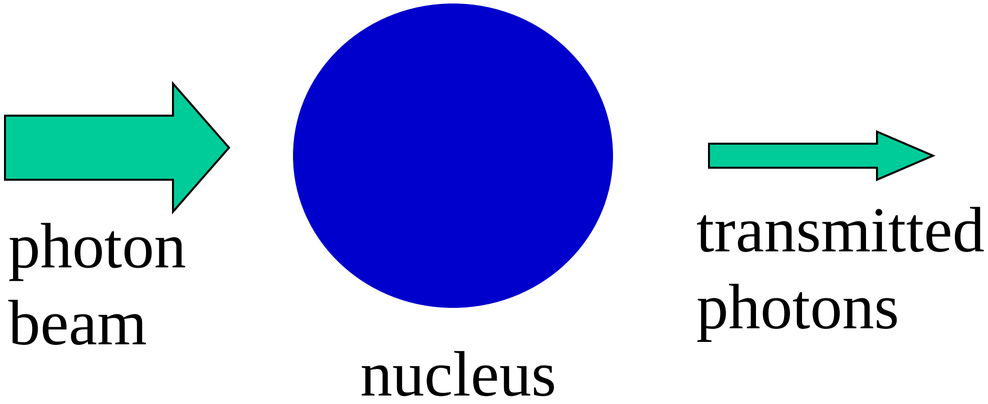
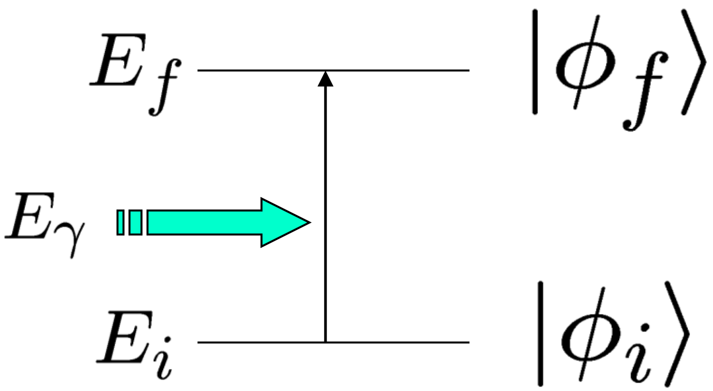
Collective Vibrations

How does a nucleus respond to an external perturbation?

i) Photo absorption cross section





The state is strongly excited when

$$E_f - E_i = E_\gamma .$$

Giant Dipole Resonance (GDR)

