


Separable RPA

Separable interaction: $\langle ph' | \bar{v} | hp' \rangle = \lambda D_{ph} D_{p'h'}^*$

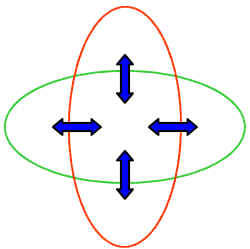

$$\frac{1}{\lambda} = \sum_{ph} \frac{|D_{ph}|^2}{E - \epsilon_{ph}} - \frac{|D_{ph}|^2}{E + \epsilon_{ph}} \quad (\text{RPA dispersion relation})$$

Vibrating Potential Model

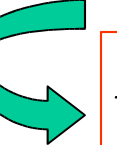
$$\hat{h}_0 = -\frac{\hbar^2}{2m} \nabla^2 + V_0(r)$$

$$V_0(r) = -V_0 / [1 + \exp((r - R)/a)]$$

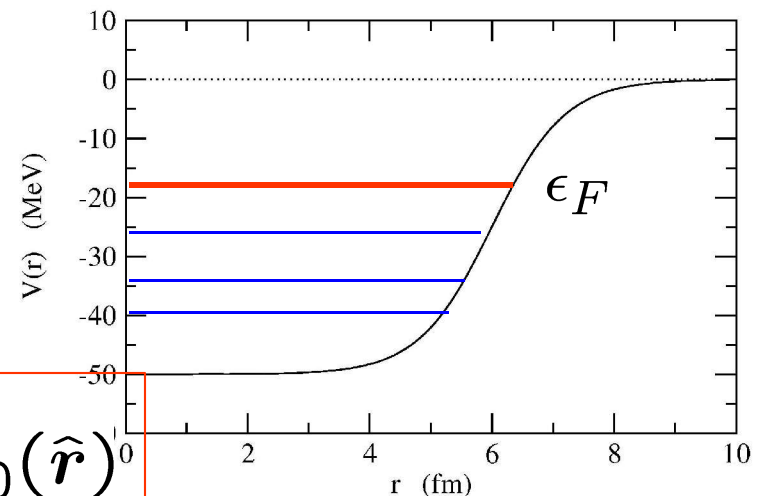
Oscillation of the surface:



$$R \rightarrow R[1 + \alpha_{\lambda 0}(t) Y_{\lambda 0}(\hat{r})]$$

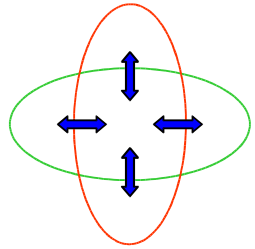

$$V_0(r) \rightarrow V_0(r) - R \frac{dV_0}{dr} \alpha_{\lambda 0} Y_{\lambda 0}(\hat{r})$$

Hartree-Fock state



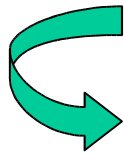
Vibrating Potential Model

$$\hat{h}_0 = -\frac{\hbar^2}{2m} \nabla^2 + V_0(r) \quad V_0(r) = -V_0/[1 + \exp((r - R)/a)]$$



Oscillation of the surface:

$$R \rightarrow R[1 + \alpha_{\lambda 0}(t) Y_{\lambda 0}(\hat{\mathbf{r}})]$$

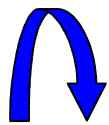


$$V_0(r) \rightarrow V_0(r) - R \frac{dV_0}{dr} \alpha_{\lambda 0} Y_{\lambda 0}(\hat{\mathbf{r}})$$

Self-Consistency:

$$\rho_0(r) \rightarrow \rho_0(r) - R \frac{d\rho_0}{dr} \alpha_{\lambda 0} Y_{\lambda 0}(\hat{\mathbf{r}})$$

