原子核物理学 II

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Nuclei: aggregate of nucleons (protons and neutrons)

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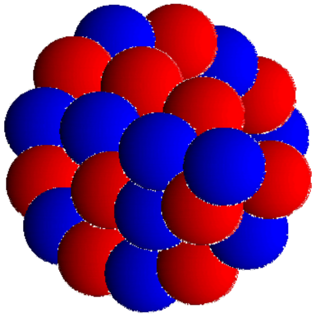

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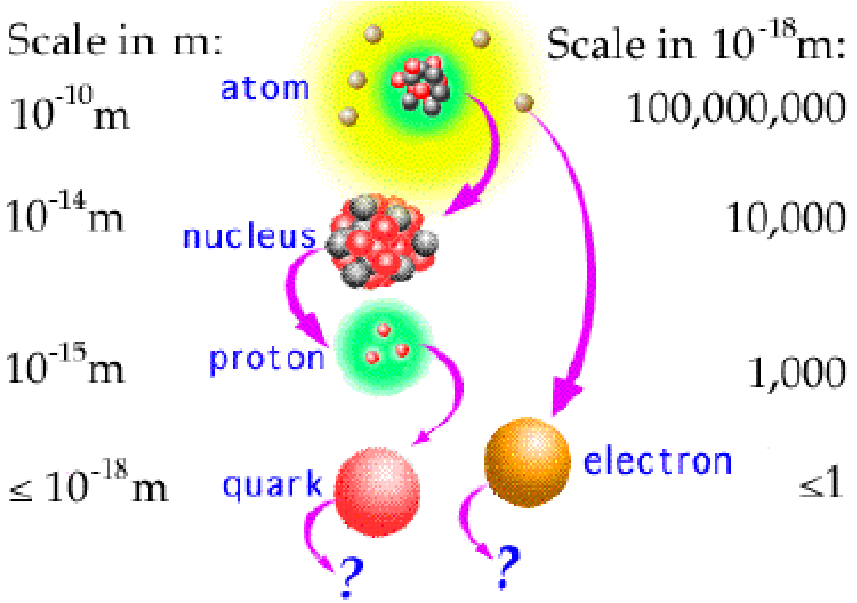
Basic Properties of Nuclei

Nuclear Physics



1 fm = 10^{-13} cm

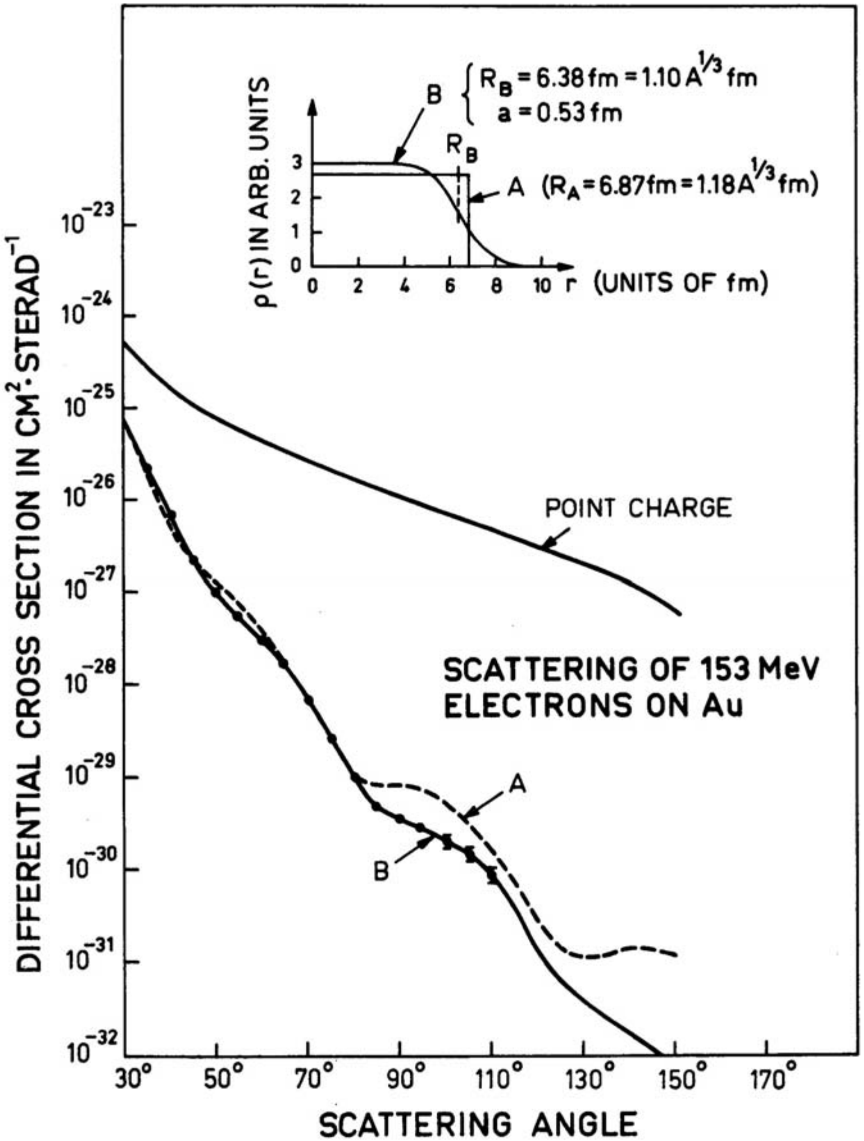
Nucleus as a *quantum many body system*

Basic ingredients:

	charge	mass (MeV)	spin
Proton	+e	938.256	½+
Neutron	0	939.550	½+

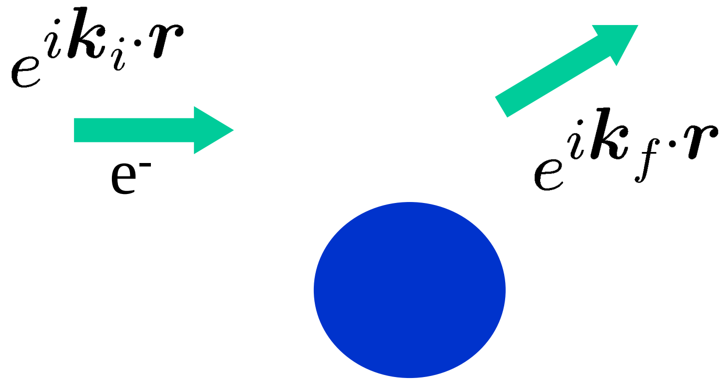
(note) $n \longrightarrow p + e^- + \bar{\nu}$ (10.4 min)

Density Distribution



High energy electron scattering

Born approximation:

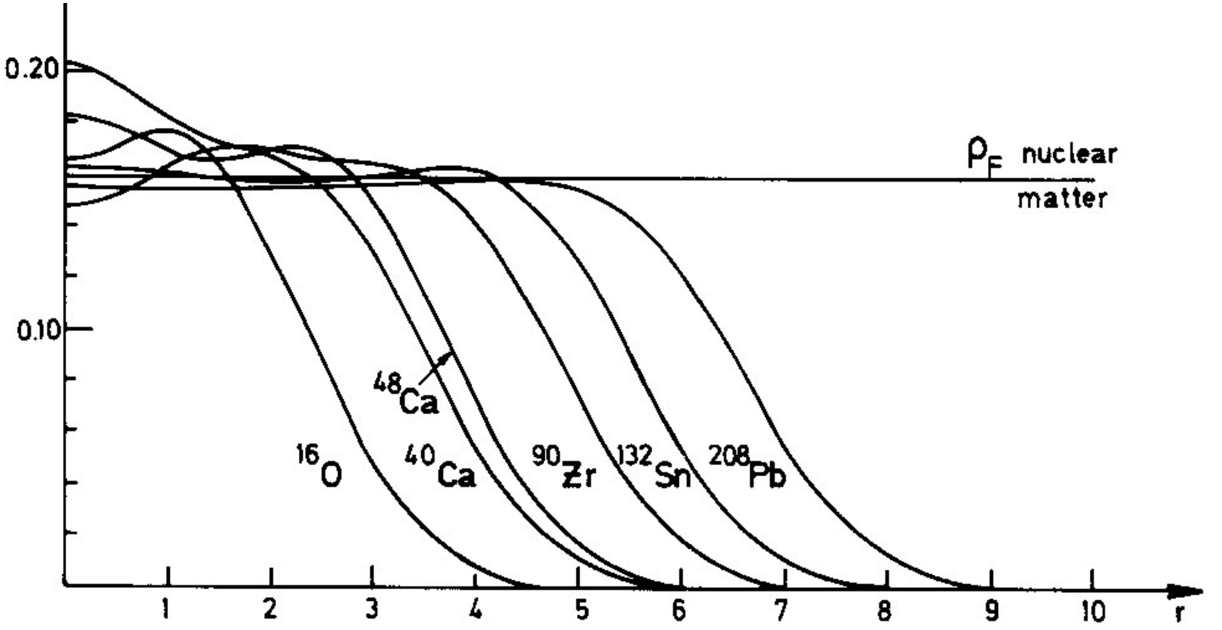


$$\frac{d\sigma}{d\Omega} = \frac{Z_P^2 e^4}{(4E\sin^2\theta/2)^2}|F(\boldsymbol{q})|^2$$

Form factor

$$F(\boldsymbol{q}) = \int e^{-i\boldsymbol{q}\cdot\boldsymbol{r}}\,\rho(\boldsymbol{r})\,d\boldsymbol{r}$$

(Fourier transform of the density)



Fermi distribution

$$\rho(r) = \rho_0/[1 + \exp((r - R_0)/a)]$$

$$\rho_0 \sim 0.17 \text{ (fm}^{-3}\text{)}$$

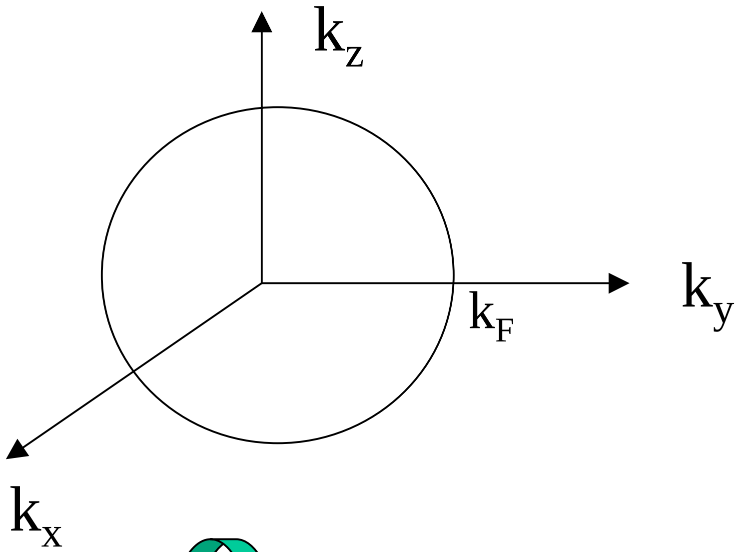
← Saturation property

$$R_0 \sim 1.1 \times A^{1/3} \text{ (fm)}$$

$$a \sim 0.57 \text{ (fm)}$$

Momentum Distribution

Fermi gas approximation



$$\rho = 2 \times 2 \times 4\pi \int_0^{k_F} \frac{k^2 dk}{(2\pi)^3} = \frac{2}{3\pi^2} k_F^3$$

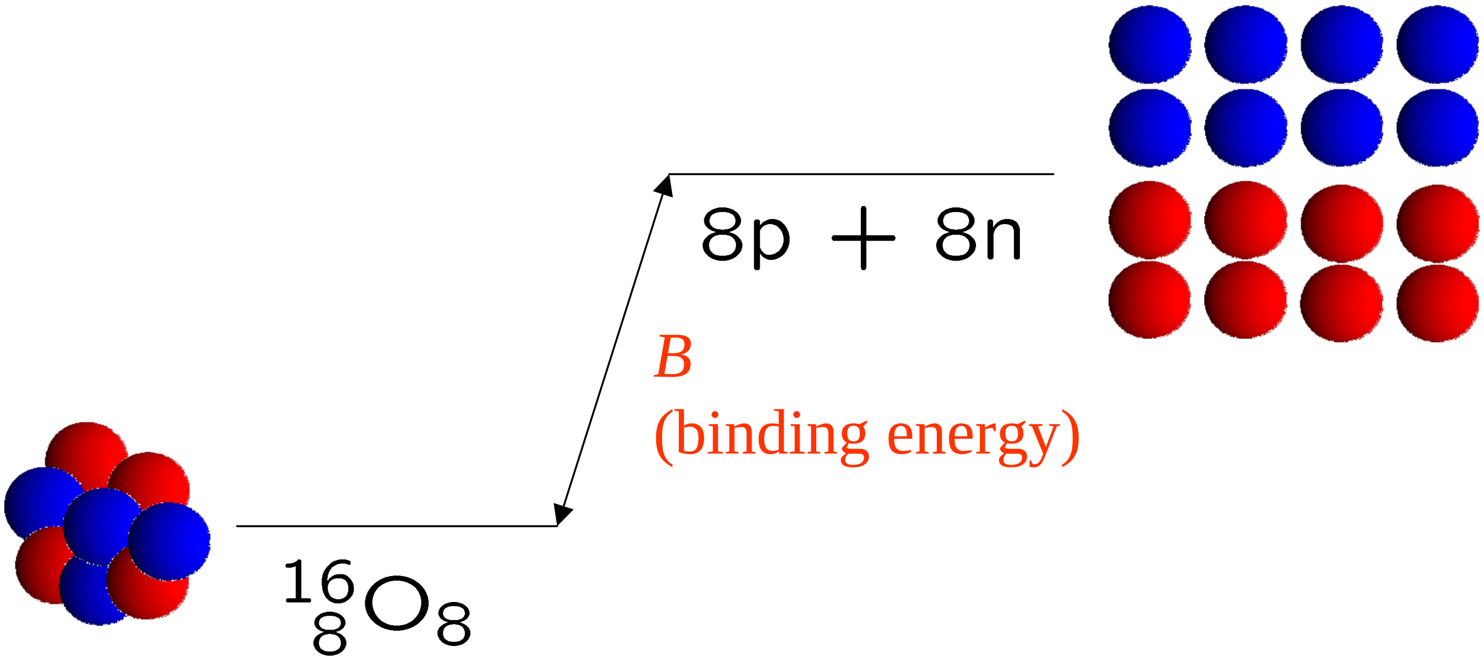
(note: spin-isospin degeneracy)