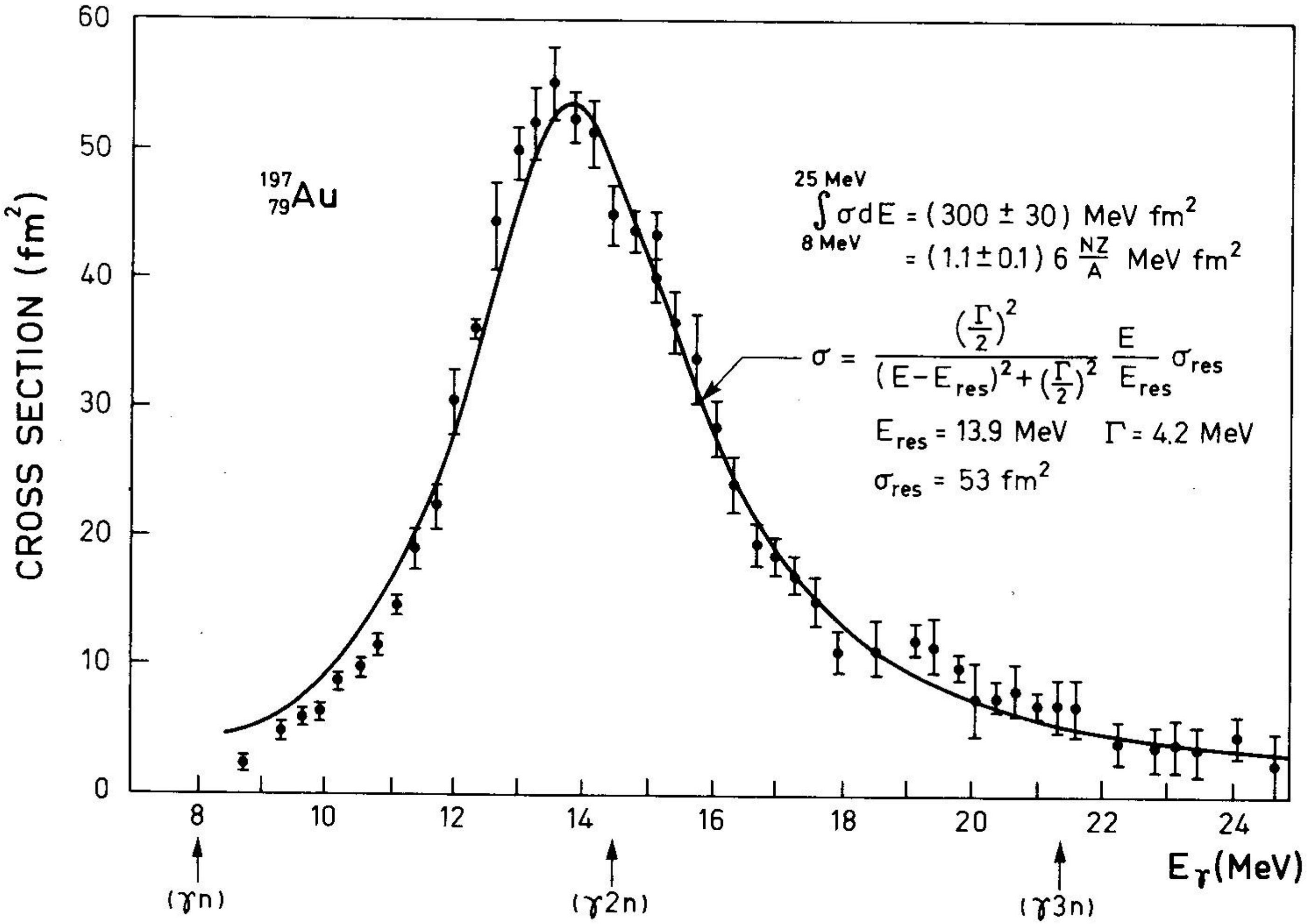


Figure 6-18 Total photoabsorption cross section for ^{197}Au. The experimental data are from S. C. Fultz, R. L. Bramblett, J. T. Caldwell, and N. A. Kerr, *Phys. Rev.* **127**, 1273 (1962). The solid curve is of Breit-Wigner shape with the indicated parameters.

Remarks

i) Photon interaction ⟷ dipole excitation

$$H_{\text{int}} = \frac{1}{2m}\frac{e}{c}(\boldsymbol{p}\cdot\boldsymbol{A}+\boldsymbol{A}\cdot\boldsymbol{p})$$

$$\boldsymbol{A}(\boldsymbol{r},t) = \sum_{\boldsymbol{k}}\sum_{\alpha=1,2}\sqrt{\frac{2\pi c^2\hbar}{\omega V}}\left(a_{\boldsymbol{k}\alpha}\,\boldsymbol{\epsilon}_\alpha\, e^{i\boldsymbol{k}\cdot\boldsymbol{r}-i\omega_k t}+h.c.\right)$$

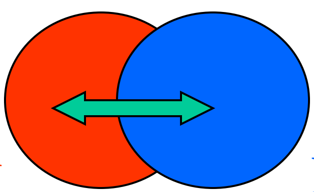
$$e^{i\boldsymbol{k}\cdot\boldsymbol{r}} \sim 1$$ (dipole approximation)

$$\sigma_{\text{abs}}(E_\gamma) = \frac{4\pi^2 e^2}{\hbar c}(E_f - E_i)|\langle\phi_f|\tilde{z}|\phi_i\rangle|^2\,\delta(E_\gamma - E_f + E_i)$$

ii) Isospin

$$\tilde{z} = \sum_p (z_p - Z_{cm})$$

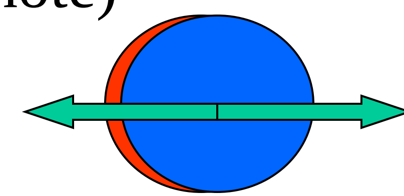
neutron



proton

Isovector type

(note)



Isoscalar dipole motion

⟷ c.m. motion (to the first order)