(note) For $v(\mathbf{r}, \mathbf{r}') = -\frac{\kappa}{2} F_{\lambda 0}(\mathbf{r}) F_{\lambda 0}(\mathbf{r}')$



$$\longrightarrow V_{MF}(\mathbf{r}) = -\kappa \left(\int d\mathbf{r}' F_{\lambda 0}(\mathbf{r}') \rho(\mathbf{r}') \right) \cdot F_{\lambda 0}(\mathbf{r})$$

$$F_{\lambda 0}(\mathbf{r}) = R \frac{dV_0}{dr} Y_{\lambda 0}(\hat{\mathbf{r}}), \quad \kappa^{-1} = -R^2 \int r^2 dr \frac{d\rho_0}{dr} \cdot \frac{dV_0}{dr}$$

Summary: the **separable interaction** that induces the surface oscillation $R \rightarrow R[1 + \sum_{\mu} \alpha_{\lambda\mu}(t) Y_{\lambda\mu}^*(\hat{r})]$ is

$$v(\mathbf{r}, \mathbf{r}') = -\frac{\kappa}{2} \sum_{\mu} F_{\lambda\mu}(\mathbf{r}) F_{\lambda\mu}^*(\mathbf{r}')$$

$$\text{with } F_{\lambda\mu}(\mathbf{r}) = R \frac{dV_0}{dr} Y_{\lambda\mu}(\hat{r})$$

$$\kappa^{-1} = -R^2 \int r^2 dr \frac{d\rho_0}{dr} \cdot \frac{dV_0}{dr}$$



Self-consistency condition
(between potential and density)

Microscopic Foundation

(note) Vibrations in quantum mechanics: *density vibration?*


Eigen states of a Hamiltonian \longrightarrow static density

$$\Psi_0(t) = e^{-iE_0 t/\hbar} \Psi_0 \rightarrow \rho_0(\mathbf{r}) = \left\langle \Psi_0(t) \left| \sum_i \delta(\mathbf{r} - \mathbf{r}_i) \right| \Psi_0(t) \right\rangle$$

Time dependent density \longrightarrow need to consider a wave packet

$$\Psi(t) = e^{-iE_0 t/\hbar} \Psi_0 + i\epsilon e^{-iE_\nu t/\hbar} \Psi_\nu$$

Ψ_ν : an excited eigenstate


$$\begin{aligned} \rho(\mathbf{r}, t) &= \left\langle \Psi(t) \left| \sum_i \delta(\mathbf{r} - \mathbf{r}_i) \right| \Psi(t) \right\rangle \\ &= \rho_0(\mathbf{r}) + 2\epsilon \sin(\omega t) \rho_{0\nu}(\mathbf{r}) + O(\epsilon^2) \end{aligned}$$

$$\rho_{0\nu}(\mathbf{r}) = \left\langle \Psi_0(t) \left| \sum_i \delta(\mathbf{r} - \mathbf{r}_i) \right| \Psi_\nu(t) \right\rangle \quad \text{transition density}$$

Collective ansatz: $\Psi_\nu = \hat{Q}\Psi_0$

$$|\Psi(t = 0_+)\rangle = e^{i\epsilon\hat{Q}} |\Psi_0\rangle$$

→ Time evolution:

$$\begin{aligned} e^{-iHt/\hbar} |\Psi\rangle &\sim (1 - iHt/\hbar)(1 + i\epsilon\hat{Q}) |\Psi_0\rangle \\ &= (1 + i\epsilon\hat{Q} + t[H, \hat{Q}]/\hbar) |\Psi_0\rangle \end{aligned}$$

Ansatz:

$$|\Psi(t)\rangle = e^{i\alpha(t)\hat{Q}} e^{\beta(t)m[H, \hat{Q}]} |\Psi_0\rangle$$


