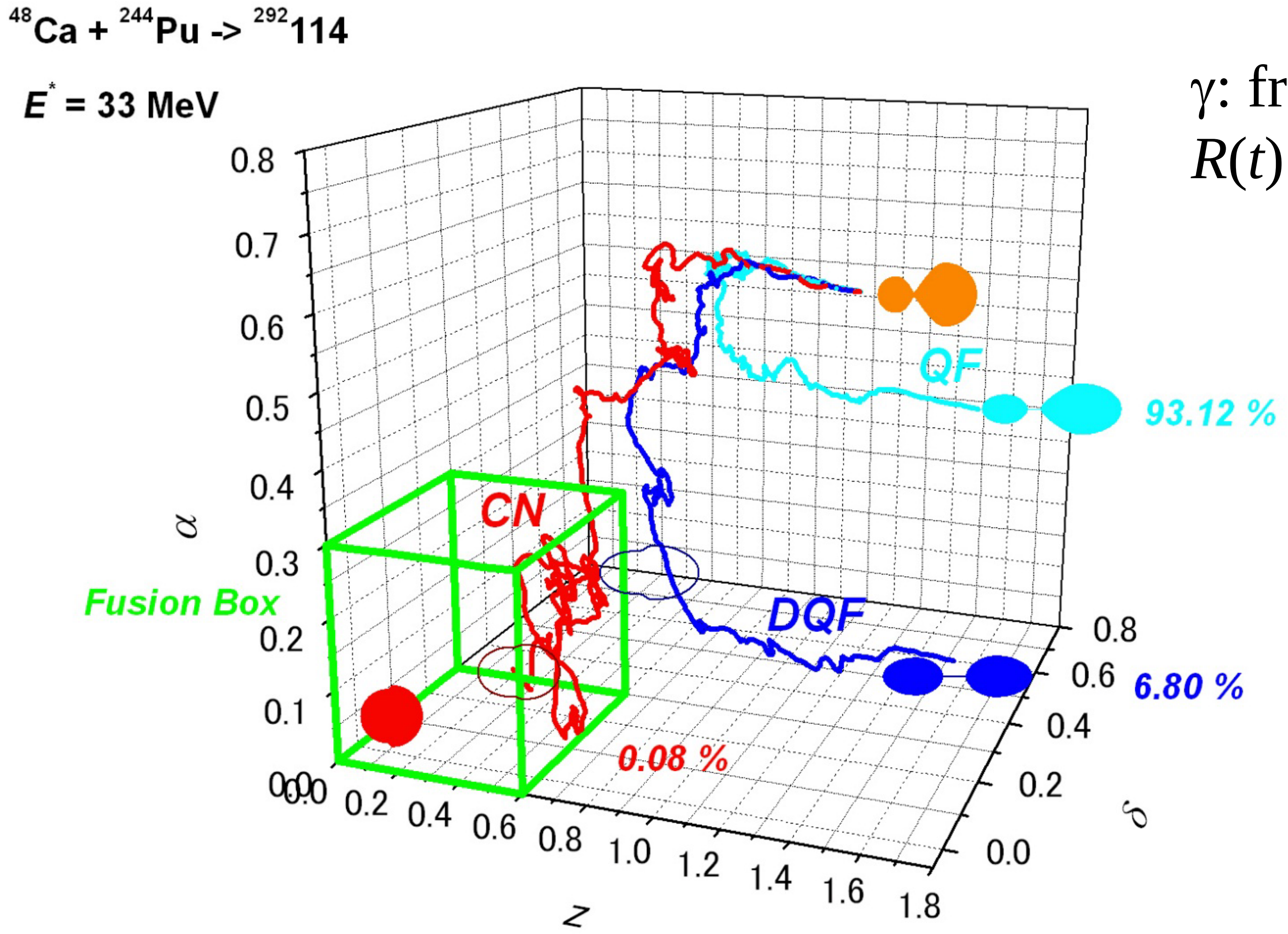
Theory: Lagenvin approach

multi-dimensional extension of:

$$m\frac{d^2q}{dt^2} = -\frac{dV(q)}{dq} - \gamma\frac{dq}{dt} + R(t)$$

γ: friction coefficient

$R(t)$: random force



Quantum friction

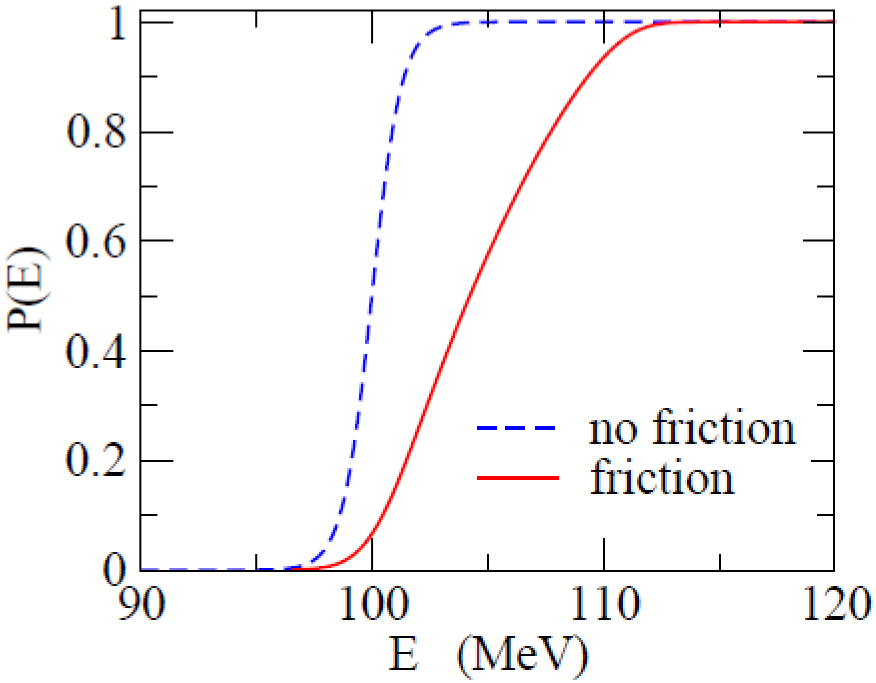
classical eq. of motion $\dot{p} = -V'(x) - \gamma p$

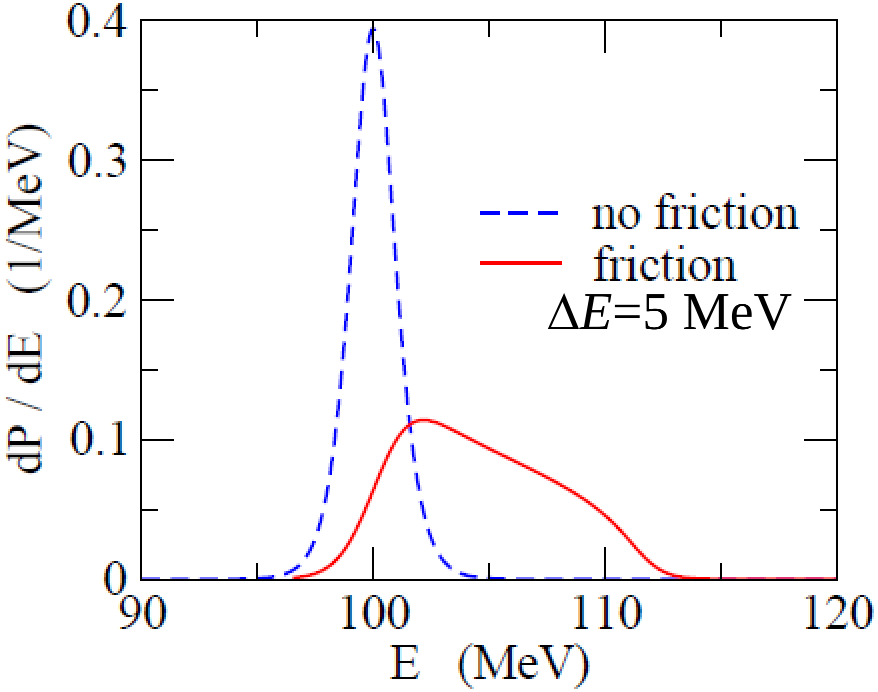
a quantization: Kanai model E. Kanai, PTP 3 (1948) 440)