iii) Collective motion

Motion of the whole nucleus rather than a single-particle motion

Deformation effect

$$\hbar\omega \sim A^{-1/3} \sim 1/R$$

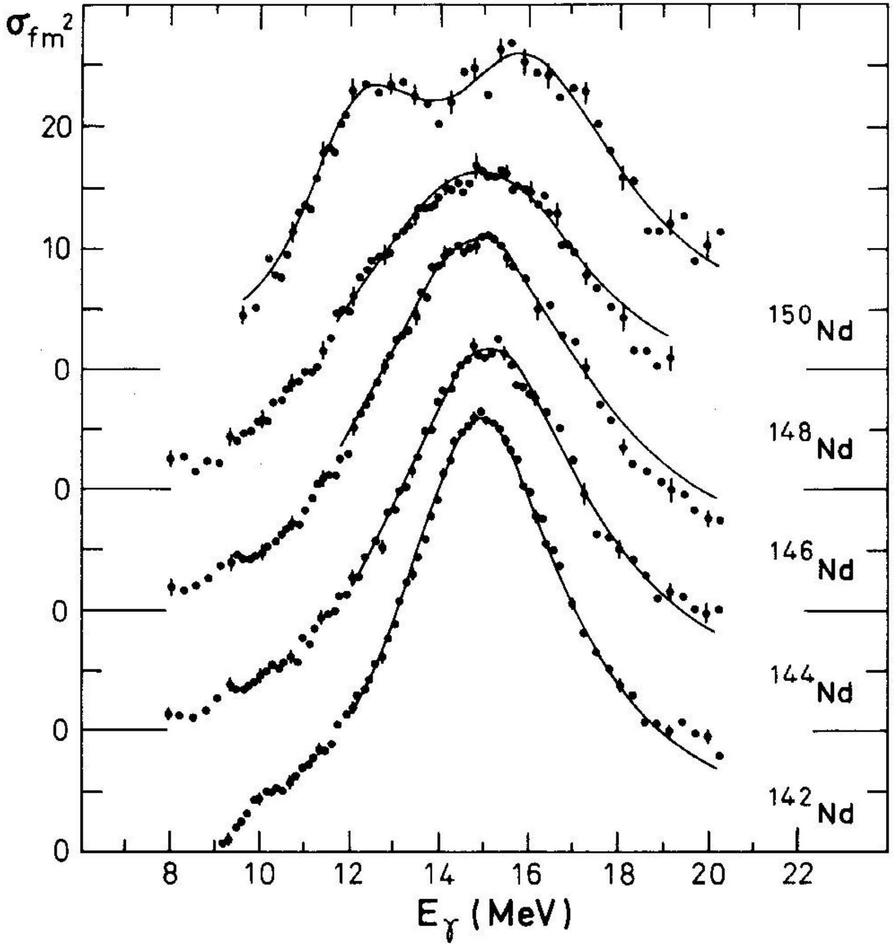
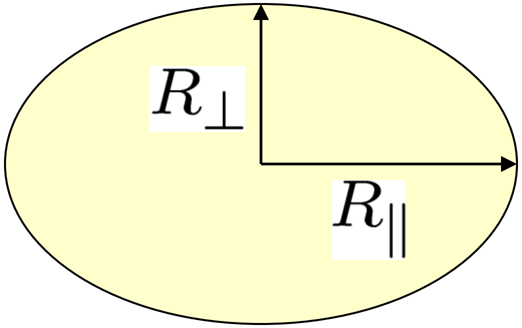
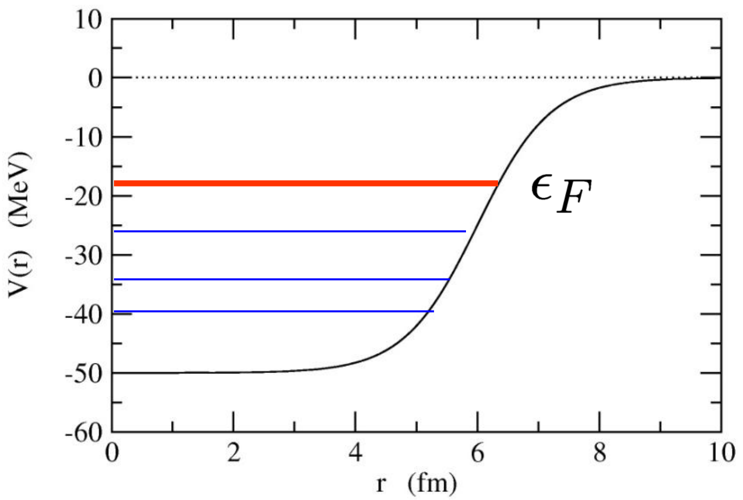


Figure 6-21 Photoabsorption cross section for even isotopes of neodymium. The experimental data are from P. Carlos, H. Beil, R. Bergère, A. Lepretre, and A. Veyssière, *Nuclear Phys.* **A172**, 437 (1971). The solid curves represent Lorentzian fits with the parameters given in Table 6-6.



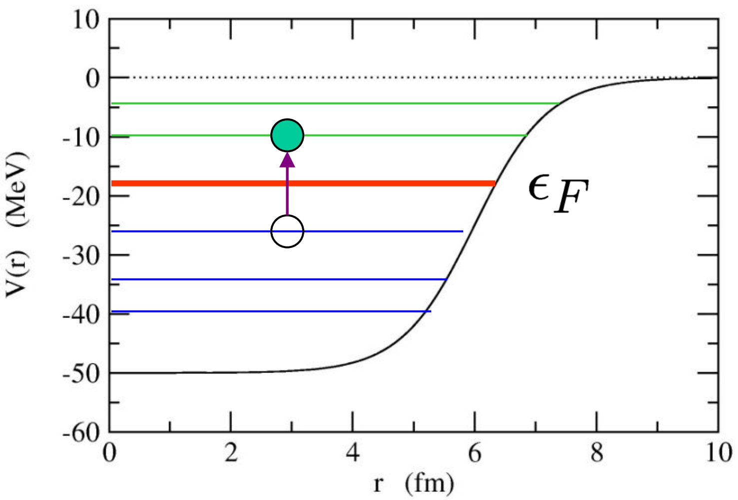
Particle-Hole excitations

Hartree-Fock state



$$|HF\rangle$$

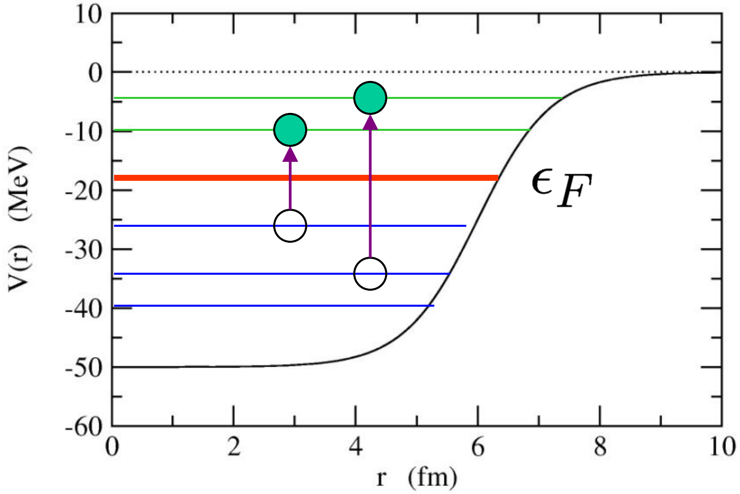
1 particle-1 hole (1p1h) state



$$a_p^\dagger a_h |HF\rangle$$

2 particle-2 hole (2p2h) state

$$a_p^\dagger a_{p'}^\dagger a_h a_{h'} |HF\rangle$$



Tamm-Dancoff Approximation

Assume: $|\nu\rangle = Q_\nu^\dagger|HF\rangle = \sum_{ph} X_{ph}\, a_p^\dagger a_h|HF\rangle$

