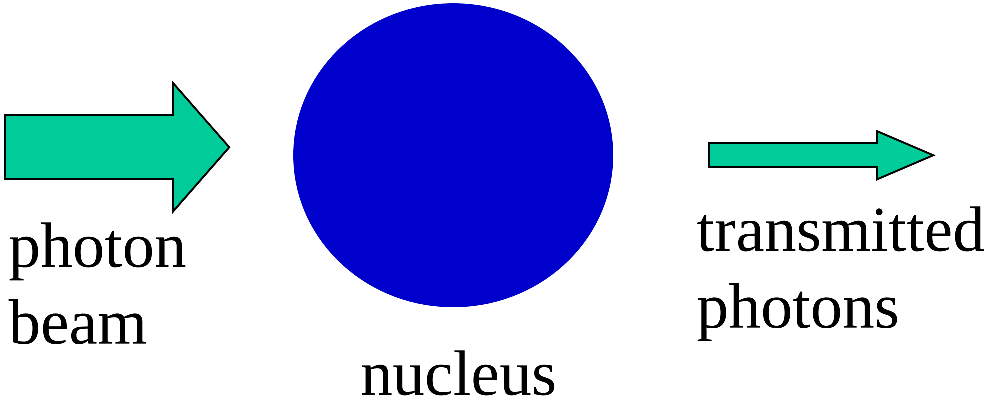
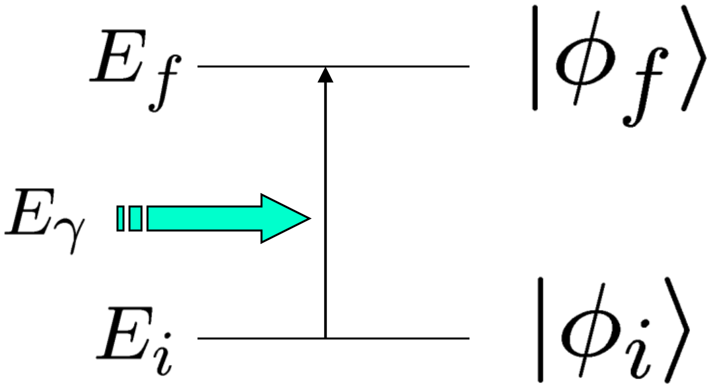
Collective Vibrations

How does a nucleus respond to an external perturbation?

i) Photo absorption cross section





The state is strongly excited when $E_f - E_i = E_\gamma$.

Giant Dipole Resonance (GDR)

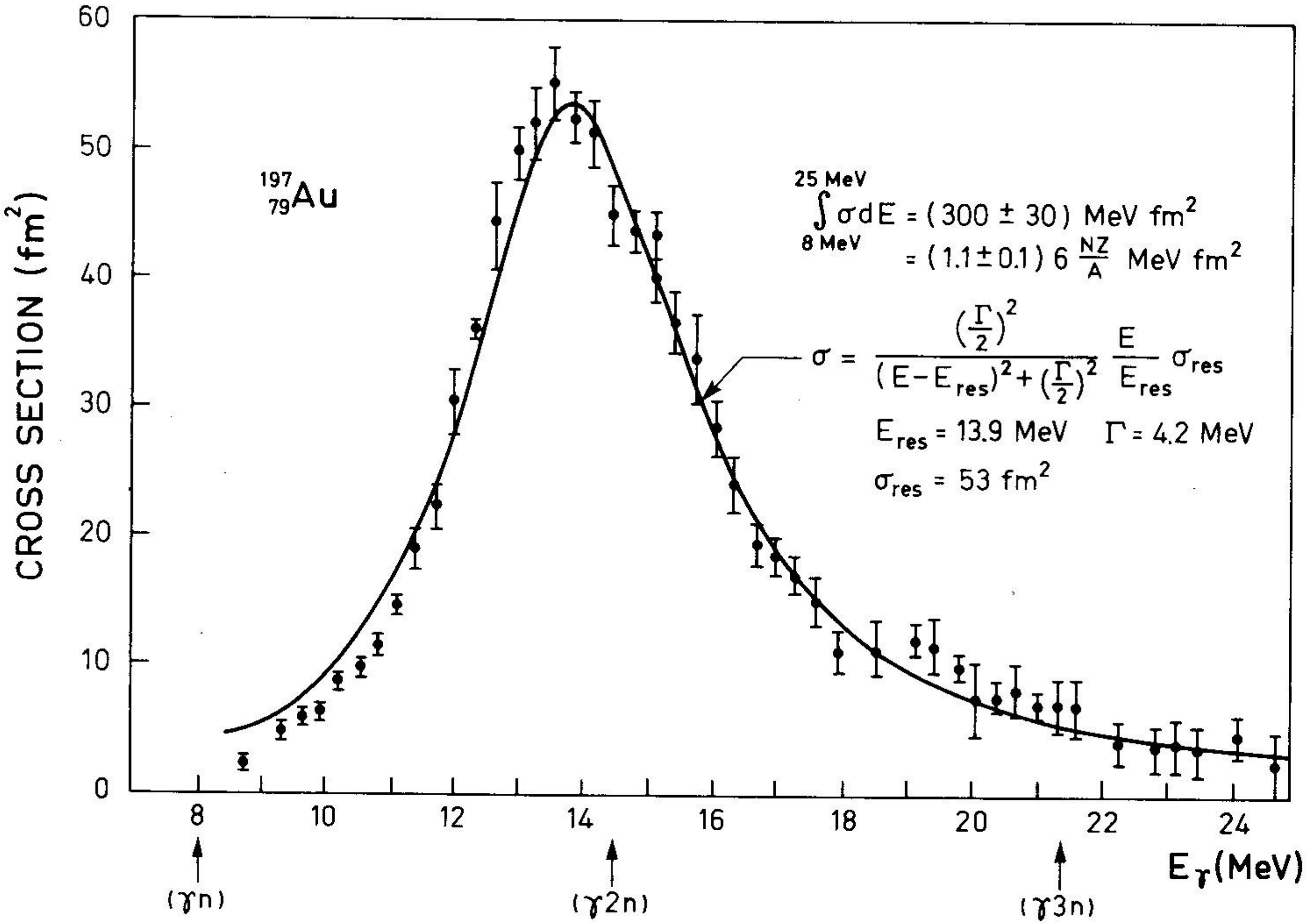


Figure 6-18 Total photoabsorption cross section for ^{197}Au. The experimental data are from S. C. Fultz, R. L. Bramblett, J. T. Caldwell, and N. A. Kerr, *Phys. Rev.* **127**, 1273 (1962). The solid curve is of Breit-Wigner shape with the indicated parameters.