(note) time dependence of $\alpha(t)$ and $\beta(t)$

← Time-dependent variational principle

Ref. G.F. Bertsch and H. Feldmeier, PRC56('97)839

(note) for a local Q and mom. independent interaction

$$\rightarrow Q_1 \equiv m[H, \hat{Q}] = -\frac{1}{2}\nabla^2\hat{Q} - \nabla\hat{Q} \cdot \nabla$$


$$\rho(\mathbf{r}, t) = \langle \Psi(t) | \sum_i \delta(\mathbf{r} - \mathbf{r}_i) | \Psi(t) \rangle$$

$$\sim \langle \Psi_0 | (1 - i\alpha\hat{Q} - \beta\hat{Q}_1) \sum_i \delta(\mathbf{r} - \mathbf{r}_i) (1 + i\alpha\hat{Q} + \beta\hat{Q}_1) | \Psi_0 \rangle$$

$$= \rho_0(\mathbf{r}) + \delta\rho(\mathbf{r}, t)$$

$$\delta\rho(\mathbf{r}, t) = -\beta \langle \Psi_0 | [\hat{Q}_1, \sum_i \delta(\mathbf{r} - \mathbf{r}_i)] | \Psi_0 \rangle$$

$$= -\beta \nabla \hat{Q} \cdot \nabla \rho_0(\mathbf{r}) \quad \leftarrow \text{Local } Q$$



$$V_{MF}[\rho(\mathbf{r})] \sim V_{MF}[\rho_0(\mathbf{r})] + \frac{\partial V_{MF}}{\partial \rho} \delta\rho$$

$$= V_{MF}[\rho_0(\mathbf{r})] - \beta \frac{\partial V_{MF}}{\partial \rho} \nabla \hat{Q} \cdot \nabla \rho_0(\mathbf{r})$$

$$= V_{MF}[\rho_0(\mathbf{r})] - \beta \nabla V_{MF} \cdot \nabla \hat{Q}$$

$$\left\{ \begin{array}{l} \delta\rho(\mathbf{r}, t) = -\beta \nabla\rho_0(\mathbf{r}) \cdot \nabla\hat{Q} \\ \delta V_{MF}(\mathbf{r}, t) = -\beta \nabla V_{MF} \cdot \nabla\hat{Q} \end{array} \right.$$

This corresponds to:

$$\begin{aligned} v(\mathbf{r}, \mathbf{r}') &= -\frac{\kappa}{2} (\nabla V_{MF} \cdot \nabla\hat{Q}) \cdot (\nabla V_{MF} \cdot \nabla\hat{Q})^* \\ \kappa^{-1} &= -\int d\mathbf{r} (\nabla V_{MF} \cdot \nabla\hat{Q}) \cdot (\nabla\rho_0 \cdot \nabla\hat{Q})^* \end{aligned}$$

Generalization:

$$\hat{Q}(\mathbf{r}) = \sum_k \hat{Q}_k(\mathbf{r})$$

$$v(\mathbf{r}, \mathbf{r}') = -\frac{1}{2} \sum_{k,k'} \kappa_{kk'} (\nabla V_{MF} \cdot \nabla\hat{Q}_k) \cdot (\nabla V_{MF} \cdot \nabla\hat{Q}_{k'})^*$$

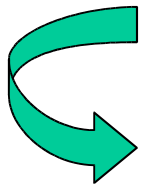
$$\kappa_{kk'}^{-1} = -\int d\mathbf{r} (\nabla V_{MF} \cdot \nabla\hat{Q}_k) \cdot (\nabla\rho_0 \cdot \nabla\hat{Q}_{k'})^*$$

(note) for a delta interaction: $v(\mathbf{r}, \mathbf{r}') = v_0(r)\delta(\mathbf{r} - \mathbf{r}')$

$$\langle ph' | v | hp' \rangle = \frac{1}{2L+1} \langle \mathcal{Y}_{j_p l_p} || Y_L || \mathcal{Y}_{j_h l_h} \rangle \langle \mathcal{Y}_{j_{p'} l_{p'}} || Y_L || \mathcal{Y}_{j_{h'} l_{h'}} \rangle^* \\ \times \int \frac{dr}{r^2} u_p(r) u_{h'}(r) u_h(r) u_{p'}(r) v_0(r)$$