$$k_F \sim 1.36 \qquad (\mathrm{fm}^{-1})$$

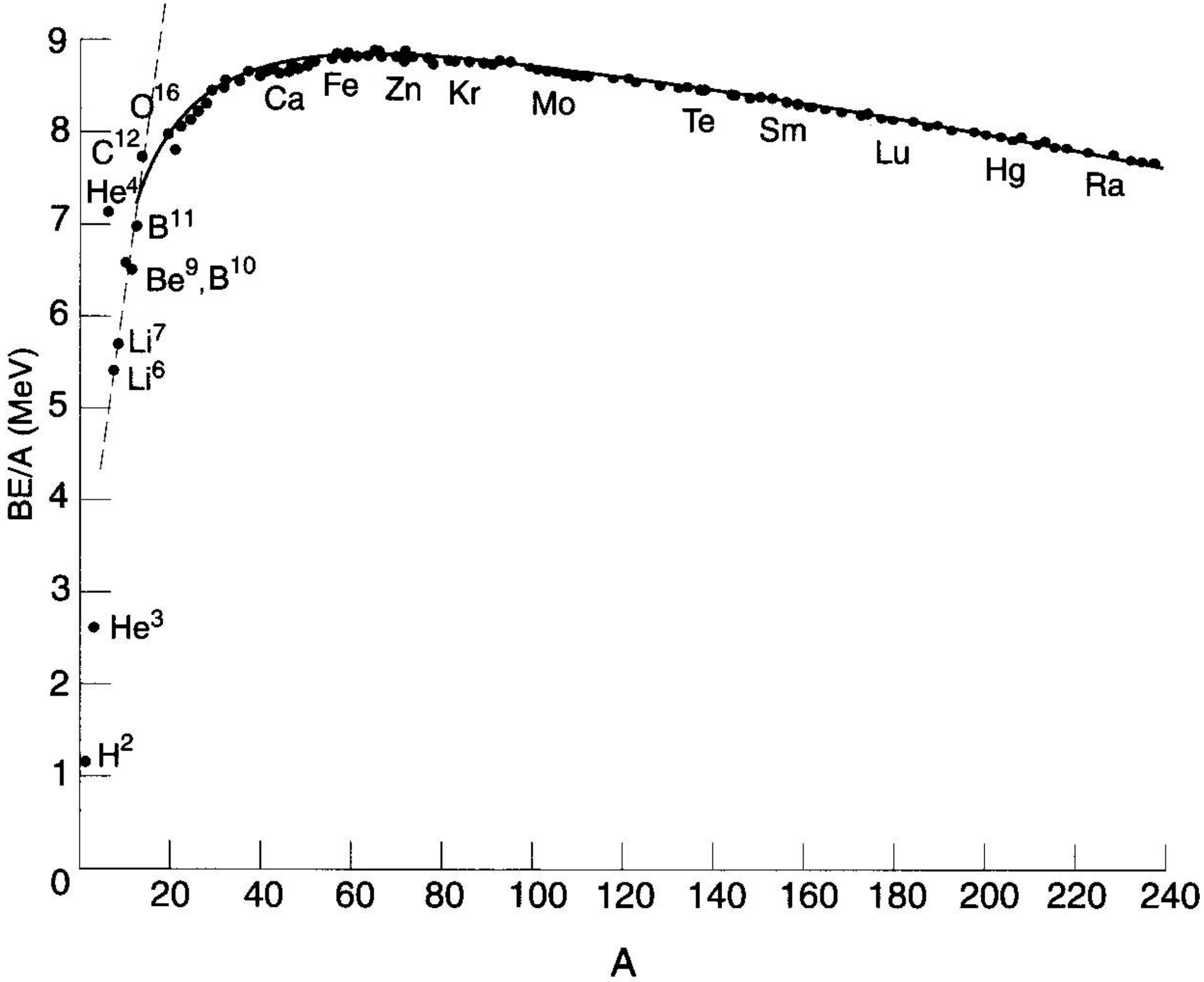
$$\Longleftrightarrow \quad \frac{v_F}{c} = \frac{k_F \cdot \hbar c}{mc^2} = 0.285$$

Fermi energy: $\epsilon_F = \dfrac{k_F^2 \hbar^2}{2m} \sim 37$ (MeV)

Nuclear Mass



$$m(N,Z)c^2 = Zm_pc^2 + Nm_nc^2 - B$$

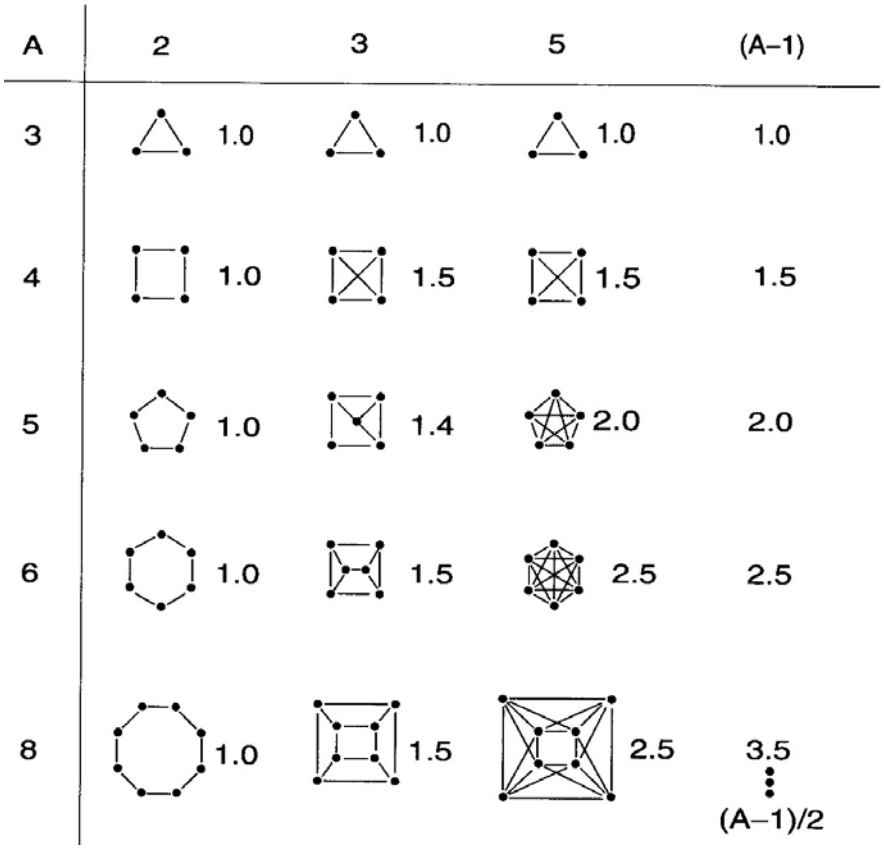


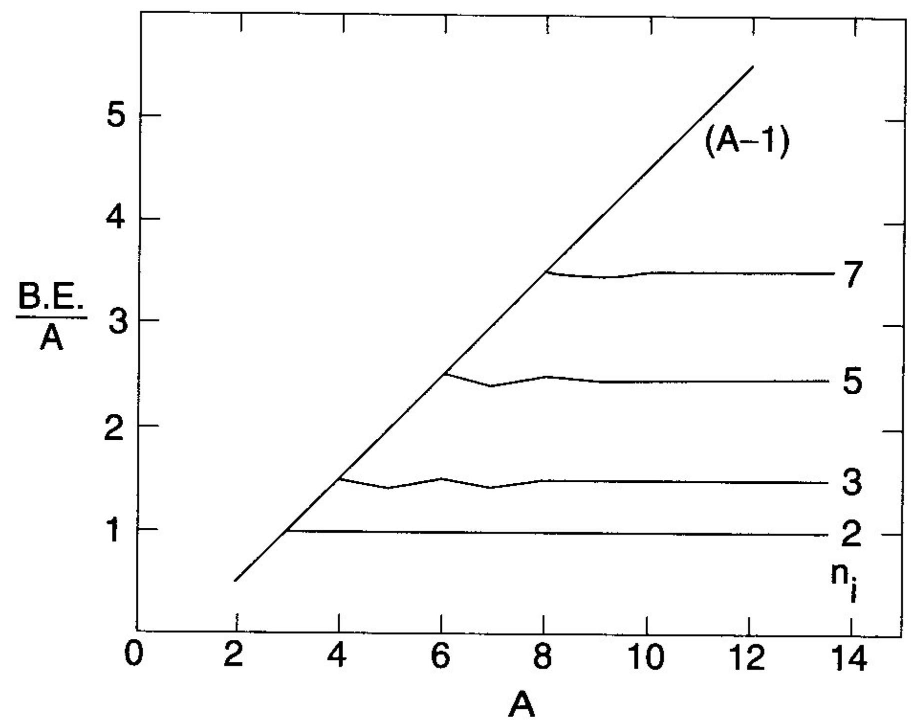
1. B(N,Z)/A ~ 8.5 MeV (A > 12) ⟺ Short range nuclear force