$$= \sum_{ph} X_{ph}|ph^{-1}\rangle$$

(superposition of 1p1h states)

$$H|\nu\rangle = E_\nu|\nu\rangle$$

$$\sum_{p'h'} H_{ph,p'h'} X_{p'h'} = E_\nu X_{ph}$$

$$H_{ph,p'h'} = (\epsilon_p - \epsilon_h)\delta_{ph,p'h'} + \langle ph'|\bar{v}|hp'\rangle$$

Tamm-Dancoff equation

TDA on a schematic model

Separable interaction: $\langle ph'|\bar{v}|hp'\rangle = \lambda D_{ph} D^*_{p'h'}$

Tamm-Dancoff equation: $\displaystyle\sum_{p'h'} A_{ph,p'h'} X_{p'h'} = E X_{ph}$

$$A_{ph,p'h'} = (\epsilon_p - \epsilon_h)\delta_{ph,p'h'} + \langle ph'|\bar{v}|hp'\rangle$$

$$(\epsilon_{ph} - E)X_{ph} + \lambda D_{ph} \cdot T = 0 \qquad T \equiv \sum_{ph} D^*_{ph} X_{ph}$$

$$X_{ph} = -\lambda \frac{D_{ph} T}{\epsilon_{ph} - E}$$

$$T = \sum_{ph} D^*_{ph} X_{ph} = -\lambda \sum_{ph} \frac{|D_{ph}|^2}{\epsilon_{ph} - E} T$$

or

$$\frac{1}{\lambda} = \sum_{ph} \frac{|D_{ph}|^2}{E - \epsilon_{ph}}$$

(TDA dispersion relation)

Graphical solutions

$$\frac{1}{\lambda} = \sum_{ph} \frac{|D_{ph}|^2}{E - \epsilon_{ph}}$$

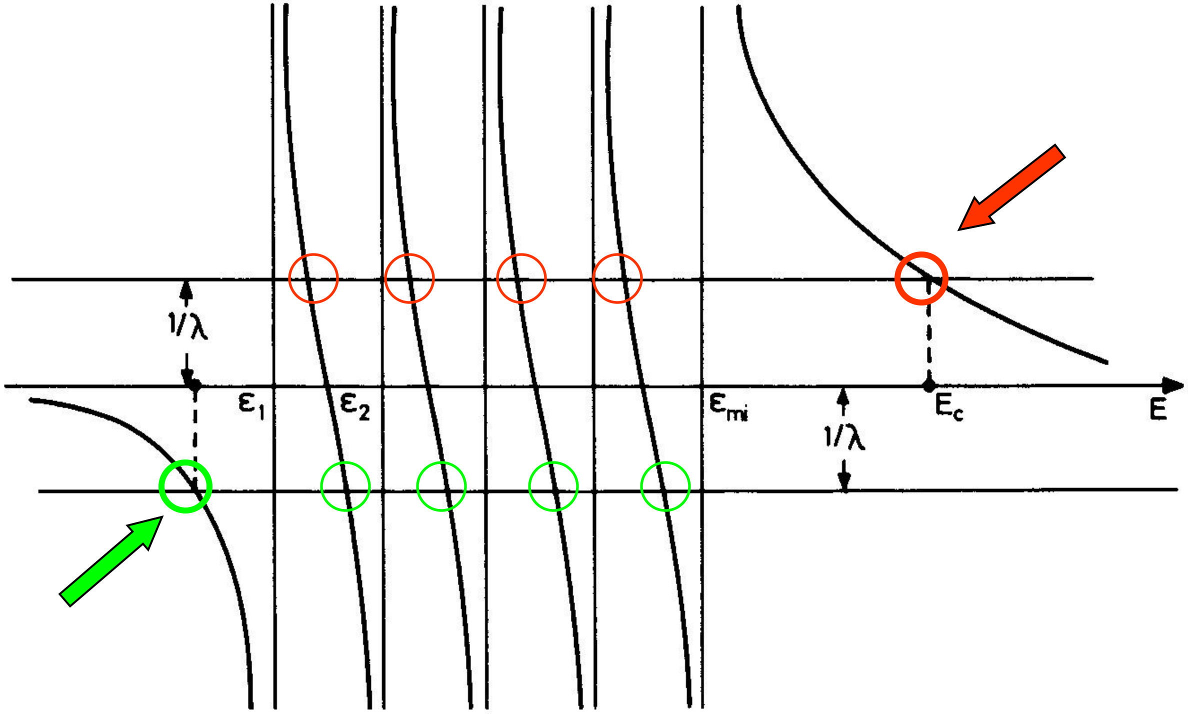


Figure 8.4. Graphical solution of Eq. (8.18).