$$\boxed{H = \frac{p^2}{2m} + V(x) \rightarrow \frac{\pi^2}{2m} e^{-\gamma t} + e^{\gamma t} V(x) \qquad (\pi = e^{\gamma t} p)}$$

(a quantal Hamiltonian which reproduces the classical eq. of motion)

time-dep. wave packet approach

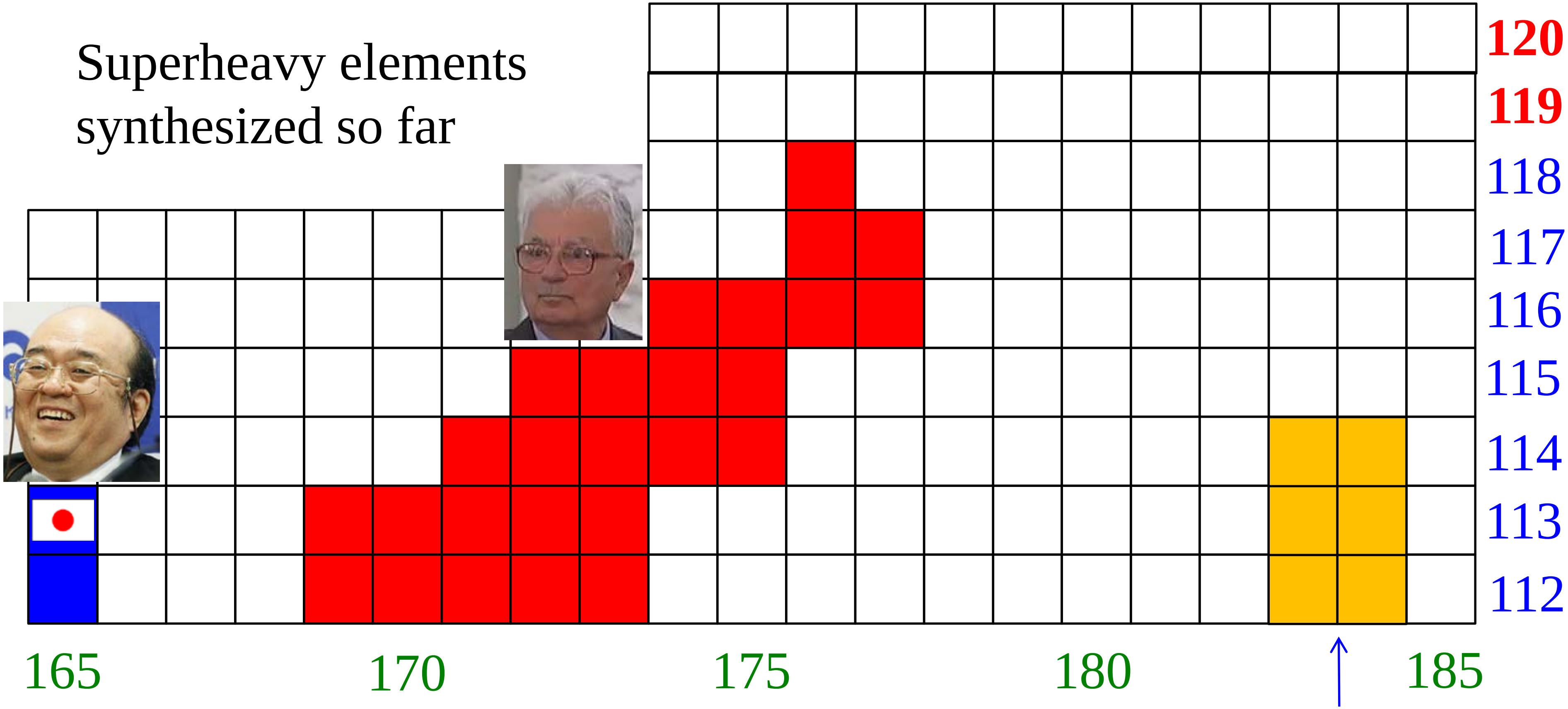




M. Tokieda and K.H., PRC95 ('17) 054604

Future directions

Superheavy elements synthesized so far



the island of stability?