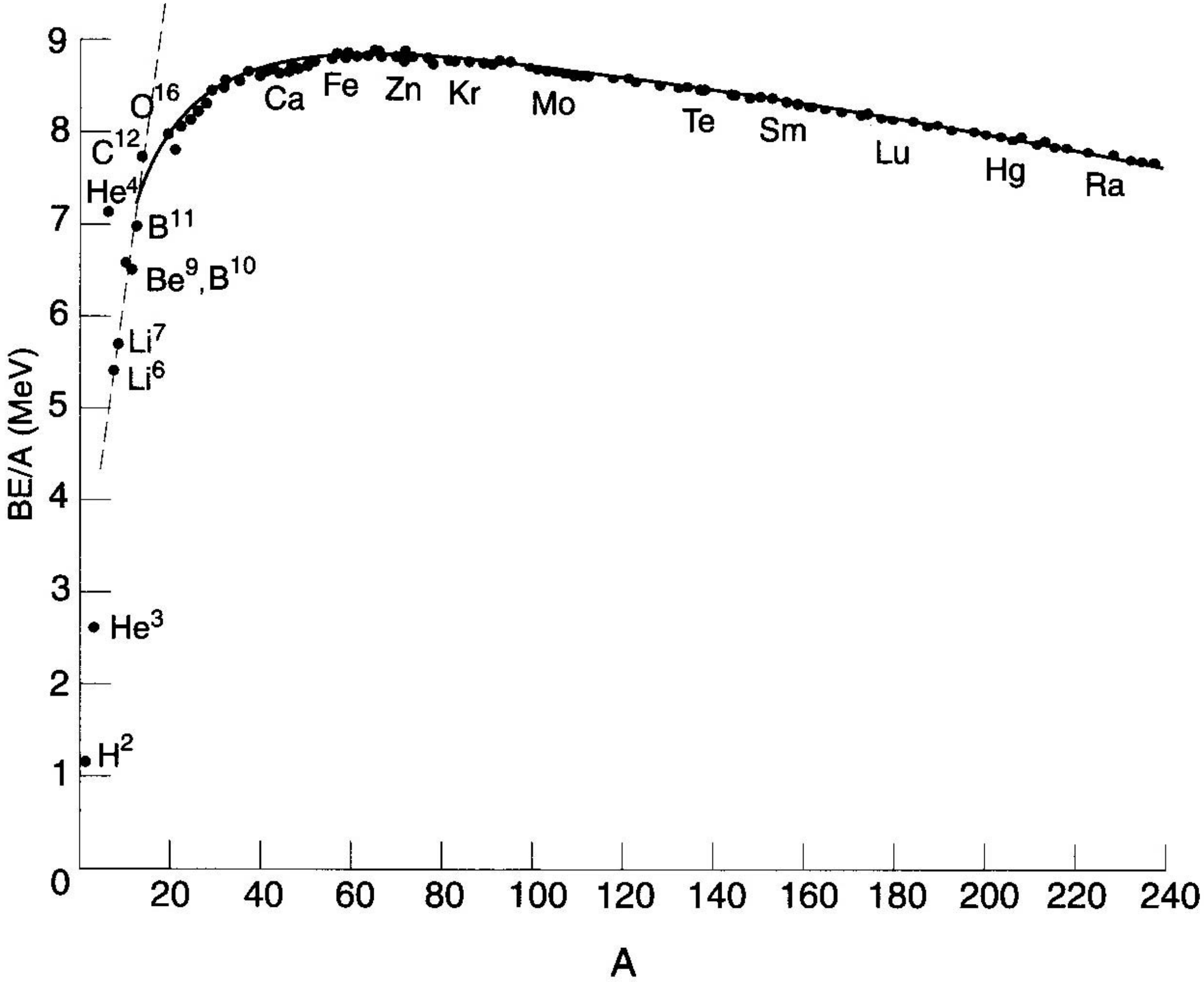
Long vs short range interaction

Long range force: $B \propto A(A-1)/2$ $B/A \propto A$

Short range force: saturation

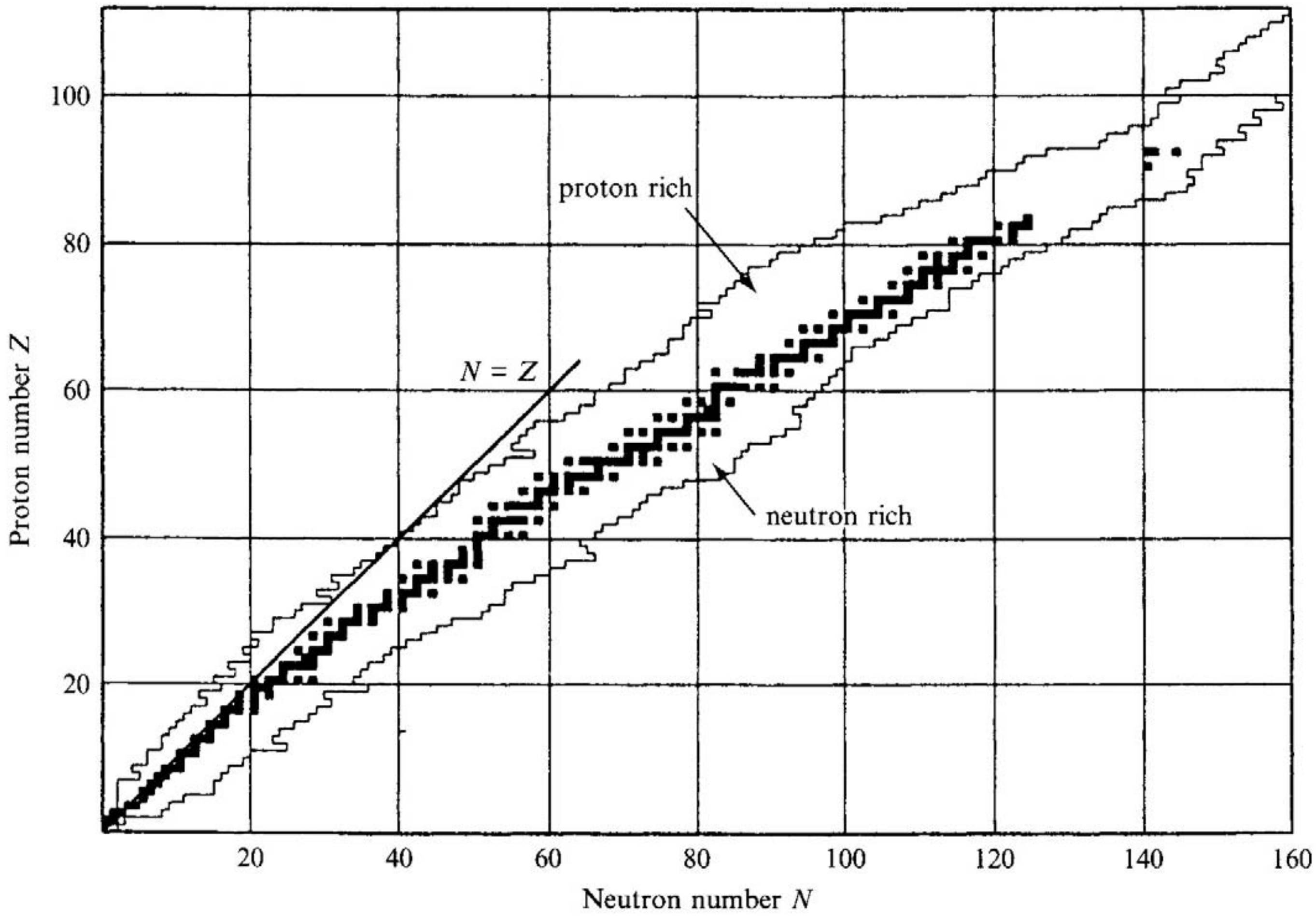




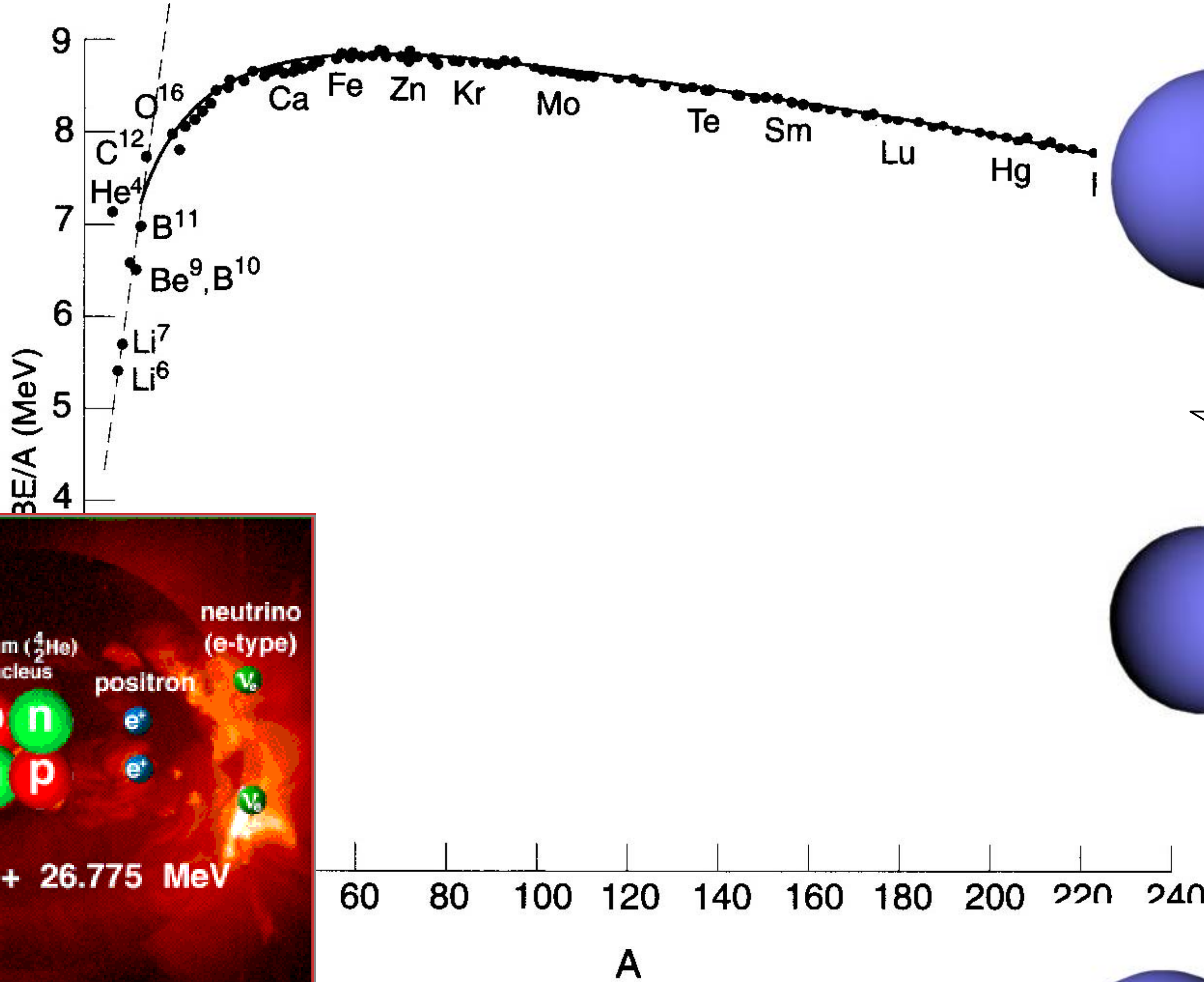


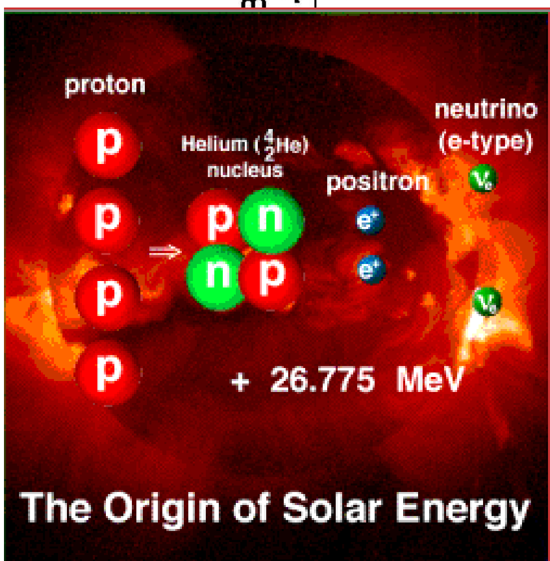
1. B(N,Z)/A ~ 8.5 MeV (A > 12) ⟺ Short range nuclear force
2. Effect of Coulomb force for heavy nuclei

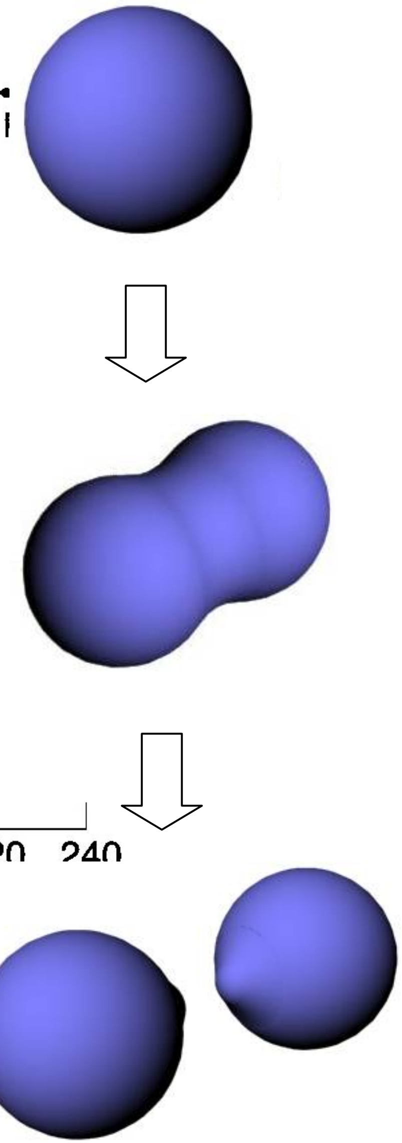
Nuclear Chart



Stable nuclei: $N \geq Z$







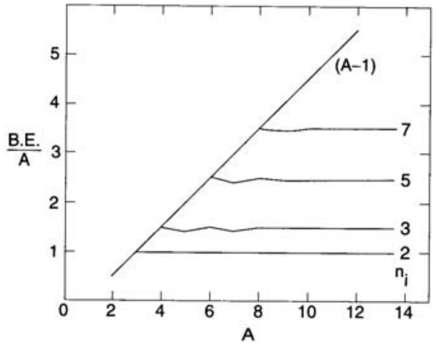
1. B(N,Z)/A ~ 8.5 MeV (A > 12) ⟺ Short range
2. Effect of Coulomb force for heavy nuclei
3. Fusion for light nuclei
4. Fission for heavy nuclei

Semi-empirical mass formula

(Bethe-Weizacker formula: Liquid-drop model)

$$B(N,Z) = a_v A - a_s A^{2/3} - a_C \frac{Z^2}{A^{1/3}} - a_{\text{sym}} \frac{(N-Z)^2}{A}$$

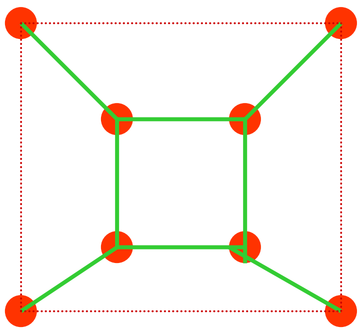
- Volume energy: $a_v A$



$$R_0 \sim 1.1 \times A^{1/3} \;\rightarrow\; V \propto A$$
$$S \propto A^{2/3}$$

- Surface energy: $-a_s A^{2/3}$

A nucleon near the surface
interacts with fewer nucleons.



$$B(N,Z) = a_v A - a_s A^{2/3} - a_C \frac{Z^2}{A^{1/3}} - a_{\text{sym}} \frac{(N-Z)^2}{A}$$