(note) in the degenerate limit: $\epsilon_{ph} \sim \epsilon$

$$E = \epsilon_{ph} + \lambda \sum_{ph} |D_{ph}|^2, \qquad |\nu\rangle = \sum_{ph} D_{ph} a_p^\dagger a_h |HF\rangle$$

coherent superpositon of 1p1h states

Iso-scalar type modes: $E < \epsilon_{ph} \rightarrow \lambda < 0$ (attractive)

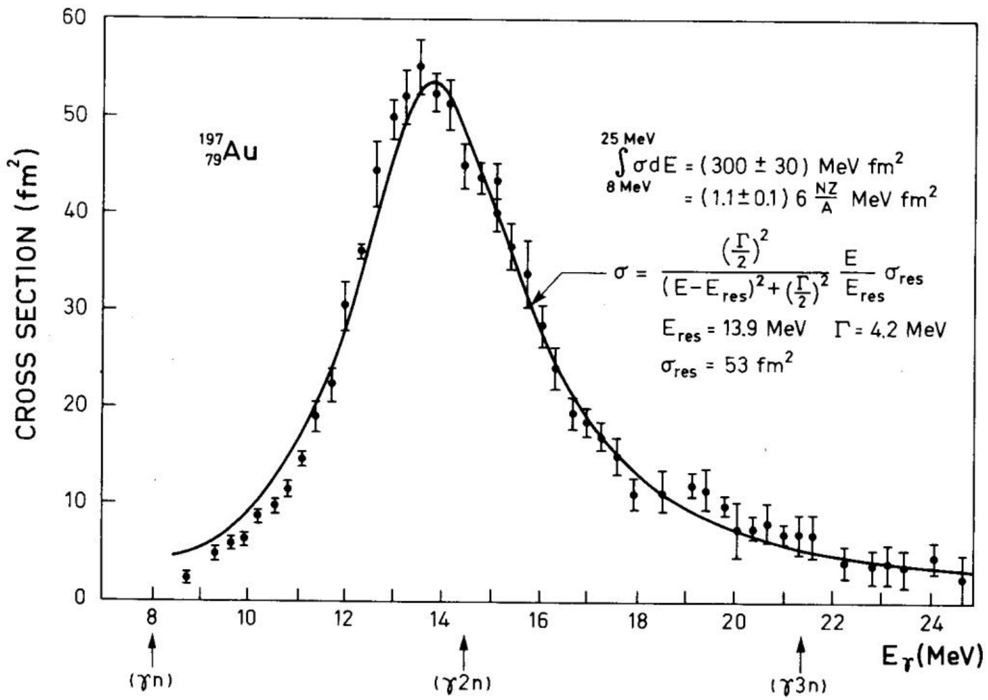
Iso-vector type modes: $E > \epsilon_{ph} \rightarrow \lambda > 0$ (repulsive)

Experimental systematics:

IV GDR: $E \sim 79 A^{-1/3}$ (MeV) ⟷ $\epsilon_{ph} \sim 41 A^{-1/3}$

IS GQR: $E \sim 65 A^{-1/3}$ (MeV) ⟷ $\epsilon_{ph} \sim 82 A^{-1/3}$

(note) single particle potential: $\hbar\omega \sim 41 A^{-1/3}$ (MeV)