- ➢Towards Z=119 and 120 nuclei

  reaction dynamics? reliable prediction of fusion cross sections?

- ➢Towards the island of stability

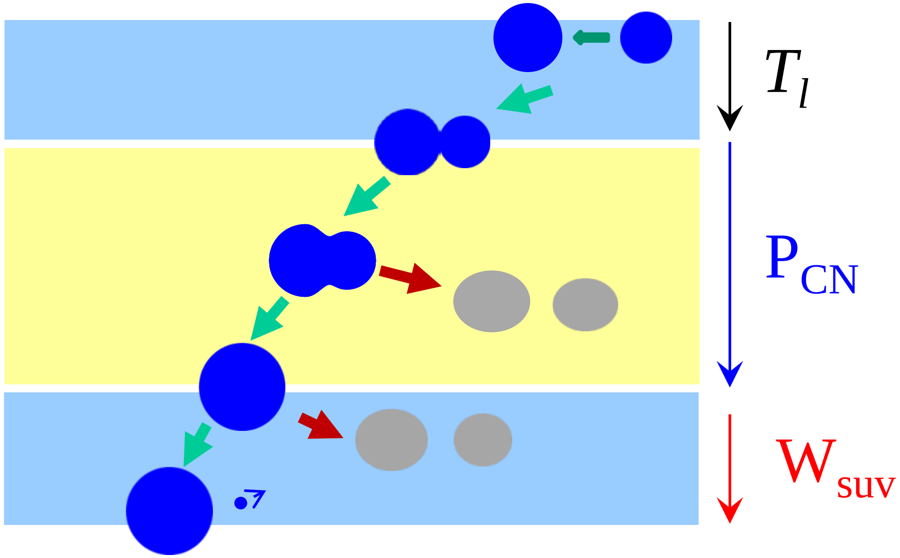
  neutron-rich beams: indispensable

Future directions -1

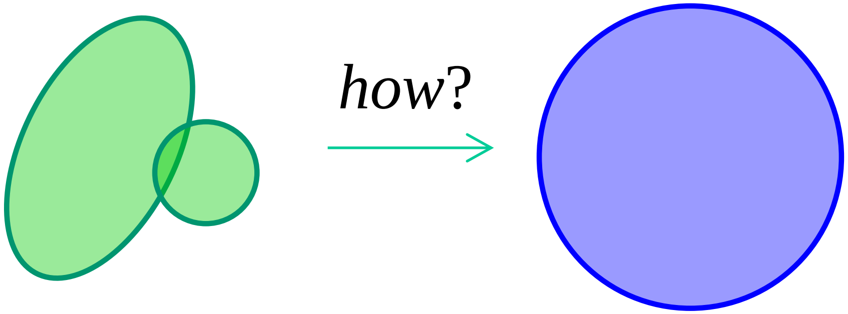
- Towards Z=119 and 120 nuclei

^{48}Ca projectile (hot fusion) → $^{50}_{22}$Ti, $^{51}_{23}$V, $^{54}_{24}$Cr projectile
+ deformed target nucleus