$$\int \frac{dr}{r^2} u_p(r) u_{h'}(r) u_h(r) u_{p'}(r) v_0(r)$$

$$\sim \sum_k \frac{\Delta r}{r_k^2} u_p(r_k) u_{h'}(r_k) u_h(r_k) u_{p'}(r_k) v_0(r_k)$$



$$\langle ph' | v | hp' \rangle = \sum_k \chi(k) D_{ph}(k) D_{p'h'}(k)^*$$

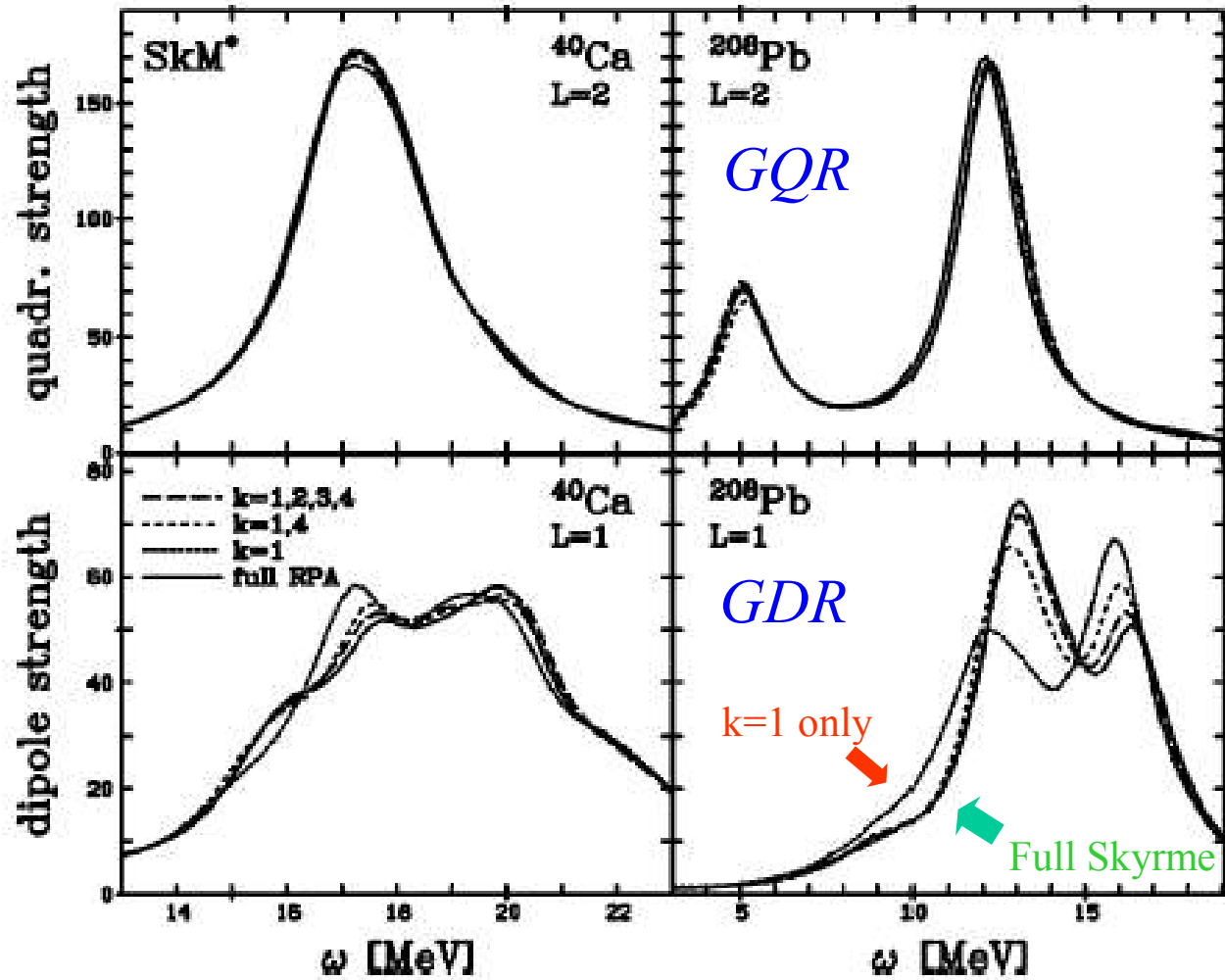
$$D_{ph}(k) = \frac{1}{\sqrt{2L+1}} \langle \mathcal{Y}_{j_p l_p} || Y_L || \mathcal{Y}_{j_h l_h} \rangle u_p(r_k) u_h(r_k)$$