Remarks

i) Photon interaction ⟷ dipole excitation

$$H_{\rm int} = \frac{1}{2m}\frac{e}{c}(\boldsymbol{p}\cdot\boldsymbol{A}+\boldsymbol{A}\cdot\boldsymbol{p})$$

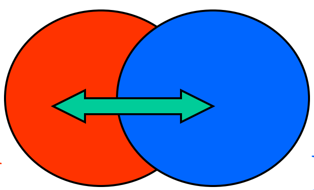
$$\boldsymbol{A}(\boldsymbol{r},t) = \sum_{\boldsymbol{k}}\sum_{\alpha=1,2}\sqrt{\frac{2\pi c^2\hbar}{\omega V}}(a_{\boldsymbol{k}\alpha}\,\boldsymbol{\epsilon}_\alpha\,e^{i\boldsymbol{k}\cdot\boldsymbol{r}-i\omega_k t}+h.c.)$$

$e^{i\boldsymbol{k}\cdot\boldsymbol{r}} \sim 1$ (dipole approximation)

$$\sigma_{\rm abs}(E_\gamma) = \frac{4\pi^2e^2}{\hbar c}(E_f-E_i)|\langle\phi_f|\tilde{z}|\phi_i\rangle|^2\,\delta(E_\gamma-E_f+E_i)$$

ii) Isospin

neutron

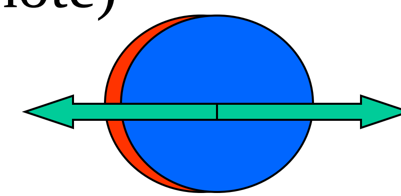


proton

Isovector type

(note)

$$\tilde{z} = \sum_p (z_p - Z_{cm})$$



Isoscalar dipole motion

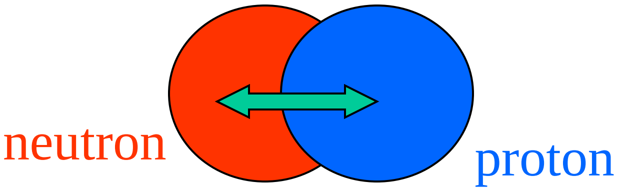
⟷ c.m. motion (to the first order)

iii) Collective motion

Motion of the whole nucleus rather than a single-particle motion

Giant Dipole Resonances

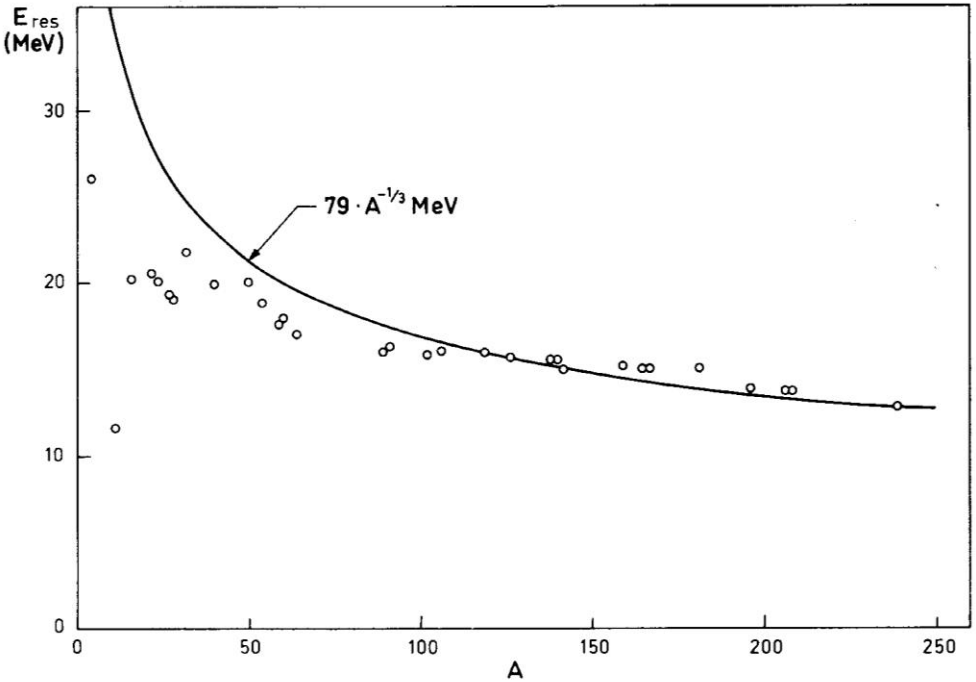
●Goldhaber-Teller type



$$\hat{Q} = rY_{1\mu}(\hat{\boldsymbol{r}})\tau_z$$

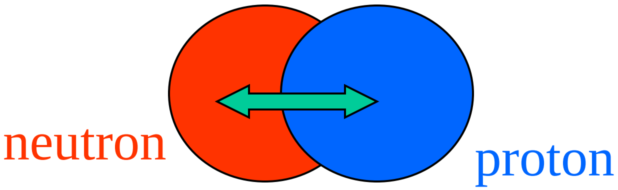
$$\Rightarrow \hbar\omega \sim A^{-1/6}$$

⇒ Inconsistent with expt. (except for light nuclei)