needs a proper understanding of deformation effects on SHE synthesis reactions



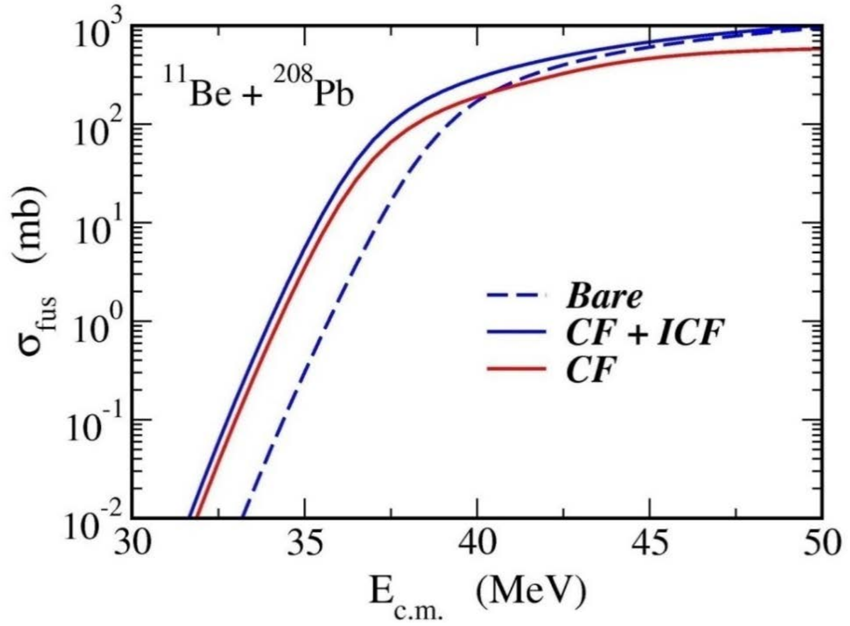
deformation effects on P_{CN} ?



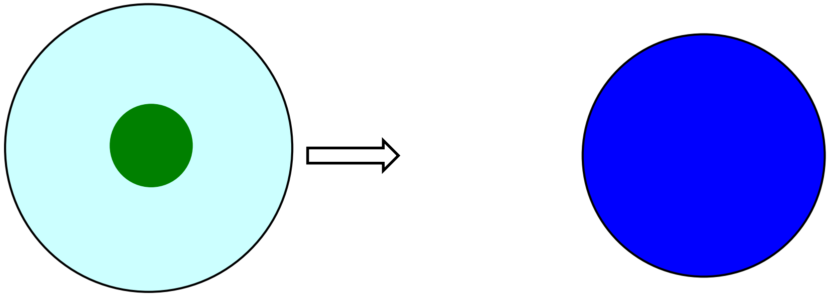
Future directions - 2

- Towards the island of stability

  neutron-rich beams: indispensable



K. Hagino, A. Vitturi, C.H. Dasso,
and S.M. Lenzi, Phys. Rev. C61 ('00) 037602



simultaneous treatment
of breakup and transfer

→ an important future problem

Summary

Heavy-ion fusion reactions around the Coulomb barrier

- ✓Strong interplay between nuclear structure and reaction
- ✓Quantum tunneling with various intrinsic degrees of freedom
- ✓coupled-channels approach

Remaining challenges

- ✓ microscopic understanding of heavy-ion fusion reactions

Future perspectives: superheavy elements

- ✓ how to reduce theoretical uncertainties?
- ✓ Towards heavier SHE (Z = 119, 120)
- ✓ Towards the island of stability

investigations of physics of SHE with neutron-rich nuclei as a keyword

ကျေးဇူးအများကြီးတင်ပါတယ်

Thank you very much!