- Coulomb energy: $-a_C\, Z^2/A^{1/3}$

$$E_C = \frac{3}{5}\frac{Z^2 e^2}{R_C}$$ for a uniformly charged sphere

- Symmetry energy: $-a_{\text{sym}}\,(N-Z)^2/A$

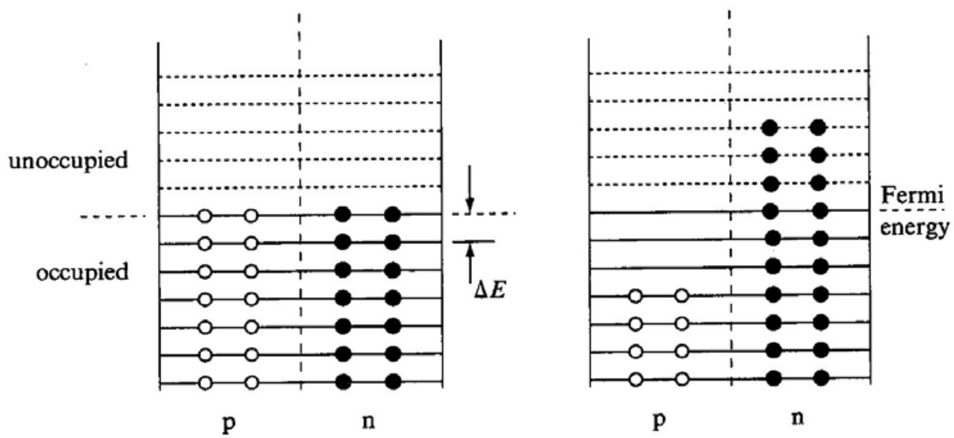
Potential energy $v_{nn} = v_{pp} = v, \quad v_{np} \sim 2v$

a nucleon interacting with nuclear matter:

$$N(v_{nn}N/A + v_{pn}Z/A) + Z(v_{pn}N/A + v_{pp}Z/A) = \frac{v}{2}(3A - (N-Z)^2/A)$$

Kinetic energy

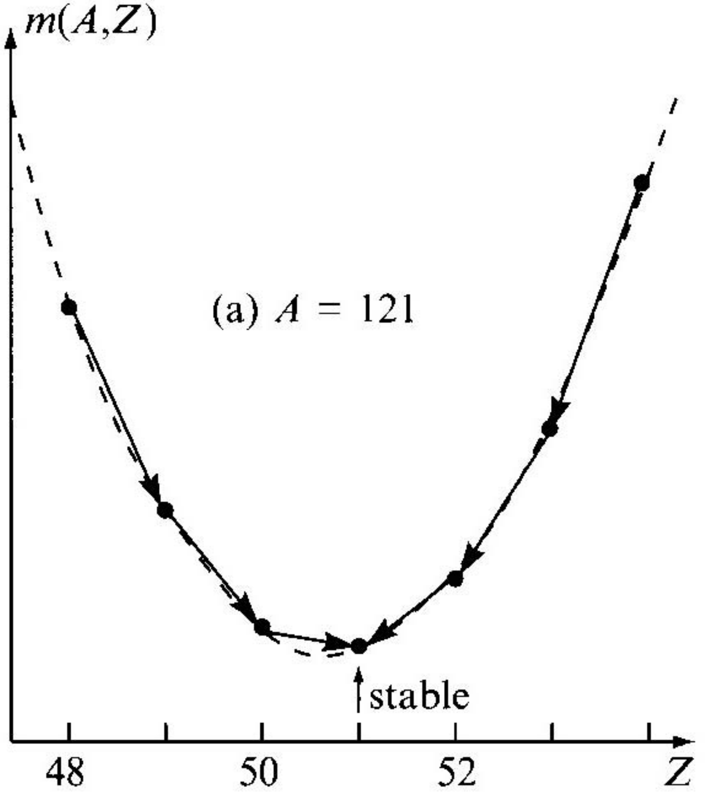
Pauli exclusion principle



β -stability line

$$B(N,Z) = a_v A - a_s A^{2/3} - a_C \frac{Z^2}{A^{1/3}} - a_{\rm sym} \frac{(N-Z)^2}{A}$$

$$m(A,Z) = f(A) + a_C \frac{Z^2}{A^{1/3}} + a_{\rm sym} \frac{(A-2Z)^2}{A}$$



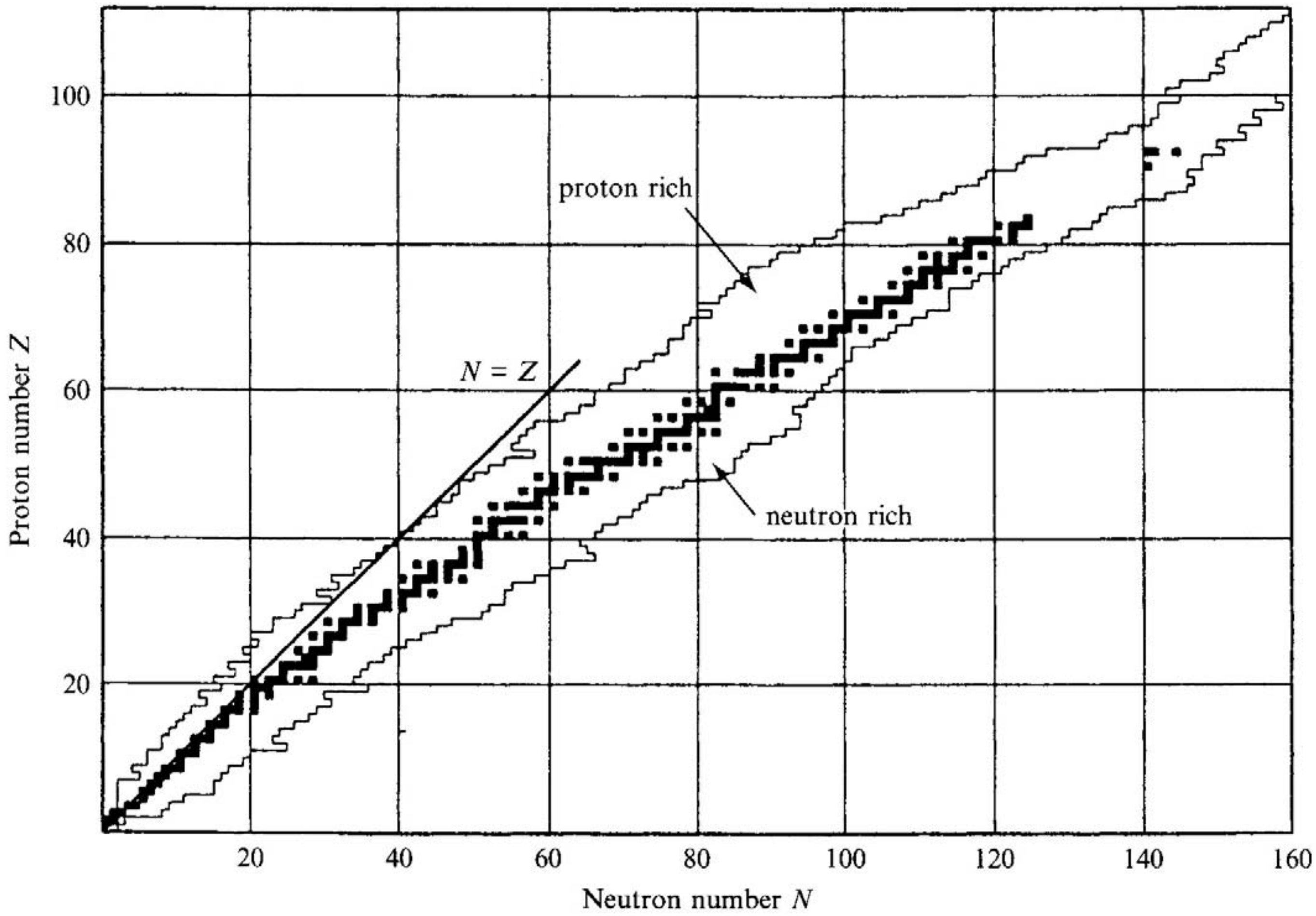
Stable nuclei (beta-stability line)

$$\left.\frac{\partial m}{\partial Z}\right|_{A=const.} = 0$$

$$Z = \frac{4a_{\rm sym}}{2a_C/A^{1/3} + 8a_{\rm sym}/A}$$

$\Rightarrow$ Z < A/2

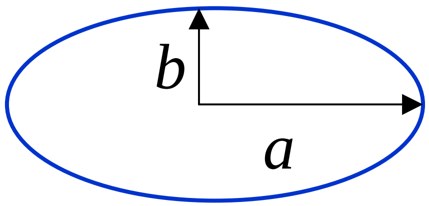
Nuclear Chart



Stable nuclei: $N \geq Z$

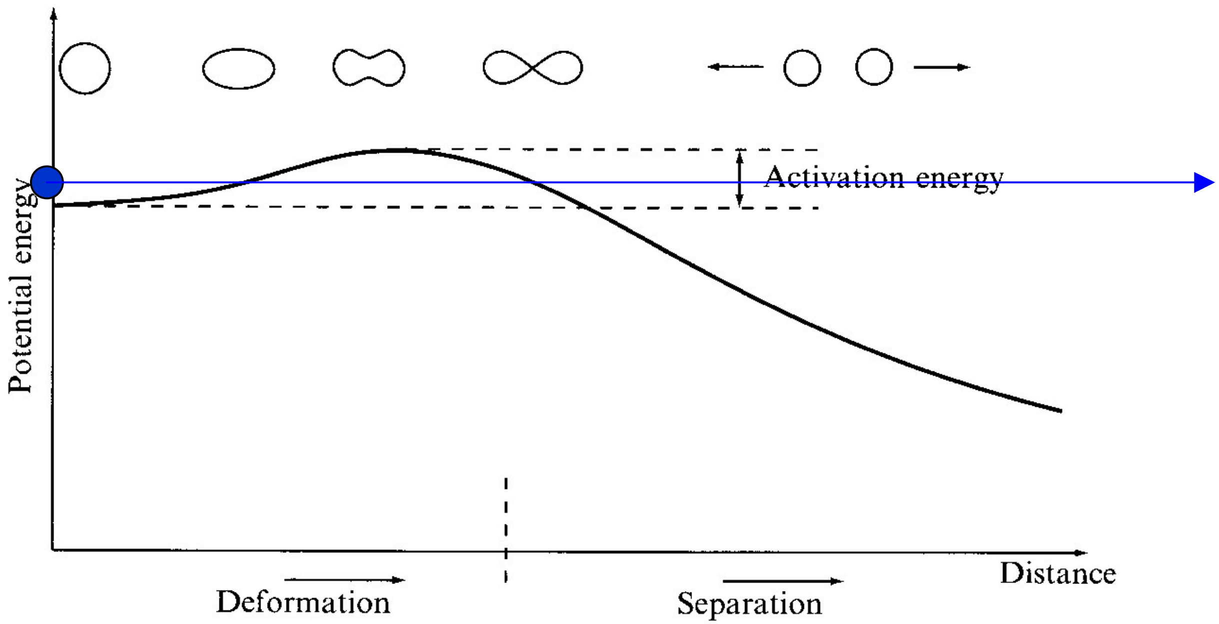
Nuclear Fission

$$B(N,Z) = a_v A - a_s A^{2/3} - a_C \frac{Z^2}{A^{1/3}} - a_{\mathrm{sym}} \frac{(N-Z)^2}{A}$$



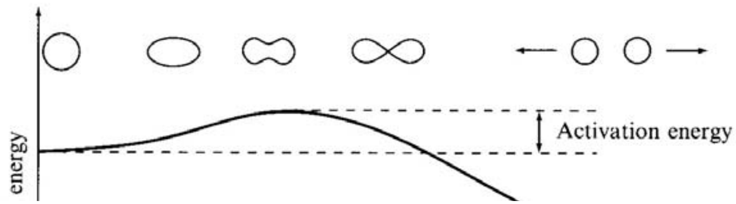
$$\begin{aligned} a &= R \cdot (1+\epsilon) \\ b &= R \cdot (1+\epsilon)^{-1/2} \end{aligned}$$

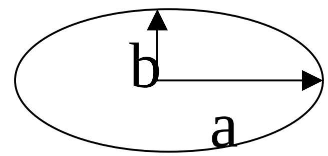
$$\begin{aligned} E_{\mathrm{surf}} &= E_{\mathrm{surf}}^{(0)} \left(1 + 2\epsilon^2/5 + \cdots\right) \\ E_C &= E_C^{(0)} \left(1 - \epsilon^2/5 + \cdots\right) \end{aligned}$$



Quantum tunneling

Collective Vibrations





$$\begin{aligned} a &= R \cdot (1+\epsilon) \\ b &= R \cdot (1+\epsilon)^{-1/2} \end{aligned}$$

In general, $R(\theta, \phi) = R_0 \left(1 + \sum_{\lambda,\mu} \alpha_{\lambda\mu} Y^*_{\lambda\mu} \right)$