Giant Dipole Resonances

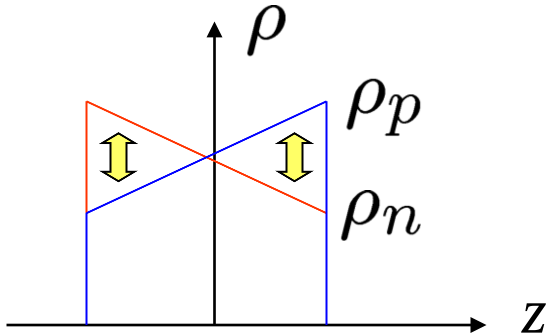
●Goldhaber-Teller type



$$\hat{Q} = rY_{1\mu}(\hat{\boldsymbol{r}})\tau_z$$

$$\longrightarrow \hbar\omega \sim A^{-1/6}$$

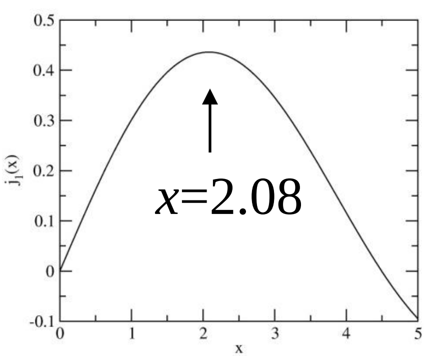
●Steinwedel-Jensen type



$$\hat{Q} = j_1(kr)Y_{1\mu}(\hat{\boldsymbol{r}})\tau_z$$

$$\longrightarrow \hbar\omega \sim A^{-1/3}$$

$$kR = 2.08$$



$$j_1(x) = (\sin x - x\cos x)/x^2$$

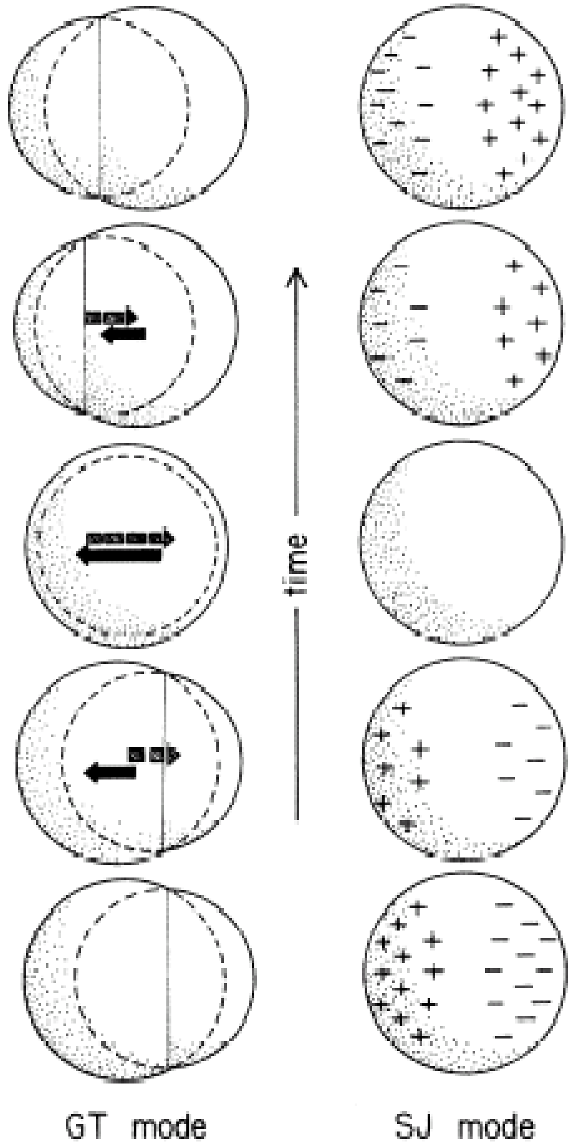
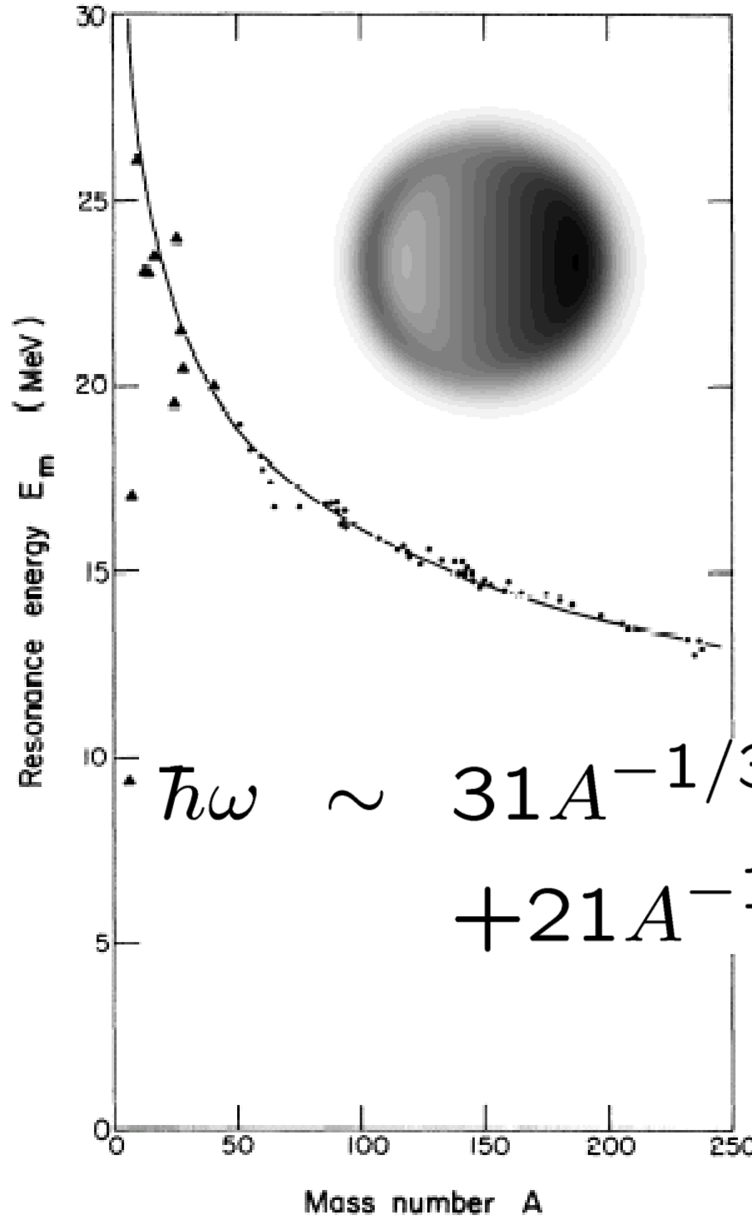


FIG. 1. Schematic drawings that serve to illustrate the the general features of the Goldhaber-Teller (Ref. 3) (GT) and Steinwedel-Jensen (Ref. 4) (SJ) dipole modes.