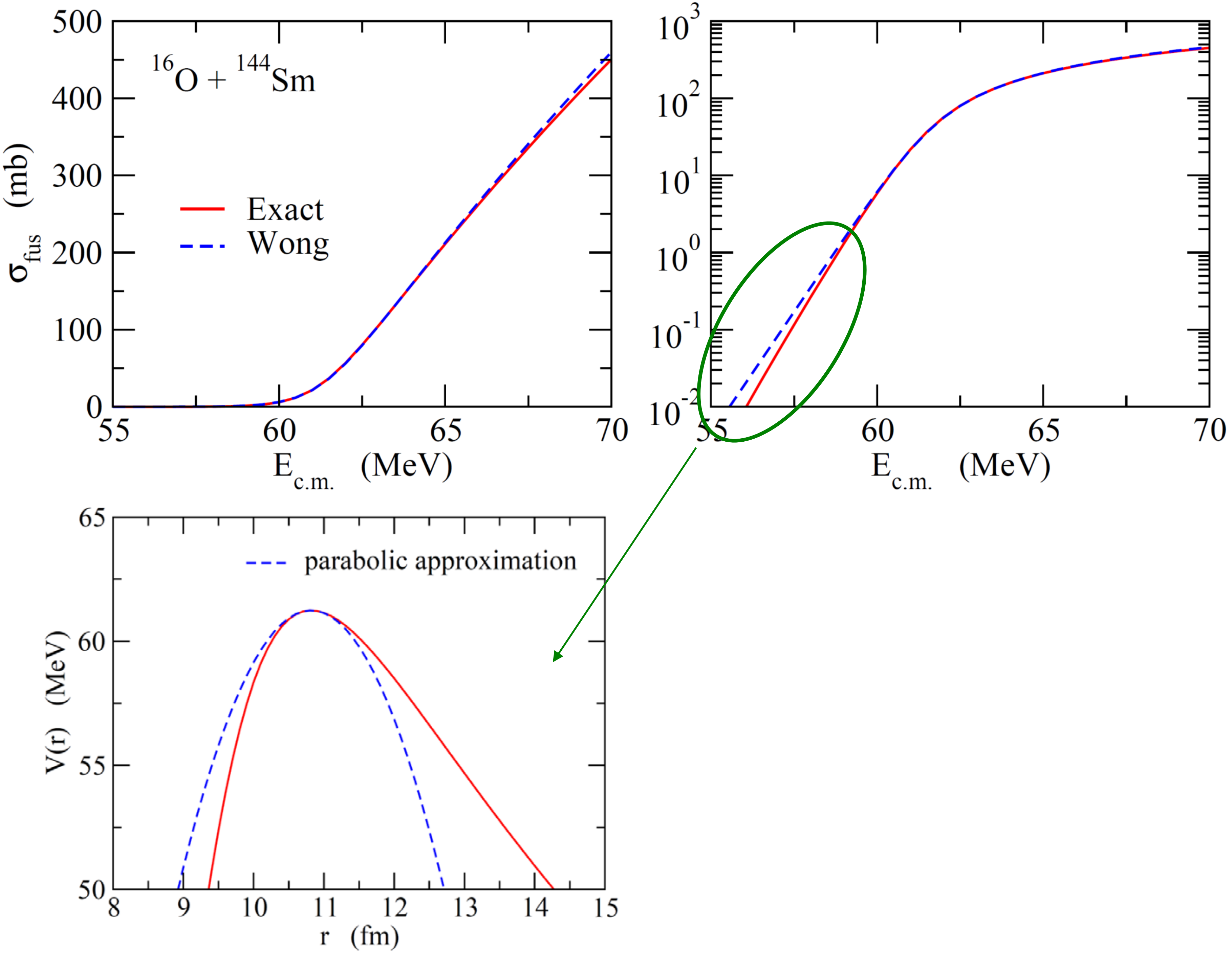
ငါမကြာမီနောက်တဖန်သင်တို့ကိုတွေ့မြင်ဖို့မျှော်လင့်