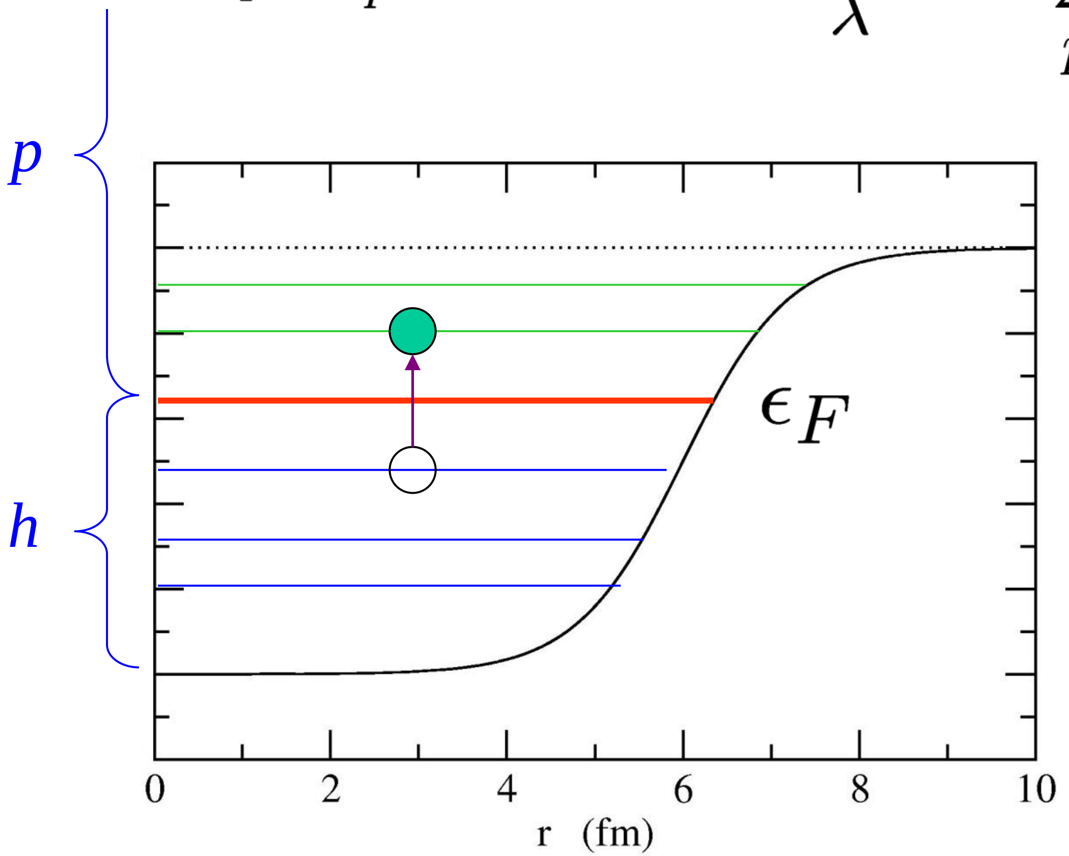
^{197}Au

$$E_{\mathrm{GDR}} = 14 \quad (\mathrm{MeV})$$

$$\epsilon_{ph} \sim 41 \cdot 197^{-1/3} \sim 7 \quad (\mathrm{MeV})$$

Continuum Excitations

$$\langle ph'|\bar{v}|hp'\rangle = \lambda D_{ph} D^*_{p'h'} \quad \Longrightarrow \quad \frac{1}{\lambda} = -\sum_{ph} \frac{|D_{ph}|^2}{\epsilon_p - \epsilon_h - E}$$



h: all the occupied (bound) states

p: the bound excited states + continuum states

$$\frac{1}{\lambda} = -\sum_{ph} \frac{|D_{ph}|^2}{\epsilon_p - \epsilon_h - E}$$

Coordinate representation: $D_{ph} = \int d\boldsymbol{r}\, \phi_p^*(\boldsymbol{r}) D(\boldsymbol{r}) \phi_h(\boldsymbol{r})$

$$\frac{1}{\lambda} = -\sum_{ph} \int d\boldsymbol{r} \int d\boldsymbol{r}'\, D(\boldsymbol{r}) D^*(\boldsymbol{r}') \frac{\phi_p^*(\boldsymbol{r})\phi_h(\boldsymbol{r})\phi_p(\boldsymbol{r}')\phi_h^*(\boldsymbol{r}')}{\epsilon_p - \epsilon_h - E}$$

(note)

$$\hat{h}\phi_p = \epsilon_p \phi_p$$

$$1 = \sum_i |\phi_i\rangle\langle\phi_i| = \sum_p |\phi_p\rangle\langle\phi_p| + \sum_h |\phi_h\rangle\langle\phi_h|$$

$$\text{rhs} = -\sum_h \int d\boldsymbol{r} \int d\boldsymbol{r}'\, D(\boldsymbol{r}) D^*(\boldsymbol{r}')\, \phi_h(\boldsymbol{r})\phi_h^*(\boldsymbol{r}')$$
$$\times \left(\left\langle \boldsymbol{r}' \left| \frac{1}{\hat{h} - \epsilon_h - E - i\eta} \right| \boldsymbol{r} \right\rangle - \sum_{h'} \frac{\phi_{h'}^*(\boldsymbol{r})\phi_{h'}(\boldsymbol{r}')}{\epsilon_{h'} - \epsilon_h - E - i\eta} \right)$$

Random Phase Approximation

Tamm-Dancoff Approximation: $|\nu\rangle = Q_\nu^\dagger |HF\rangle = \sum_{ph} X_{ph}\, a_p^\dagger a_h |HF\rangle$

(superposition of 1p1h states)

$$[H, Q_\nu^\dagger] \approx E_\nu Q_\nu^\dagger$$

$$\Longleftrightarrow \quad \langle HF|[\delta Q, [H, Q_\nu^\dagger]]|HF\rangle = E_\nu \langle HF|[\delta Q, Q_\nu^\dagger]|HF\rangle$$

Drawbacks:

- No influence of v in the ground state

$$E_{coll} = \epsilon + \lambda \sum_{ph} |D_{ph}|^2$$ ⇐ Interaction is essential in describing collective excitations

- Energy Weighted Sum Rule is violated in TDA
- Admixture of the spurious modes with the physical excitation modes

HF ⟺ Broken Symmetries (CM localization, rotation,…….)

Restoration of broken symmetries ⟹ Goldstone mode (spurious motion)

A better approximation: **the random phase approximation (RPA)**

$$|\nu\rangle = Q_\nu^\dagger|0\rangle = \sum_{ph}\left(X_{ph}\, a_p^\dagger a_h - Y_{ph}\, a_h^\dagger a_p\right)|0\rangle$$

(superposition of 1p1h states)

$$[H, Q_\nu^\dagger] \approx E_\nu Q_\nu^\dagger$$

$$\Longleftrightarrow \quad \langle 0|[\delta Q, [H, Q_\nu^\dagger]]|0\rangle = E_\nu \langle 0|[\delta Q, Q_\nu^\dagger]|0\rangle$$

δQ : arbitrary operator

(note) Harmonic oscillator: $H_{\rm HO} = \hbar\omega(a^\dagger a + 1/2)$

$$\Longrightarrow \quad [H_{\rm HO}, a^\dagger] = \hbar\omega a^\dagger$$

RPA: describes a harmonic motion

- RPA ground state: $Q_\nu|0\rangle = 0$
- Normalization: $\langle\nu|\nu\rangle = 1 \rightarrow \sum_{ph}|X_{ph}|^2 - |Y_{ph}|^2 = 1$

$$Q^\dagger_\nu = \sum_{ph} X_{ph}\, a^\dagger_p a_h - Y_{ph}\, a^\dagger_h a_p$$

$$\langle 0|[\delta Q, [H, Q^\dagger_\nu]]|0\rangle = E_\nu \langle 0|[\delta Q, Q^\dagger_\nu]|0\rangle$$

$$\begin{cases} H = \sum_{1,2} t_{12}\, a^\dagger_1\, a_2 + \frac{1}{4} \sum_{1,2,3,4} \bar{v}_{1234}\, a^\dagger_1\, a^\dagger_2\, a_4\, a_3 \\ \delta Q = a^\dagger_h a_p, \qquad a^\dagger_p a_h \end{cases}$$