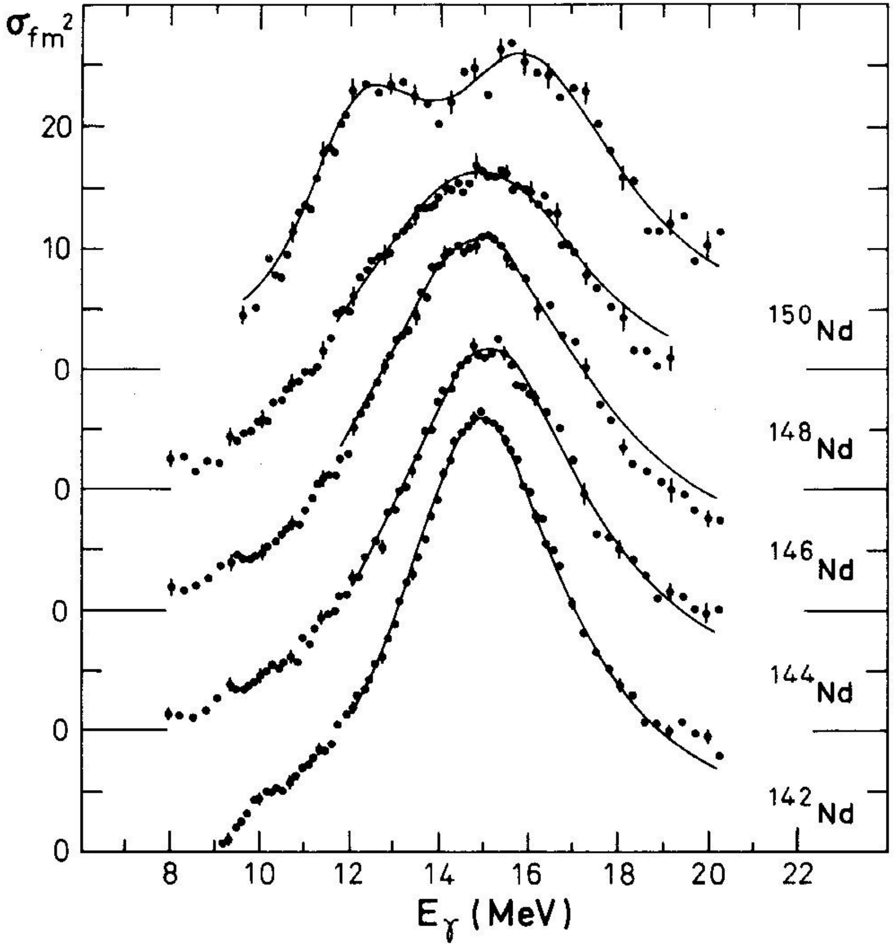


$$\hbar\omega \sim 31A^{-1/3} + 21A^{-1/6}$$

J.D. Myers et al., PRC15('77)2032

Deformation effect

$$\hbar\omega \sim A^{-1/3} \sim 1/R$$



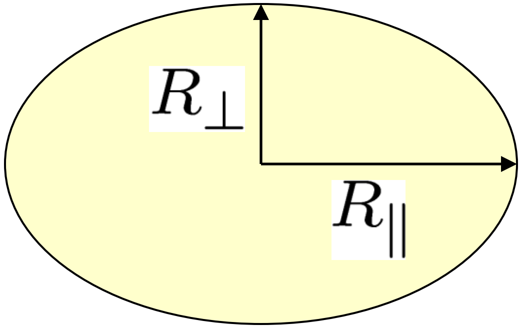
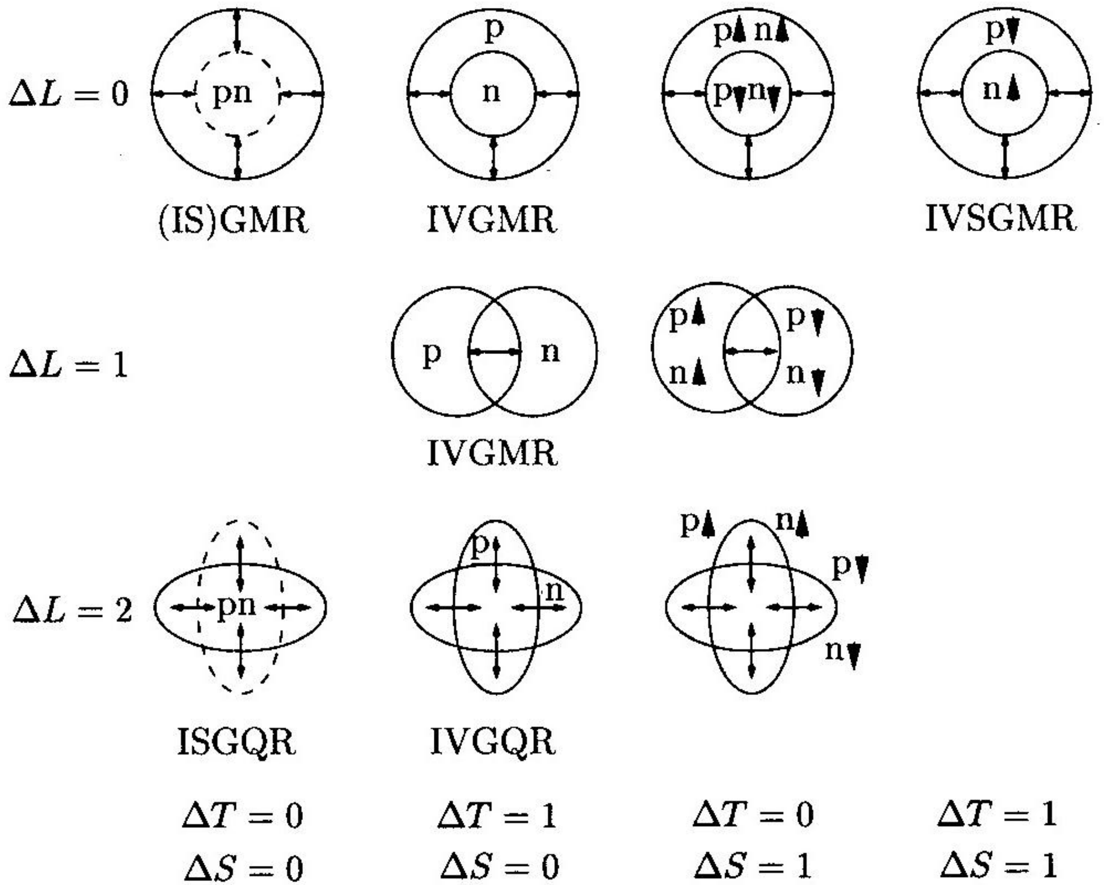


Figure 6-21 Photoabsorption cross section for even isotopes of neodymium. The experimental data are from P. Carlos, H. Beil, R. Bergère, A. Lepretre, and A. Veyssière, *Nuclear Phys.* **A172**, 437 (1971). The solid curves represent Lorentzian fits with the parameters given in Table 6-6.

ii) Inelastic scattering

(e,e'), (p,p'), Heavy-ion ⟶ Higher multipolarities



(note) $\Delta L = 2 \longrightarrow \Delta N = 2$ Giant Resonance (GQR)

$\Delta N = 0$ Low-lying state

Discovery of Giant Quadrupole Resonance (GQR)

VOLUME 29, NUMBER 16 PHYSICAL REVIEW LETTERS 16 OCTOBER 1972

Giant Multipole Resonances in ^{90}Zr Observed by Inelastic Electron Scattering

S. Fukuda and Y. Torizuka
Laboratory of Nuclear Science, Tohoku University, Tomizawa, Sendai, Japan
(Received 24 August 1972)

Inelastic electron scattering from the giant dipole resonance region in ^{90}Zr was measured. In addition to the usual dipole resonance we have found new resonances at 14.0 MeV and around 28 MeV. The spins and parities and transition strengths of these states are discussed.

VOLUME 30, NUMBER 21 PHYSICAL REVIEW LETTERS 21 MAY 1973

Electroexcitation of Giant Resonances in ^{208}Pb

M. Nagao and Y. Torizuka
Laboratory of Nuclear Science, Tohoku University, Tomizawa, Sendai, Japan
(Received 27 February 1973)

The giant-resonance region in ^{208}Pb was observed by inelastic electron scattering. We present evidence for the existences of a 2^+ (or 0^+) state at ~ 22 MeV and a 3^- state at ~ 19 MeV with giant-resonance character. The resonance states between 8.6 and 11.6 MeV are confirmed to be 2^+ (or 0^+) and the sum of their strengths exhausts about 50% of the $E2$ sum rule or 100% of $E0$.

Sum Rule

Strength function:

$$S(E) = \sum_{\nu} |\langle \nu | F | 0 \rangle|^2 \, \delta(E_\nu - E)$$

Energy weighted sum rule:

$$\begin{aligned} S_1 &= \sum_{\nu} (E_\nu - E_0) |\langle \nu | F | 0 \rangle|^2 \\ &= \frac{1}{2} \langle 0 | [F, [H, F]] | 0 \rangle \end{aligned}$$

(note)

$$\begin{aligned} \frac{1}{2} \langle 0 | [F, [H, F]] | 0 \rangle &= \frac{1}{2} \langle F(HF - FH) - (HF - FH)H \rangle \\ &= \langle FHF - E_0 F^2 \rangle \\ &= \sum_{\nu} E_\nu \, |\langle 0 | F | \nu \rangle|^2 - E_0 \langle 0 | F^2 | 0 \rangle \\ &= \sum_{\nu} (E_\nu - E_0) |\langle \nu | F | 0 \rangle|^2 \end{aligned}$$

Energy weighted sum rule:

$$S_1 = \sum_\nu (E_\nu - E_0)|\langle\nu|F|0\rangle|^2 = \frac{1}{2}\langle 0|[F,[H,F]]|0\rangle$$

For $F = F(\boldsymbol{r})$ (local operator)

$$[H,F] = \left[-\frac{\hbar^2}{2m}\nabla^2, F\right] = -\frac{\hbar^2}{2m}(\nabla^2 F + 2\nabla F\cdot\nabla)$$

$$[F,[H,F]] = \frac{\hbar^2}{m}(\nabla F)^2$$

$$S_1 = \frac{\hbar^2}{2m}\int d\boldsymbol{r}\,\rho(\boldsymbol{r})\cdot(\nabla F)^2$$

$$S_1 = \sum_\nu (E_\nu - E_0)|\langle\nu|F|0\rangle|^2 = \frac{\hbar^2}{2m}\int d\boldsymbol{r}\,\rho(\boldsymbol{r})\cdot(\nabla F)^2$$

For *F=z*

$$S_1 = \sum_\nu (E_\nu - E_0)|\langle\nu|z|0\rangle|^2 = \frac{\hbar^2 N_{sys}}{2m}$$

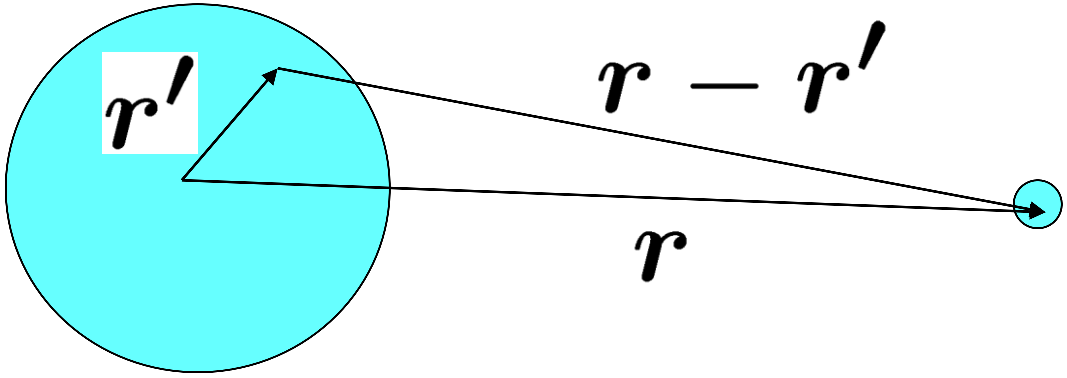
[TRK (Thomas-Reiche-Kuhn) Sum Rule]