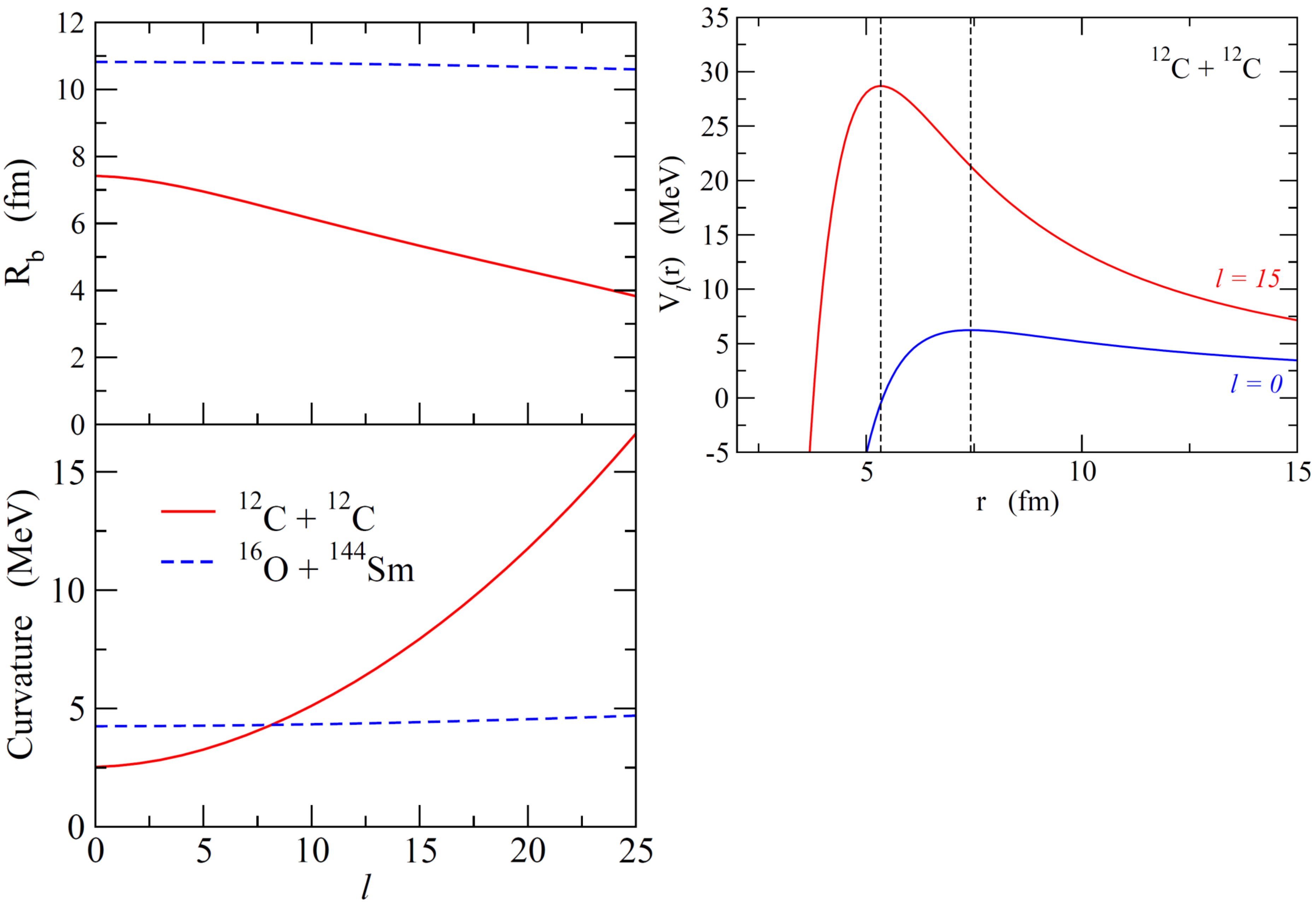
I hope to see you again soon!

ကျေးဇူးအများကြီးတင်ပါတယ်

Thank you very much

....especially to Swe-san, Nyein-san,
Than-san, and Kalyar-san


^{16}O + ^{144}Sm
σ_{fus} (mb)
Exact
Wong
$E_{c.m.}$ (MeV)
500
400
300
200
100
0
55
60
65
70
10^3
10^2
10^1
10^0
10^{-1}
10^{-2}
parabolic approximation
V(r) (MeV)
r (fm)
65
60
55
50
8
9
10
11
12
13
14
15



12
10
8
6
4
2
0
R_b (fm)
15
10
5
0
Curvature (MeV)
^{12}C + ^{12}C
^{16}O + ^{144}Sm
0
5
10
15
20
25
l
35
30
25
20
15
10
5
0
-5
V_l(r) (MeV)
^{12}C + ^{12}C
l = 15
l = 0
5
10
15
r (fm)