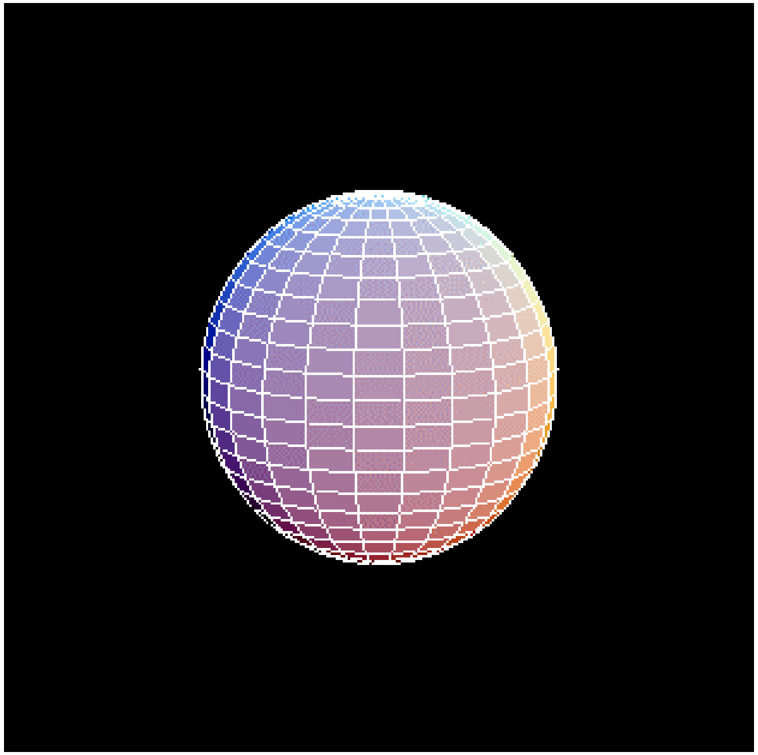
$$V = \frac{1}{2} \sum_{\lambda,\mu} C_\lambda \left|\alpha_{\lambda\mu}\right|^2$$

Quantization: Harmonic Vibrations

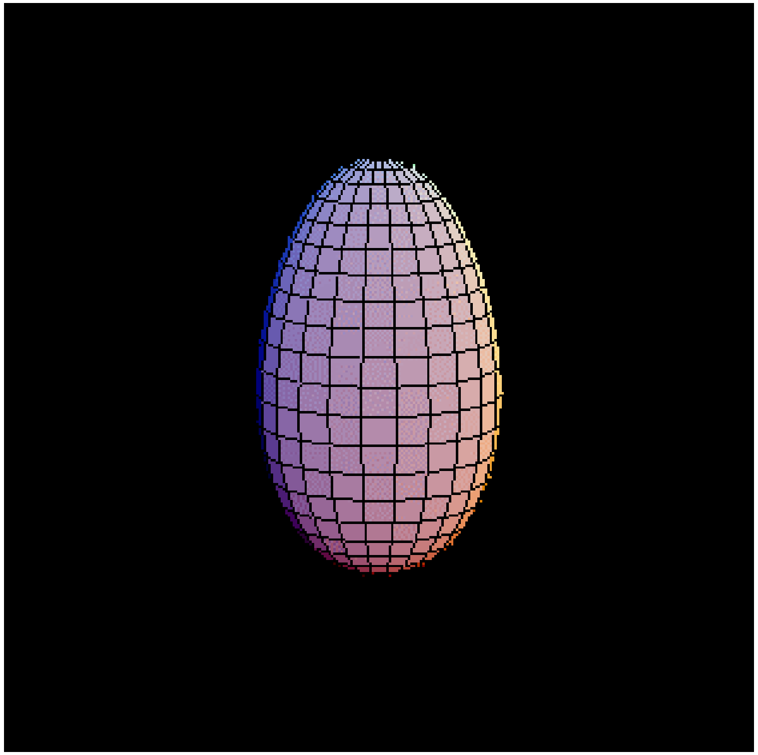
(note) moment of inertia $\Leftarrow$ incompressible and irrotational flow

$$R(\theta,\phi) = R_0\left(1 + \sum_{\lambda,\mu} \alpha_{\lambda\mu} Y^*_{\lambda\mu}\right)$$

$$V = \frac{1}{2}\sum_{\lambda,\mu} C_\lambda \left|\alpha_{\lambda\mu}\right|^2$$



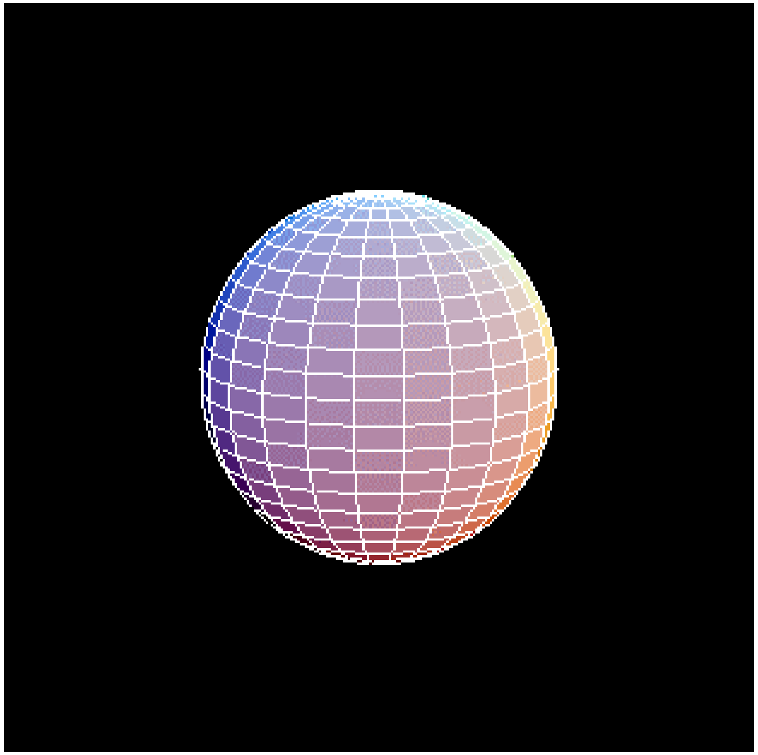
λ =2: Quadrupole vibration



λ =3: Octupole vibration

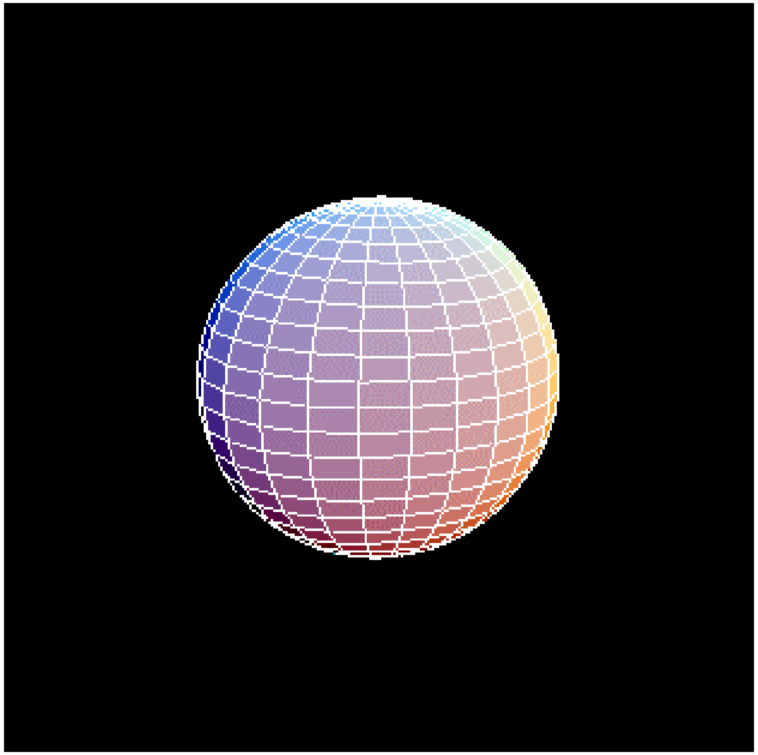
$$R(\theta,\phi) = R_0\left(1 + \sum_{\lambda,\mu} \alpha_{\lambda\mu} Y^*_{\lambda\mu}\right)$$

$$V = \frac{1}{2}\sum_{\lambda,\mu} C_\lambda \left|\alpha_{\lambda\mu}\right|^2$$



Y_{20} 型振動

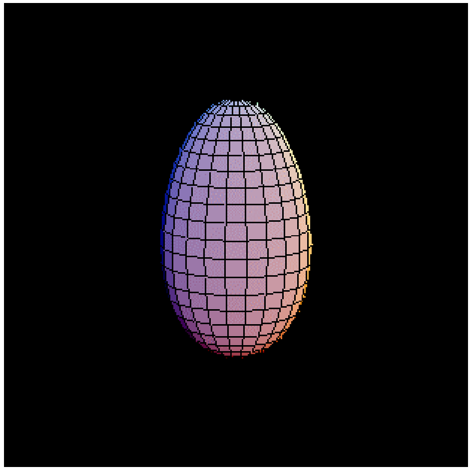
λ =2, μ = 0



Y_{22} 型振動

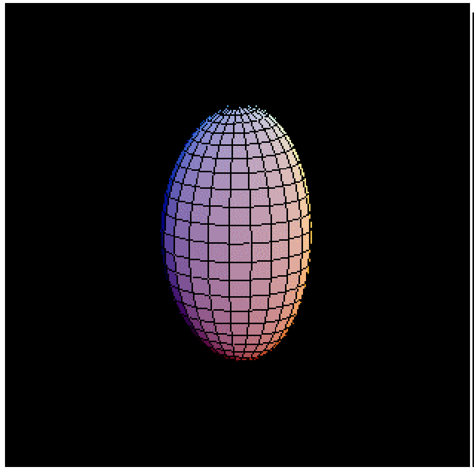
λ =2, μ = +/- 2

$$R(\theta,\phi) = R_0 \left(1 + \sum_{\lambda,\mu} \alpha_{\lambda\mu} Y^*_{\lambda\mu}\right) \qquad V = \frac{1}{2}\sum_{\lambda,\mu} C_\lambda \left|\alpha_{\lambda\mu}\right|^2$$