Model independent

For $F = r^\lambda Y_{\lambda\mu}(\hat{\boldsymbol{r}})$

$$S_1 = \frac{\lambda(2\lambda+1)\hbar^2}{8\pi m} A \left\langle r^{2\lambda-2}\right\rangle$$

(note)



$$V_C(\boldsymbol{r}) = e^2 \int \frac{\rho_C(\boldsymbol{r}')}{|\boldsymbol{r}-\boldsymbol{r}'|} d\boldsymbol{r}'$$

$$\frac{1}{|\boldsymbol{r}-\boldsymbol{r}'|} = \sum_{\lambda,\mu} \frac{4\pi}{2\lambda+1} \frac{Y^*_{\lambda\mu}(\hat{\boldsymbol{r}})}{r^{\lambda+1}} \cdot \underline{\underline{r'^{\lambda} Y_{\lambda\mu}(\hat{\boldsymbol{r}}')}}$$

Photo absorption cross section:

$$\sigma_{\mathrm{abs}}(E_\gamma) = \frac{4\pi^2 e^2}{\hbar c}(E_f - E_i)|\langle\phi_f|\tilde{z}|\phi_i\rangle|^2\,\delta(E_\gamma - E_f + E_i)$$

$$\begin{aligned}
\tilde{z} = \sum_p (z_p - Z_{cm}) &= \sum_p \left\{ z_p - \frac{1}{A}\left(\sum_{p'} z_{p'} + \sum_n z_n\right)\right\} \\
&= \frac{NZ}{A}\left(\frac{1}{Z}\sum_p z_p - \frac{1}{N}\sum_n z_n\right)
\end{aligned}$$

$$\begin{aligned}
\int \sigma_{\mathrm{abs}}(E_\gamma)dE_\gamma &= \frac{4\pi^2 e^2}{\hbar c}\cdot\frac{\hbar^2}{2m}\cdot\frac{NZ}{A} \\
&= \frac{2\pi^2 e^2 \hbar}{mc}\cdot\frac{NZ}{A}
\end{aligned}$$

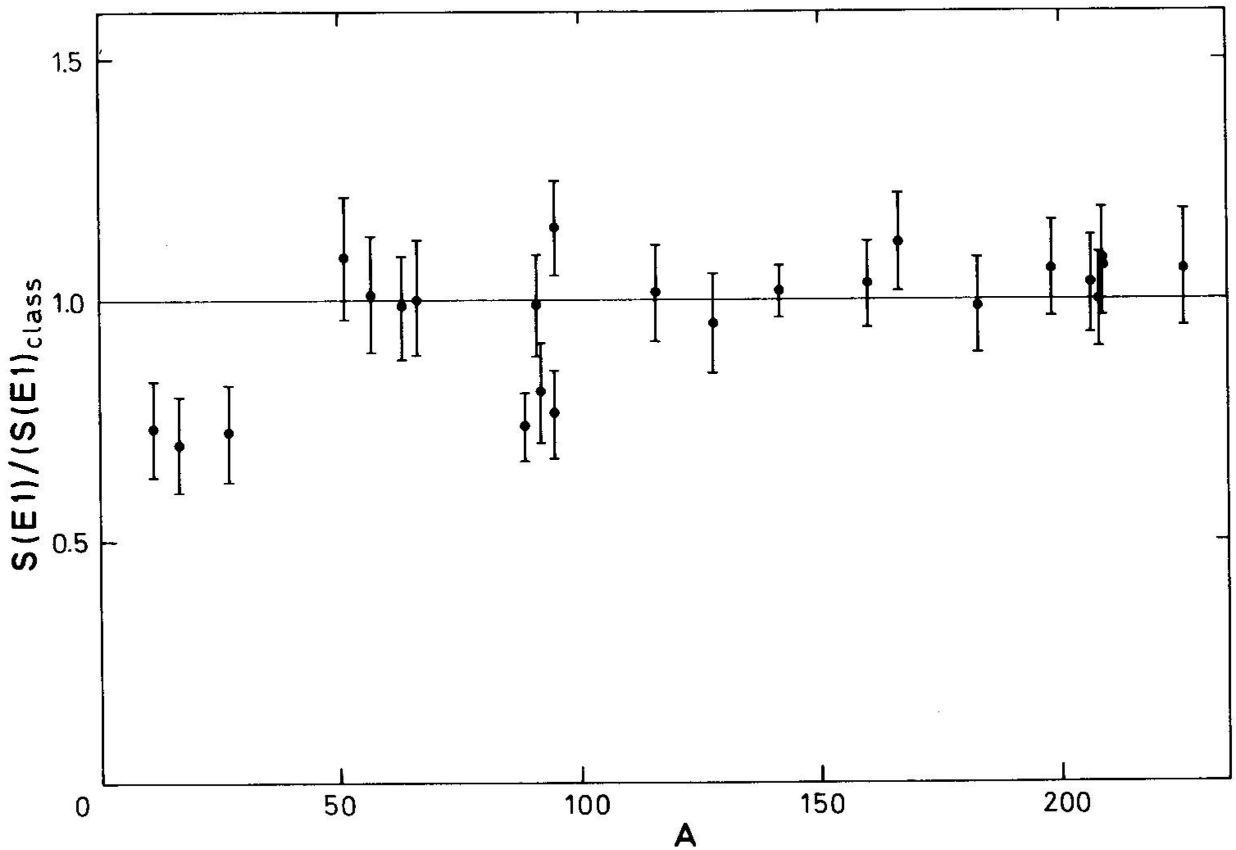
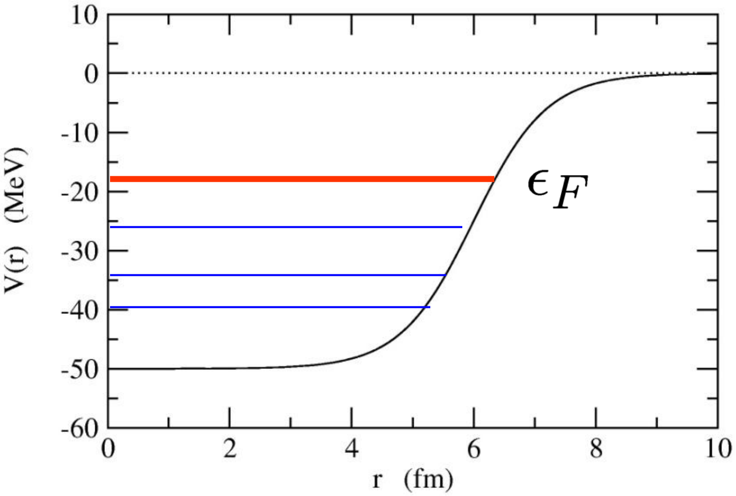


Figure 6-20 Total oscillator strength for dipole resonance. The observed total oscillator strength for energies up to 30 MeV is given in units of the classical sum rule value. For the nuclei with $A > 50$, the integrated oscillator strengths have been obtained from measurements of neutron yields produced by monochromatic γ rays (S. C. Fultz, R. L. Bramblett, B. L. Berman, J. T. Caldwell, and M. A. Kelly, in *Proc. Intern. Nuclear Physics Conference*, p. 397, ed.-in-chief R. L. Becker, Academic Press, New York, 1967). The photoscattering cross sections have been ignored, since they contribute only a very small fraction of the total cross sections. For the lighter nuclei, the yield of (γp) processes must be included and the data are from: ^{12}C and ^{27}Al (S. C. Fultz, J. T. Caldwell, B. L. Berman, R. L. Bramblett, and R. R. Harvey, *Phys. Rev.* **143**, 790, 1966); ^{16}O (Dolbilkin *et al.*, *loc.cit.*, Fig. 6-26). For the heavy nuclei ($A > 50$), other measurements have yielded total oscillator strengths that are about 20% larger than those shown in the figure (see, for example, Veyssière *et al.*, 1970).