$$\chi(k) = \Delta r \cdot v_0(r_k) / r_k^2$$

Application of separable RPA

V.O. Nesterenko, J. Kvasil, P.-G. Reinhard
 PRC66('02)044307

$$\text{Strength function: } S_\lambda(E) = \sum_\nu |\langle \nu | r^\lambda Y_{\lambda 0} | 0 \rangle|^2 \delta(E_\nu - E)$$



$$Q_1 = r^\lambda Y_{\lambda 0}$$

$$Q_k = j_\lambda(q_k r) Y_{\lambda 0} \quad (k = 2, 3, 4)$$

$$q_k = a_k z_\lambda / R_0$$

$$a_2 = 0.6$$

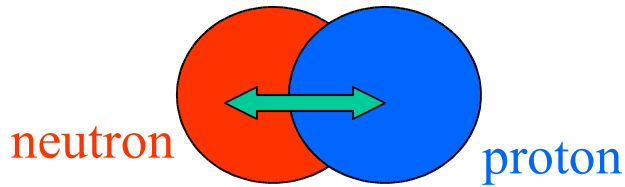
$$a_3 = 0.9$$

$$a_4 = 1.2$$

$$j_\lambda(z_\lambda) = 0$$

Giant Dipole Resonances

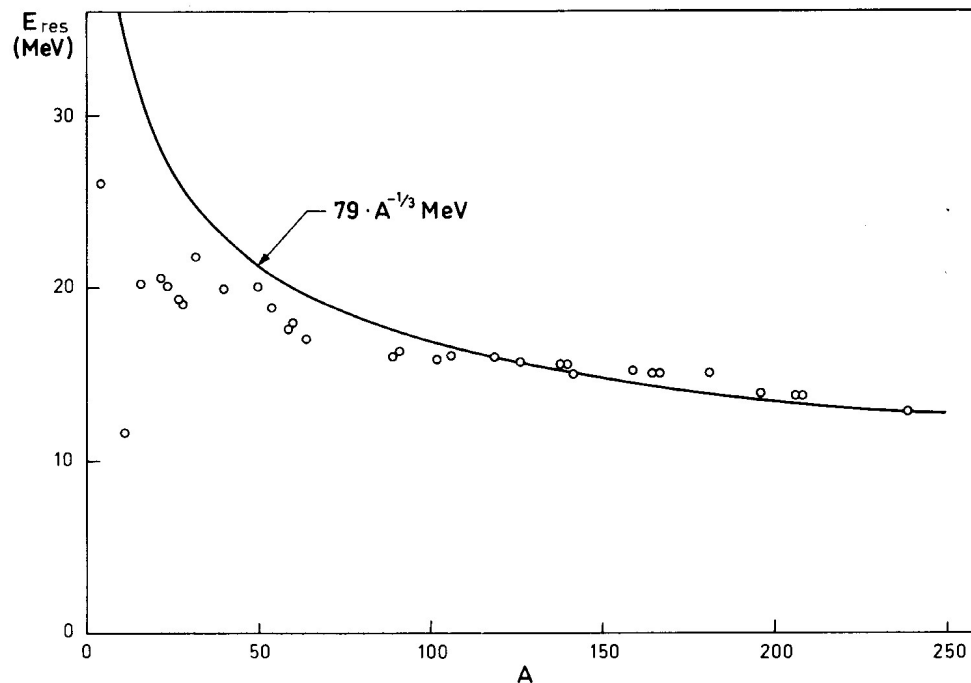
• Goldhaber-Teller type



$$\hat{Q} = r Y_{1\mu}(\hat{r}) \tau_z$$

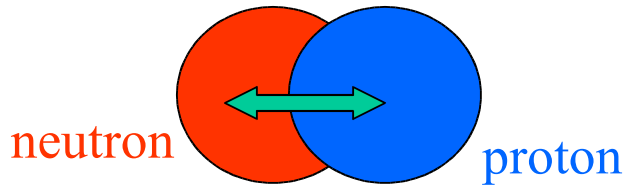
→ $\hbar\omega \sim A^{-1/6}$

→ Inconsistent with expt.
(except for light nuclei)