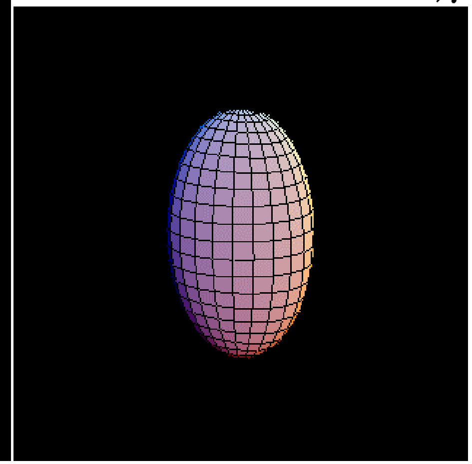
Y_{30} 型振動

$\lambda=3,\ \mu = 0$



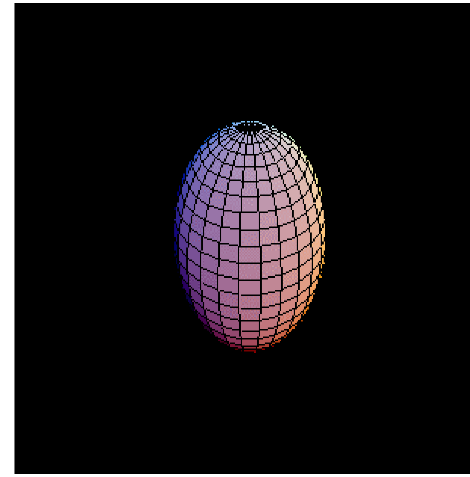
Y_{31} 型振動

$\lambda=3,\ \mu = +/- 1$



Y_{32} 型振動

$\lambda=3,\ \mu = +/- 2$



Y_{33} 型振動

$\lambda=3,\ \mu = +/- 3$

どのくらいのエネルギーを与えれば原子核は振動しはじめるのか？

$\Longleftrightarrow$ 振動の励起エネルギー

ムービー：在田謙一郎氏（名古屋工大）

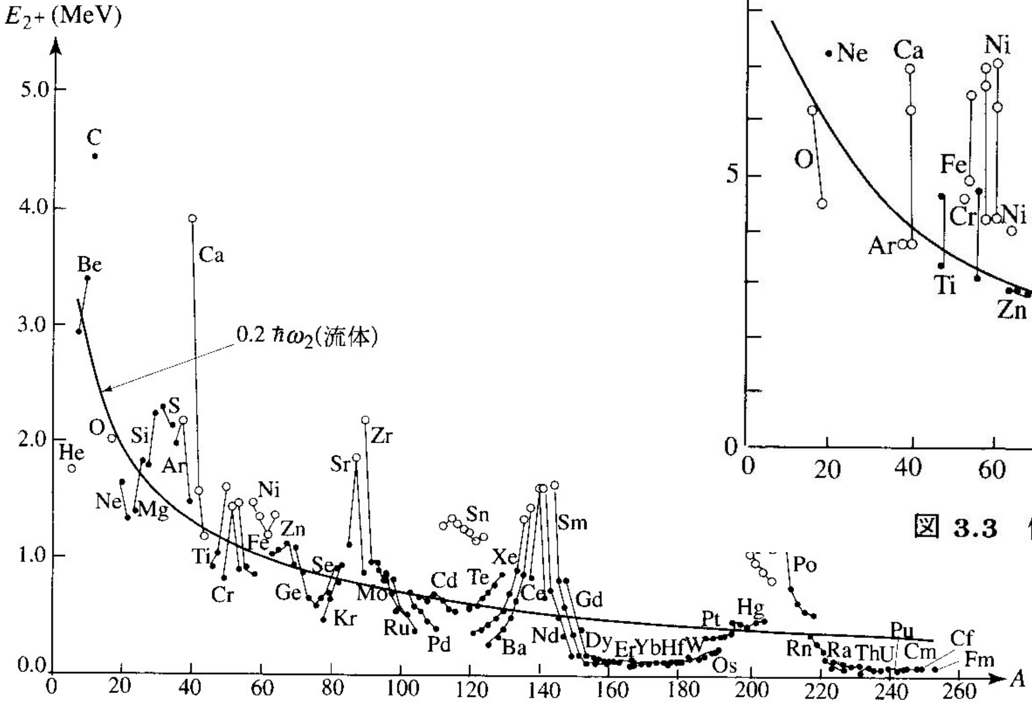


図 3.2 偶々核の第 1 励起 2^+ 状態の励起エネルギー

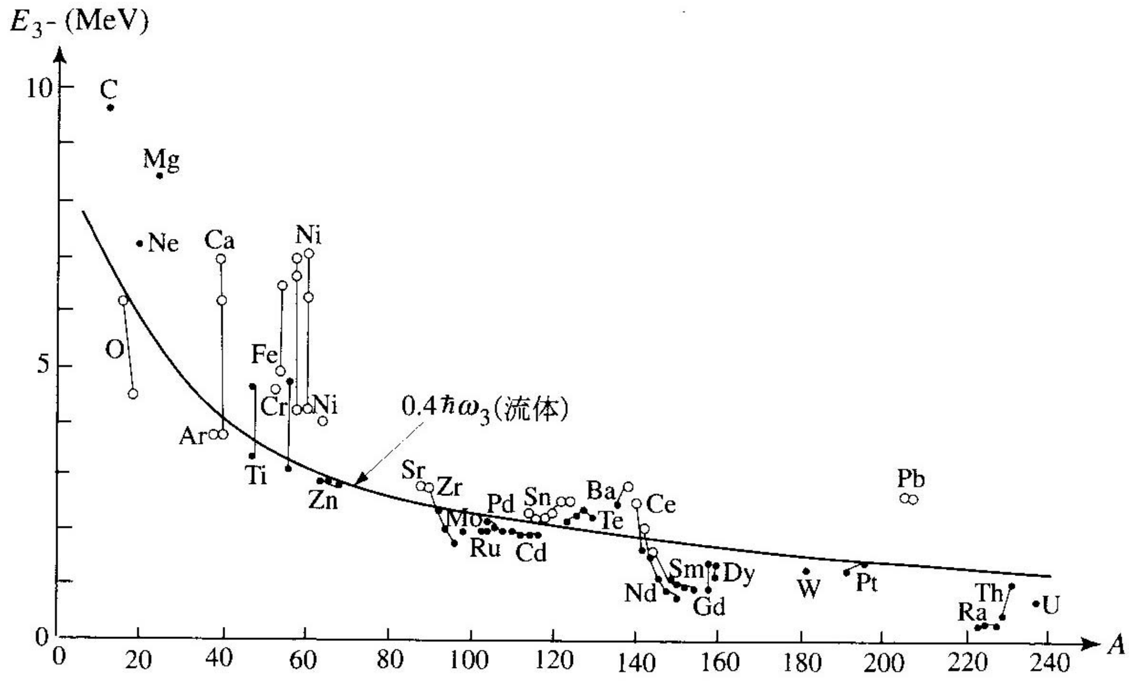
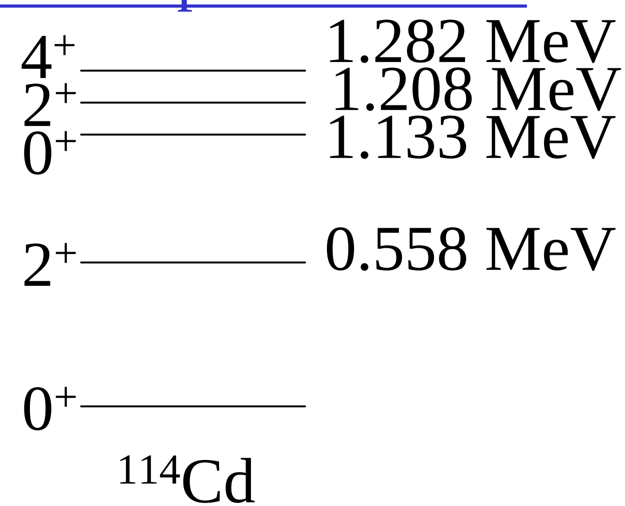
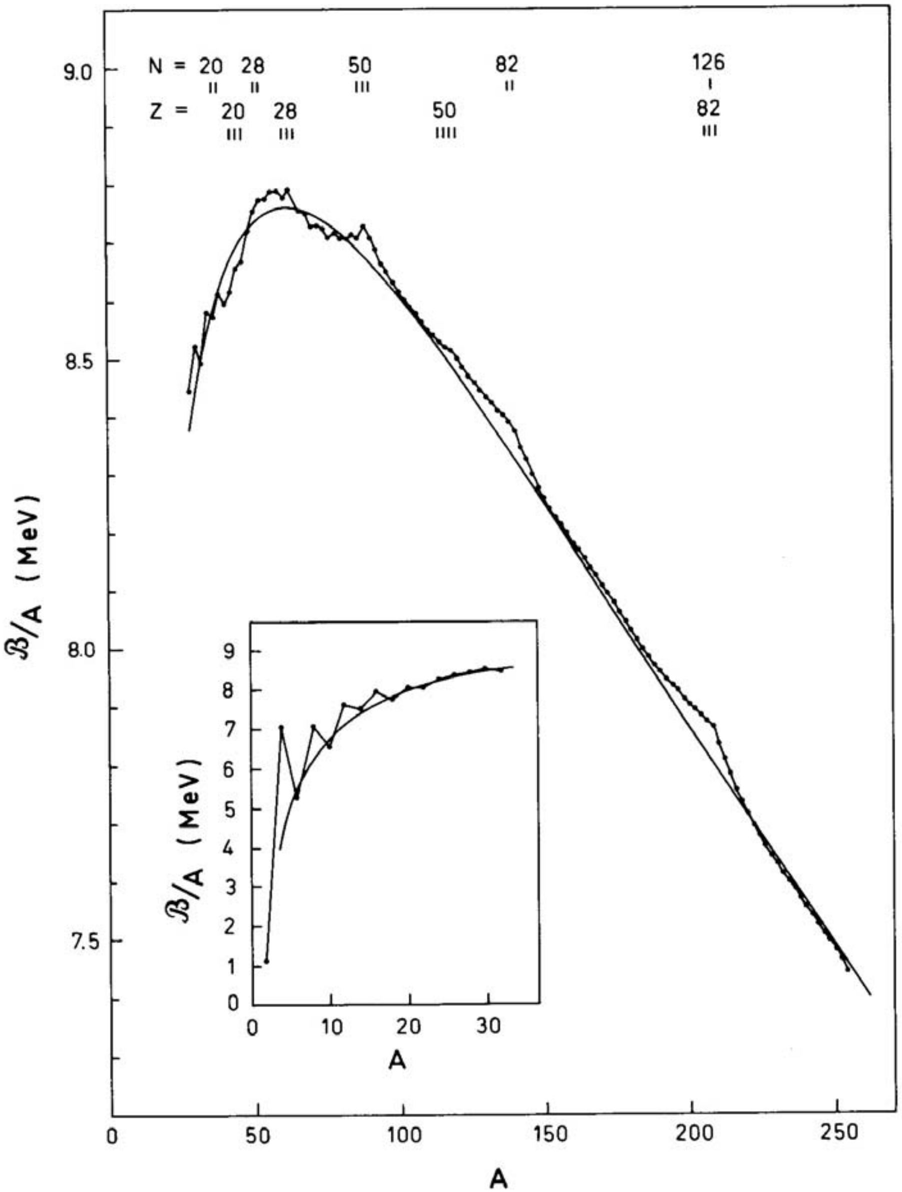


図 3.3 偶々核の第 1 励起 3^- 状態の励起エネルギー

Double phonon states



Shell Structure $B(N,Z) = B_{\text{macro}}(N,Z) + B_{\text{micro}}(N,Z)$



- Smooth part

$$B_{\text{macro}}(N,Z) = a_v A - a_s A^{2/3} - a_C \frac{Z^2}{A^{1/3}} - a_{\text{sym}} \frac{(N-Z)^2}{A}$$

- Fluctuation part

$$B_{\text{micro}} = B_{\text{pair}} + B_{\text{shell}}$$

Liquid drop model:

$$B_{\text{LDM}} = B_{\text{macro}} + B_{\text{pair}}$$

Pairing Energy

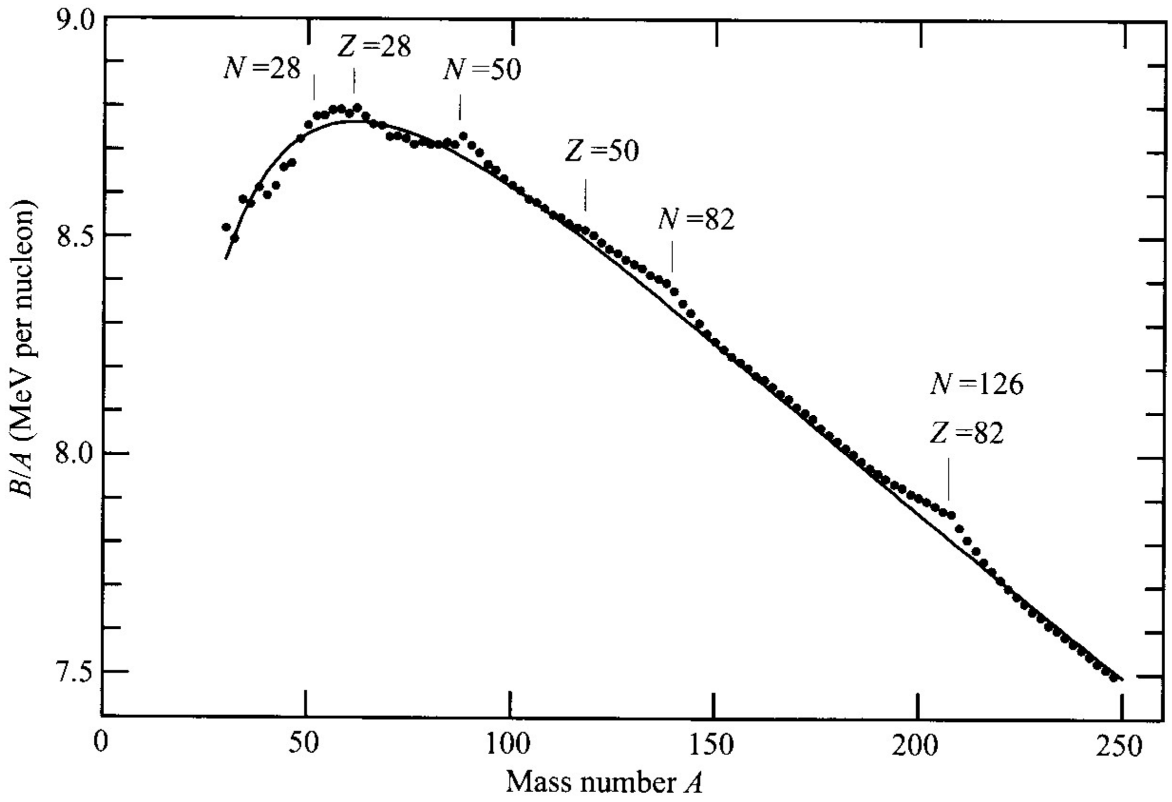
Extra binding when like nucleons form a spin-zero pair

Example:

	Binding energy (MeV)
${}^{210}_{82}\mathrm{Pb}_{128} = {}^{208}_{82}\mathrm{Pb}_{126} + 2n$	1646.6
${}^{210}_{83}\mathrm{Bi}_{127} = {}^{208}_{82}\mathrm{Pb}_{126} + n + p$	1644.8
${}^{209}_{82}\mathrm{Pb}_{127} = {}^{208}_{82}\mathrm{Pb}_{126} + n$	1640.4
${}^{209}_{83}\mathrm{Bi}_{126} = {}^{208}_{82}\mathrm{Pb}_{126} + p$	1640.2

$$
\begin{aligned}
B_{\mathrm{pair}} &= \Delta && \text{(for even} - \text{even)} \\
&= 0 && \text{(for even} - \text{odd)} \\
&= -\Delta && \text{(for odd} - \text{odd)}
\end{aligned}
$$

Shell Energy



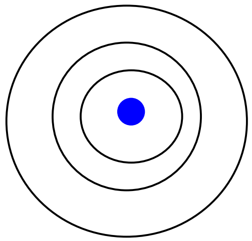
Extra binding for N,Z = 2, 8, 20, 28, 50, 82, 126 (magic numbers)

⇨ Very stable

${}^{4}_{2}He_{2}, {}^{16}_{8}O_{8}, {}^{40}_{20}Ca_{20}, {}^{48}_{20}Ca_{28}, {}^{208}_{82}Pb_{126}$

(note) Atomic magic numbers (Noble gas)

He (Z=2), Ne (Z=10), Ar (Z=18), Kr (Z=36), Xe (Z=54), Rn (Z=86)