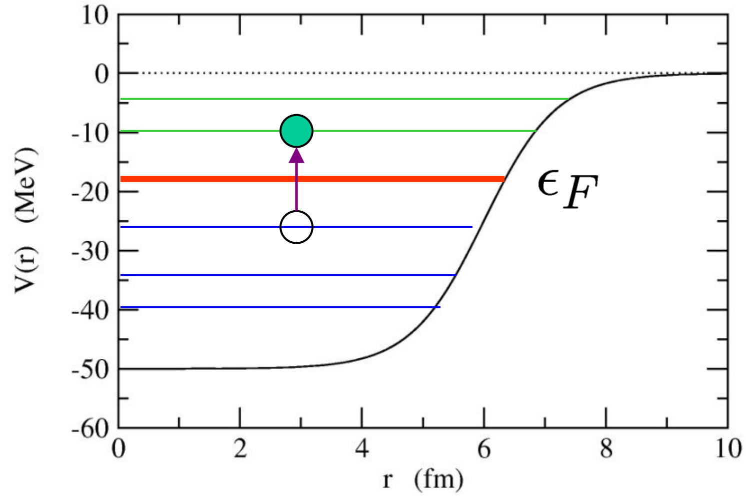
Particle-Hole excitations

Hartree-Fock state



$$|HF\rangle$$

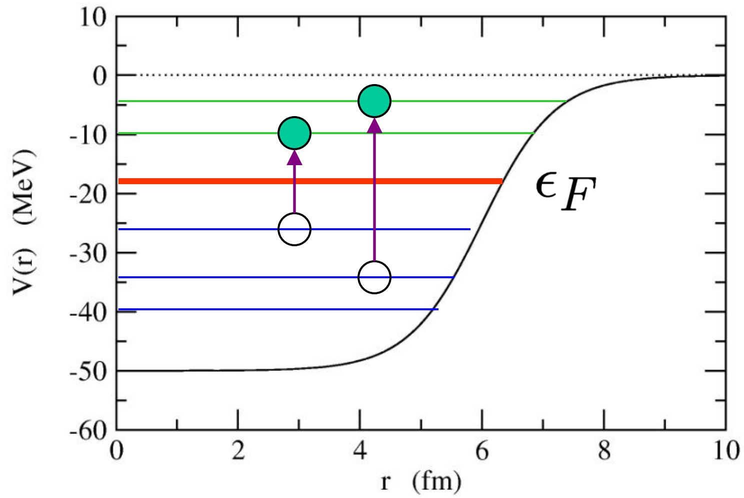
1 particle-1 hole (1p1h) state



$$a_p^\dagger a_h |HF\rangle$$

2 particle-2 hole (2p2h) state

$$a_p^\dagger a_{p'}^\dagger a_h a_{h'} |HF\rangle$$



Tamm-Dancoff Approximation

Assume: $|\nu\rangle = Q_\nu^\dagger |HF\rangle \;=\; \sum_{ph} X_{ph}\, a_p^\dagger a_h |HF\rangle$

$$= \sum_{ph} X_{ph} |ph^{-1}\rangle$$

(superposition of 1p1h states)

$$H|\nu\rangle = E_\nu |\nu\rangle$$

$$\sum_{p'h'} H_{ph,p'h'} X_{p'h'} = E_\nu X_{ph}$$

$$H_{ph,p'h'} = (\epsilon_p - \epsilon_h)\delta_{ph,p'h'} + \langle ph'|\bar{v}|hp'\rangle$$

Tamm-Dancoff equation

Tamm-Dancoff Approximation

Assume: $|\nu\rangle = Q_\nu^\dagger|HF\rangle = \sum_{ph} X_{ph}\, a_p^\dagger a_h|HF\rangle$

(superposition of 1p1h states)

and $H|\nu\rangle = E_\nu|\nu\rangle$

(note) $Q_\nu|HF\rangle = 0$

$$[H, Q_\nu^\dagger]|HF\rangle = E_\nu Q_\nu^\dagger|HF\rangle$$

$E_0 = 0$

$$\langle HF|[\delta Q, [H, Q_\nu^\dagger]]|HF\rangle = E_\nu\langle HF|[\delta Q, Q_\nu^\dagger]|HF\rangle$$

δQ : arbitrary operator

$$Q_\nu^\dagger = \sum_{ph} X_{ph}\, a_p^\dagger a_h$$

$$\langle HF|[\delta Q,[H,Q_\nu^\dagger]]|HF\rangle = E_\nu \langle HF|[\delta Q,Q_\nu^\dagger]|HF\rangle$$

$$\begin{cases} H = \sum_{1,2} t_{12}\, a_1^\dagger\, a_2 + \frac{1}{4}\sum_{1,2,3,4} \bar{v}_{1234}\, a_1^\dagger\, a_2^\dagger\, a_4\, a_3 \\ \delta Q = a_h^\dagger a_p \end{cases}$$

Tamm-Dancoff equation:

$$\sum_{p'h'} A_{ph,p'h'} X_{p'h'} = E_\nu X_{ph}$$

$$A_{ph,p'h'} = (\epsilon_p - \epsilon_h)\delta_{ph,p'h'} + \langle ph'|\bar{v}|hp'\rangle$$

TDA on a schematic model

Separable interaction: $\langle ph'|\bar{v}|hp'\rangle = \lambda D_{ph} D^*_{p'h'}$

Tamm-Dancoff equation: $\sum_{p'h'} A_{ph,p'h'} X_{p'h'} = E X_{ph}$

$$A_{ph,p'h'} = (\epsilon_p - \epsilon_h)\delta_{ph,p'h'} + \langle ph'|\bar{v}|hp'\rangle$$

$$(\epsilon_{ph} - E)X_{ph} + \lambda D_{ph} \cdot T = 0 \qquad T \equiv \sum_{ph} D^*_{ph} X_{ph}$$

$$X_{ph} = -\lambda D_{ph} T/(\epsilon_{ph} - E)$$

$$T = \sum_{ph} D^*_{ph} X_{ph} = -\lambda \sum_{ph} \frac{|D_{ph}|^2}{\epsilon_{ph} - E} T$$

or

$$\frac{1}{\lambda} = \sum_{ph} \frac{|D_{ph}|^2}{E - \epsilon_{ph}}$$

(TDA dispersion relation)