Giant Dipole Resonances

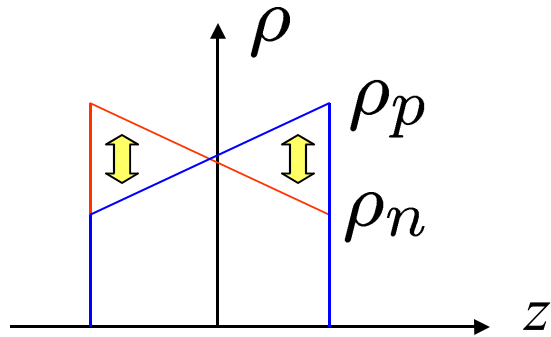
- Goldhaber-Teller type



$$\hat{Q} = r Y_{1\mu}(\hat{r}) \tau_z$$

→ $\hbar\omega \sim A^{-1/6}$

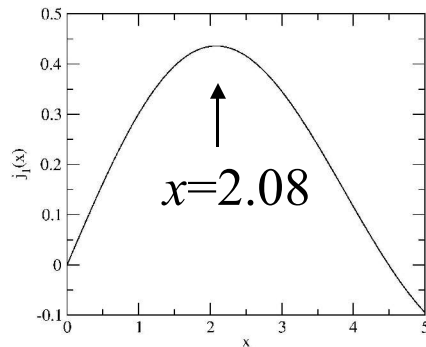
- Steinwedel-Jensen type



$$\hat{Q} = j_1(kr) Y_{1\mu}(\hat{r}) \tau_z$$

→ $\hbar\omega \sim A^{-1/3}$