→ Equations for X_{ph} and Y_{ph}

Quasi-boson approximation:

$$\langle 0|[\delta Q, [H, Q^\dagger_\nu]]|0\rangle \approx \langle HF|[\delta Q, [H, Q^\dagger_\nu]]|HF\rangle$$

$$\langle 0|[\delta Q, Q^\dagger_\nu]|0\rangle \approx \langle HF|[\delta Q, Q^\dagger_\nu]|HF\rangle$$

(note)

$$\langle 0|[Q_\nu, Q^\dagger_\nu]|0\rangle \approx \langle HF|[Q_\nu, Q^\dagger_\nu]|HF\rangle = 1$$

$$\langle HF|[\delta Q,[H,Q_\nu^\dagger]]|HF\rangle = E_\nu\langle HF|[\delta Q,Q_\nu^\dagger]|HF\rangle$$

$$Q_\nu^\dagger = \sum_{ph} X_{ph}\,a_p^\dagger a_h - Y_{ph}\,a_h^\dagger a_p \qquad \delta Q = a_h^\dagger a_p, \quad a_p^\dagger a_h$$

RPA equation:

$$\begin{aligned}\sum_{p'h'} A_{ph,p'h'}X_{p'h'} + B_{ph,p'h'}Y_{p'h'} &= E_\nu X_{ph}\\ \sum_{p'h'} B^*_{ph,p'h'}X_{p'h'} + A^*_{ph,p'h'}Y_{p'h'} &= -E_\nu Y_{ph}\end{aligned}$$

$$\begin{aligned}A_{ph,p'h'} &= (\epsilon_p-\epsilon_h)\delta_{ph,p'h'} + \langle ph'|\bar{v}|hp'\rangle\\ B_{ph,p'h'} &= \langle pp'|\bar{v}|hh'\rangle\end{aligned}$$

or

$$\begin{pmatrix} A & B \\ -B^* & -A^* \end{pmatrix}\begin{pmatrix} X \\ Y \end{pmatrix} = E_\nu \begin{pmatrix} X \\ Y \end{pmatrix}$$

RPA on a schematic model

Separable interaction: $\langle ph'|\bar{v}|hp'\rangle = \lambda D_{ph} D^*_{p'h'}$

$$\frac{1}{\lambda} = \sum_{ph} \frac{|D_{ph}|^2}{E - \epsilon_{ph}} - \frac{|D_{ph}|^2}{E + \epsilon_{ph}}$$

(RPA dispersion relation)

Cf. TDA dispersion relation: $\frac{1}{\lambda} = \sum_{ph} \frac{|D_{ph}|^2}{E - \epsilon_{ph}}$