Graphical solutions

$$\frac{1}{\lambda} = \sum_{ph} \frac{|D_{ph}|^2}{E - \epsilon_{ph}}$$

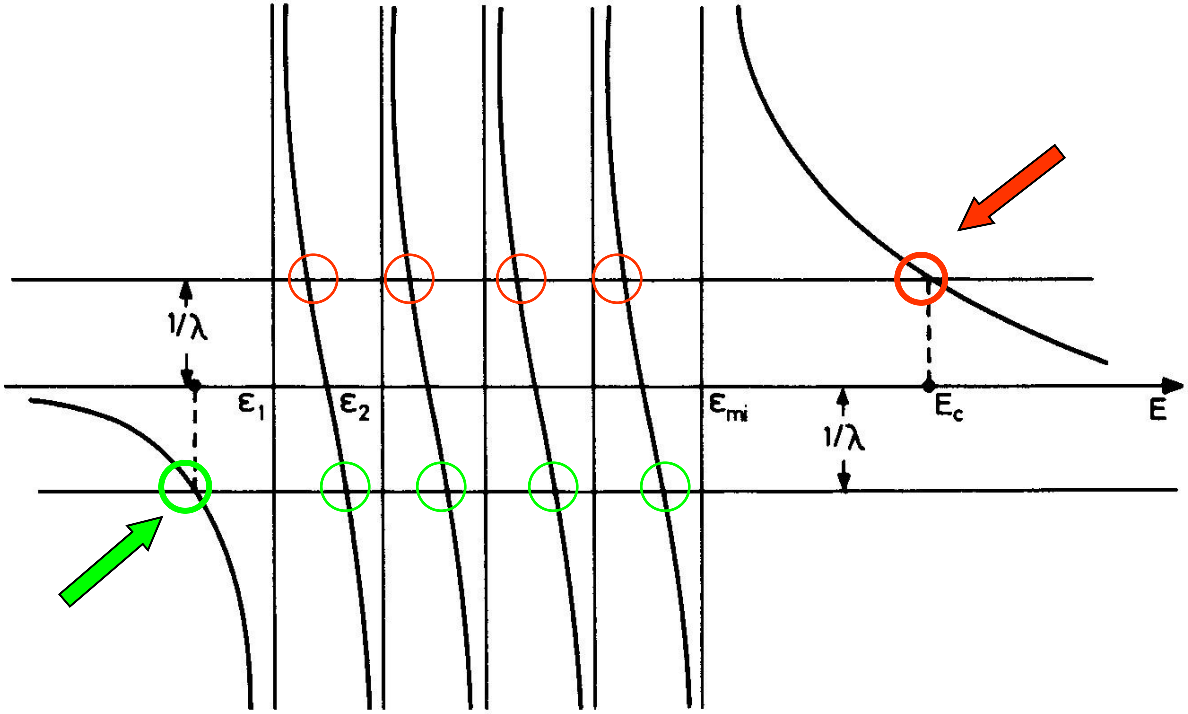


Figure 8.4. Graphical solution of Eq. (8.18).

(note) in the degenerate limit: $\epsilon_{ph} \sim \epsilon$

$$E = \epsilon_{ph} + \lambda \sum_{ph} |D_{ph}|^2, \qquad |\nu\rangle = \sum_{ph} D_{ph} a_p^\dagger a_h |HF\rangle$$

coherent superpositon of 1p1h states

Iso-scalar type modes: $E < \epsilon_{ph} \rightarrow \lambda < 0$ (attractive)

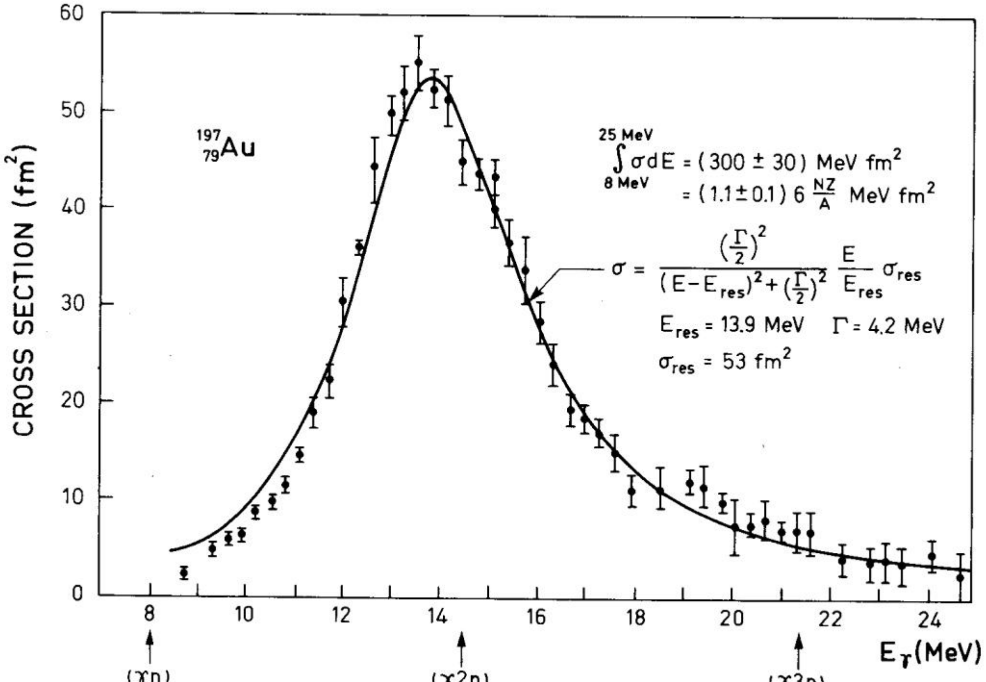
Iso-vector type modes: $E > \epsilon_{ph} \rightarrow \lambda > 0$ (repulsive)

Experimental systematics:

IV GDR: $E \sim 79A^{-1/3}$ (MeV) $\longleftrightarrow$ $\epsilon_{ph} \sim 41A^{-1/3}$

IS GQR: $E \sim 65A^{-1/3}$ (MeV) $\longleftrightarrow$ $\epsilon_{ph} \sim 82A^{-1/3}$

(note) single particle potential: $\hbar\omega \sim 41A^{-1/3}$ (MeV)



^{197}Au

$$E_{GDR} = 14 \text{ (MeV)}$$

$$\epsilon_{ph} \sim 41 \cdot 197^{-1/3}$$

$$\sim 7 \text{ (MeV)}$$

(note) contact interaction $v(\boldsymbol{r}, \boldsymbol{r}') = t_0 \delta(\boldsymbol{r} - \boldsymbol{r}')$

Energy Functional:

$$
\begin{aligned}
E[\rho_p, \rho_n] &= \langle \Psi_{\mathrm{HF}} | \sum_{i<j} v_{ij} | \Psi_{HF} \rangle \\
&= \int d\boldsymbol{r} \left\{ \frac{t_0}{2}(\rho_p^2 + 2\rho_p\rho_n + \rho_n^2) - \frac{t_0}{4}(\rho_p^2 + \rho_n^2) \right\}
\end{aligned}
$$

Mean-Field potential:

$$
V_n(\boldsymbol{r}) = \frac{\delta E}{\delta \rho_n} = \frac{t_0}{2}\rho_n + t_0\rho_p, \qquad V_p(\boldsymbol{r}) = \frac{\delta E}{\delta \rho_p} = \frac{t_0}{2}\rho_p + t_0\rho_n
$$

or

$$
V(\boldsymbol{r}) = \frac{3}{4} t_0\, \rho_{IS} - \frac{t_0}{4} \tau_z\, \rho_{IV}
$$

$$
\left\{
\begin{aligned}
\rho_{IV} &= \rho_n - \rho_p \\
\rho_{IS} &= \rho_n + \rho_p
\end{aligned}
\right.
$$

Residual interaction:

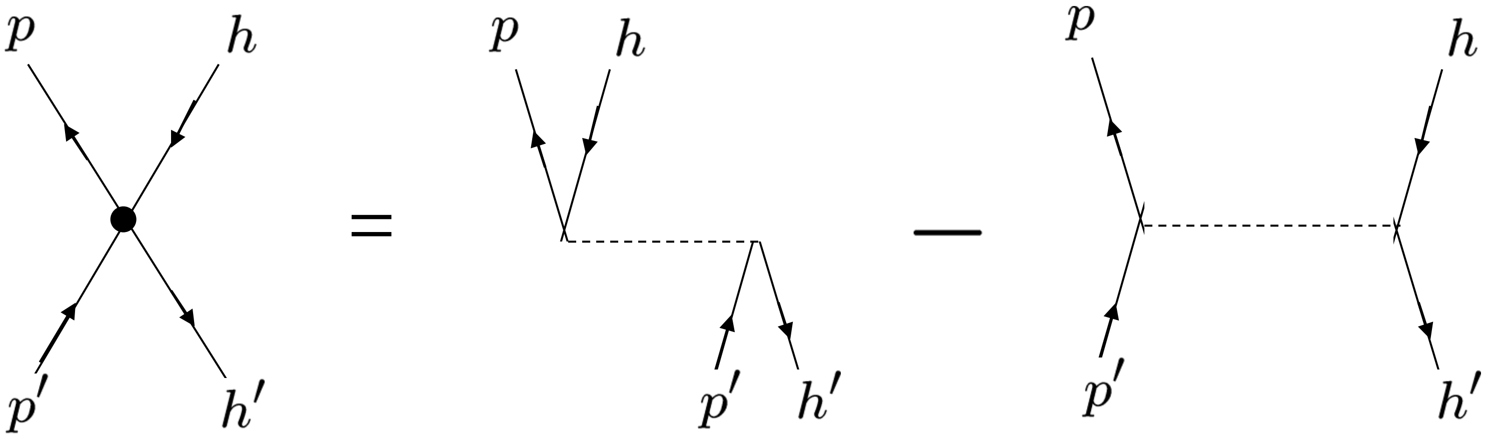
$$
\begin{aligned}
v_{IS}(\boldsymbol{r}, \boldsymbol{r}') &= 3t_0/4 \cdot \delta(\boldsymbol{r} - \boldsymbol{r}') \\
v_{IV}(\boldsymbol{r}, \boldsymbol{r}') &= -t_0/4 \cdot \delta(\boldsymbol{r} - \boldsymbol{r}')
\end{aligned}
$$

(note)

$t_0 < 0$

Another argument

$$\langle ph^{-1}|\bar{v}|p'h'^{-1}\rangle = \langle ph'|\bar{v}|hp'\rangle = \langle ph'|v|hp'\rangle - \langle ph'|v|p'h\rangle$$



Direct term Exchange term

$$\langle PP^{-1}|\bar{v}|PP^{-1}\rangle \sim \langle NN^{-1}|\bar{v}|NN^{-1}\rangle = D - E$$

$$\langle PP^{-1}|\bar{v}|NN^{-1}\rangle = D \quad \text{(no charge exchange)}$$

$$\langle IS|\bar{v}|IS\rangle = 2D - E \sim D$$

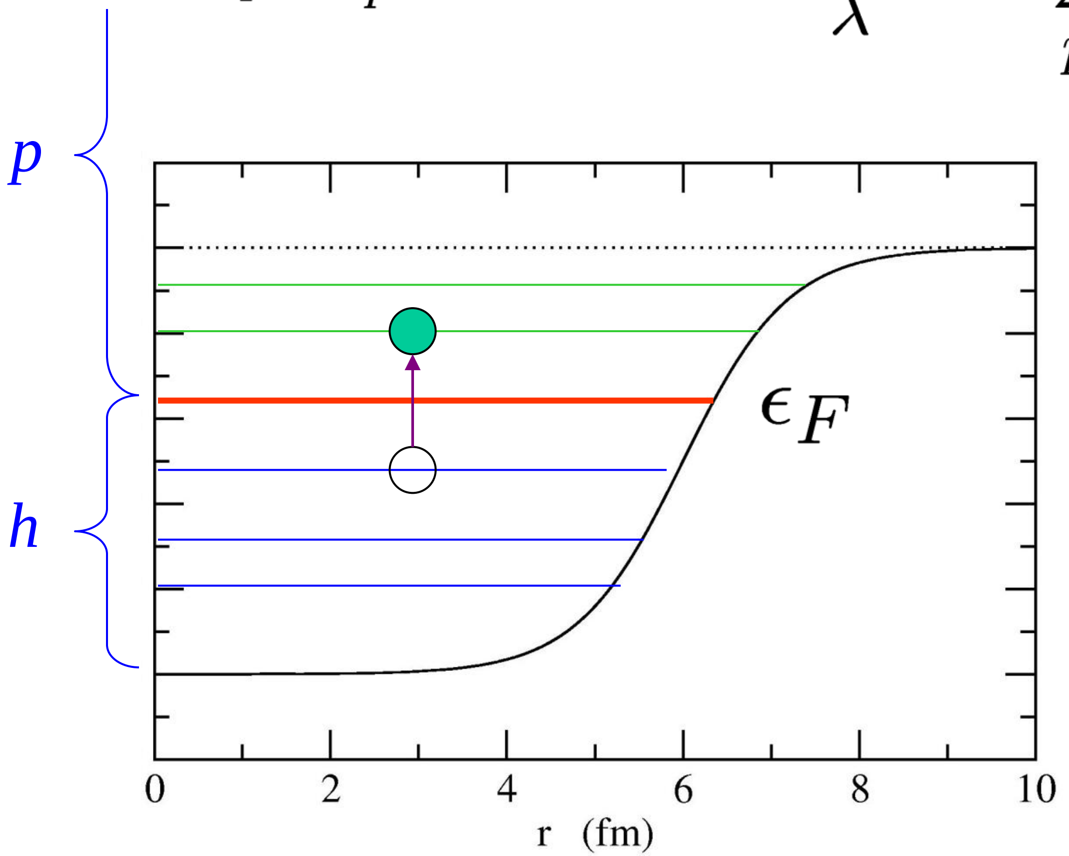
$$\langle IV|\bar{v}|IV\rangle = -E \sim -D$$

$$|IS\rangle = |NN^{-1}\rangle + |PP^{-1}\rangle$$

$$|IV\rangle = |NN^{-1}\rangle - |PP^{-1}\rangle$$

Continuum Excitations

$$\langle ph'|\bar{v}|hp'\rangle = \lambda D_{ph} D^*_{p'h'} \quad \Rightarrow \quad \frac{1}{\lambda} = -\sum_{ph} \frac{|D_{ph}|^2}{\epsilon_p - \epsilon_h - E}$$



h: all the occupied (bound) states

p: the bound excited states + continuum states

$$\frac{1}{\lambda} = -\sum_{ph} \frac{|D_{ph}|^2}{\epsilon_p - \epsilon_h - E}$$

Coordinate representation: $D_{ph} = \int d\boldsymbol{r}\, \phi_p^*(\boldsymbol{r}) D(\boldsymbol{r}) \phi_h(\boldsymbol{r})$

$$\frac{1}{\lambda} = -\sum_{ph} \int d\boldsymbol{r} \int d\boldsymbol{r}'\, D(\boldsymbol{r}) D^*(\boldsymbol{r}') \frac{\phi_p^*(\boldsymbol{r})\phi_h(\boldsymbol{r})\phi_p(\boldsymbol{r}')\phi_h^*(\boldsymbol{r}')}{\epsilon_p - \epsilon_h - E}$$

(note)

$$\hat{h}\phi_p = \epsilon_p \phi_p$$

$$1 = \sum_i |\phi_i\rangle\langle\phi_i| = \sum_p |\phi_p\rangle\langle\phi_p| + \sum_h |\phi_h\rangle\langle\phi_h|$$

$$\text{rhs} = -\sum_h \int d\boldsymbol{r} \int d\boldsymbol{r}'\, D(\boldsymbol{r}) D^*(\boldsymbol{r}')\, \phi_h(\boldsymbol{r})\phi_h^*(\boldsymbol{r}') \times \left(\left\langle \boldsymbol{r}' \left| \frac{1}{\hat{h} - \epsilon_h - E - i\eta} \right| \boldsymbol{r} \right\rangle - \sum_{h'} \frac{\phi_{h'}^*(\boldsymbol{r})\phi_{h'}(\boldsymbol{r}')}{\epsilon_{h'} - \epsilon_h - E - i\eta} \right)$$