$$kR = 2.08$$



$$j_1(x) = (\sin x - x \cos x) / x^2$$

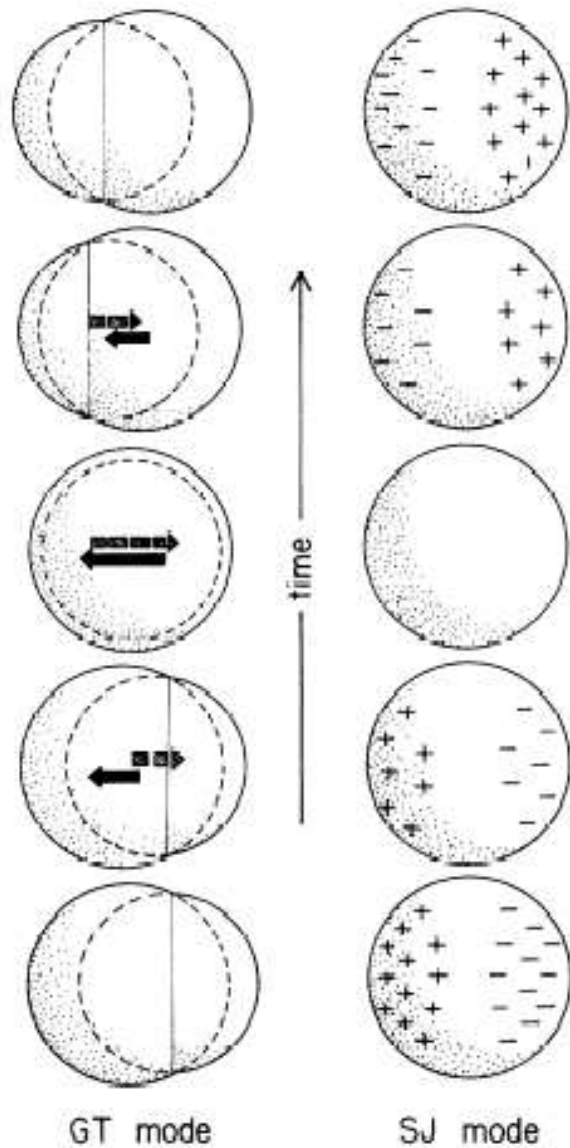
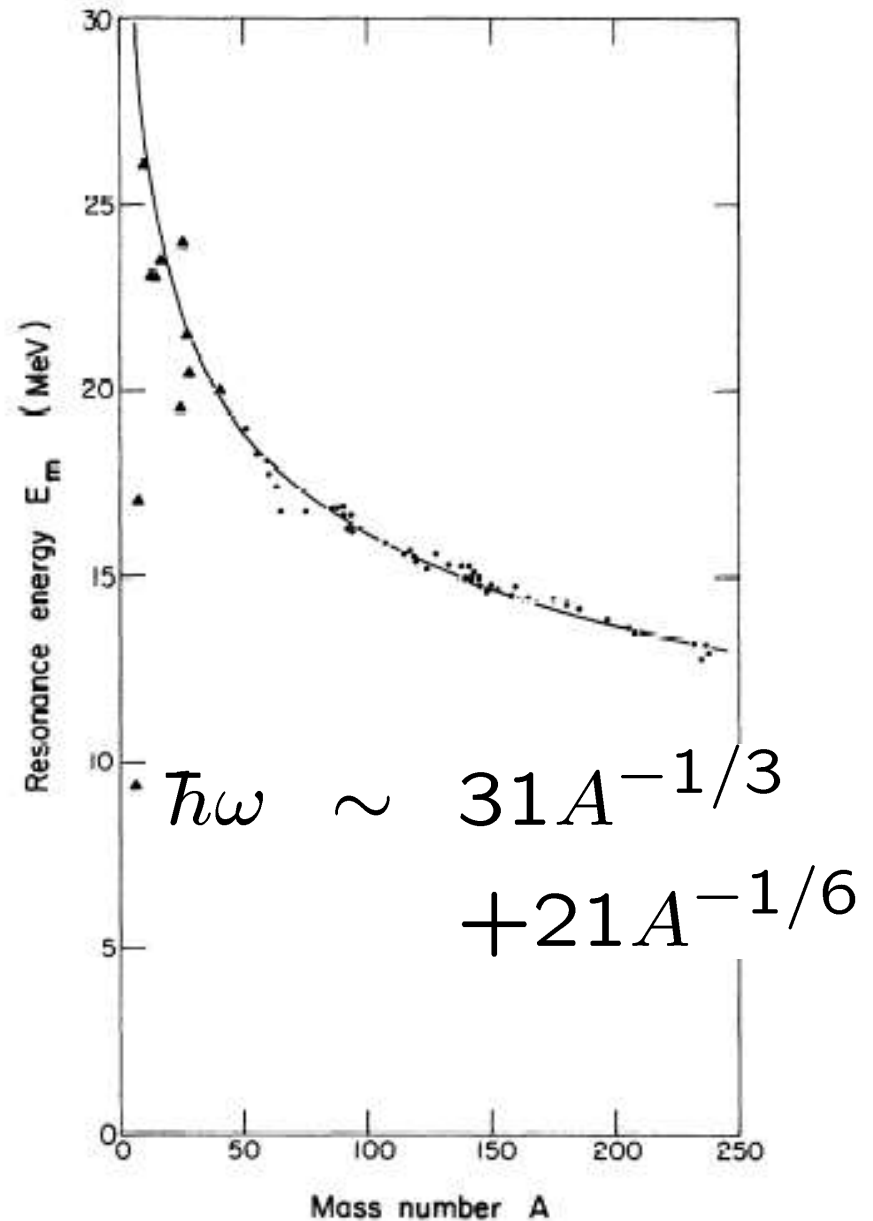


FIG. 1. Schematic drawings that serve to illustrate the general features of the Goldhaber-Teller (Ref. 3) (GT) and Steinwedel-Jensen (Ref. 4) (SJ) dipole modes.