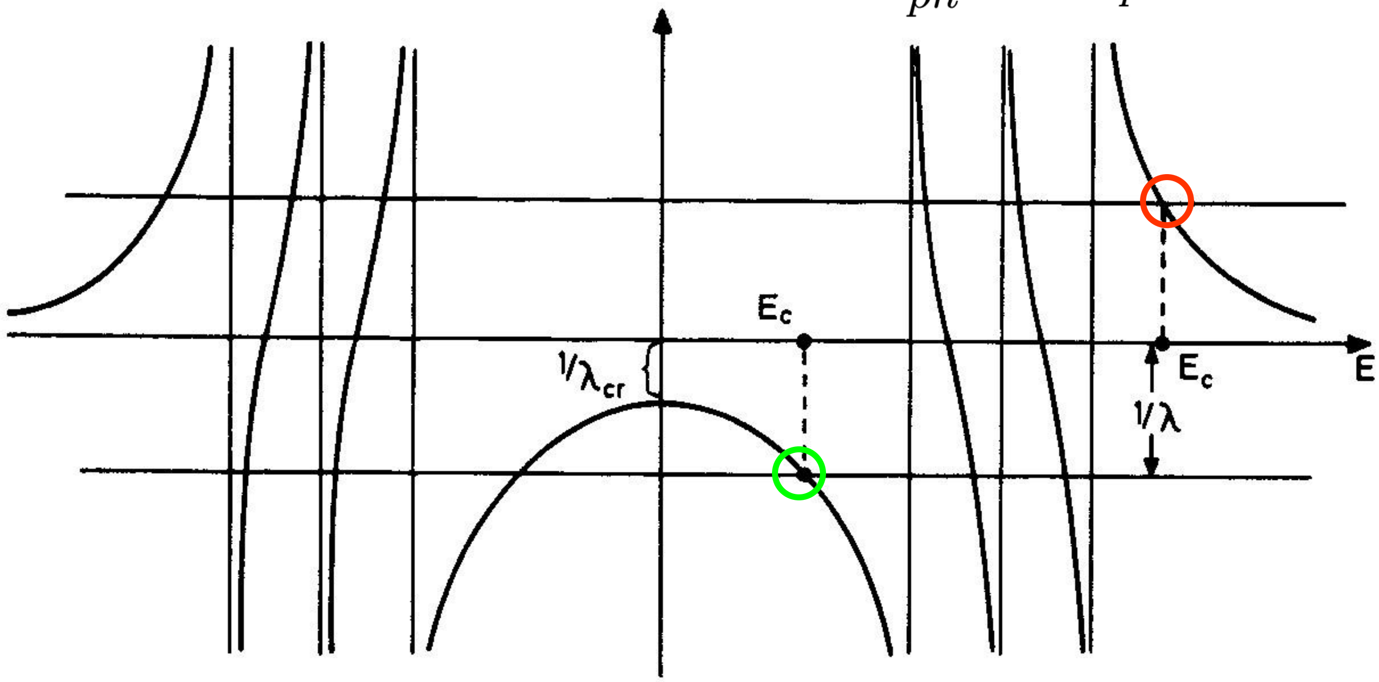
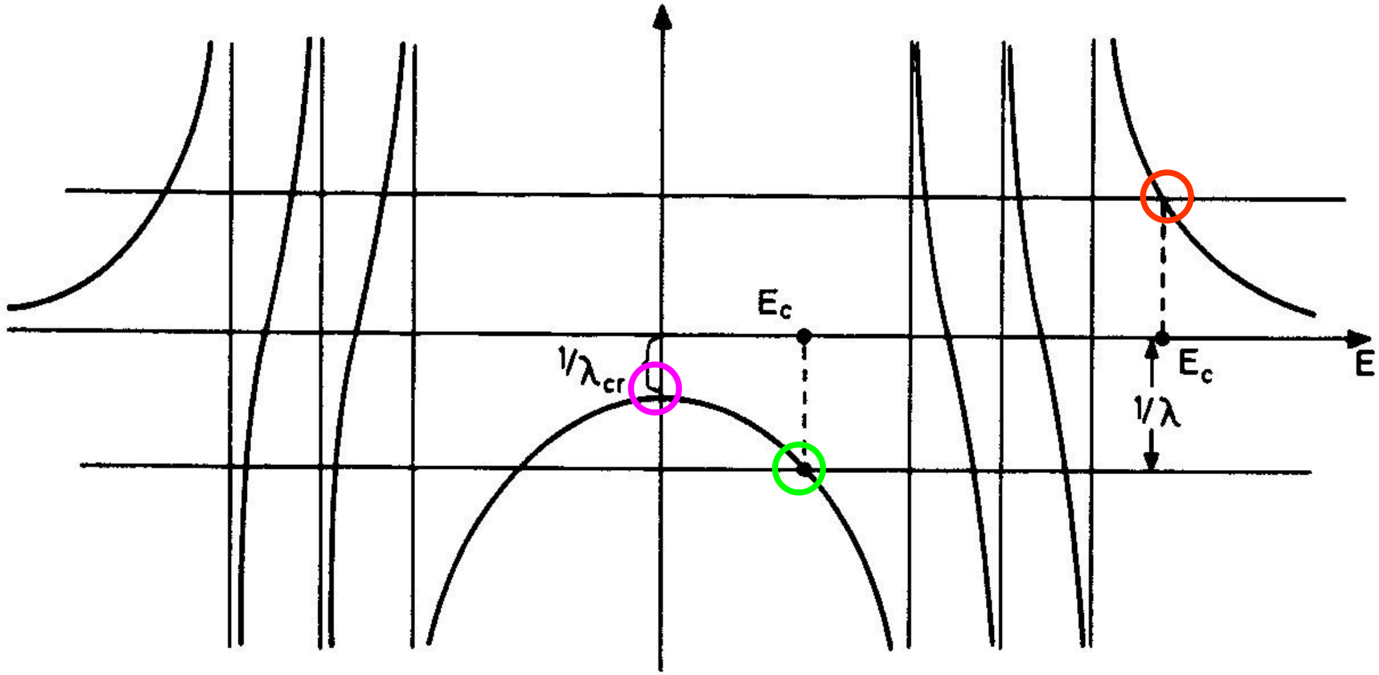


Figure 8.11. Graphical solution of the dispersion relation (8.135).

$$\frac{1}{\lambda} = \sum_{ph} \frac{|D_{ph}|^2}{E - \epsilon_{ph}} - \frac{|D_{ph}|^2}{E + \epsilon_{ph}}$$

(RPA dispersion relation)



i) Critical strength for attractive interaction

$$\lambda > \lambda_{crit} \rightarrow E^2 < 0$$ ⟷ Instability of the HF state

ii) Symmetric between *E* and –*E*

iii) In the degenerate limit $$E^2 = \epsilon^2 + 2\epsilon\lambda \sum_{ph} |D_{ph}|^2$$

Spurious motion in RPA

Mean-Field Approximation ⟷ Broken symmetiries

- Center of mass localization (single center)
- Rotational motion

Restoration of broken symmetries

⇒ Zero mode (Nambu-Goldstone mode)

RPA $\langle HF|[\delta Q,[H,Q_\nu^\dagger]]|HF\rangle = E_\nu \langle HF|[\delta Q, Q_\nu^\dagger]|HF\rangle$

if $[H,\hat{O}]=0$

Then $\hat{O}$ is a solution of RPA with *E=0*

$$\hat{O}=\sum_{ph}(O_{ph}a_p^\dagger a_h + O_{hp}a_h^\dagger a_p)$$

The physical solutions are exactly separated out from the spurious modes.