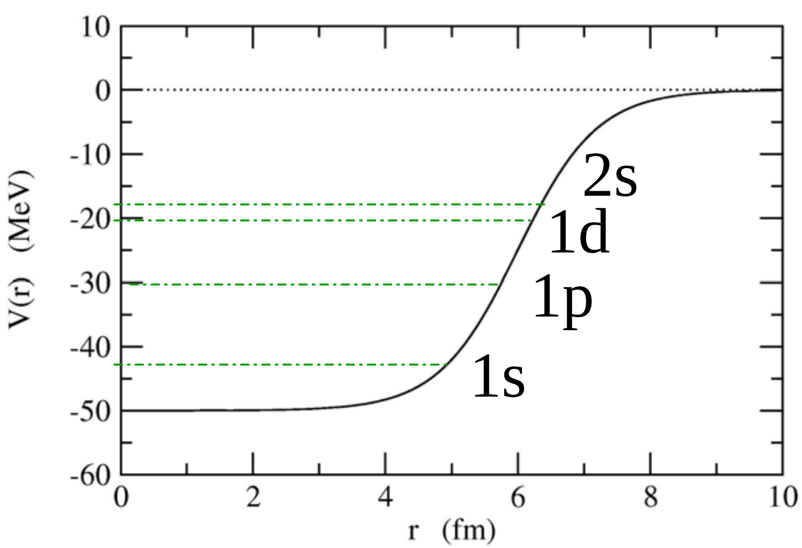


Shell structure

Similar attempt in nuclear physics: independent particle motion in a potential well

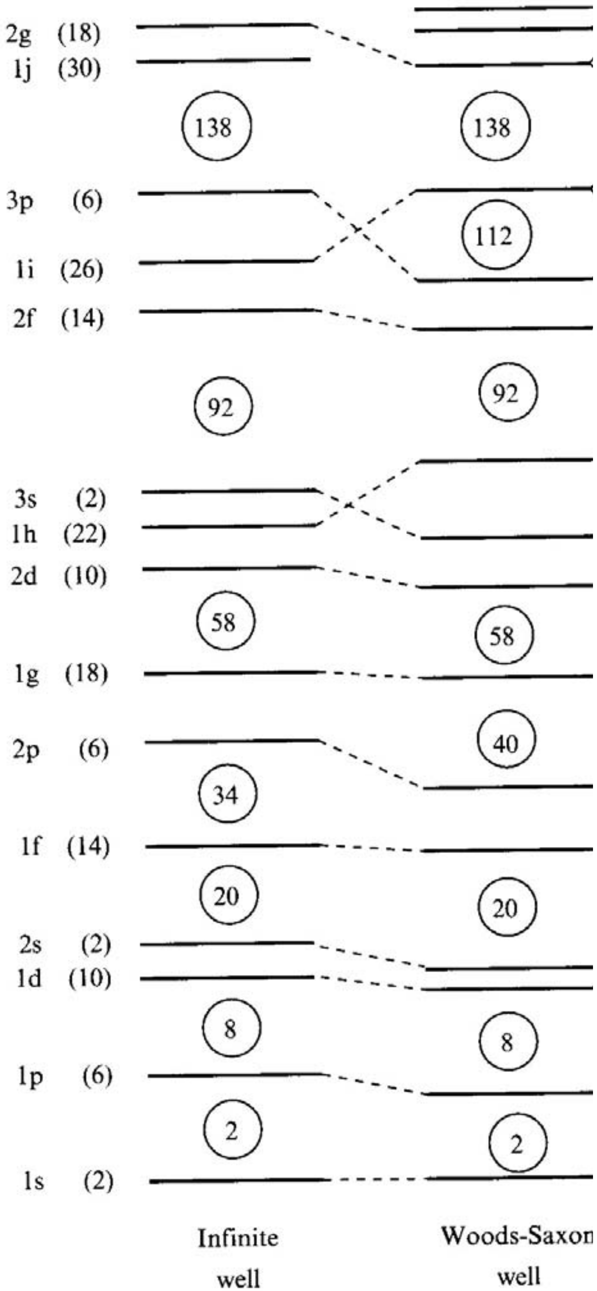
Woods-Saxon potential

$$V(r) = -V_0/[1+\exp((r-R_0)/a]$$



$$\left[-\frac{\hbar^2}{2m}\nabla^2 + V(r) - \epsilon\right]\psi(\boldsymbol{r}) = 0$$

$$\psi(\boldsymbol{r}) = \frac{u_l(r)}{r} Y_{lm}(\hat{\boldsymbol{r}}) \cdot \chi_{m_s}$$



Woods-Saxon itself does not provide the correct magic numbers (2,8,20,28, 50,82,126).

Meyer and Jensen (1949):

Strong spin-orbit interaction

$$\left[-\frac{\hbar^2}{2m}\nabla^2 + V(r) + V_{ls}(r)\boldsymbol{l}\cdot\boldsymbol{s} - \epsilon\right]\psi(\boldsymbol{r}) = 0$$

$$V_{ls}(r) \sim -\lambda \frac{1}{r}\frac{dV}{dr} \qquad (\lambda > 0)$$

jj coupling shell model

$$\left[-\frac{\hbar^2}{2m}\nabla^2 + V(r) - \epsilon\right]\psi(\boldsymbol{r}) = 0 \quad \Longrightarrow \quad \psi_{lmm_s}(\boldsymbol{r}) = \frac{u_l(r)}{r} Y_{lm}(\hat{\boldsymbol{r}}) \cdot \chi_{m_s}$$

Spin-orbit interaction

$$\left[-\frac{\hbar^2}{2m}\nabla^2 + V(r) + V_{ls}(r)\boldsymbol{l}\cdot\boldsymbol{s} - \epsilon\right]\psi(\boldsymbol{r}) = 0$$

(note) $\boldsymbol{j} = \boldsymbol{l} + \boldsymbol{s}$ $\quad \Longrightarrow \quad$ $\boldsymbol{l}\cdot\boldsymbol{s} = (\boldsymbol{j}^2 - \boldsymbol{l}^2 - \boldsymbol{s}^2)/2$

$$\psi_{jlm}(\boldsymbol{r}) = \frac{u_{jl}(r)}{r}\mathcal{Y}_{jlm}(\hat{\boldsymbol{r}})$$

$$\mathcal{Y}_{jlm}(\hat{\boldsymbol{r}}) = \sum_{m_l, m_s} \langle l\ m_l\ 1/2\ m_s | j\ m\rangle Y_{lm_l}(\hat{\boldsymbol{r}})\chi_{m_s}$$

jj coupling shell model

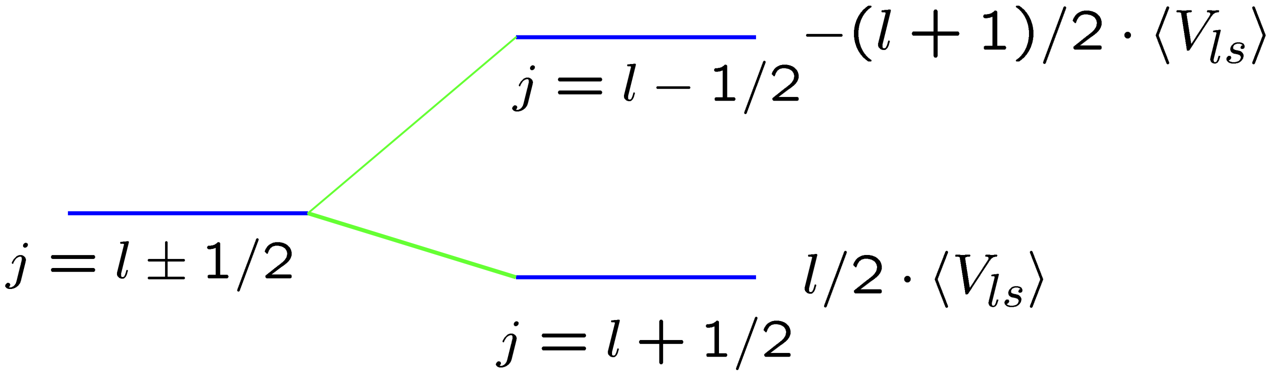
$$\left[-\frac{\hbar^2}{2m}\nabla^2 + V(r) + V_{ls}(r)\boldsymbol{l}\cdot\boldsymbol{s} - \epsilon\right]\psi(\boldsymbol{r}) = 0$$

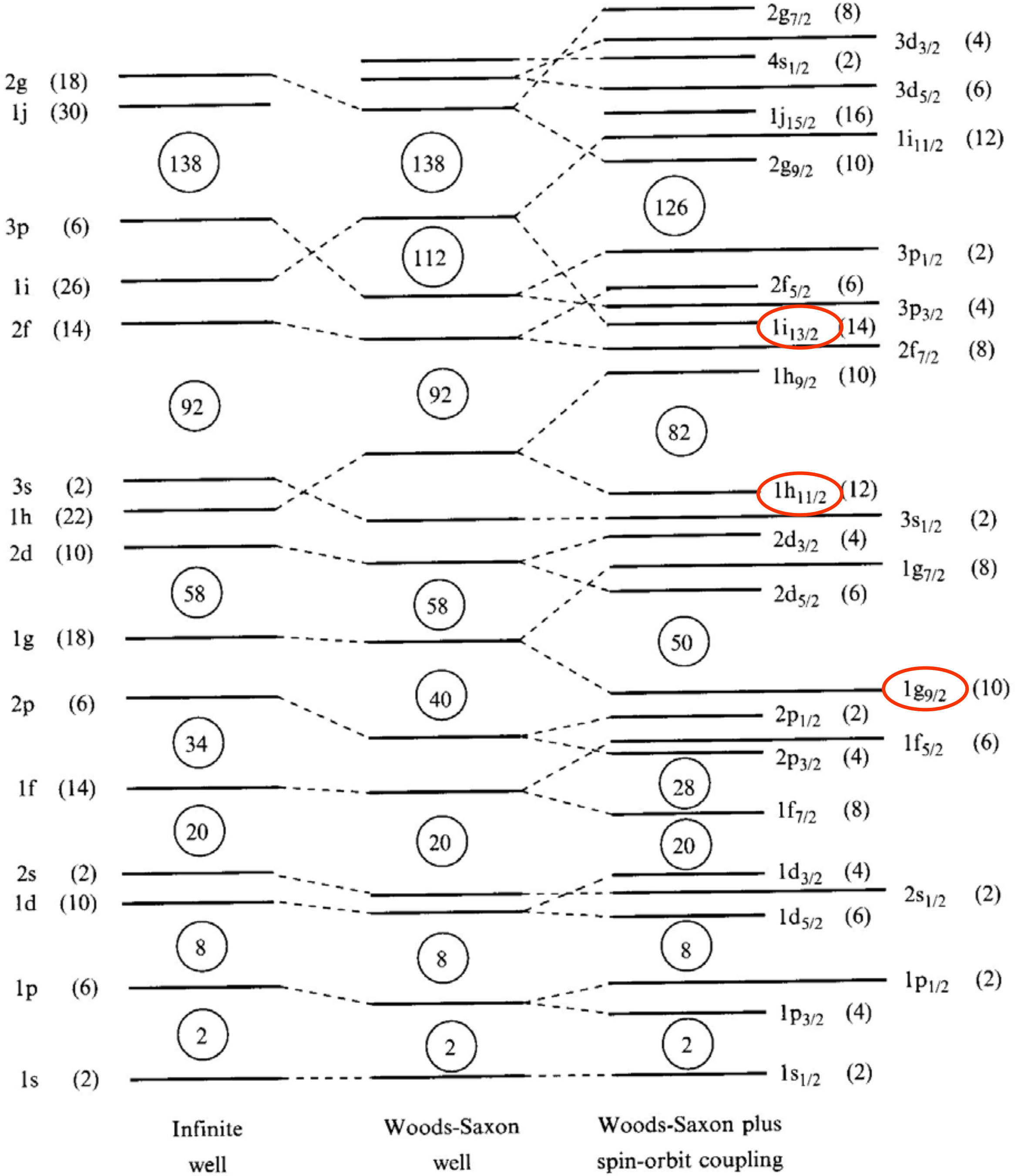
(note) $\boldsymbol{j} = \boldsymbol{l} + \boldsymbol{s} \quad \Longrightarrow \quad \boldsymbol{l}\cdot\boldsymbol{s} = (\boldsymbol{j}^2 - \boldsymbol{l}^2 - \boldsymbol{s}^2)/2$

$$\psi_{jlm}(\boldsymbol{r}) = \frac{u_{jl}(r)}{r}\mathcal{Y}_{jlm}(\hat{\boldsymbol{r}})$$

$$\mathcal{Y}_{jlm}(\hat{\boldsymbol{r}}) = \sum_{m_l, m_s}\langle l\ m_l\ 1/2\ m_s | j\ m\rangle Y_{lm_l}(\hat{\boldsymbol{r}})\chi_{m_s}$$

$$\boldsymbol{l}\cdot\boldsymbol{s} = l/2\ (j = l + 1/2), \quad -(l+1)/2\ (j = l - 1/2)$$