Deformation effect

$$\hbar\omega \sim A^{-1/3} \sim 1/R$$

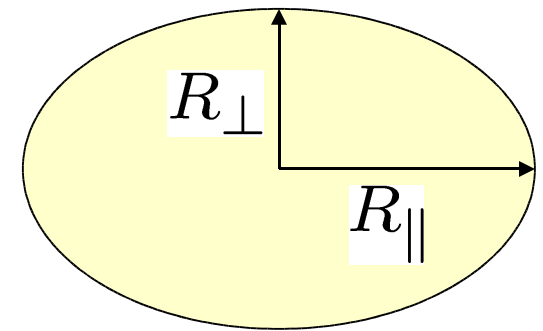
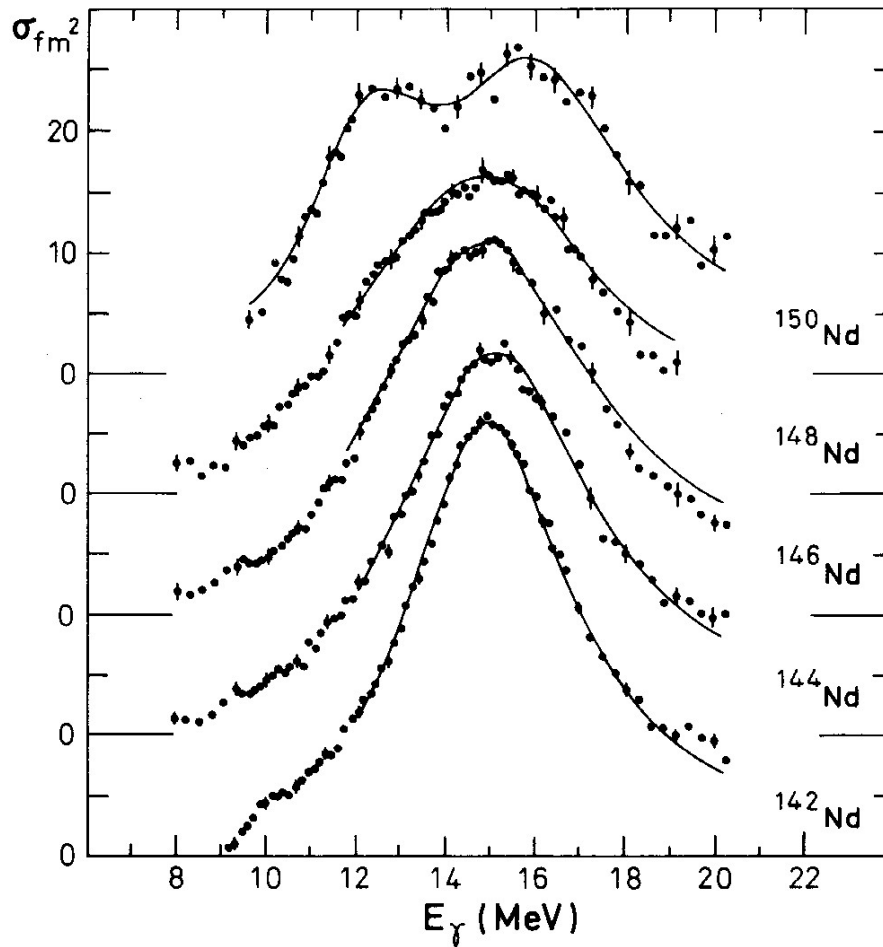


Figure 6-21 Photoabsorption cross section for even isotopes of neodymium. The experimental data are from P. Carlos, H. Beil, R. Bergère, A. Lepretre, and A. Veyssièrre, *Nuclear Phys. A172*, 437 (1971). The solid curves represent Lorentzian fits with the parameters given in Table 6-6.