intruder states
unique parity states

Single particle spectra

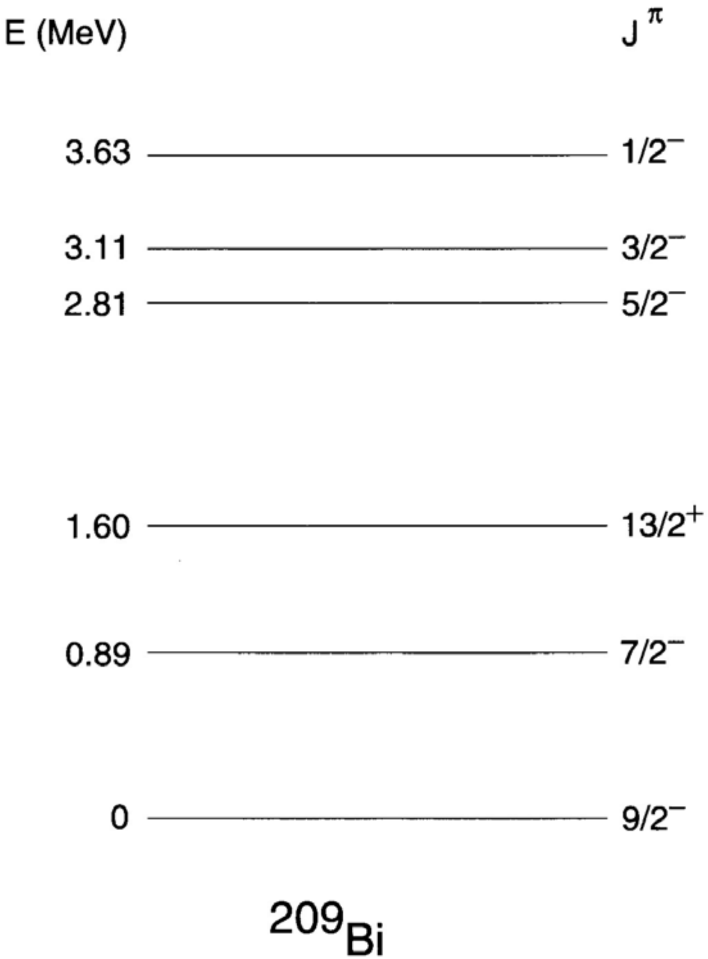
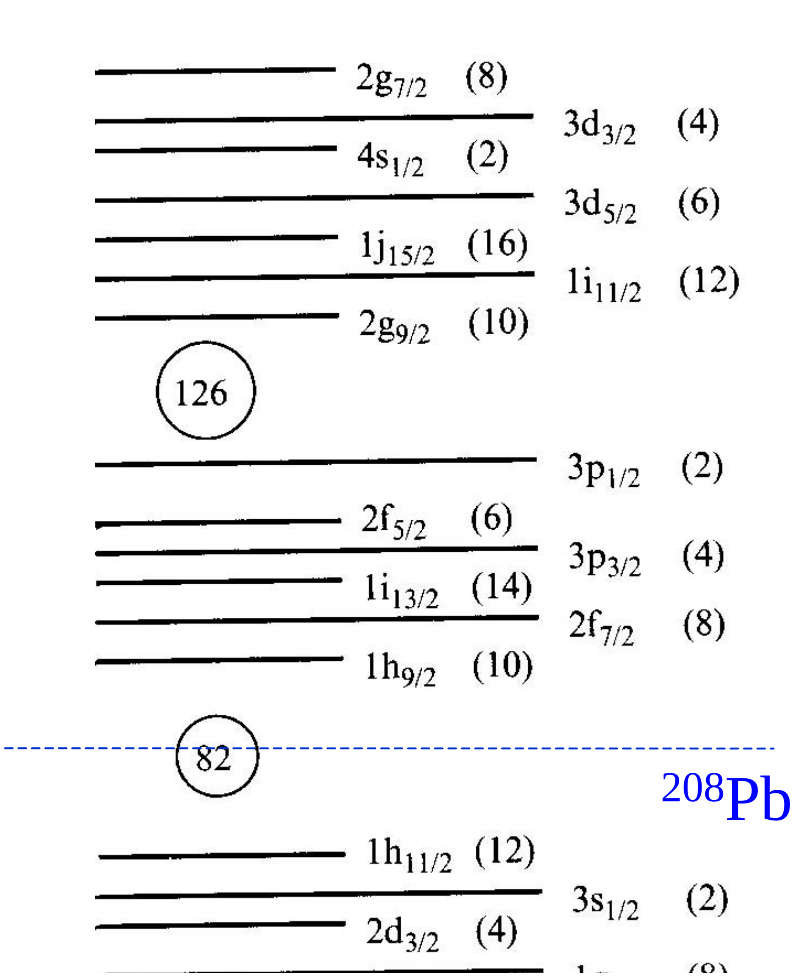
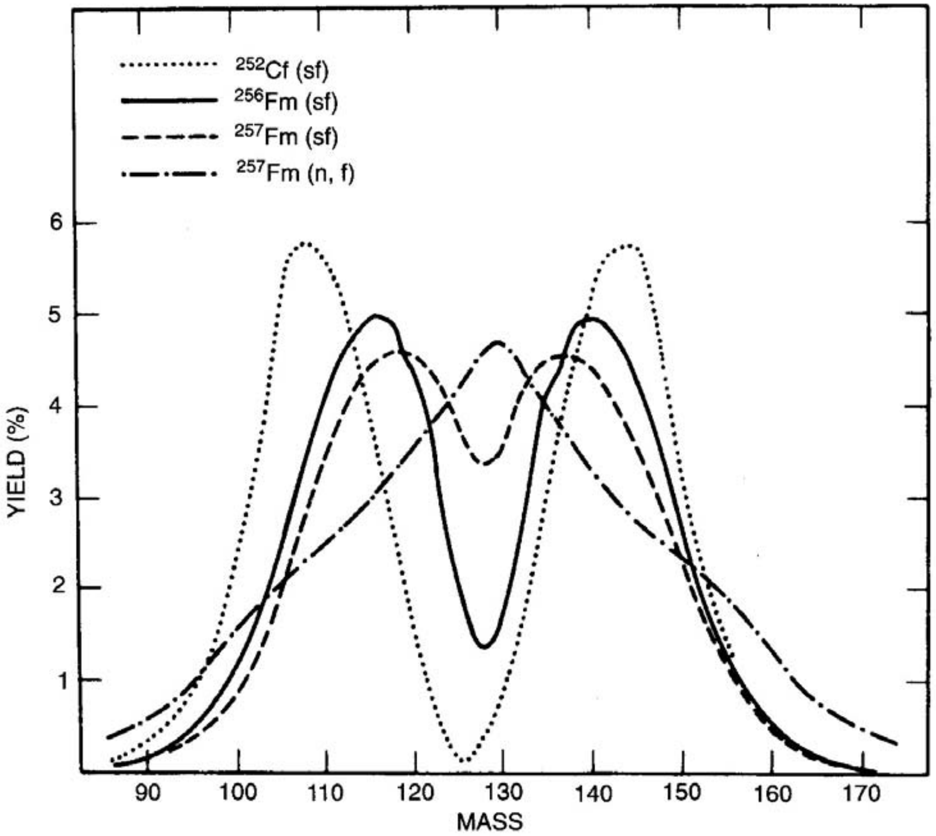


FIG. 3.6. Low-lying single-particle levels of ^{209}Bi.



核分裂片の質量分布



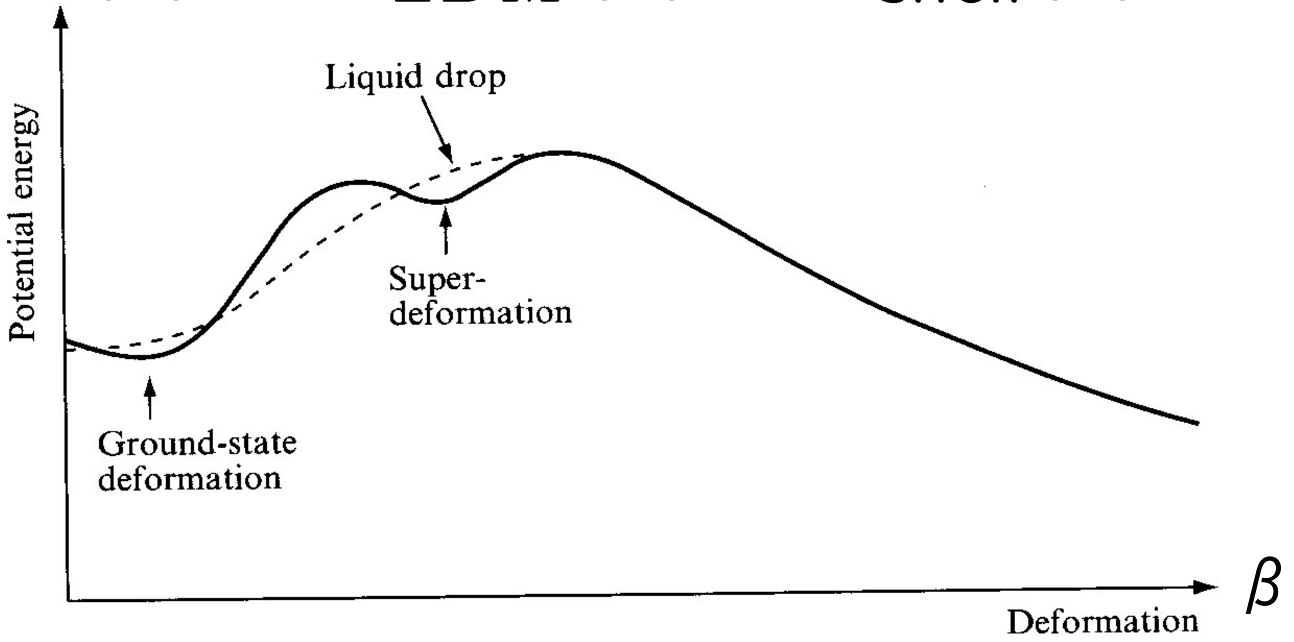
cf. $^{120}_{50}Sn$

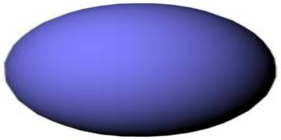
Fig. 4.1. Mass distributions in terms of the fission fragment masses for spontaneous fission of $^{252}_{98}$Cf, $^{256}_{100}$Fm and $^{257}_{100}$Fm and for neutron-induced fission of ^{257}Fm. Note the trend toward symmetric fission with increasing mass and in addition the larger number of symmetric events for neutron-induced than for spontaneous fission (from R. Vandenbosch and J.R. Huizenga, *Nuclear Fission* (Academic Press, New York and London, 1973)).

原子核の変形

Deformed energy surface for a given nucleus

$$E(\beta) = E_{LDM}(\beta) + E_{\rm shell}(\beta)$$



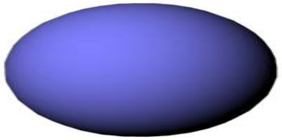


LDM only → always spherical ground state

Shell correction → may lead to a deformed g.s.

* Spontaneous Symmetry Breaking

原子核の変形



^{154}Sm の励起スペクトル

0.903 (MeV)	8^+
0.544	6^+
0.267	4^+
0.082	2^+
0	0^+

^{154}Sm

$$E_I \sim \frac{I(I+1)\hbar^2}{2\mathcal{J}}$$

Cf. 剛体の回転エネルギー(古典力学)

$$E = \frac{1}{2}\mathcal{J}\omega^2 = \frac{I^2}{2\mathcal{J}}$$

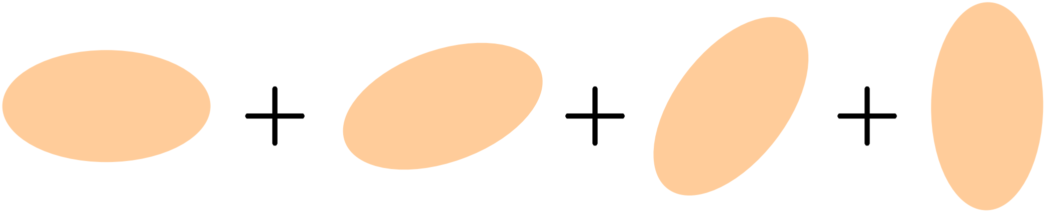
$$(I = \mathcal{J}\omega, \ \ \omega = \dot{\theta})$$

^{154}Sm は変形している

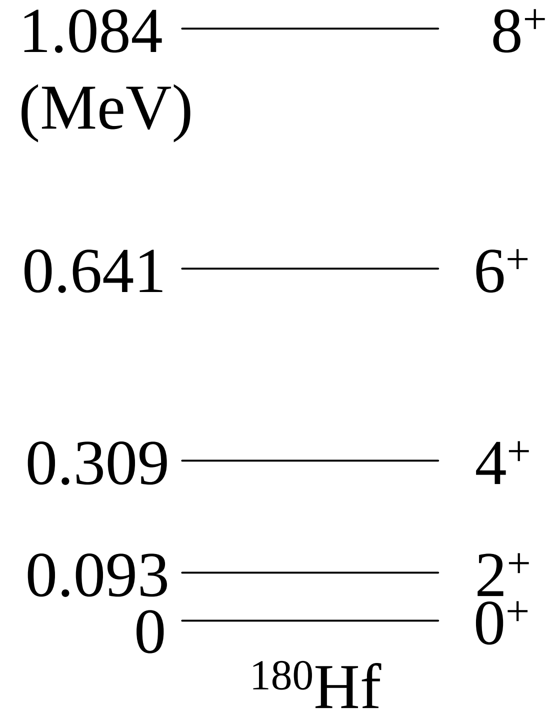
(note) 0^+ 状態とは(量子力学)？

0^+: 空間の異方性がない

色々な向きが等確率で混ざっている



Evidences for nuclear deformation



- The existence of rotational bands

$$E_I = \frac{I(I+1)\hbar^2}{2\mathcal{J}}$$

- Very large quadrupole moments
  (for odd-A nuclei)

$$Q = e\sqrt{\frac{16\pi}{5}}\langle\Psi_{II}|r^2Y_{20}|\Psi_{II}\rangle$$

- Strongly enhanced quadrupole transition probabilities
- Hexadecapole matrix elements ⟷ β_4
- Single-particle structure
- Fission isomers